Dynamics of Fluids without Boost Symmetries

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as well as earlier work with Stefan Vandoren

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Introduction

- Standard treatments of fluid dynamics use a notion of rest frame.
- What if a fluid moves through a medium that breaks boost symmetries?
- Often translations get broken as well but we will assume that happens at a different (smaller) scale.
- Example: relativistic fluid in EM background: $\partial_\mu T^\mu{}_\nu = J^\mu F_{\mu\nu} \Longrightarrow \partial_\mu \left(T^\mu{}_\nu - J^\mu A_\nu\right) \sim \lambda^{-1}\partial_\nu \lambda$ where $A_\nu = \lambda \delta^0_\nu$ with $\partial_\mu J^\mu = 0$ for $\lambda = \text{cst}$.
- For slowly varying λ we recover translation invariance at the expense of boost symmetry as $T_{\text{new}}{}^\mu{}_\nu = T^\mu{}_\nu - J^\mu A_\nu$ is no longer symmetric.

Introduction

- From an algebraic perspective (if there is time translation invariance) translation breaking implies boost breaking as $[H\,,B_i] = P_i$ but the converse is not true.
- We see that translations break at smaller scales than boosts.
- So there can exist a regime in which we effectively recover translation invariance but without boost symmetry.
- This includes cases with Lifshitz scale symmetry.
- Earlier work: [Hoyos, Kim, Oz, 2013], [de Boer, JH, Obers, Sybesma, Vandoren, 2017] and [Novak, Sonner, Withers, 2019].

Introduction

- A thermodynamic system with Lifshitz scale invariance is only compatible with boost symmetries when the dynamical critical exponent $z = 1, 2$ [de Boer, JH, Obers, Sybesma, Vandoren, 2017].
- This is despite the existence of algebras with Galilean boosts and dilatations for all z (Schroedinger algebras).
- The UIR of the associated group do not form a discrete spectrum when placing the system in finite volume. See also [Grinstein, Pal, 2018].
- The nogo result can be generalised to systems with hyperscaling violation exponent θ and charge anomalous dimension α in that boosts can only exist when $z = \alpha + 2$ (Galilei) or $z = 1$ (Lorentz).

Introduction

- What is the medium in the case of a Lifshitz invariant system that is responsible for the boost breaking?
- Example of a domain wall separating two superfluids: capillary waves with $z = 3/2$ [Watanabe, Murayama, 2014]:

$$L = \frac{1}{2}\int d^2x d^2y \frac{\dot{\phi}(t,\vec{x})\dot{\phi}(t,\vec{y})}{|\vec{x}-\vec{y}|} - \frac{\sigma}{2}\int d^2x \frac{\partial\phi(t,\vec{x})}{\partial\vec{x}} \cdot \frac{\partial\phi(t,\vec{x})}{\partial\vec{x}}$$

- Results from integrating out gapless superfluid modes. ϕ is displacement field of 2D domain wall.
- Notice this Lagrangian has $z = 3/2$ but no fractional powers of derivatives.

Outline

- Thermodynamics, perfect fluids and geometry of absolute spacetime
- Entropy current formalism
- Hydrostatic partition function and action for non-dissipative transport
- Constitutive relations
- Dissipative transport
- Comments on holographic realisations of Lifshitz fluids
- Outlook

Thermodynamics

- Consider a thermodynamic system with grandcanonical partition function

$$\mathcal{Z}(T,V,\mu,\vec{v}) = \mathrm{Tr}\, e^{-\beta\left(\hat{H}-\mu\hat{N}-\vec{v}\cdot\hat{\vec{P}}\right)}$$

- We treat velocity as a chemical potential to describe systems in different inertial frames.
- Grand potential: $\Omega = -T \log \mathcal{Z}(T,V,\mu,\vec{v})$
- Extensivity: $\Omega = -PV$
- First law: $d\Omega = -SdT - PdV - P_i dv^i - N d\mu$
- Here P_i is the (ensemble averaged) momentum

