SL(2,R) Lattices and Networks as Information Processors

A model for an old black hole and perhaps other fantastic systems

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Credits

Paper to appear soon with

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The figures on these slides have been prepared by Tanay Kibe

Thanks to the moon and the sea for inspiration

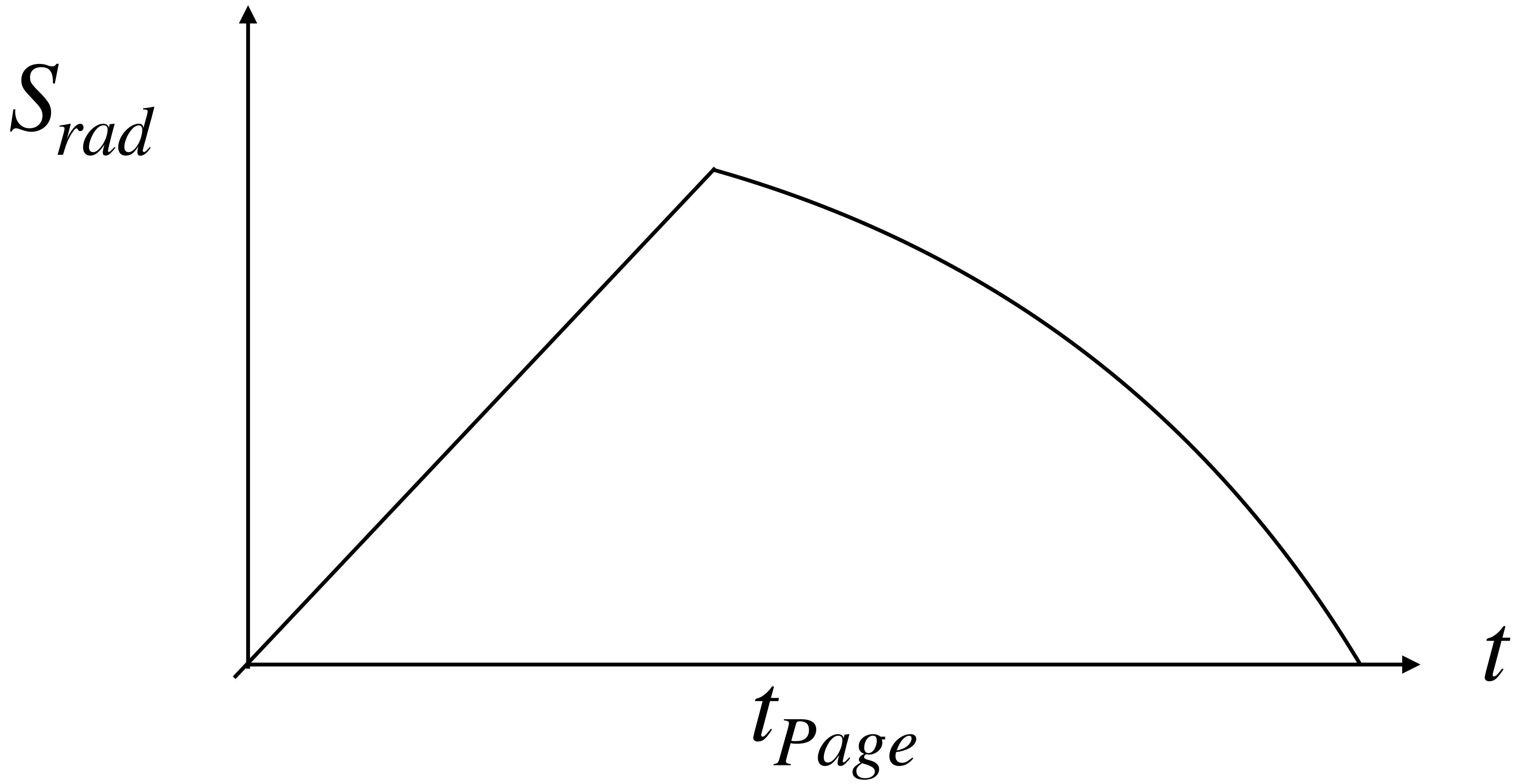
Plan

- Introduction and Motivation
- The Model
- The Microstates and the arrow of time
- Shock and Verify
- The Hayden-Preskill Protocol
- Fragmented Holography
- Conclusions and Outlook

Introduction and Motivation

"That old black hole"

- The old BH is past its Page time — half of the Hawking quanta have been emitted (10^{67} years for a Sun-sized black hole)
- Assuming a typical initial state, the reduced density matrix of the Hawking radiation should be maximally mixed at Page time and then approach purity. Information of the black hole interior starts coming out after Page time.
- How to reconcile with Hawking's original computation?



