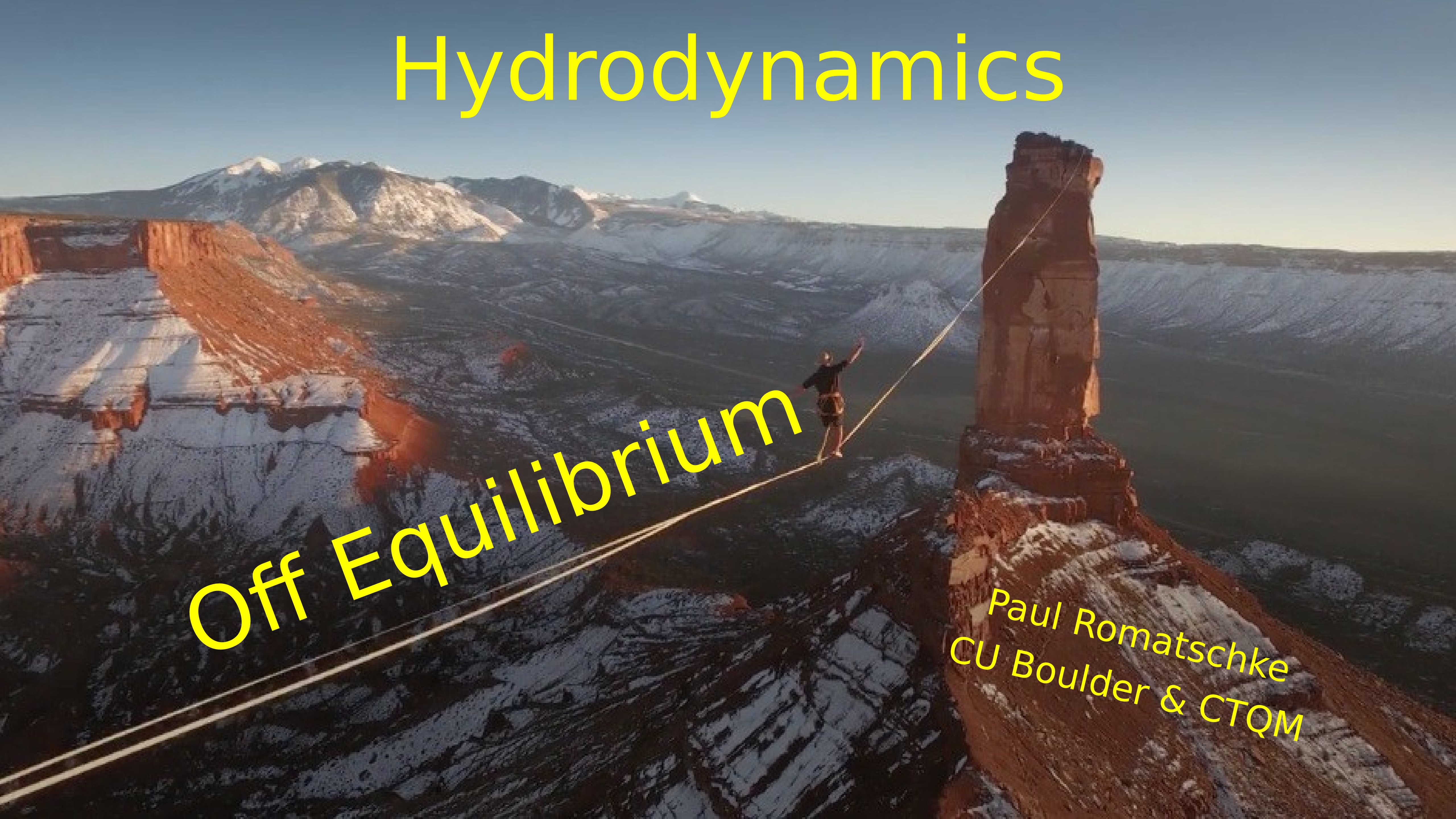

Hydrodynamics
Off Equilibrium
Paul Romatschke
CU Boulder & CTQM

(many) Textbooks:

Hydrodynamics requires near-equilibrium system to be applicable
“Thermalization”/”Equilibration”

→ Lots of work in context of heavy-ion collisions
→ We now know heavy-ion collisions don’t ever thermalize
→ (but that’s another story: 1609.02820)

(many) Textbooks:

Hydrodynamics requires near-equilibrium system to be applicable

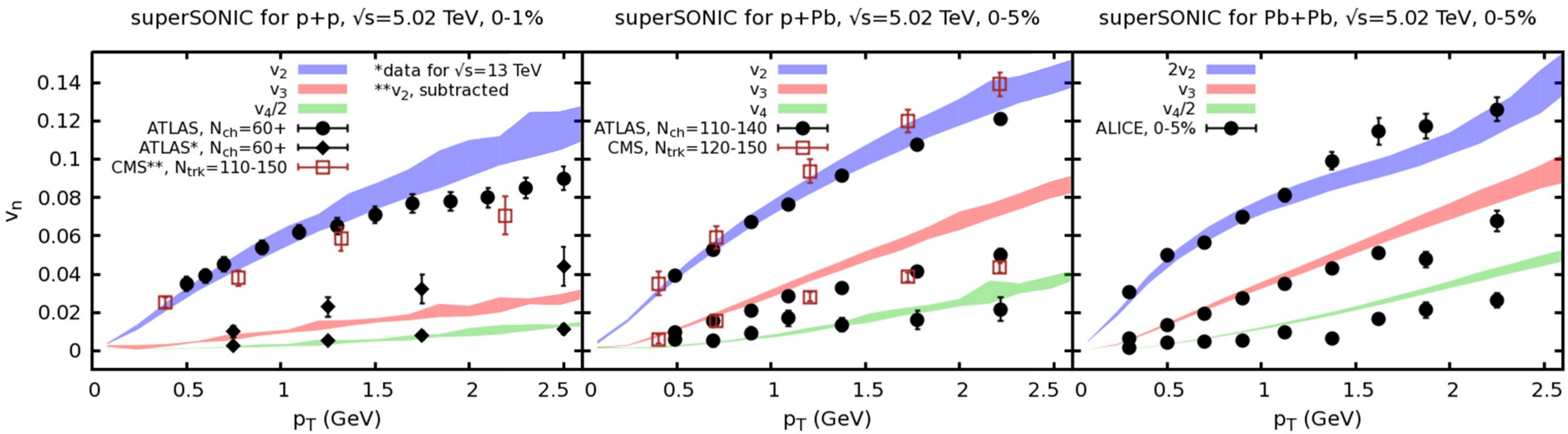
“Thermalization”/”Equilibration”

Is that really true? Or are appearances deceiving?

Nuclear Collision Experiments

- Heavy-Ion Collisions: designed to learn about finite temperature QCD “Quark Gluon Plasma” (QGP)
- Light-Ion Collisions (p+Pb) collisions designed as “control experiment”: no QGP
- Proton-Proton collisions: ultimate control, thought of as too small, too short-lived to equilibrate and show hydro-like behavior

Nuclear Collision Experiments



[Weller, PR, 1701.07145]

"One fluid to rule them all"

…apparently hydro works well quite far away from equilibrium…

why?

...lots of progress in the past 10yrs...