Thermodynamics

- Energy density: $\mathcal{E} = Ts + \mu n + \mathcal{P}_i v^i - P$
- First law: $\delta\mathcal{E} = T\delta s + \mu\delta n + v^i\delta\mathcal{P}_i$
- Momentum density: $\mathcal{P}_i = \rho v^i$ by rotational symmetry
- Gibbs-Duhem: $\delta P = s\delta T + n\delta\mu + \frac{1}{2}\rho\delta v^2$
- Boost symmetries fix ρ. For example for Galilean boosts $\rho = n$ so that $dP = sdT + nd\hat{\mu}$ with $\hat{\mu} = \mu + \frac{1}{2}v^2$ and all explicit v^2-dependence has disappeared.
- Scaling: $\delta P = (d - \theta + z)\lambda P\,,\ \delta T = z\lambda T\,,\ \delta\mu = (z + \alpha)\lambda\mu\,,$ $\delta v^2 = 2(z-1)\lambda v^2$
- Scaling dimensions $[n] = d - \theta + \alpha$ and $[\rho] = d - \theta - z + 2$ are equal (which they must for Galilean invariance) iff $z = \alpha + 2$

Perfect Fluids

- Based on rotational symmetry only, the energy-momentum tensor is more general than allowed by thermodynamics.

- Charges flow with velocity v^i:

$$T^0{}_0 = -\mathcal{E} \qquad T^i{}_0 = -\left(\mathcal{E} + P\right) v^i \qquad T^0{}_j = \rho v^j \qquad T^i{}_j = P\delta^i_j + \rho v^i v^j$$

$$J^0 = n \qquad J^i = n v^i$$

- momentum flow: $\rho v^i v^j$, energy flux: $\left(\mathcal{E} + P\right) v^i$, charge flow: $n v^i$

- Equations of motion are conservation of energy, momentum and charge: $\partial_\mu J^\mu = 0\,, \qquad \partial_\mu T^\mu{}_\nu = 0$

- Lorentz and Galilei boosts:
  $T^0{}_i = -T^i{}_0 \Longrightarrow \rho = \mathcal{E} + P \qquad\qquad T^0{}_i = J^i \Longrightarrow \rho = n$

Geometry of absolute spacetime

- Fluids on curved backgrounds useful because:
  - Hydrostatic partition function and generalisations (non-dissipative transport)
  - Kubo formulas
  - General structure of derivative terms in constitutive relations and transport coefficients
- Metric structure of absolute spacetime:
  - Clock 1-form τ_μ and spatial metric $h_{\mu\nu}$ with signature $(0, 1, ..., 1)$.
  - Also known as Aristotelian geometry [Penrose, 1968].
- Appears in Hořava–Lifshitz gravity and Lifshitz field theories etc. See e.g. the works: [Griffin, Grosvenor, Hořava, Yan, 2013-15].

Geometry of absolute spacetime

- Due to signature of $h_{\mu\nu}$ we can write $h_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu$ where e^a_μ with $a = 1, \dots, d$ are spatial vielbeins.

- The spatial vielbeins transform under local $SO(d)$ transformations.

- Inverse vielbeins:

$$v^\mu \tau_\mu = -1 \qquad v^\mu e^a_\mu = 0 \qquad e^\mu_a \tau_\mu = 0 \qquad e^\mu_a e^b_\mu = \delta^b_a$$

- Integration measure: $e = \text{det}(\tau_\mu, e^a_\mu)$

- It is possible to define a metric compatible connection but we will not need it for our purposes: first order derivative corrections can be treated with Lie derivatives only.

Perfect fluids on curved space

- Remainder of this talk: no charge current.
- Energy-momentum tensor can be decomposed as $T^{\mu}{}_{\nu} = -T^{\mu}\tau_{\nu} + T^{\mu\rho}h_{\rho\nu}$
- energy current: $T^{\mu} = \mathcal{E}u^{\mu} + Ph^{\mu\rho}h_{\rho\nu}u^{\nu}$
- momentum-stress tensor: $T^{\mu\nu} = Ph^{\mu\nu} + \rho u^{\mu}u^{\nu}$
- Covariant velocity u^{μ} normalised as $\tau_{\mu}u^{\mu} = 1$
- Curved space EM conservation (diffeo Ward identity):

$$e^{-1}\partial_{\mu}\left(eT^{\mu}{}_{\rho}\right) + T^{\mu}\partial_{\rho}\tau_{\mu} - \frac{1}{2}T^{\mu\nu}\partial_{\rho}h_{\mu\nu} = 0$$

- This is the analogue of $\nabla_{\mu}T^{\mu}{}_{\nu} = 0$ in the relativistic case.

