Diffusion in a magnetic field

- in $(2+1)$-dimensions -

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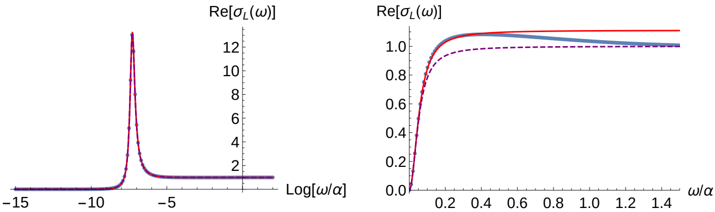
Let me set the scene...

- Why do I care about magnetic fields in $(2+1)$-dimensions?
  - Quite a few interesting materials are effectively planar.
  - A useful (and one of the few) experimental parameters available to explore a material.
  - Interesting effects: Hall effects, suppressing superconductivity, anyons etc.
  - Theoretically fun: no broken rotational invariance - yay!
- How can holography help us?
  - Useful testbed for ideas.
  - Easy to explore rather large magnetic fields (and still get answers).

In the beginning... well, 2007 anyway



- Strong coupling - use hydro!
- Constitutive relation for charge current[1]

$$\vec{J} = \sigma_0 \left(\vec{E} - T\vec{\nabla}\frac{\mu}{T} \right) - \alpha_0 \vec{\nabla} T \, .$$

- Relativistic invariance implies $\alpha_0 = -(\mu/T)\sigma_0$ and $\kappa_0 = (\mu/T)^2\sigma_0$.
- $\sigma_{\mathrm{L}}^{\mathrm{DC}} = \alpha_{\mathrm{L}}^{\mathrm{DC}} = 0 \, , \quad \sigma_{\mathrm{H}}^{\mathrm{DC}} = \frac{n}{B} \, , \quad \alpha_{\mathrm{H}}^{\mathrm{DC}} = \frac{s}{B} \text{ or } \frac{s}{B} - \frac{m}{T} \, .$

[1] Hartnoll, Kovtun, et al., "Theory of the Nernst effect near quantum phase transitions in condensed matter and in dyonic black holes"; Hartnoll and Herzog, "Ohm's law at strong coupling: S-duality and the cyclotron resonance".

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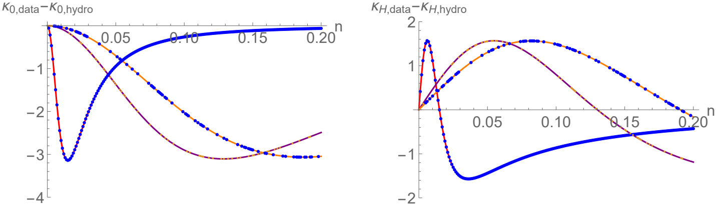

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Some (non-exhaustive) examples...

- Non-linear fluids (with parity violation):[2]
  - extra transport coefficients from broken parity invariance,
  - new transport coefficients are dissipationless,
  - Hall viscosity.[3]
- Various investigations in probe brane models:
  - understanding of how diffusion becomes zero sound,[4]
  - used as models for anyons.[5]
- Plus way too many more to list, but what about an actual systems?

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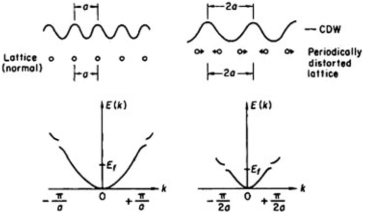
[2] Jensen et al., "Parity-violating hydrodynamics in 2 + 1 dimensions".

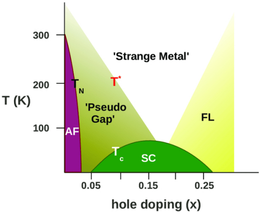
[3] Hoyos et al., "Hall viscosity in a strongly coupled magnetized plasma".

[4] Goykhman et al., "Fluctuations in finite density holographic quantum liquids"; Brattan, Davison, et al., "Collective Excitations of Holographic Quantum Liquids in a Magnetic Field"; Chen et al., "Origin of the Drude peak and of zero sound in probe brane holography".

[5] Brattan and Lifschytz, "Holographic plasma and anyonic fluids"; Brattan, "A strongly coupled anyon material"; Ihl et al., "Holographic anyonization: a systematic approach".

Bi-2201 and CDWs





Left: A cartoon of a high$-T_c$ superconductor phase diagram from Banerjee et al. (2018). **Right:** 1D CDW distortion and energy band gaps from Bhadeshia et al. (2014)

- Charge-density wave (CDW) order appears to be a ubiquitous feature of cuprate superconductors.
- Our material,[6] $Bi_2Sr_2CuO_6$:
  - 2D CDW confirmed (by X-ray diffraction) to extend to optimal and over-doped region,
  - low critical temperature ($T_c \sim 10 - 20$ K).