In this talk, I’ll focus on the basics rather than very recent developments

Off-Equilibrium Hydro Textbook

arXiv.org > nucl-th > arXiv:1712.05815

Search or A

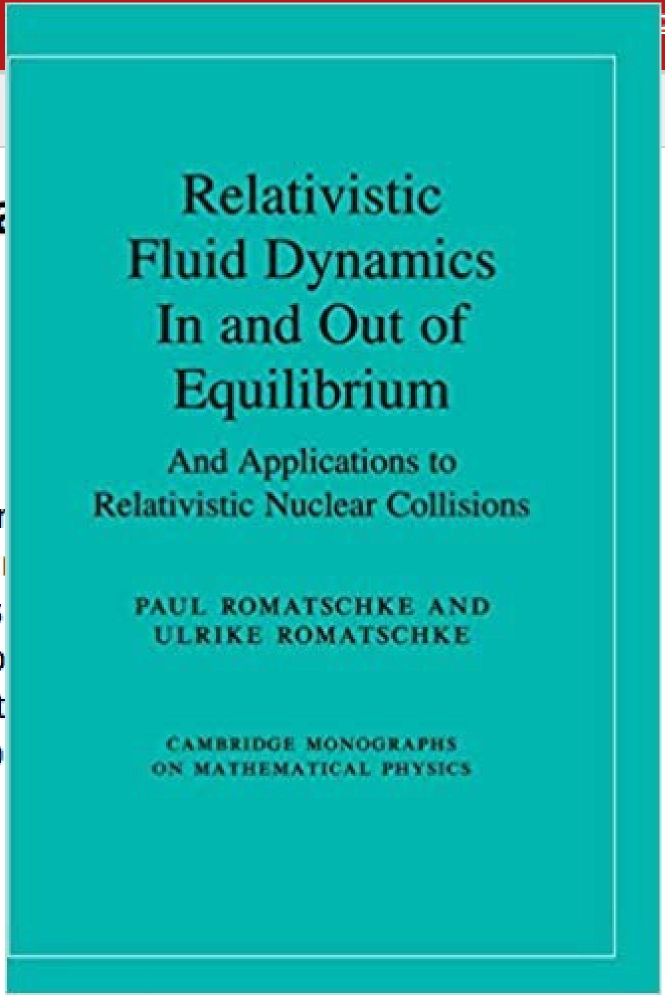
Nuclear Theory

Relativistic Fluid Dynamics In and Out of Equilibrium -- Ten Yea Theory and Numerical Simulations of Nuclear Collisions

Paul Romatschke, Ulrike Romatschke

(Submitted on 15 Dec 2017 (v1), last revised 2 Mar 2018 (this version, v2))

Ten years ago, relativistic viscous fluid dynamics was formulated from first principles in an effective field theor knowledge of symmetries and long-lived degrees of freedom. In the same year, numerical simulations for the collision experiments became first available, providing constraints on the shear viscosity in QCD. The field has present the current status of the theory of non-equilibrium fluid dynamics in 2017, including the divergence o resurgence, non-equilibrium attractor solutions, the inclusion of thermal fluctuations as well as their relation t we review the theory basis for numerical fluid dynamics simulations of relativistic nuclear collisions, and comp experimental data for nucleus-nucleus, nucleus-proton and proton-proton collisions.




…getting started:
Hydro Near Equilibrium

Hydro as an EFT

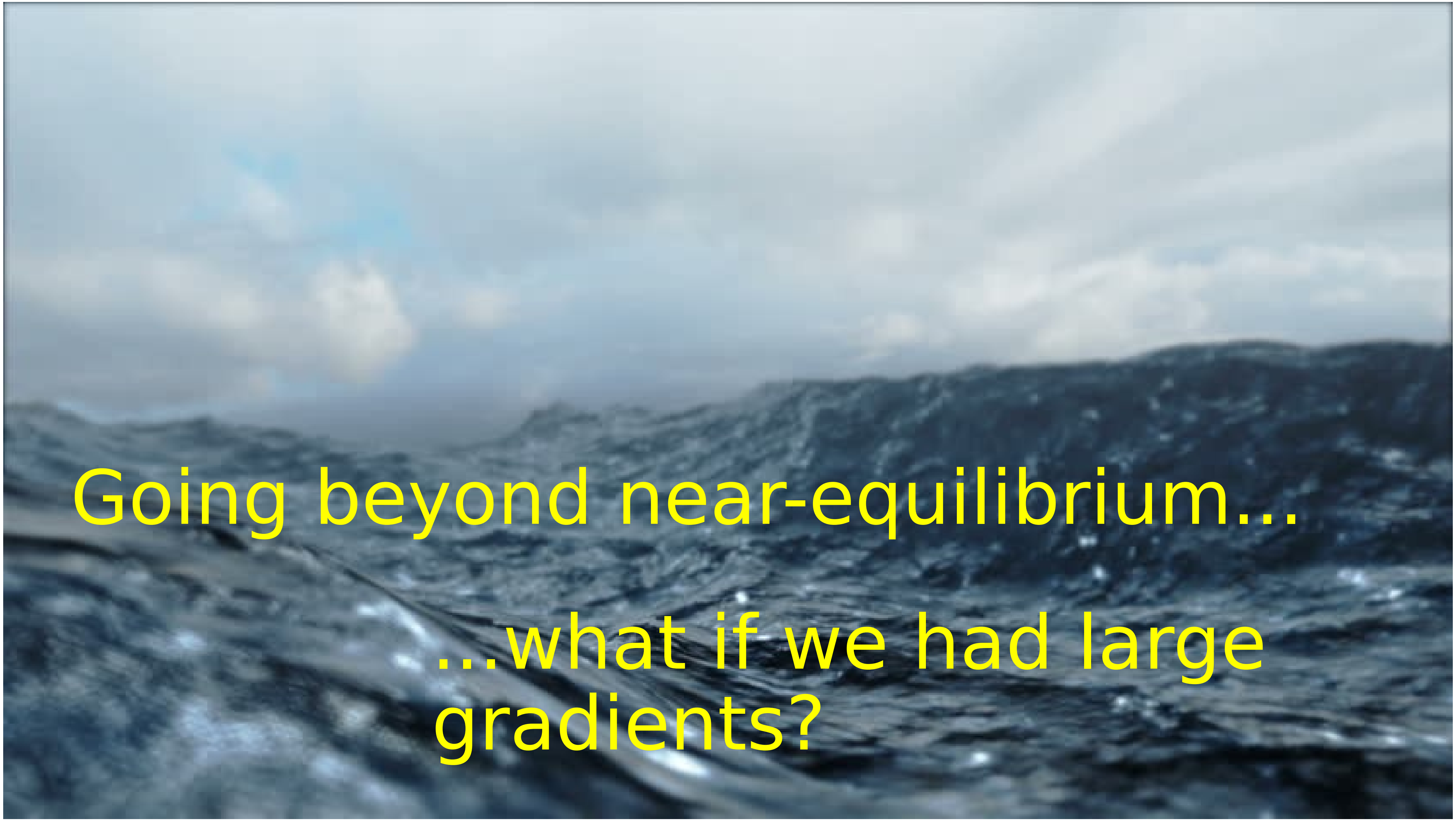
- Many derivations of hydro equations
- Most general approach: Effective Field Theory (EFT)
- Hydro = EFT of long-lived, long-wavelength excitations
- EFT variables: pressure, energy density, fluid velocity

Hydro as an EFT

- Write down quantities using EFT variables and their gradients
- Energy-Momentum Tensor for relativistic fluid

$$T^{ab} = (\epsilon + P)u^a u^b + P g^{ab} - 2\eta \nabla^{\langle a} u^{b\rangle} + \dots$$

- Universal (structure fixed, only values depend on microphysics)
- No thermal equilibrium or particle description needed
- EFT Expansion: Seems we need small gradients!


Going beyond near-equilibrium...
...what if we had large gradients?

Gradient expansion example

- What if we had LARGE gradients?
- Example: $f(x)=e^x$, for $x \sim 1$
- $f(x) \sim 1+x+x^2/2+x^3/3!+x^4/4! \ldots$
- $f(1) \sim 1+1+1/2!+1/3!+1/4!=2.70833 \sim 2.71828=e^1$
- Works for any value of x because gradient expansion converges (but may need high gradient order)

Hydro as an EFT

- What if we had LARGE gradients?
- Try to improve description by including higher orders in EFT gradient series
- E.g. Bjorken flow, go to order 240 (AdS/CFT)

$$T(\tau) = \hat{\tau}^{-1/3} \left(1 + \sum_{n=1}^{240} \alpha_n \hat{\tau}^{-2n/3}\right)$$

- Find: $\alpha_n \sim n!$, gradient series diverges

[1302.0697, 1503.07514, 1603.05344, 1608.07869, 1609.04803]

Hydro as an EFT

- Gradient series diverges
- But it is Borel-summable! [Heller et al, 1302.0697]
- Borel-resumming AdS/CFT gradient series:

$$T(\tau) = T_{\rm hydro}(\tau) + \gamma \exp\left[-i \int d\hat{\tau} \left(\hat{\omega}_{\rm Borel}\hat{\tau}^{-1/3} + \sum_{n=1} \hat{\omega}_n \hat{\tau}^{-(2n+1)/3}\right]\right) + \ldots$$

