Effective Field Theory for Chaotic CFTs

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Based on: Felix Haehl and MR , arXiv:1808.02898, Felix myself and Wyatt Reeves 1909.05847, and another to appear soon(-ish).

Outline:

1. Introduction and Motivation
2. Scrambling and OTOCs
3. Paradigm for Soft Mode EFT: the Schwarzian
   - Jackiw-Teitelboim Gravity
   - Schwarzian Action
   - Coupling to Matter
4. Theory of Soft Modes in 2D CFTs
   - Quadratic Action
   - Coupling to External Sources
   - Correlation Functions
5. Higher Dimensional CFTs
6. Future Directions

Quantifying Chaos in Many-Body Quantum Systems

- Classical chaos has short-time manifestations (divergence of trajectories) as well as long-time ones (ergodicity). Classical ergodicity makes connection with the physics of thermalization or transport.
- In quantum chaos, short-time manifestations have to do with the much discussed out-of-time-ordered correlators (OTOCs), whereas long times are usually associated with spectral statistics (or ETH). Quantum-Classical correspondence holds for early times, but late time and classical limits do not commute.
- Quantum ergodicity is aimed to provide the foundation of statistical mechanics. Simple observables in a typical state "look thermal". More tricky to explain the dynamics towards the final thermal state.
- Not just an academic exercise, for example some many-body systems do not thermalize (MBL).

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Quantifying Chaos in Many-Body Quantum Systems

- In this talk, we focus on short time chaos and OTOCs, exploring some intriguing connections to long time physics (i.e. hydrodynamics).
- In the context of CFTs we discuss an effective "scramblon" theory which reproduces the chaotic behavior, in the maximally chaotic case.
- This theory also calculates the series of increasingly more "fine-grained" OTOCs that probe aspects of thermalization beyond the initial scrambling (see paper with Felix in December 2017).
- I start with results are for 2D CFTs, see also work by Kristan Jensen and Jordan Cottler on AdS_3 gravity. This is useful for other purposes, e.g. calculating conformal blocks. In a precise sense this is the EFT for the identity sector in large c CFTs.

Quantifying Chaos in Many-Body Quantum Systems

- I then discuss work on higher dimensional CFTs on hyperbolic spaces, which is still maximally chaotic. Some of this is published and some in progress.
- The higher dimensional case makes connection to the shadow formalism, and other results from the bootstrap program.
- I end with ongoing work, including some quarter-baked ideas on moving away from the maximally chaotic case.

Out-of-Time-Order Correlators

Quick reminder on this much discussed topic. The OTOCs are close cousins of

- Semi-classical extension of the classical divergence of trajectories and Lyapunov exponent.
- Growth of localized (or simple) operators under unitary time evolution (but at finite temperature).
- Loschmidt echo.

Those all lead to the idea of looking at growth of commutators of general Hermitian operators $[V(t), W(0)]$. The expectation value of the commutator is related to linear response, to a perturbation V as measured subsequently by W.

Out-of-Time-Order Correlators

To quantify the "size" of the commutator look at the second moment

$$C[t] = -\langle [V(0), W(t)]^2 \rangle_\beta$$

Where the expectation value is evaluated at inverse temperature β.

The non-trivial part of this is the OTOC

$$F(t) = \langle V(0)W(t)V(0)W(t) \rangle_\beta \sim \frac{1}{N} e^{\lambda t}$$

As the other operator orderings are determined by factorization at large t. The (connected) OTOC grows exponentially at sufficient early time, then saturates at the "scrambling" or Ehrnfest time.

Comments:

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- Note this has two operators and two "switchbacks" in both Lorentzian time and operator ordering (=Euclidean time).
- The OTOC starts low and experiences exponential growth until it becomes order one (where higher order corrections cause it to saturate). The time it is allowed to grow is determined by how low it can get initially, or how sensitive it is to the special starting point. This defines the scrambling time $t_* \sim LogN$
- The rate of growth λ is bounded, $\lambda_L \leq \frac{2\pi}{\beta}$

Next: review and generalize the calculation of the OTOCs in AdS_2, related to the SYK model. This is a paradigm for an "effective" rather than micrsocopic calculation.