Perfect fluids on curved space

- The equation for energy-momentum conservation can be rewritten as

$$\mathcal{L}_\beta \tau_\rho = \frac{1}{2} X_\rho{}^{\mu\nu}(\mathcal{L}_\beta h_{\mu\nu} - h_{\mu\sigma}u^\sigma \mathcal{L}_\beta \tau_\nu - h_{\nu\sigma}u^\sigma \mathcal{L}_\beta \tau_\mu)$$

  where $\beta^\mu = u^\mu/T$ and $X_\rho{}^{\mu\nu}$ is a tensor that is constructed from thermodynamic objects.

- On flat space this relates temperature derivatives to the velocity derivatives: $\partial_t v_i$ and $\partial_i v_j + \partial_j v_i$.

- The Lie derivatives along β of the metric objects τ_μ and $h_{\mu\nu}$ can be viewed as derivatives of the fluid variables.

Entropy current formalism

- There must exist an entropy current S^μ built from the fluid variables such that $e^{-1}\partial_\mu\left(eS^\mu\right) \geq 0$.
- We split this into a canonical and non-canonical part:
  $S^\mu = S^\mu_{\text{can}} + S^\mu_{\text{non}}\,, \qquad S^\mu_{\text{can}} = -T^\mu{}_\nu\beta^\nu + P\beta^\mu$
- The canonical part obeys: $\tau_\mu S^\mu_{\text{can}} = s = \frac{1}{T}\left(\mathcal{E} + P - \rho v^2\right)$
- Using EOM and first law:

$$e^{-1}\partial_\mu\left(eS^\mu\right) = \left(T^\mu - T^\mu_{(0)}\right)\mathcal{L}_\beta\tau_\mu - \frac{1}{2}\left(T^{\mu\nu} - T^{\mu\nu}_{(0)}\right)\mathcal{L}_\beta h_{\mu\nu} + e^{-1}\partial_\mu\left(eS^\mu_{\text{non}}\right)$$

- (0) denotes the perfect fluid part, i.e. the part at zeroth order in derivatives.

Entropy current formalism

$$e^{-1}\partial_\mu\left(eS^\mu\right) = \left(T^\mu - T^\mu_{(0)}\right)\mathcal{L}_\beta\tau_\mu - \frac{1}{2}\left(T^{\mu\nu} - T^{\mu\nu}_{(0)}\right)\mathcal{L}_\beta h_{\mu\nu} + e^{-1}\partial_\mu\left(eS^\mu_{\text{non}}\right)$$

- We decompose the EMT as:

  $$T^\mu - T^\mu_{(0)} = T^\mu_{\text{D}} + T^\mu_{\text{HS}} + T^\mu_{\text{NHS}}\,, \qquad T^{\mu\nu} - T^{\mu\nu}_{(0)} = T^{\mu\nu}_{\text{D}} + T^{\mu\nu}_{\text{HS}} + T^{\mu\nu}_{\text{NHS}}$$

  where the three parts are defined as:

  - Dissipative transport (equality iff currents vanish):
    $e^{-1}\partial_\mu\left(eS^\mu\right) = T^\mu_{\text{D}}\mathcal{L}_\beta\tau_\mu - \frac{1}{2}T^{\mu\nu}_{\text{D}}\mathcal{L}_\beta h_{\mu\nu} \geq 0$
  - Non-hydrostatic non-dissipative transport:
    $T^\mu_{\text{NHS}}\mathcal{L}_\beta\tau_\mu - \frac{1}{2}T^{\mu\nu}_{\text{NHS}}\mathcal{L}_\beta h_{\mu\nu} = 0$
  - Hydrostatic non-dissipative transport:
    $e^{-1}\partial_\mu\left(eS^\mu_{\text{non}}\right) = -T^\mu_{\text{HS}}\mathcal{L}_\beta\tau_\mu + \frac{1}{2}T^{\mu\nu}_{\text{HS}}\mathcal{L}_\beta h_{\mu\nu}$

Hydrostatic partition function

- Consider a weakly curved background with time translation symmetry generated by a Hamiltonian with Killing vector β [Jensen, Kaminski, Kovtun, Meyer, Ritz, 2012].

- Killing equations: $\mathcal{L}_\beta \tau_\mu = 0 = \mathcal{L}_\beta h_{\mu\nu}$. These conditions define global thermal equilibrium.

- Analytically continue time (affine parameter along integral curves of β) and impose periodicity.