- $T_{\rm hydro}$ is “hydrodynamic attractor” (more later)
- Extra pieces non-analytic in gradient expansion; this is why grad series diverges! (Would be like $f(x)=e^x+e^{-1/x}$ in our earlier example)

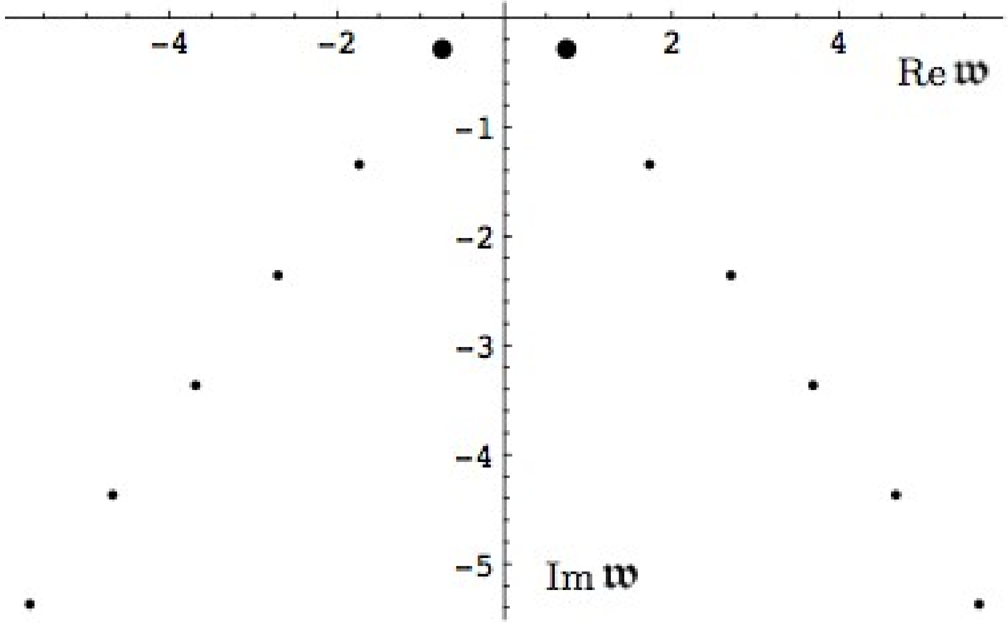
Hydro as an EFT

- Borel resummation gives Hydro part and other (“Non-Hydro”) part
- Non-hydro part:

$$\gamma \exp\left[-i\int d\hat{\tau}\left(\hat{\omega}_{\text{Borel}}\hat{\tau}^{-1/3}\right)\right]$$

- $w_{\text{Borel}}=\pm 3.1193-2.7471\, i$ [Heller et al, 1302.0697]
- $w_{\text{QNM}}=\pm 3.119-2.747i$ [Starinets, hep-th/0207133]

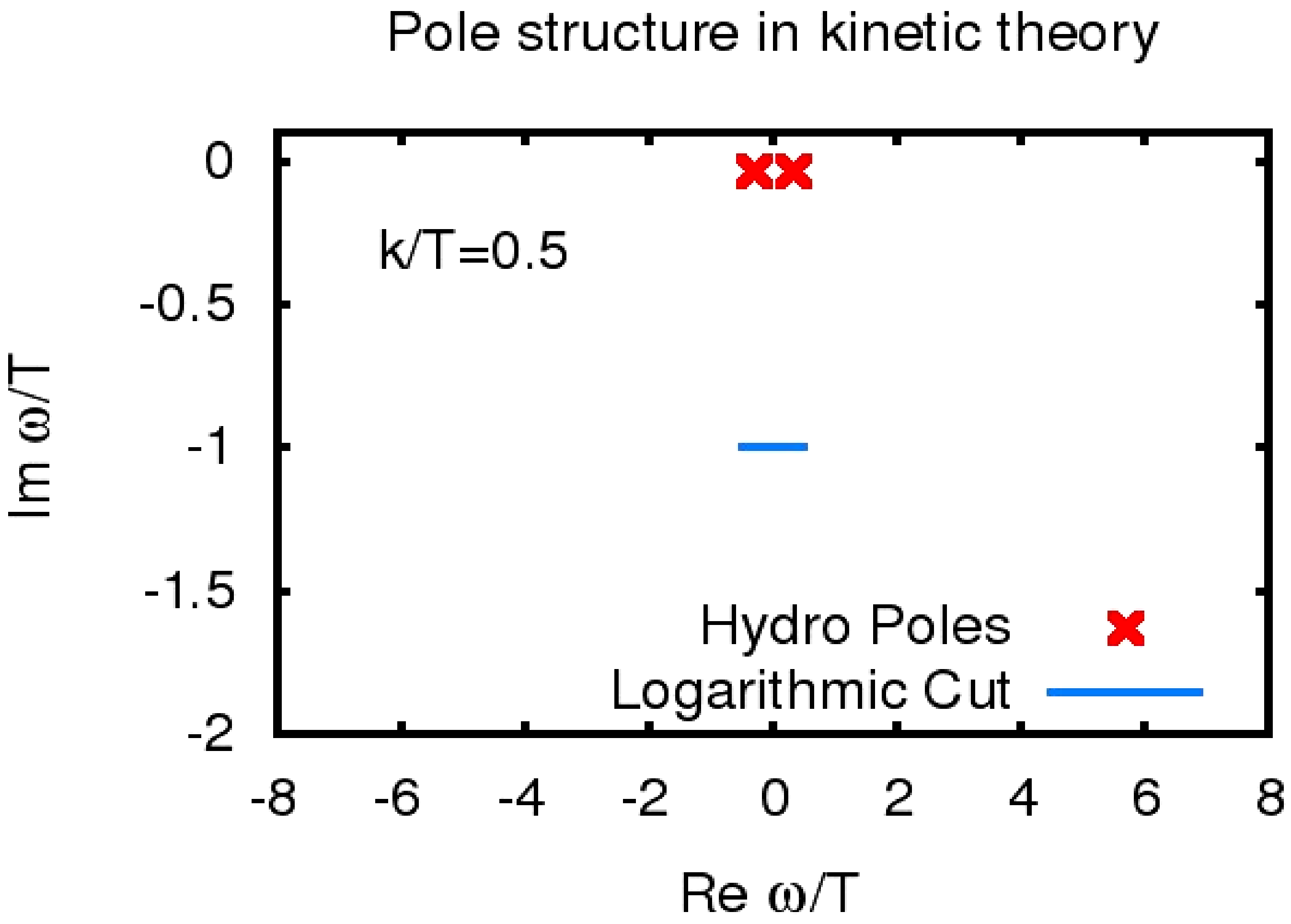
Hydro as an EFT: strong coupling perspective



: Quasinormal spectrum of gravitational fluctuations in the sound channel,

[Kovtun&Starinets, hep-th/0506184]

