Superluminal chaos after a quantum quench

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based on arXiv:1908.08955
in collaboration with V. Balasubramanian, B. Craps and M. De Clerck

Holotube Webinars

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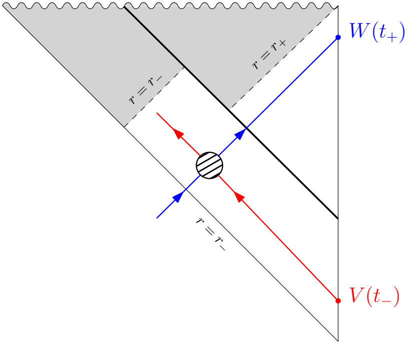
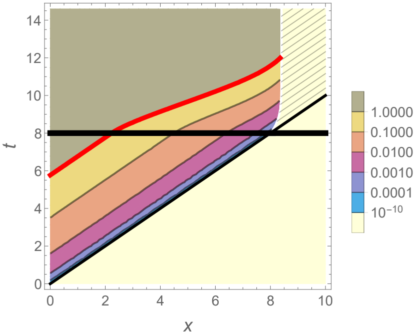
HARVARD UNIVERSITY

Overview

$$\langle [W(t,x),V(0)]^2\rangle \approx \langle WW\rangle\langle VV\rangle - \text{Re}\,\underbrace{\langle V(0)W(t,x)V(0)W(t,x)\rangle}_{OTOC(t,x)} \approx e^{\lambda_L(t-|x|/v_B)}$$

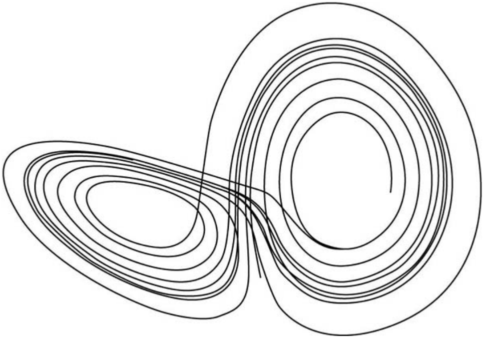




- ▶ WKB/geodesic approximation
- ▶ Superluminal butterfly velocity

Introduction

Butterfly effect in classical mechanics



$$\{q(t), p(0)\} = \frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda_L t}$$ Lyapunov growth

Out-of-time-order correlators

Commutator squared as a quantum analogue

$$\langle [W(t), V(0)]^2 \rangle_\beta \sim e^{2\lambda_L t}$$ [Larkin & Ovchinikov '69]

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After the dissipation timescale, it simplifies to

$$\langle [W(t), V(0)]^2 \rangle_\beta \approx \text{Re}\ \underbrace{\langle V(0) W(t) V(0) W(t) \rangle_\beta}_{OTOC(t)} - \langle WW \rangle_\beta \langle VV \rangle_\beta \qquad (t \gg \beta)$$

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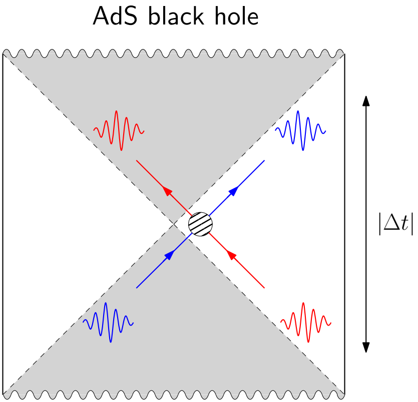
At the scrambling time t_*, dissipation eventually wins over Lyapunov growth

$$OTOC(t) \approx 0 \qquad CS(t) \equiv -\frac{\langle [W(t), V(0)]^2 \rangle_\beta}{\langle WW \rangle_\beta \langle VV \rangle_\beta} \approx 1 \qquad (t \gg t_*)$$

Butterfly effect in AdS/CFT

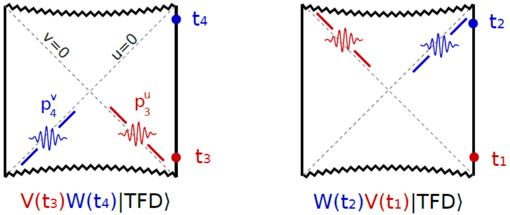


$$E^2_{collision} \sim e^{\lambda_L |\Delta t|} E_{in} E_{out} \qquad \text{with} \qquad \lambda_L = 2\pi T_{Hawking}$$