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Dilaton Gravity in 1+1 Dimensions

Gravity in 1+1 dimensions is trivial, to get something interesting we need at least a dilaton. So look at the theory with a negative cosmological constant (JT gravity)

$$I = -\frac{1}{16\pi G_N}\int \sqrt{-g}\,\phi(R+2) + 2\int_{bdy}\phi_{bdy}\,K$$

Possibly coupled to matter fields.

Locally any solution is trivial, it can be mapped to AdS_2 and a fixed dilaton profile, in some coordinates $\phi(z) \sim \frac{1}{z}$, where z=0 is the boundary of AdS_2.

Importantly, this diverges so this theory only makes sense with a cutoff and the boundary condition $\phi(z=\epsilon) \sim \frac{\phi_r(u)}{\epsilon}$, for some fixed $\phi_r(u)$.

Now different solutions to the equations of motion are identical locally but could be different globally. They are characterized by the shape of the curve on which the prescribed boundary condition is satisfied.

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Schwarzian Action

In the end, the gravitational action reduces to a boundary term, which describes the dynamics of the soft mode $t(u)$, which represents the shape of the boundary curve, or equivalently a diffeomorphism of the circle.

$$-I_{grav} = \frac{1}{\kappa^2} \int du \left[-\frac{1}{2} \left(\frac{t''}{t'} \right)^2 + \left(\frac{t''}{t'} \right)' \right]$$

This is the Schwarzian action, which is determined by a pattern of spontaneous and explicit conformal symmetry breaking. As such, it is also the low energy EFT for the SYK model, for example.

To compute correlators perturbatively in a black hole background, we transform $t(u) = \tan \frac{\tau(u)}{2}$, corresponding to working with temperature $\beta = 2\pi$, and expand around the saddle: $\tau(u) = u + \kappa\, \varepsilon(u)$. Our expansion parameter is κ, the gravitational constant. We are taking backreaction into account pertrurbatively in κ.

Soft Mode Propagator

$\epsilon(u)$ is our soft mode or "scramblon".

To leading order in κ the Schwarzian action gives a quadratic term, and hence a propagator for the mode $\varepsilon(u)$

$$\langle \varepsilon(u)\varepsilon(0)\rangle = \frac{1}{2\pi}\left[\frac{2\sin\, u - (\pi + u)}{2}(\pi + u) + 2\pi\Theta(u)(u - \sin\, u)\right]$$

Though there are also non-linearities, for our purposes we don't need to discuss them.

Matter Action

We couple the gravity theory to a matter action which represents external massless particles:

$$-I_{matter} = \frac{1}{2\pi}\int du_1 du_2 \frac{t'(u_1)t'(u_2)}{(t(u_1)-t(u_2))^2} j(u_1)j(u_2)$$

where j is a source for the (primary) operator whose correlator we are calculating. This reproduces, via the usual AdS/CFT. the boundary correlators of chiral primary field of dimension we choose for simplicity to be $\Delta = 1$.
We write the expansion in κ as

$$-I_{matter} = \frac{1}{2\pi}\int du_1 du_2 \frac{j(u_1)j(u_2)}{4\sin^2(\frac{u_{12}}{2})} \sum_{p\geq 0} \kappa^p \mathcal{B}^{(p)}(u_1, u_2)$$

where $u_{12} \equiv u_1 - u_2$ and $\mathcal{B}^{(0)} = 1$. The interaction vertices are explicit functions of the soft mode ϵ.

Interaction Vertices

We need the first and second order expansions, corresponding to the way the matter sources the soft mode $\varepsilon(u)$ to orders κ and κ^2

$$\mathcal{B}^{(1)}(u_1, u_2) = \varepsilon'(u_1) + \varepsilon'(u_2) - \frac{\varepsilon(u_1) - \varepsilon(u_2)}{\tan(\frac{u_{12}}{2})}$$

$$\begin{aligned}\mathcal{B}^{(2)}(u_1, u_2) = \frac{1}{4\sin^2(\frac{u_{12}}{2})}\Big[&(2 + \cos u_{12})\left(\varepsilon(u_1) - \varepsilon(u_2)\right)^2 \\ &+ 4\sin^2\left(\frac{u_{12}}{2}\right)\varepsilon'(u_1)\varepsilon'(u_2) \\ &- 2\sin u_{12}\left(\varepsilon(u_1) - \varepsilon(u_2)\right)\left(\varepsilon'(u_1) + \varepsilon'(u_2)\right)\Big]\end{aligned}$$

Effective Field Theory of the Soft Mode

For the SYK model (at infinite βJ), the microscopic calculation of the OTOCs is reproduced by the Schwarzian theory, which is the theory of the soft mode, Goldstone mode for the breaking of time diffeomorphisms. This is despite the fact we are not discussing asymptotically late time (or low energies).