Hydro as an EFT: weak coupling perspective

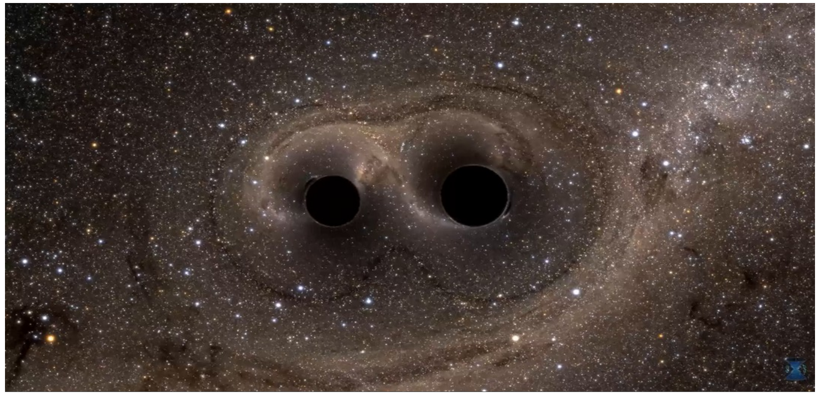


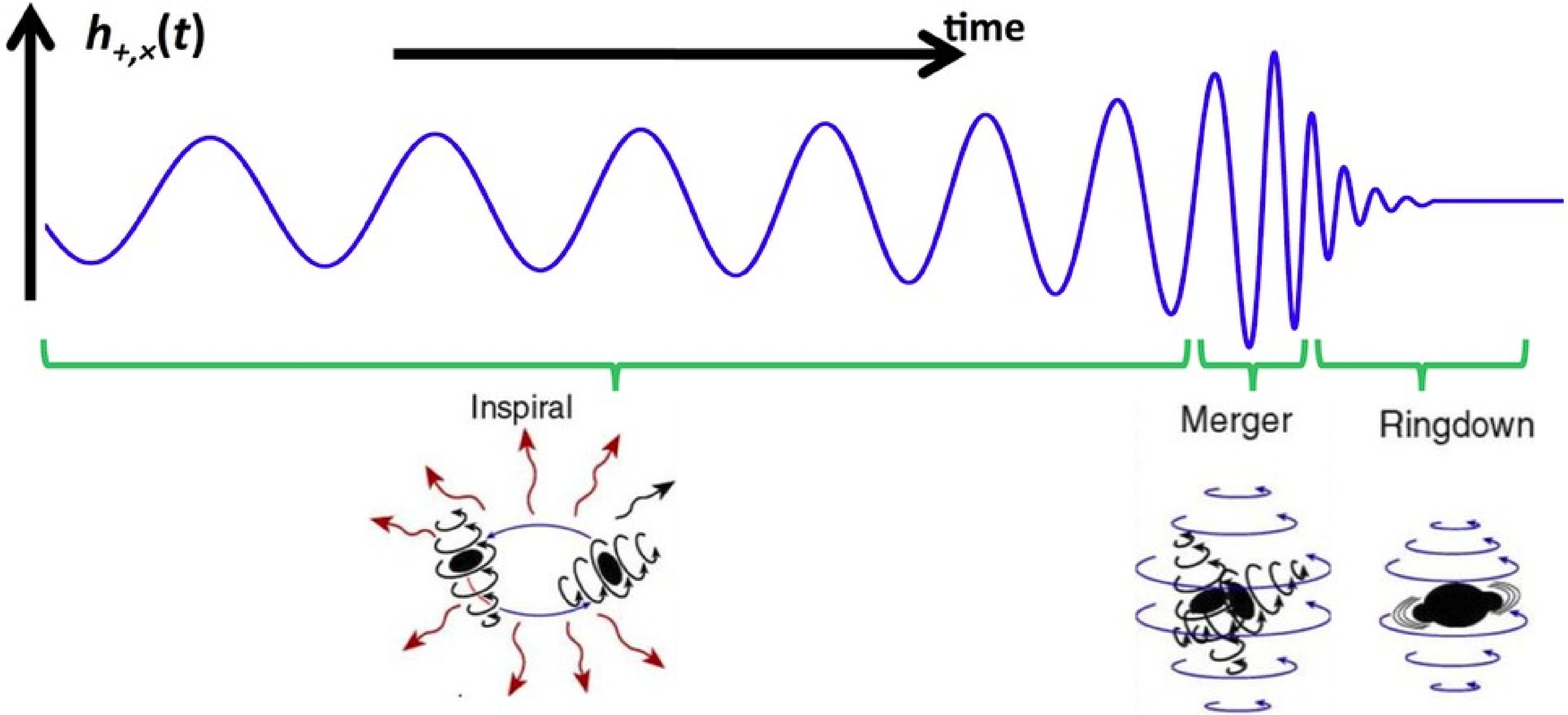
[1512.02641; 1712.04376]

Finding

Hydrodynamic Gradient Series Diverges because of the Presence of Non-Hydrodynamic Modes

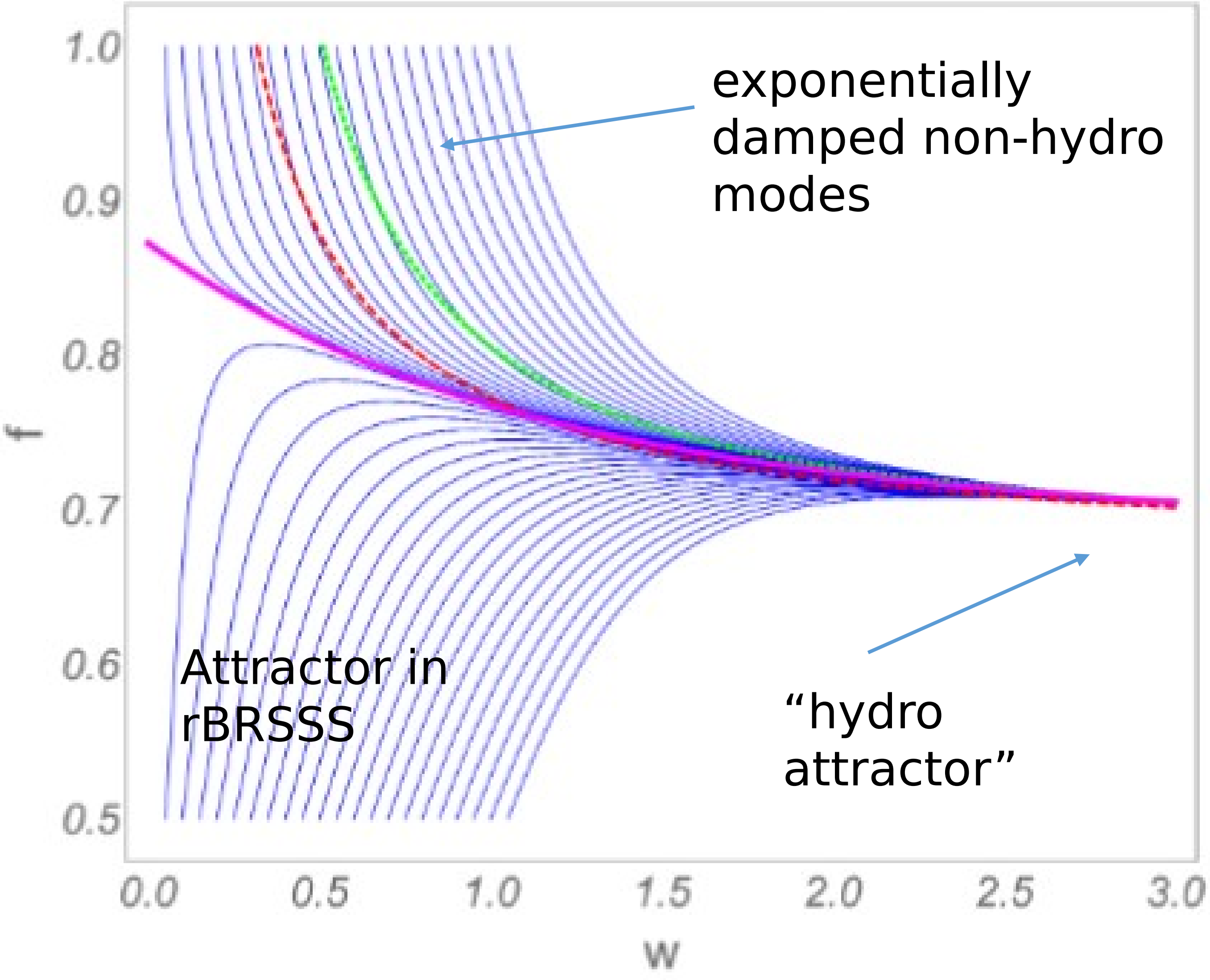
Aside: What are Non-Hydrodynamic Modes?