Original gravity formula

[Shenker & Stanford '14]

$$OTOC = \langle W(t,x)V(0)W(t,x)V(0)\rangle_\beta$$



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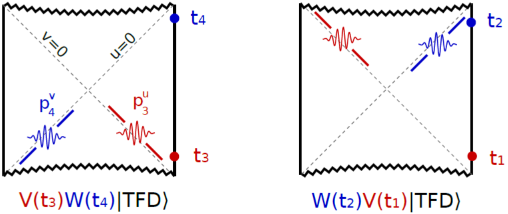


$$OTOC = \int e^{i\delta(t,x'-x'')}\left[p^U\Psi_1^*(p^U,x')\Psi_3(p^U,x')\right]\left[p^V\Psi_2^*(p^V,x'')\Psi_4(p^V,x'')\right]$$

- Ψ: Fourier transform of the boundary-to-horizon propagator
- $e^{i\delta}$: eikonal scattering amplitude along the horizon $(t \gg \beta)$
- evaluation by saddle point approximation

Static BTZ black hole

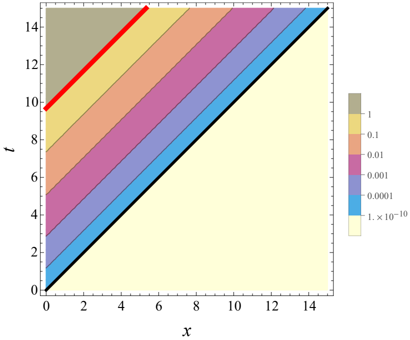
[Shenker & Stanford '14, Roberts & Stanford '14]

$$CS(t,x) \approx G_N \; e^{2\lambda_L(t - x/v_B)} \qquad (\beta \ll t \ll t_*)$$

$\lambda_L = 2\pi\beta^{-1}$ Lyapunov exponent

$v_B = c$ butterfly velocity

$t_* \sim \lambda_L^{-1} \ln G_N^{-1} \sim \lambda_L^{-1} \ln N$ scrambling time



Geodesic approximation

Limitations of the previous OTOC formula:

- relies on the Fourier transform
- valid for $t \gg \beta$

Derivation of an alternative OTOC formula:

1. geodesic approximation to bulk propagators
2. time-ordered bulk 4-point function ($2 \to 2$ eikonal scattering)
3. out-of-time-order correlator

Geodesic approximation

Klein-Gordon field

$$\Box\phi = m^2\phi$$

WKB ansatz

$$\phi(X) = A(X)\ e^{imS(X)}, \qquad m \gg 1$$

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At leading order in $1/m$

$$k^2 = -1, \qquad k_\mu \equiv \partial_\mu S$$

with

$$k^\nu \nabla_\nu k_\mu = 0.$$

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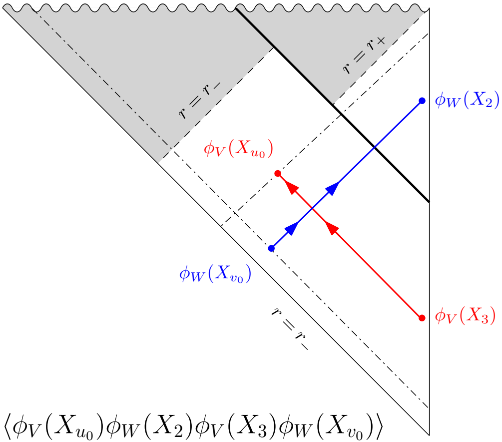
with

$$k^\nu \nabla_\nu k_\mu = 0.$$

Hence, the phase $S(X)$ is simply the action of a timelike geodesic,

$$S(X) = \int^X k_\mu dx^\mu = -\int^X d\tau$$

Time-ordered correlator



$$\langle \phi_V(X_{u_0}) \phi_W(X_2) \phi_V(X_3) \phi_W(X_{v_0}) \rangle$$

Action

$$S\left[\phi_W, \phi_V, g_{\mu\nu}\right] = S_{KG}\left[\phi_W, g_{\mu\nu}\right] + S_{KG}\left[\phi_V, g_{\mu\nu}\right] + S_{EH}\left[g_{\mu\nu}\right]$$