[6] Amoretti et al., "Hydrodynamical description for magneto-transport in the strange metal phase of Bi-2201".

Our assumptions

- Maybe CDW is enough to describe strange metal phase?
- General assumptions: magnetic field can be treated as very small, relativistic fixed point, no fundamental Hall conductivities ...
- The (non-) conservation equations

$$\partial_t (n, s) + \partial_i \left(J^i, Q^i / T \right) = 0 \,, \tag{1}$$

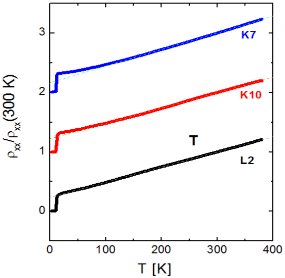
$$\partial_t \pi^i + \partial_j T^{ji} = F^{ij} J_j - \Gamma^{ij} \pi_j - k_0^2 G \phi^i \,, \tag{2}$$

$$\partial_t \lambda_a + \partial_i J^i_a = -\Omega_a{}^b \lambda_b = -\Omega_1 \lambda_a - B \Omega_2 \epsilon_{ab} \lambda^b \,. \tag{3}$$

- Four unknowns: $n, s, \sigma_0, \tilde{\sigma}$.

$$\sigma_{\mathrm{DC}} = \sigma_0 + \tilde{\sigma} \,, \qquad \tilde{\sigma} = \frac{n^2}{\chi_{\pi\pi}} \frac{\Omega_1}{\Omega_1 \Gamma + \omega_0^2} \,. \tag{4}$$

Matching data to experiment



Data:

- $\underline{\rho_{xx} \sim (T, B^0)}$,
- $\Delta\rho/\rho \sim (T^{-4}, B^2)$,
- $\cot(\Theta_{\mathrm{H}}) \sim (T^{\frac{3}{2}}, B^{-1})$
- and $\kappa_{xy} \sim (T^{-3}, B)$.

Variables:

- n,
- s,
- σ_0,
- and $\tilde{\sigma}$.

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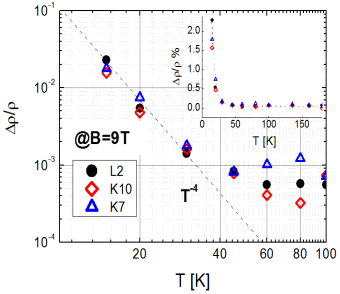

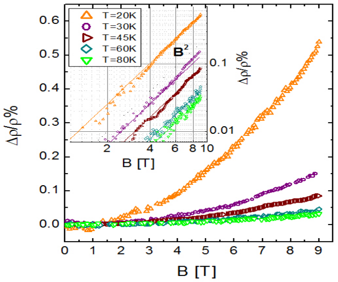

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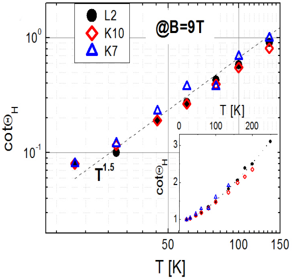

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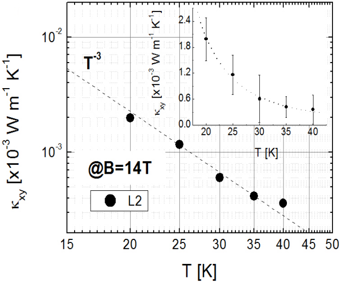

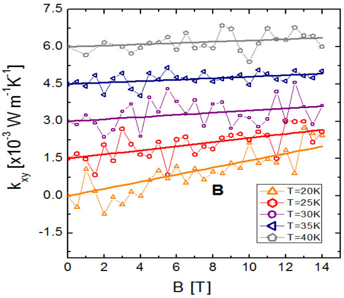

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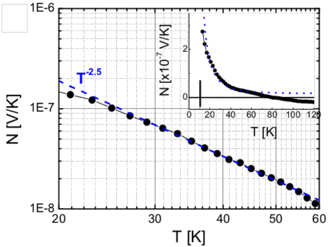
Recovering the Nernst behaviour



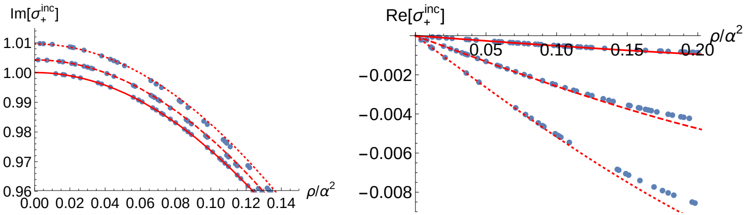
- The Nernst coefficient behaves as

$$N \sim \frac{\mu B \tilde{\sigma}}{nT} \sim \frac{\mu}{T \cot \Theta_H} \sim T^{-2.5} \, . \tag{5}$$

- We have complete consistency with experiment!
- N dominated by CDW relaxation, anomalously large Nernst signal can be explained.[7]

[7] Cyr-Choinière et al., "Enhancement of the Nernst effect by stripe order in a high-Tc superconductor".