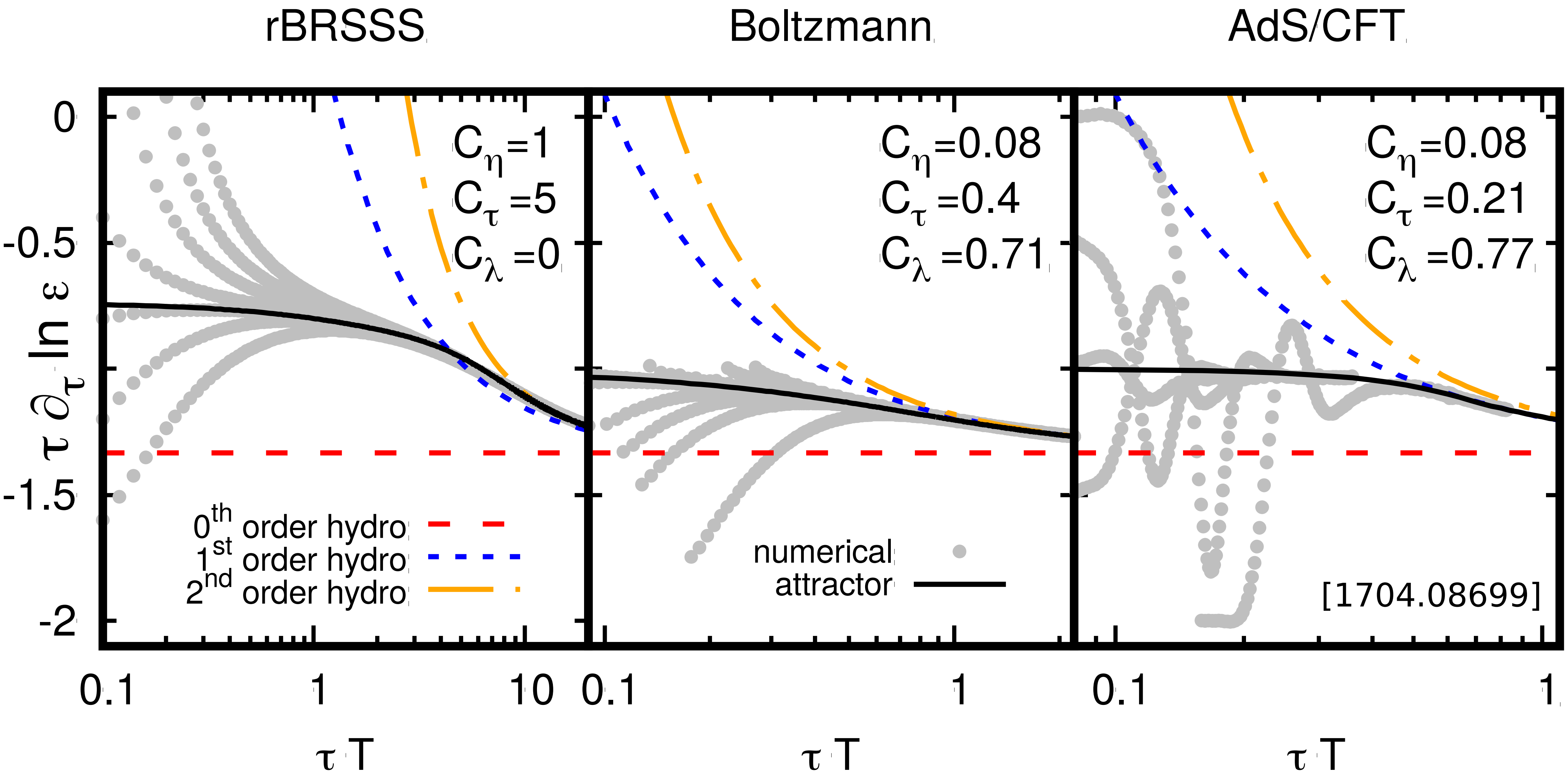
M. Favata/SXS/K. Thorne

Back to main story: off-equilibrium hydro



[Heller and Spalinski, 1503.07514]

Attractors as definitions for off-equilibrium hydro



[cf. 1708.01921, 1709.06644, 1710.03234, 1710.07095, 1711.01657, 1711.01745, 1712.02772, 1712.03856, ...]

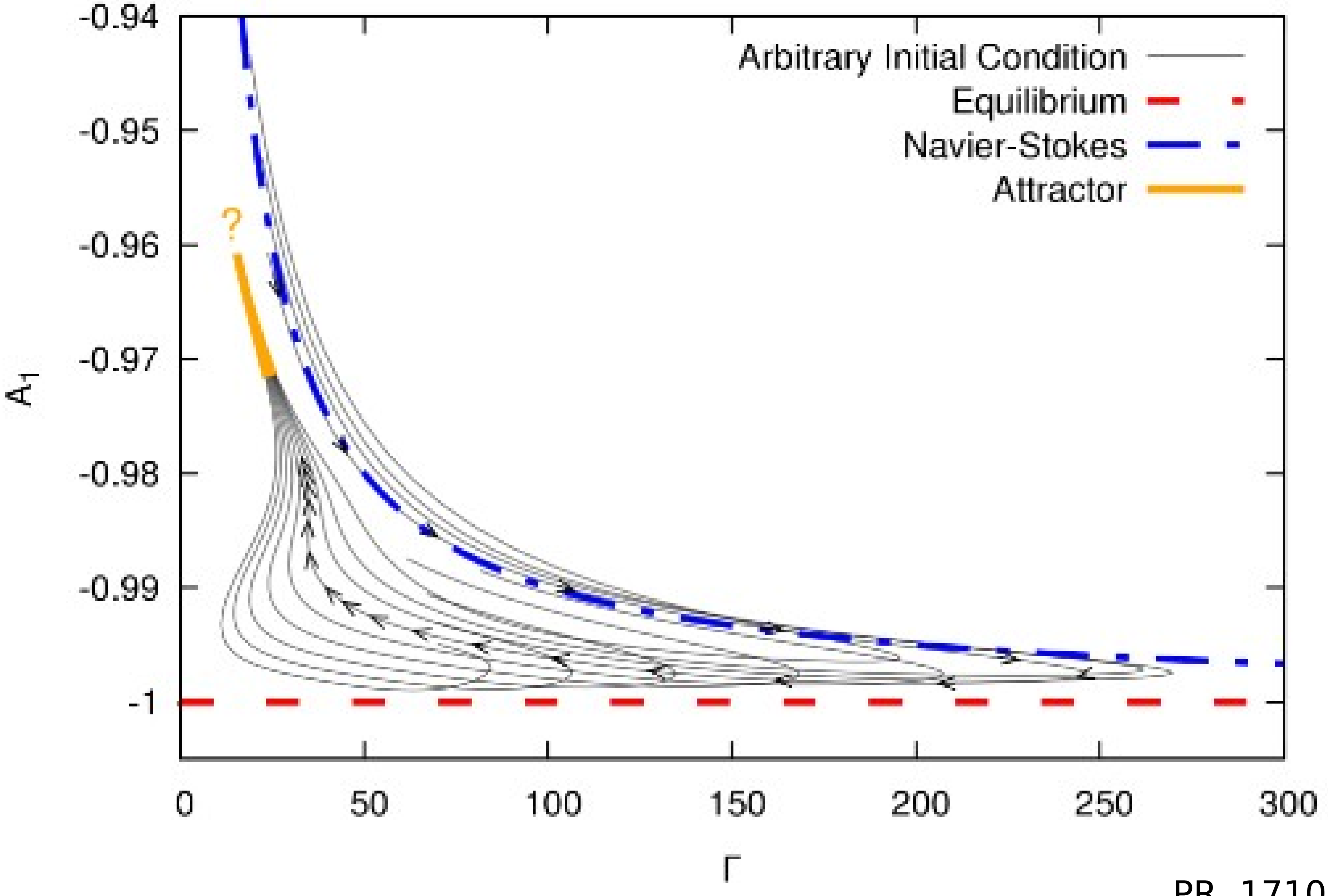
Finding (& Take-Home Message)

Attractor offers quantitatively reliable out-of-equilibrium description as long as contribution from non-hydro modes can be neglected[1].

No need of thermal equilibrium
No need of isotropy

[1] If a local rest frame exists.