Path integral representation

$$\begin{aligned}
&\langle \mathcal{T}\phi_V\phi_W\phi_V\phi_W\rangle \\
&= \int Dg_{\mu\nu} D\phi_W D\phi_V \;\phi_V\phi_W\phi_V\phi_W \;\; e^{iS[\phi_W,\phi_V,g_{\mu\nu}]} \\
&= \int Dg_{\mu\nu} \;\langle \mathcal{T}\phi_W\phi_W\rangle_{g_{\mu\nu}} \langle \mathcal{T}\phi_V\phi_V\rangle_{g_{\mu\nu}} \;\; e^{iS_{EH}[g_{\mu\nu}]} \\
&\approx \int Dg_{\mu\nu} \;A_W e^{im_W S_W} \;A_V e^{im_V S_V} \;\; e^{iS_{EH}[g_{\mu\nu}]} \approx A_W A_V e^{iS_{\text{on-shell}}}
\end{aligned}$$

with

$$S_{\text{on-shell}}\left[g_{\mu\nu}\right] = S_{EH} + m_W S_W + m_V S_V\Big|_{\text{on-shell}}$$

On-shell metric includes gravitational radiation emitted by the scattered particles

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}^{W} + h_{\mu\nu}^{V} + \mathcal{O}(h^2)$$

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$$S_{\text{on-shell}}\left[g_{\mu\nu}\right] = S_{\text{on-shell}}\left[g_{\mu\nu}^{AdS}\right] + S^{(1)}\left[h_{\mu\nu}\right] + \mathcal{O}(h^2)$$

with the eikonal phase shift

$$S^{(1)}\left[h_{\mu\nu}\right] = \int \left(h_{\mu\nu}^{V}\, T_{W}^{\mu\nu} + h_{\mu\nu}^{W}\, T_{V}^{\mu\nu}\right) \equiv \delta$$

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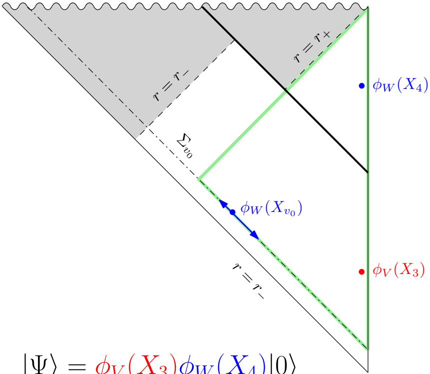
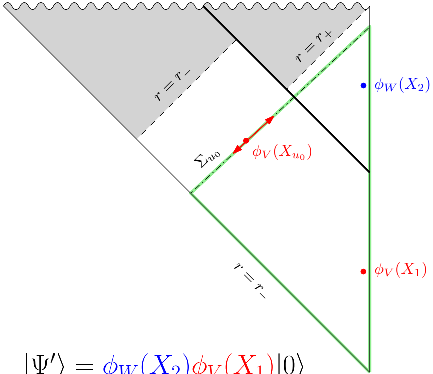
In summary, we find

$$\begin{aligned}\langle \mathcal{T}\phi_V\phi_W\phi_V\phi_W\rangle &\approx A_W e^{im_W S_W}\ A_V e^{im_V S_V}\ e^{iS_{EH}\left[g^{AdS}_{\mu\nu}\right]}\ e^{i\delta} \\ &\approx \langle \mathcal{T}\phi_W\phi_W\rangle\ \langle \mathcal{T}\phi_V\phi_V\rangle\ e^{iS_{EH}\left[g^{AdS}_{\mu\nu}\right]}\ e^{i\delta}\end{aligned}$$

Out-of-time-order correlator

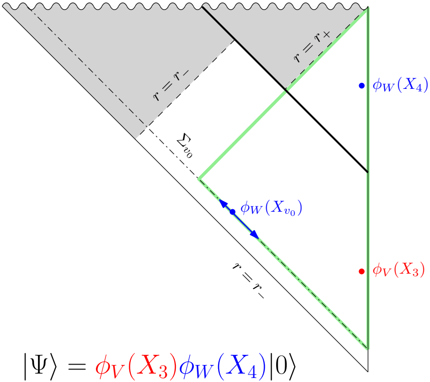


$$|\Psi\rangle = \phi_V(X_3)\phi_W(X_4)|0\rangle$$



$$|\Psi'\rangle = \phi_W(X_2)\phi_V(X_1)|0\rangle$$

Out-of-time-order correlator



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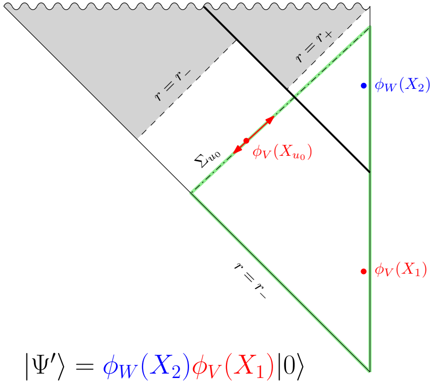