Perhaps this is a general phenomena? Blake, Lee and Liu formulated a theory of "quantum hydrodynamics": a theory of the energy mode extrapolated away from the traditional long wavelength limit. With the aid of a phenomenological "shift symmetry" this reproduces the behaviour of correlation functions, but only for the maximally chaotic case.

Our goal is to systematically derive such theory for large c CFTs, where perhaps some of the underlying assumptions and surprises are more natural. we now proceed to discuss the required ingredients, following the analogous case of the SYK model just reviewed.

The Soft Mode

We follow standard approach to theory of Goldstone bosons, e.g. called "external field methods" in Weinberg's book.

To obtain the effective action for the soft modes, we consider performing the CFT path integral in the presence of sources for conformal transformations $\delta z = \epsilon(z)$, which are generated by the currents $J(z) = \epsilon(z)T(z)$. We explictly discuss only the left movers but same applies to the right movers .

Whereas such transformations are a global symmetry for holomorphic parameters $\epsilon(z)$, they become gapless modes for general transformation parameter $\epsilon(z, \bar{z})$. That is, the action for the soft mode corresponding to holomorphic reparametrizations stems from its dependence on the anti-holomorphic coordinate $\bar{z}$.

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The Soft Mode

Those "local" conformal transformations are implemented, via the conformal Ward identities, by the addition of a term $\int d^2z\, \bar{\partial}(\epsilon^\alpha(z,\bar{z})T^\alpha(z))$ to the action. The results of that path integral define the effective action as $Z(\epsilon) = e^{iW(\epsilon)}$.

While the soft mode ϵ starts its life as an external source, conjugate to the conformal currents, performing a Legendre transform trades energy-momentum fluctuations with fluctuations of the soft mode ϵ. We now have a theory of the soft mode which summarizes energy-momentum exhcnages in a large c CFT.

There is a systematic procedure to building this theory perturbatively in fluctuations of ϵ, but we will need only the quadratic action. The expansion parameter is $1/c$ and not the low frequency as is customary in soft mode actions.

Note that the mode $\epsilon(z,\bar{z})$ formally has conformal weights $(h,\bar{h}) = (-1,0)$. This formal non-unitarity is the reason ϵ will be able to exhibit an exponentially growing evolution.

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Quadratic Action

From the above definition it is then clear that the quadratic term in the effective action is

$$W_2 = -\frac{1}{2}\int d^2z_1\, d^2z_2\, \bar{\partial}\epsilon(z_1,\bar{z}_1)\, \bar{\partial}\epsilon(z_2,\bar{z}_2)\, \langle T(z_1)T(z_2)\rangle_C + (\text{anti-holo.})$$

Tu put the theory at finite temperature we map the plane to the cyllinder

$$z = e^{-iu} \equiv e^{-i(\tau+i\sigma)}\,, \qquad \bar{z} = e^{i\bar{u}} \equiv e^{i(\tau-i\sigma)}\,.$$

Getting

$$W_2 = -\frac{c}{64}\int d^2u_1 d^2u_2\, \frac{\bar{\partial}\epsilon(u_1,\bar{u}_1)\, \bar{\partial}\epsilon(u_2,\bar{u}_2)}{\sin^4\left(\frac{u_1-u_2}{2}\right)} + (\text{anti-holo.})\,.$$

The quadratic action is of order c, thus in perturbative calculations each factor of the soft mode is accompanied by $1/\sqrt{c}$.

Soft Mode Propagator

This results in the Euclidean propagator

$$\langle \epsilon(\omega_E, k)\, \epsilon(-\omega_E, -k) \rangle = -\frac{24\pi}{c} \frac{1}{\omega_E\,(\omega_E^2 - 1)\,(\omega_E + ik)}\,.$$

This is analogous ito the propagator coming from the Schwarzian, and has similar behaviour with respect to $SL(2, R)$, as is discussed in the paper. For our purposes, calculating Lorentzian correlators including OTOCs, we need the theory formulated on the Schwinger-Keldyish contour. The Lorentzian propagators (retarded, advanced, Keldyish) are displayed in the paper.