Attractor in Conformal 2+1d rBRSSS

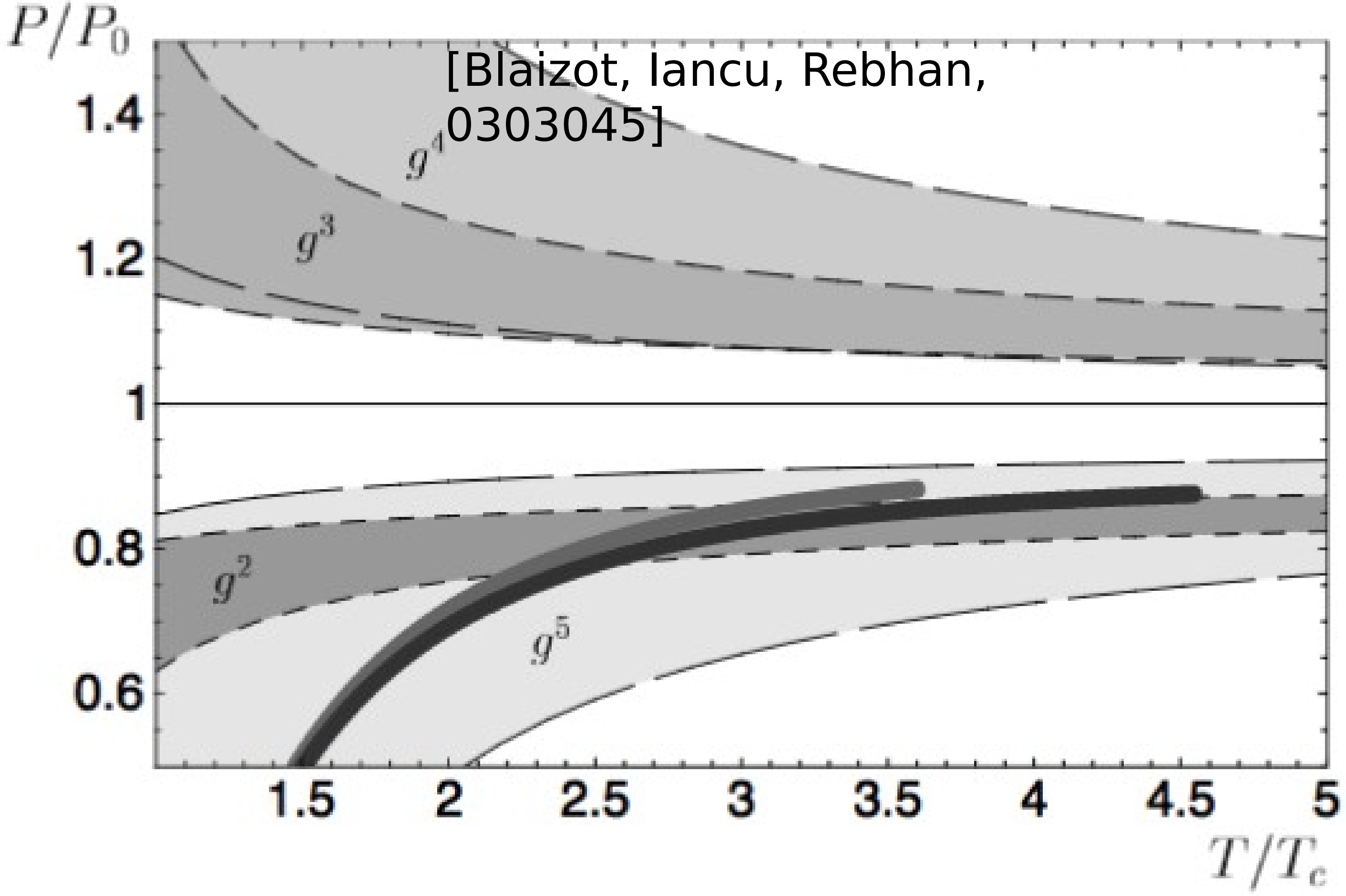


PR, 1710.03234

Attractors only known for a couple of cases/theories.
“Standard” Hydro solutions easier to come by

When does “standard” hydro apply?

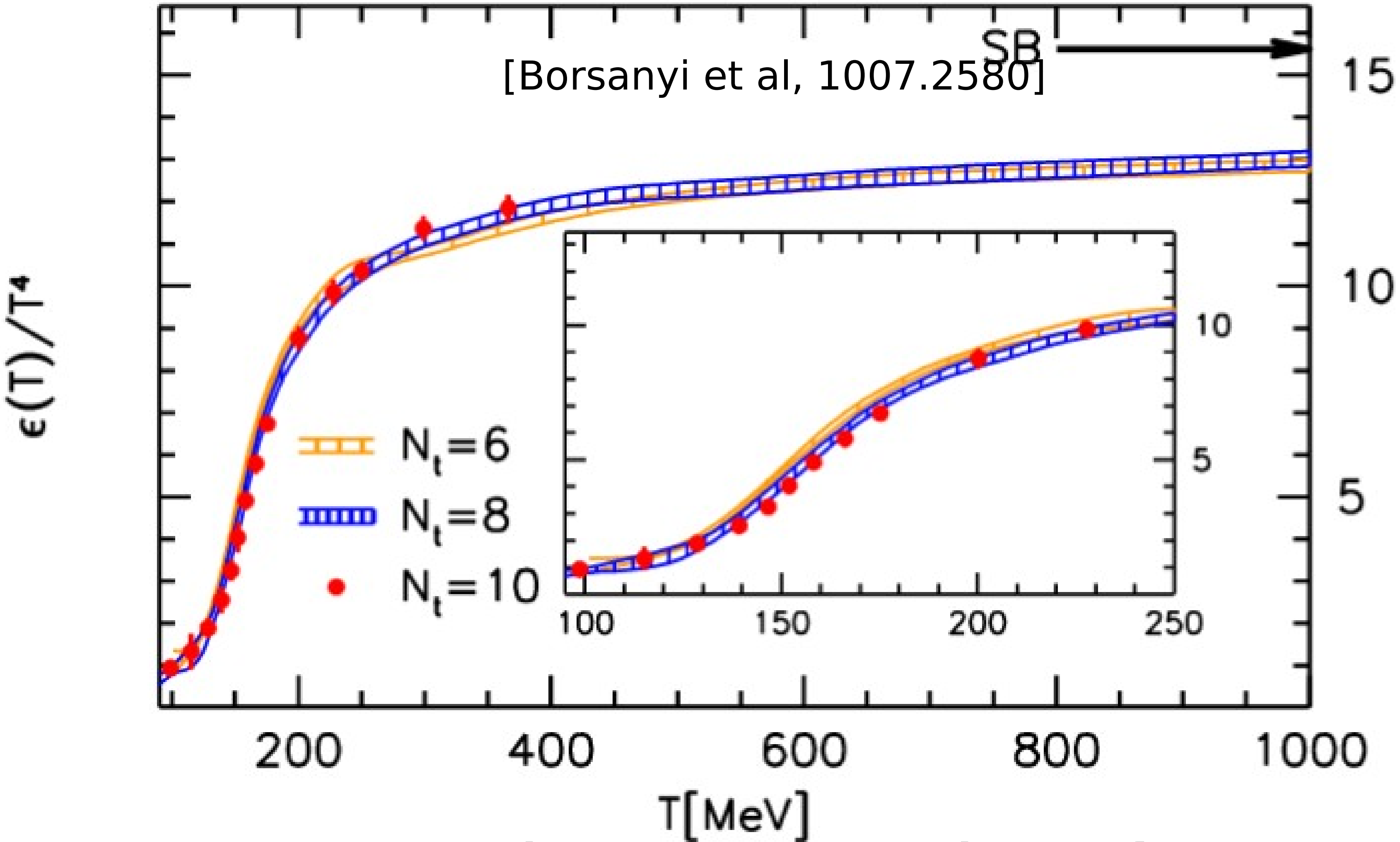
Hints from QCD thermodynamics



[Blaizot, Iancu, Rebhan, 0303045]