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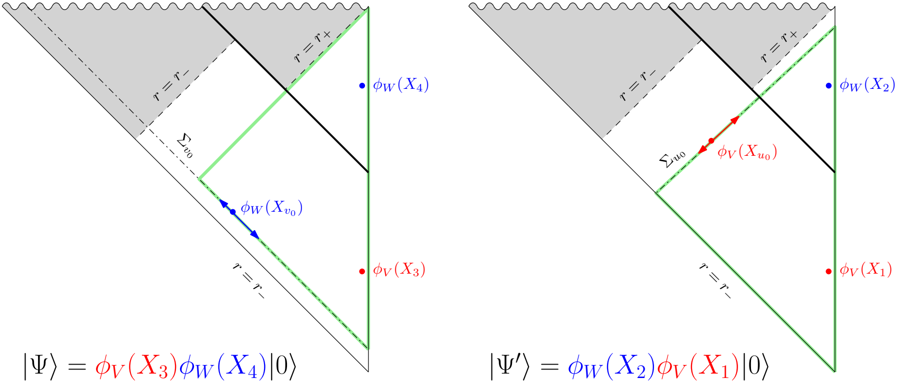
$$\hat{\phi}_V(X_1) = \int d\Sigma^v \left(\hat{\phi}_V(X_{u_0}) \overleftrightarrow{\partial_v} \, G_V^R(X_{u_0}, X_1) \right)$$

Out-of-time-order correlator



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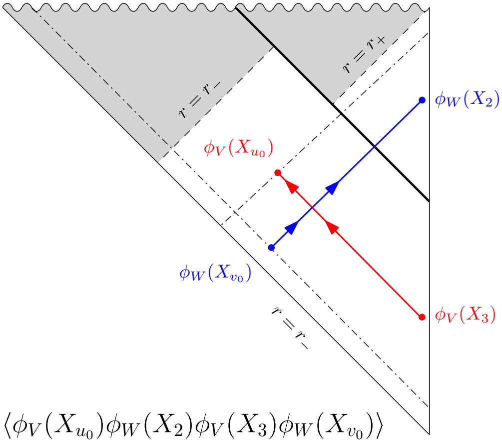
$$\langle \Psi' | \Psi \rangle \sim \int d\Sigma^u \, d\Sigma^v \, G^R_V(X_{u_0}, X_1) \, G^R_W(X_4, X_{v_0}) \, \langle \phi_V(X_{u_0}) \phi_W(X_2) \phi_V(X_3) \phi_W(X_{v_0}) \rangle$$



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Geodesic approximation

$$\mathrm{OTOC} = \langle \Psi' | \Psi \rangle \sim \int d\Sigma^u d\Sigma^v \; |A_W|^2 \; e^{-i m_W \epsilon_+ e_{out}} |A_V|^2 \; e^{-i m_V \epsilon_- e_{in}} \; e^{i\delta}$$

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Saddle point approximation

$$\frac{\text{OTOC}}{\langle \phi_V \phi_V \rangle \langle \phi_W \phi_W \rangle} \approx e^{i\delta} \Big|_{\text{saddle}}$$

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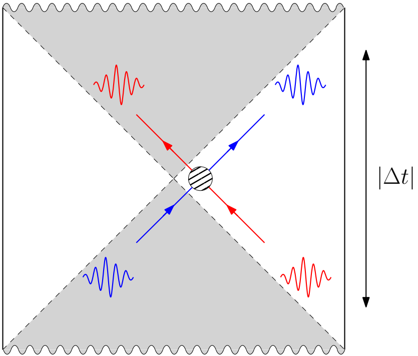
During the early-time Lyapunov growth, we simply have

$$\frac{\langle [\phi_W, \phi_V]^2 \rangle}{\langle \phi_V \phi_V \rangle \langle \phi_W \phi_W \rangle} \approx \delta^2 \sim (E^2_{collision})^2 \qquad (\delta \ll 1 \text{ or } t \ll t_*)$$

AdS black hole



$$E^2_{collision} \sim e^{\lambda_L |\Delta t|} E_{in} E_{out} \qquad \text{with} \qquad \lambda_L = 2\pi T_{Hawking}$$

Slow scrambling at low temperature

The original computational method of [Shenker & Stanford '14] assumes $t \gg \beta$, and is therefore not suited to study the zero temperature limit.