- Hydrostatic PF: $S_{\text{HPF}} = -i \log \mathcal{Z} = \sum_n S_{\text{HPF}}^{(n)}\,, \quad \mathcal{Z} = \text{Tr}\left[\text{e}^{-H/T}\right]$

- Up to first order in derivatives the allowed terms, taking into account conditions of thermal equilibrium, are $(u^2 = h_{\mu\nu}u^\mu u^\nu)$

$$S_{\text{HPF}} = \int d^{d+1}x e\left(P(T,u^2) + F_1(T,u^2)v^\mu\partial_\mu T + F_2(T,u^2)v^\mu\partial_\mu u^2\right) + \mathcal{O}(\partial^2)$$

Non-dissipative transport

- We drop the condition that β is a Killing vector and vary the geometric objects to obtain an EMT. This is possible because varying β under diffeos leads to the fluid EOM [Haehl, Loganayagam, Rangamani, 2014].

$$\delta_\xi S_{\text{HS}} = \int_{\mathcal{M}} d^{d+1}x\, e\left(-T^\mu \delta_\xi \tau_\mu + \frac{1}{2} T^{\mu\nu}\delta_\xi h_{\mu\nu} + F_\mu \delta_\xi \beta^\mu\right) = 0$$

- This leads to the on shell diffeomorphism Ward identity:

$$e^{-1}\partial_\mu\left(e T^\mu{}_\rho\right) + T^\mu \partial_\rho \tau_\mu - \frac{1}{2} T^{\mu\nu}\partial_\rho h_{\mu\nu} = 0$$

- This is not an action principle in the usual sense because we do not view $\beta^\mu = \frac{u^\mu}{T}$ as a fundamental variable.
- The HS action is the same as the hydrostatic partition function but without the restriction that β is Killing.

Hydrostatic non-dissipative transport

- Varying the HS action and going to Landau frame $T^{\mu}{}_{\nu}u^{\nu} = -\left(\mathcal{E} - \rho u^2\right)u^{\mu}$ we find at first order in derivatives:

$$T^{\mu\nu}_{(1)\text{HS}} = \frac{T}{2}\eta^{\mu\nu\alpha\beta}_{\text{HS}}\left(\mathcal{L}_\beta h_{\alpha\beta} - u_\alpha \mathcal{L}_\beta \tau_\beta - u_\beta \mathcal{L}_\beta \tau_\alpha\right) + \frac{1}{2}\eta^{\mu\nu\alpha\beta}_{\text{rot}}\left(\partial_\alpha u_\beta - \partial_\beta u_\alpha\right) + \ldots$$

- Here $u_\mu = h_{\mu\nu}u^\nu$. Dots contain terms that are only nonzero on curved backgrounds.

- The viscosity tensors η are decomposed in terms of $u^\mu, v^\mu, h^{\mu\nu}$ with coefficients depending on the functions appearing in the action.

- For details see [de Boer, JH, Have, Obers, Sybesma, 2020].

Hydrostatic non-dissipative transport

- It can be checked that (in Landau frame) the HS part of the EMT obeys:

$$e^{-1}\partial_\mu\left(eS^\mu_{(1)\text{non}}\right) = \frac{1}{2}T^{\mu\nu}_{(1)\text{HS}}\left(\mathcal{L}_\beta h_{\mu\nu} - u_\nu\mathcal{L}_\beta\tau_\mu - u_\mu\mathcal{L}_\beta\tau_\nu\right)$$

$$S^\mu_{(1)\text{non}} = -\frac{1}{T}\left(v^\mu u^\rho - v^\rho u^\mu\right)\left(F_1\partial_\rho T + F_2\partial_\rho u^2\right)$$

- This can also be derived from constitutive relations for the non-canonical entropy current, and the definition of the HS part of the EMT.

- Scale invariance: only one free function in the HS sector ($\alpha = u^2 T^{\frac{2}{z}-2}$):

$$S_{\text{HS(Lif)}} = \int d^{d+1}x e\left(T^{1+\frac{d}{z}}p(\alpha) + T^{\frac{d}{z}}q(\alpha)v^\mu\partial_\mu\alpha\right) + \mathcal{O}(\partial^2)$$

Non-hydrostatic non-dissipative transport

- When we drop the condition that β is Killing we can add 4 additional independent terms to the action.

$$S_{\text{NHS}} = \int d^{d+1}xe\left(F_3 u^\mu \partial_\mu T + F_4 u^\mu \partial_\mu u^2 - TF_5 v^\mu \mathcal{L}_\beta \tau_\mu - 2TF_6 u^\mu v^\nu \mathcal{L}_\beta h_{\mu\nu}\right)$$