Something's amiss...



- Laurent expansion about hydrodynamic pole

$$\begin{aligned} \sigma_+(\omega) &= \sigma_{xy}(\omega) + i\sigma_{xx}(\omega) \\ &= \frac{r_{\mathrm{hydro}}}{\omega - \omega_{\mathrm{hydro}}} + \sigma_+^{\mathrm{inc}} + \mathcal{O}(\omega - \omega_{\mathrm{hydro}}) \,. \qquad (6) \end{aligned}$$

- Original result, $\sigma_+^{\mathrm{inc}} = i\sigma_0$ with $\sigma_0 \leq 1$.
- $\mathrm{Re}(\sigma_+^{\mathrm{inc}}) \propto B$, hence we have an incoherent Hall conductivity.

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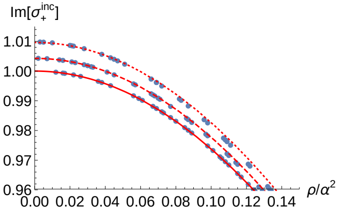

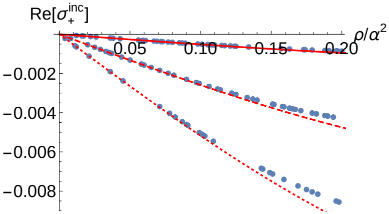

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$$\begin{aligned}\sigma_+(\omega) &\rightarrow \sigma_+(\omega)|_{\omega\to\omega-i\Gamma}\\ &= \frac{r_{\mathrm{hydro}}}{\omega - i\Gamma - \omega_{\mathrm{hydro}}} + \sigma_+^{\mathrm{inc}} + \ldots \,.\end{aligned}$$

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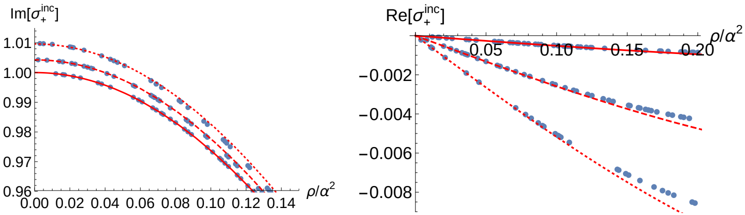

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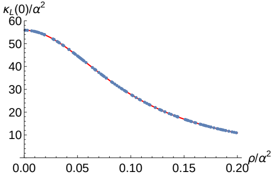
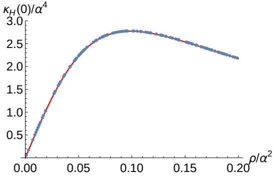
Incoherent conductivities (I) - Ward identities





- Ward identities[8] e.g.

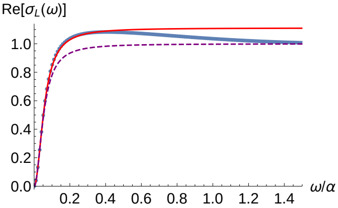
$$\langle \mathcal{J}^i \mathcal{Q}^j \rangle \;=\; -n\delta^{ij} - \left(\mu\delta^i{}_k - \frac{iB}{\omega}\epsilon^i{}_k\right)\langle \mathcal{J}^k \mathcal{J}^j \rangle \;.$$

- Low frequency expansion of $\sigma_+(\omega)$ from Ward identities

$$\frac{n}{B} + \frac{sT + \mu n - mB}{B^2}\omega + \left[\frac{2\left(\kappa_{\mathrm{H}}(0) + \mu^2 n + 2\mu sT - 2\mu mB\right)}{2B^3} + i\frac{\kappa_{\mathrm{L}}(0)}{B^2}\right]\omega^2$$

[8] Hartnoll and Herzog, "Ohm's law at strong coupling: S-duality and the cyclotron resonance".