Perturbative Expansion for pressure is also divergent series

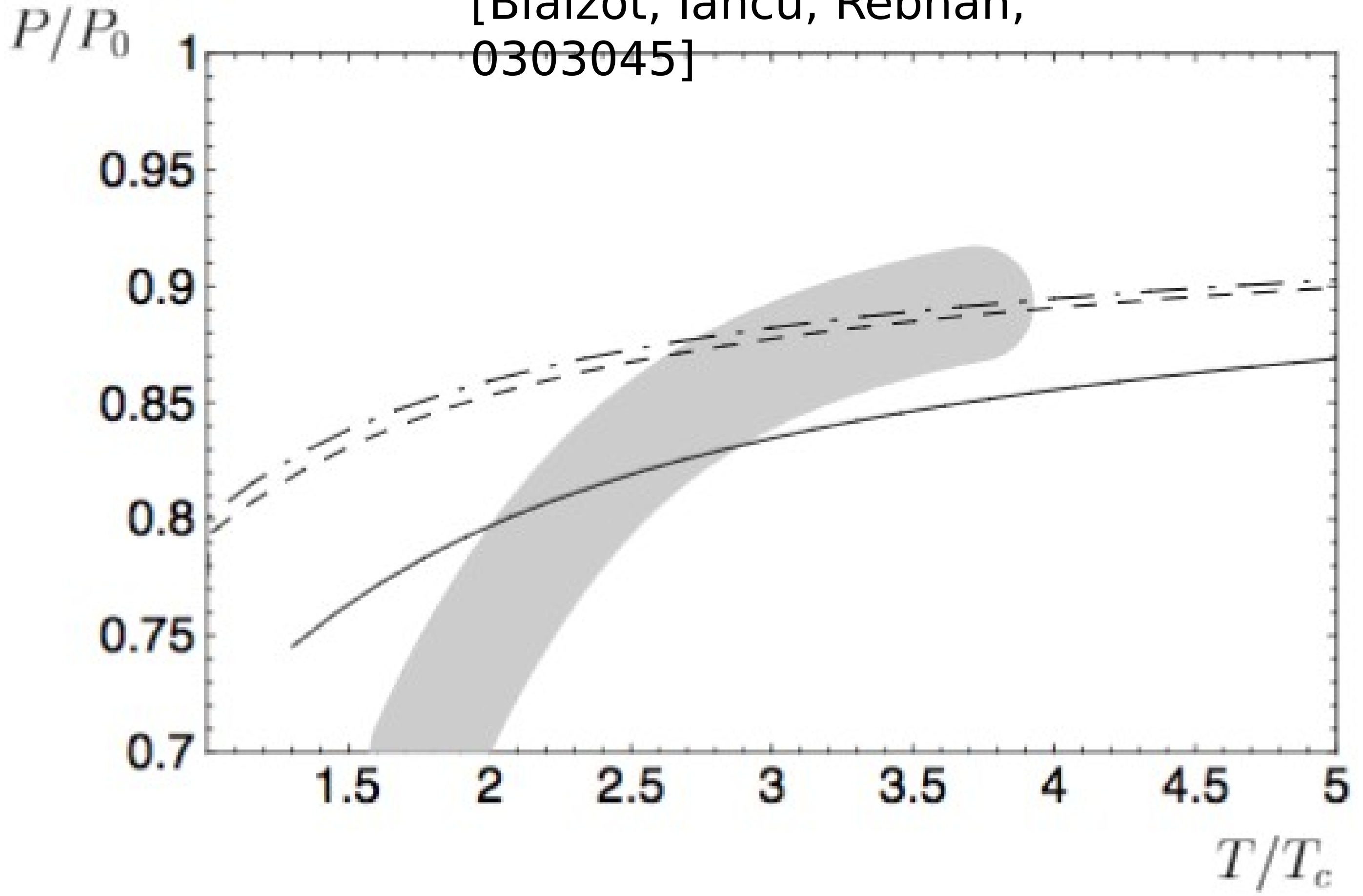
Hints from QCD thermodynamics



But a non-analytic result exists for all T

Hints from QCD thermodynamics

[Blaizot, Iancu, Rebhan, 0303045]



…and low-order pQCD is close to the full result

Finding

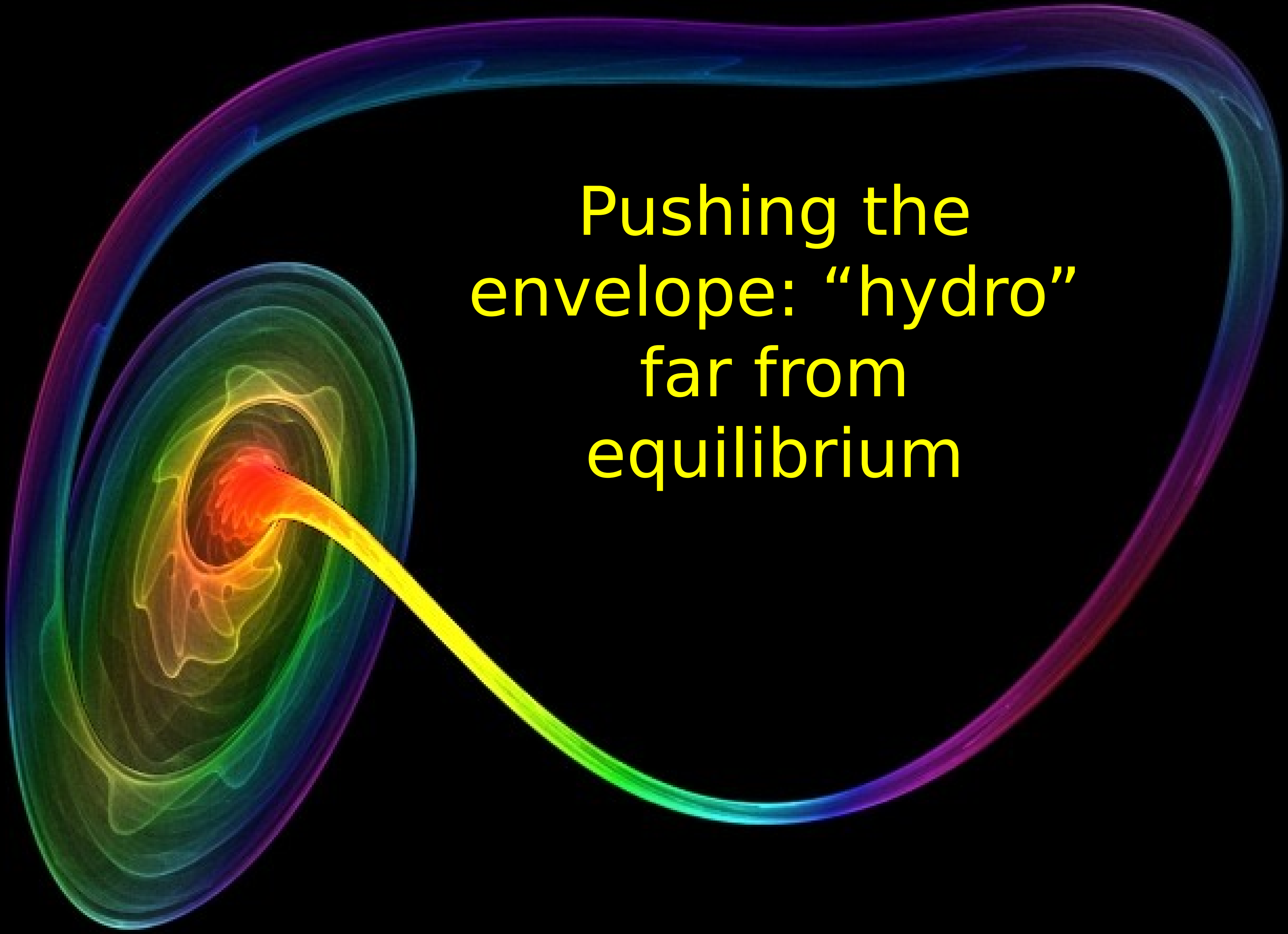
(Low order) Hydrodynamics coincides with attractor solution at moderate gradients.

Suggestive explanation why low-order hydrodynamics works quantitatively out-of-equilibrium