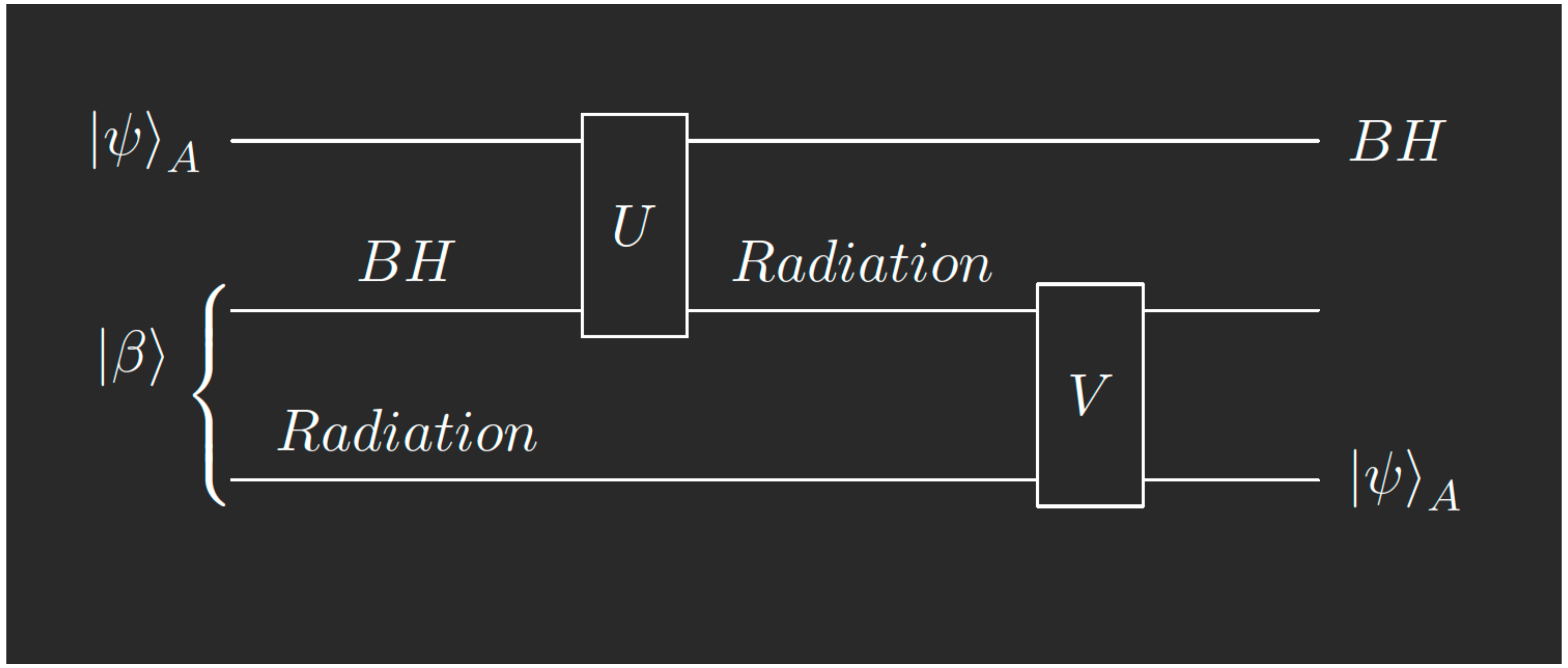
To avoid cloning and AMPS paradoxes without requiring us to give up smooth horizons and the equivalence principle, it is likely that we need the Harlow-Hayden resolution.

Harlow & Hayden resolution (2013): To process the information of the BH interior it will take computational time which scales exponentially with the entropy of the black hole at Page time. (The evaporation time scales polynomially.)

Kim, Tang and Preskill (2020): This indeed works out if the post-time Hawking radiation is pseudo-random

The magical information processor

- Old black holes are fast scramblers — information in an infalling qubit is mixed rapidly with the interior degrees of freedom
- A (unitary) computational model of the black hole preserving unitarity is that of a quantum erasure channel working at maximal capacity



- Hayden and Preskill (2007) argued that the old BH is an information mirror! Any infalling information leaks out in time $\mathcal{O}(r_S \ln r_S)$ where r_S is the horizon radius at infall time. This is also the scrambling time.

What falls in after Page time is revealed much faster than the interior of the black hole.

See Kitaev and Yoshida (2017) for a computational model of the Hayden-Preskill protocol (how to construct the V for a given U).