Crucially, the retarded propagator (defined as vanishing in the infinite past) has a pole in the upper half plane, indicating an instability.

$$G^R(t, \sigma) = -\frac{12\pi}{c}\,\Theta(t)\left[\Theta(-\sigma)\,e^{t+\sigma} - \Theta(\sigma)\left(e^{-(t+\sigma)} - 2\,e^{-(t+\sigma)\varepsilon}\right)\right].$$

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OTOcs in Chaotic CFTs

Next we need the "matter action" which is the reparametrized bi-local action corresponding to 2-point function of chiral primary operators. The expression is similar to the SYK result.

$$-I^{(h,\bar{h})}_{matter} = \frac{\mathcal{C}_X}{2^{2(h+\bar{h})}} \int d^2u_1\, d^2u_2\, \frac{J(u_1,\bar{u}_1)\, J(u_2,\bar{u}_2)}{\sin^{2h}\left(\frac{u_1-u_2}{2}\right)\, \sin^{2\bar{h}}\left(\frac{\bar{u}_1-\bar{u}_2}{2}\right)} \sum_{p\geq 0} \mathcal{B}^{(p)}_{(h,\bar{h})}$$

where the interaction vertices $\mathcal{B}^{(p)}_{(h,\bar{h})}(u_1, u_2, \bar{u}_1, \bar{u}_2)$ are almost indetical to the 1D case.

Using those ingredients we can calculate correlation functions, in analogy to the 1D case.

For the 4-point OTOCs we obtain the previous results (Roberts and Stanford) in a fraction of the work. We also calculate the chaos exponent and butterfly velocity from the "pole skipping" phenomena in one (short) line.

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Comments

This summarizes a sketch of the calculation for 2D CFTs. More comments on the EFT:

In the spirit of hydrodynamical EFTs, we formulated the EFT on the SK contour. This is more significant in the dissipative case. In this formulation we also identified our "fine-grained" correlators, which previously had a Euclidean definition, in the Lorentzian setting.

The SL(2) gauge symmetries generalize those of SYK, and include the shift symmetry of Blake et. al. They are also reminiscent of the mysterious $U(1)_T$ which appear in some formulations of hydro EFT. They are crucial to the chaos story hanging together. More on this below.

The Euclidean action is useful as a the theory of the identity conformal block in large c CFTs. For example it is useful to calculating conformal blocks in large c, including $1/c$ corrections and higher point blocks. See paper by Cotler and Jensen (and also recent one by Felix and Tarek Anous) for details.

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Easiest to describe CFTs on hyperbolic spaces since the basic ingredients (2-point functions of stress-energy and primaries) are all known – related to same quantities in the vacuum.

Pole skipping reveals the theory is maximally chaotic, and gives the butterfly "velocity".

The soft mode is the generator of ()space-time dependent) conformal transformations. It gets an action (in even dimensional spaces) from the conformal anomaly.

One can also understand the soft mode using the shadow formalism. The shadow of the energy-momentum tensor is a total derivative, it can be written as the derivative of the soft mode, which then becomes local.

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One can also understand the soft mode using the shadow formalism. The shadow of the energy-momentum tensor is a total derivative, it can be written as the derivative of the soft mode, which then becomes local.

Higher Dimensional CFTs

The analysis is more techincal so I will be brief. We have a constriuctive way to develop a perturbation theory for the soft mode. This can be applied in principle to any CFT.

Easiest to describe CFTs on hyperbolic spaces since the basic ingredients (2-point functions of stress-energy and primaries) are all known – related to same quantities in the vacuum.

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Conclusions

Possible Directions for further research

- Most ambitiously: what new ingredient is needed for non-maximal case (dissipation? breaking of SL(2)).
- Non-linear action in higher dimension, and its applications.
- How effective the EFT is in calculating conformal blocks (higher point, 1/c corrections). Sometimes easier and more systematic than existing methods.
- Bulk analysis, in higher dimensions.
- deSitter.
- Deformations: massive or $T\bar{T}$.
- ...