Our new method allows to recover *slow scrambling* at zero temperature

$$CS(t, x) \sim G_N \left(t - |x|\right)^2$$

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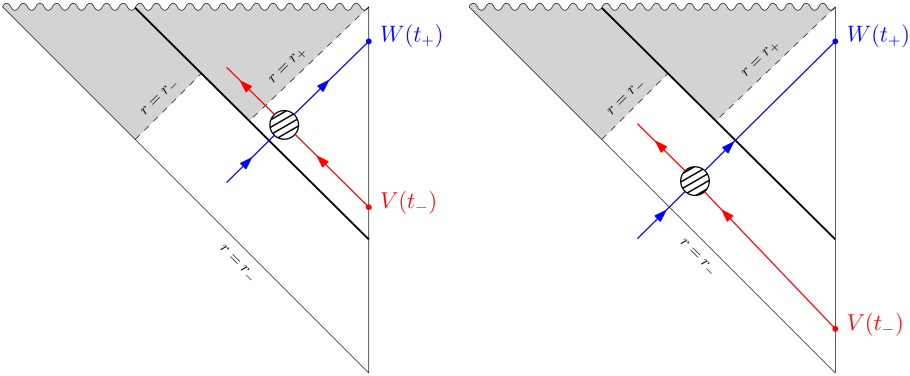
Limitation: this approximation does not describe the late-time saturation $(t \sim t_*)$ at finite temperature

OTOC with global energy injection

OTOC in BTZ-Vaidya



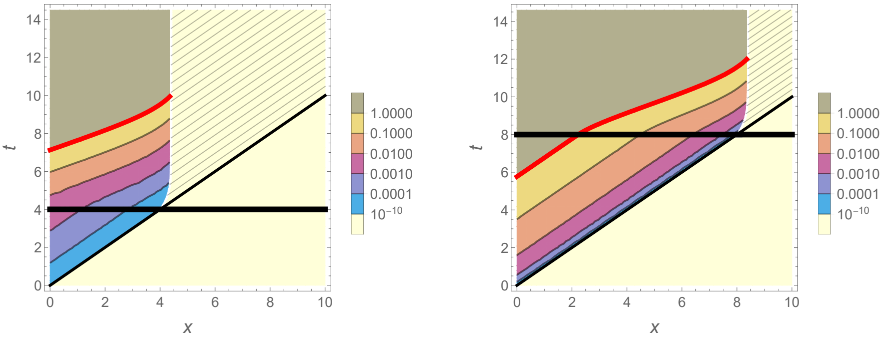
$$ds^2 = -\left(r^2 - \theta(v_s - v)r_-^2 - \theta(v - v_s)r_+^2\right) dv^2 + 2dvdr + r^2 dx^2$$

Eikonal phase in BTZ-Vaidya

$$\delta(t,x) \approx G_N m_V m_W \begin{cases} e^{r_-(t-|x|)} & (t < v_s) \\ e^{r_- v_s} e^{r_+(t-v_s)} e^{-r_-|x|} & (t > v_s) \end{cases}$$

We observe saturation of the *local* chaos bound

$$\lambda_L = r_h = 2\pi T_H$$



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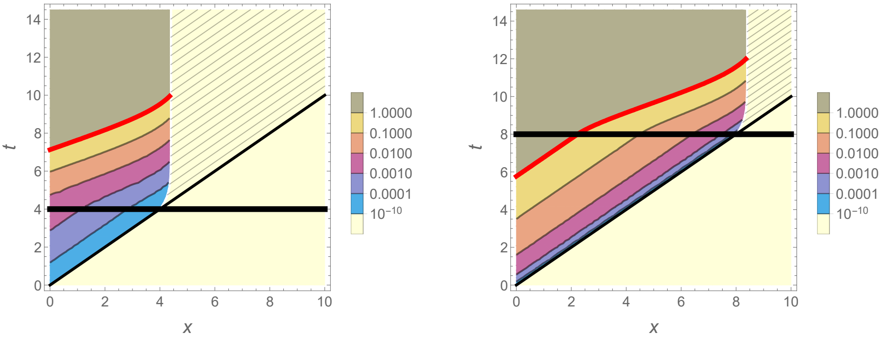
We observe a *transient superluminal* butterfly velocity

$$\frac{v_B}{c} = \frac{r_+}{r_-} = \frac{T_+}{T_-} > 1$$



Summary and open problems

OTOC by geodesic approximation

- *pros*: position space, applicable to any AlAdS spacetime, behavior at finite time (slow scrambling)
- *cons*: no account for late-time dissipation (OTOC late-time decay)

OTOC in BTZ-Vaidya

- local saturation of chaos bound
- transient superluminal butterfly velocity
- behavior at large spatial separations? [in progress]
- CFT computation? [see Anous & Hartman & Rovai & Sonner '16 '17]

Open problems

- inclusion of dissipation [see Festuccia & Liu '08]
- extremal BTZ, black hole microstates [in preparation]
- ...