- The EMT obeys: $T^\mu_{\text{NHS}} \mathcal{L}_\beta \tau_\mu - \frac{1}{2} T^{\mu\nu}_{\text{NHS}} \mathcal{L}_\beta h_{\mu\nu} = 0$. In Landau frame this means:

$$T^{\mu\nu}_{\text{NHS}} = \frac{T}{2} \eta^{\mu\nu\alpha\beta}_{\text{NHS}} \left(\mathcal{L}_\beta h_{\alpha\beta} - u_\alpha \mathcal{L}_\beta \tau_\beta - u_\beta \mathcal{L}_\beta \tau_\alpha\right), \qquad \eta^{\mu\nu\alpha\beta}_{\text{NHS}} = -\eta^{\alpha\beta\mu\nu}_{\text{NHS}}$$

- The HS terms in the action belong to the coset space of non-dissipative terms modulo the NHS terms.
- When we start from the most general constitutive relations for the EMT and demand that it corresponds to the NHS sector we also find 4 free functions.

Constitutive relations

- Divergence of the entropy (after rewriting the non-can. part):

$$e^{-1}\partial_\mu\left(eS^\mu\right) = -\frac{1}{2}\left(T^{\mu\nu}_{(1)} - T^{\mu\nu}_{(1)\text{HS}}\right)\left(\mathcal{L}_\beta h_{\mu\nu} - h_{\rho\nu}u^\rho\mathcal{L}_\beta\tau_\mu - h_{\mu\rho}u^\rho\mathcal{L}_\beta\tau_\nu\right)$$

- Non-negative entropy production dictates that:

$$T^{\mu\nu}_{(1)} - T^{\mu\nu}_{(1)\text{HS}} = \frac{T}{2}\eta^{\mu\nu\rho\sigma}\left(\mathcal{L}_\beta h_{\rho\sigma} - h_{\kappa\sigma}u^\kappa\mathcal{L}_\beta\tau_\rho - h_{\rho\kappa}u^\kappa\mathcal{L}_\beta\tau_\sigma\right)$$

- What is left is to classify all the allowed terms in $\eta^{\mu\nu\rho\sigma}$.
- Allowed terms must obey the $SO(d)$ Ward identities and all terms must be invariant under the symmetries of the thermal state around which we expand.
- Thermal state spontaneously breaks $SO(d)$ down to $SO(d-1)$ consisting of all rotations that leave $h^{\mu\rho}h_{\rho\sigma}u^\sigma$ invariant.

Dissipative transport

- The NHS terms correspond to the antisymmetric part of $\eta^{\mu\nu\rho\sigma}$. Linearised these are set to zero by the Onsager relations. There are 4 such coefficients (2 for Lifshitz).
- For the dissipative terms on flat space we have

$$
\begin{aligned}
(T_{(1)\mathsf{D}})^{0j} &= \frac{1}{2}\eta_{jkl}\left(\partial_k v^l + \partial_l v^k\right) + \kappa_{jk}\partial_t v^k \\
(T_{(1)\mathsf{D}})^{ij} &= \frac{1}{2}\eta^{ijkl}\left(\partial_k v^l + \partial_l v^k\right) + \kappa^{ijk}\partial_t v^k
\end{aligned}
$$

- The η and κ tensors are expanded in terms of the $SO(d-1)$ invariants: $n^i = \frac{v^i}{\sqrt{v^2}}, \qquad P^{ij} = \delta^i_j - n^i n^j$
- There are 10 dissipative transport coefficients (obeying inequalities to ensure positive entropy production). This reduces to 7 when we impose the z-deformed tracelessness condition for scale invariance: $zT^0{}_0 + T^i{}_i = 0$.

Holographic realisations

- Bulk theory admitting Lifshitz solutions:

$$\mathcal{L}_{(4)} = \sqrt{-g}\left(R - \frac{1}{4}Z(\Phi)F^2 - W(\Phi)B^2 - \frac{1}{2}(\partial\Phi)^2 - V(\Phi)\right) + \mathcal{L}_{\text{ct}}$$

- For examples of counterterm actions and near-boundary analysis see [Christensen, JH, Obers, Rollier, 2013] and [JH, Kiritsis, Obers, 2014/15] as well as [Chemissany, Papadimitriou, 2014].

- Answer for general actions and general z still elusive.