Incoherent conductivities (II) - Linearised magnetohydro



- Linearised hydrodynamics

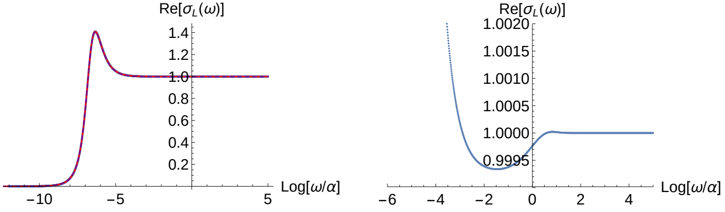
$$\partial_t \delta\mathcal{P}^i = F^{i\mu}\delta\mathcal{J}_\mu = n\delta E^i + F^{ij}\delta\delta\mathcal{J}_j \,, \tag{8}$$

$$\delta\mathcal{J}^i = \hat{\sigma}_0^{ij}\delta E_j + \hat{\chi}^{ij}\delta\mathcal{P}_j \,. \tag{9}$$

- Generic form of the correlators

$$\sigma_+(\omega) = \alpha_1 + \frac{\alpha_2}{\omega - \alpha_3} \,. \tag{10}$$

Further confirmation - features of the charge conductivity



$B \ll n$

- From hydrodynamics

$$\sigma_{\mathrm{L}}(\omega) \overset{\omega/T \gg 1}{\longrightarrow} \sigma_0 \; . \tag{11}$$

- For a conformal theory

$$\sigma_{\mathrm{L}}(\omega) \overset{\omega/T \gg 1}{\longrightarrow} 1 \; . \tag{12}$$

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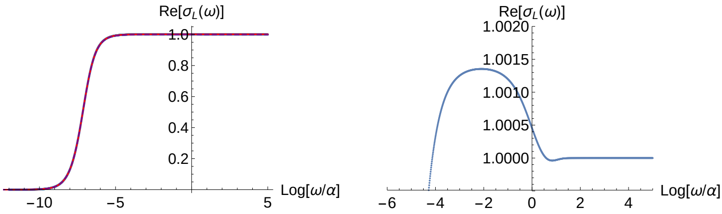

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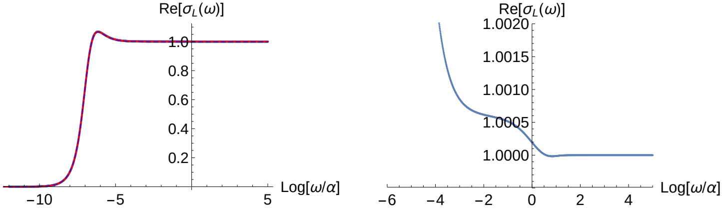

$B \sim n$

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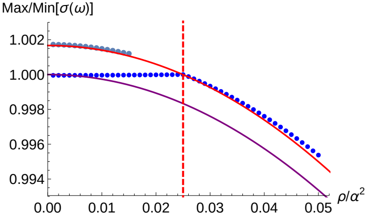

Flow of the feature.

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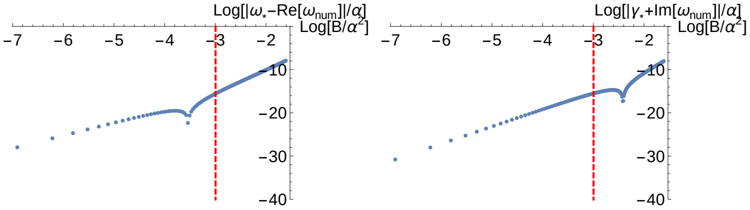
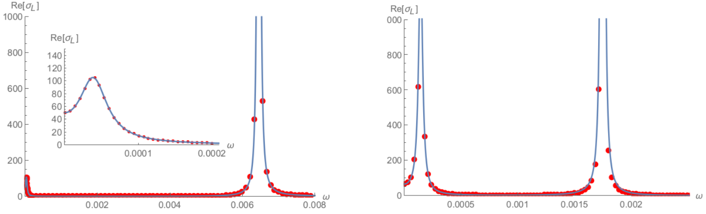
A remaining question - quasinormal modes



- Resummation approach[9] - exact quasinormal mode.
  - Does failure of series to converge indicate limit of hydrodynamics?
  - What is the series expansion in B?

[9] Withers, "Short-lived modes from hydrodynamic dispersion relations".

Returning to charge density waves



- There is a new Ward identity relating $\langle \mathcal{J}^i O^j \rangle$ to $\langle \mathcal{Q}^i O^j \rangle$.
- There are two hydrodynamic peaks!
- Can everything be determined in terms of DC data again?

What does our result hint for quasihydrodynamics?

- Quasihydrodynamic - hydrodynamics with some sources for the charges:
  1. broken translation invariance $\partial_t P^i = \Gamma^{ij} P_j$,
  2. anyonic systems,
  3. probe branes again and more...
- What other quasihydrodynamic systems can handle a constant breaking parameter? How do these parameters mix?

Thanks for listening!