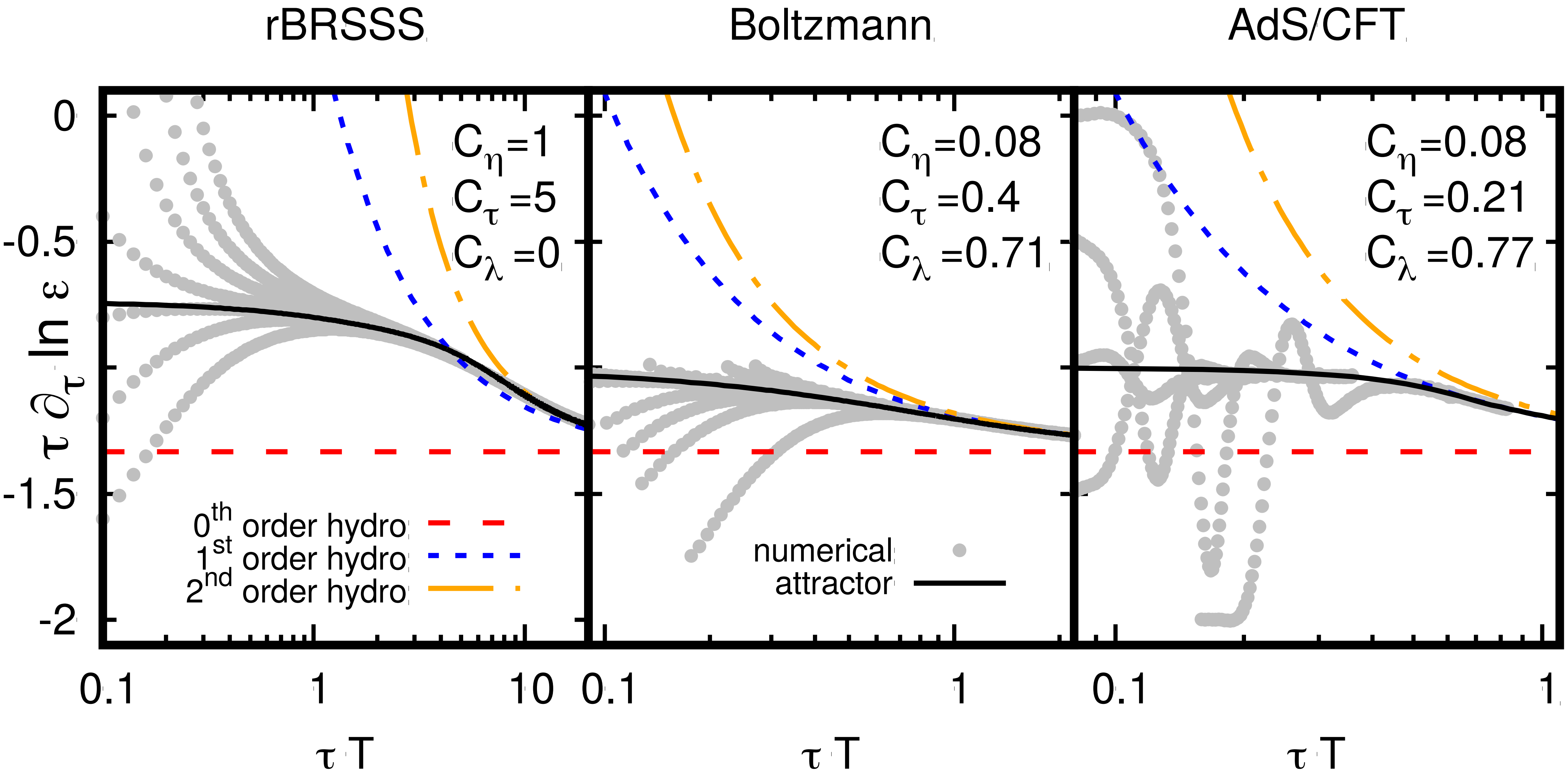
[1609.02820]

So far:
Why hydro models apply *moderately* out of equilibrium

Pushing the envelope: “hydro” far from equilibrium



'Borel-resummation' : attractor solutions far from equilibrium



Far-from-equilibrium ("Borel") Hydro

- Normal hydro:

$$T^{ab} = (\epsilon + P)u^a u^b + P g^{ab} - 2\eta \nabla^{\langle a} u^{b\rangle} + \ldots$$

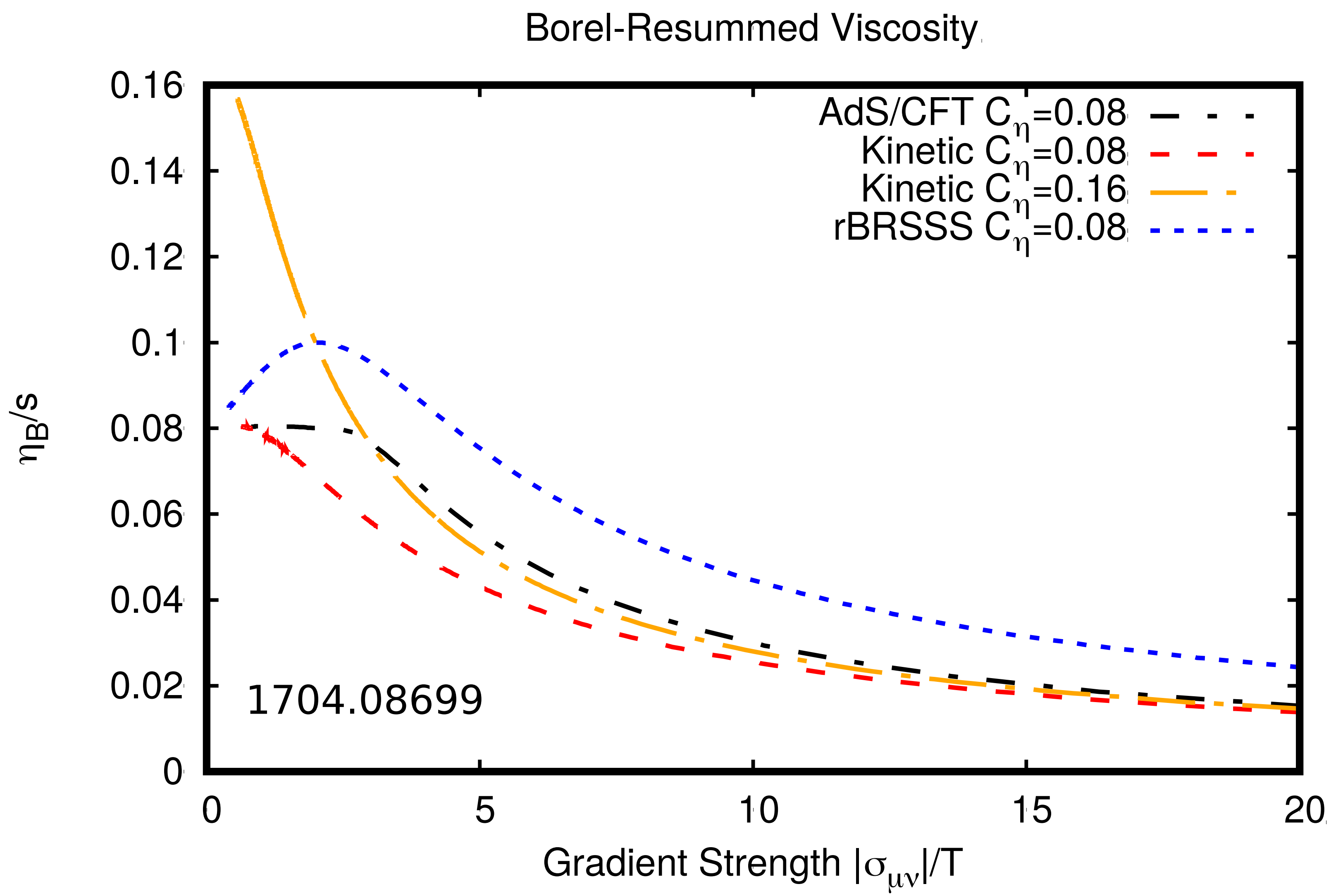
- Far-from equilibrium hydro:

$$T^{ab} = (\epsilon_B + P_B)u^a u^b + P_B g^{ab} - 2\eta_B \nabla^{\langle a} u^{b\rangle}$$

where $\eta_B = \eta_B(|\nabla^{\langle a} u^{b\rangle}|)$ depends on gradient strength

- For conformal system, $\varepsilon_B = 3 P_B$ even far from equilibrium!
- Non-conformal systems, attractor similarly tracks EoS [1710.03234]

Effective (Borel-Resummed) Viscosity



Qualitatively similar to "All-orders hydro" idea by Shuryak & Lublinsky

Far-From-Equilibrium Hydro

- Robust attractor solution far from equilibrium
- Effective viscosity typically smaller than equilibrium viscosity
- Possible hint for effective description for far from equilibrium strongly coupled problems?

Conclusions

- Standard equilibrium requirement for hydro may be too restrictive
- Numerical+Experimental evidence that hydro applies out-of-equilibrium
- Theory developments in past 10 years:
- Hydrodynamic gradient series diverges, can be resummed, gives rise to attractor solution
- Attractor may be thought of as “off equilibrium hydro” generalization
- Low order hydro coincides with attractor at moderate gradient strength
- Off-equilibrium hydro can be characterized by equilibrium EoS and non-equilibrium transport coefficients

Future Directions

- What is the hydro attractor (if it exists) for QCD?
- Attractor analysis only for classical fluids so far (no fluctuations, long time tails): how about resurgence of hydro action?
- Attractor analysis intertwined with non-hydro modes: can we study non-hydro modes in nature?
- For others, see 1712.05184, chapter 6.3

Bonus Material