- Can we do fluid/gravity for moving Lifshitz black branes beyond the EMD model studied in [Kiritsis, Matsuo, 2015/16]?

Holographic realisations

- Ansatz for moving black brane solutions ($z = 2$) [JH, Obers, Sanchioni, 2016]:

$$
\begin{aligned}
ds_4^2 &= -r^{-4}F_1 dt^2 + \frac{dr^2}{r^2 F_2} + \frac{F_3}{r^2}dx^2 + \frac{F_4}{r^2}\left(dy + N^y dt\right)^2 \\
B &= r^{-2}G dt + A_y\left(dy + N^y dt\right) , \qquad \Phi = \Phi(r)
\end{aligned}
$$

- Near boundary ($r = 0$) exp.: $ds_4^2 = e^{\Phi}\frac{dr^2}{r^2} + h_{\mu\nu}dx^\mu dx^\nu$

$$
\begin{aligned}
\Phi &= -\frac{1}{8}r^2\rho - r^4\left(\frac{1}{6}T^t{}_t + \frac{1}{64}\rho^2\right) + \mathcal{O}(r^6) \\
B_t &= r^{-2} + \frac{1}{4}\rho + r^2\left(\frac{1}{12}T^t{}_t + \frac{1}{16}\rho^2\right) + \mathcal{O}(r^4) \\
B_i &= -\frac{1}{4}r^2 T^t{}_i + \mathcal{O}(r^4) \qquad \text{likewise for } h_{\mu\nu}
\end{aligned}
$$

Holographic realisations

- Ansatz for full solution (all functions depend on r only):

$$
\begin{aligned}
ds_4^2 &= -r^{-4}F_1 dt^2 + \frac{dr^2}{r^2 F_2} + \frac{F_3}{r^2}dx^2 + \frac{F_4}{r^2}\left(dy + N^y dt\right)^2 \\
B &= r^{-2}G dt + A_y\left(dy + N^y dt\right)\,, \qquad \Phi = \Phi(r)
\end{aligned}
$$

- Effective action for this ansatz has two scale symmetries:

$$1). \quad [G] = 1\,, \quad [F_1] = 2\,, \quad [F_{3,4}] = 1\,, \quad [A_y] = -1/2\,, \quad [N^y] = 3/2$$

$$2). \quad [F_3] = 2\,, \quad [F_4] = -2\,, \quad [A_y] = -1\,, \quad [N^y] = 1$$

- Associated Noether charges Q_1 and Q_2 are 1st int. of motion along r (similar to) [Bertoldi, Burrington, Peet, 2009].

- Near horizon regularity: G, F_1, F_2 first order zeros at $r = r_h$, the rest are nonzero.

Holographic realisations

- Horizon generator $X = \partial_t - N^y(r_h)\partial_y$ gives temperature and chemical potential ($N^y(r_h) = -v$).

- Conserved Noether charges: $Q_1 - \frac{3}{2}Q_2 = Ts$ at the horizon and $\tilde{\mathcal{E}} + P$ at the boundary.

- 1st law from on-shell action, grand potential as a function of temperature T and chemical potential v^2:
  $\tilde{\mathcal{E}} + P = Ts\,, \qquad \delta P = s\delta T + \frac{1}{2}\rho\delta v^2$.

- Open problems:
  - Can we do fluid/gravity when the 0th order solution is not analytically known?
  - What is the holographic renormalisation for generic actions admitting Lifshitz solutions?

Outlook

- Adding a $U(1)$ current for charge or particle no. see e.g. [de Boer, JH, Obers, Sybesma, Vandoren, 2017] and [Novak, Sonner, Withers, 2019].
- Kubo formulas and higher point correlation functions.
- In [de Boer, JH, Obers, Sybesma, Vandoren, 2017] we studied the hydrodynamic modes for fluctuations of a charged fluid around a fluid at rest. This can now be generalised to perturbations around a moving fluid.
- We now understand the role of boost breaking for speed of sound, transport coefficients, hydro modes but can we measure such effects?

Outlook

- Find examples of systems with a hierarchy in boost and translation symmetry breaking.
- Expand fluid/gravity for Lifshitz fluids beyond EMD models studied in [Kiritsis, Matsuo, 2015/16] and Hořava–Lifshitz gravity [Garbiso, Kaminski, 2019]?
- Can we find holographic realisations of non-dissipative transport?
- Find new universal bounds involving the new transport coefficients?