Motivation

- A breakthrough has been recently achieved in computing the Page curve of the Hawking radiation via AdS/CFT. The entanglement entropy has been computed via the **quantum extremal surface (QES)** of Negelhardt and Wall in the **semi-classical geometry** of 2-dimensional JT gravity coupled to a CFT — the QES acquires a first order phase transition at Page time. [*Penington; Almheiri, Engelhardt, Marolf, Maxfield; Almheiri, Mahajan, Maldacena, Zhao*]

- Can we construct a simple phenomenological model for the BH as an information processor, in which we can explicitly demonstrate **scrambling, information mirroring and the Page curve**? Also realize the Hayden-Preskill protocol explicitly?

Disclaimers

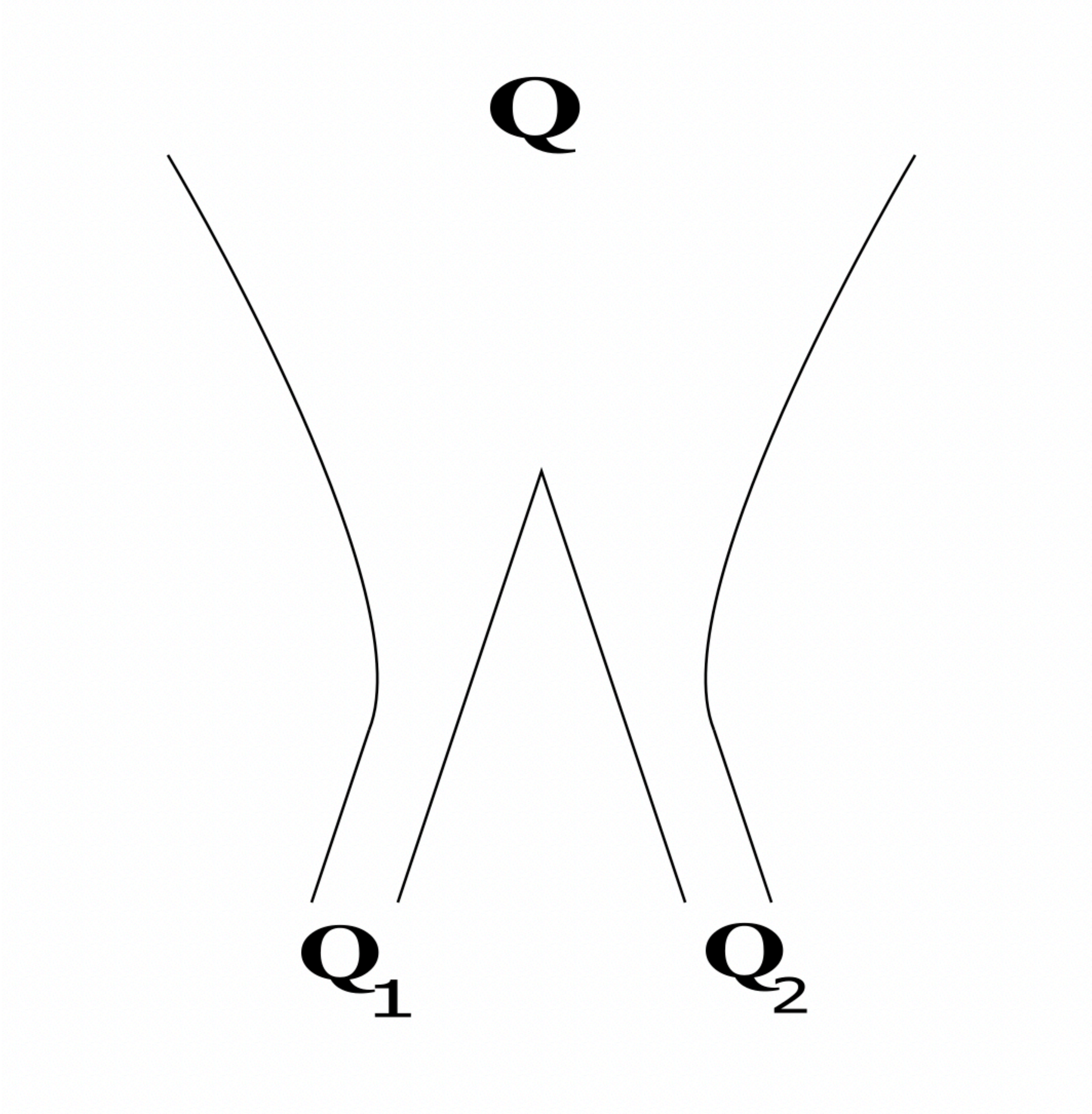
We do not wish to be quantitatively exact in terms of reproducing the BH entropy or derive the model from first principles although we do construct it from the known non-perturbative dynamics of the semi-classical BH.

Even in terms of phenomenological features, our model likely needs improvements. We discuss reasons and possibilities later.

Even with reduced ambition, it is still challenging to come up with a model that can describe the BH microstates, and which walks and talks like a BH.

The model

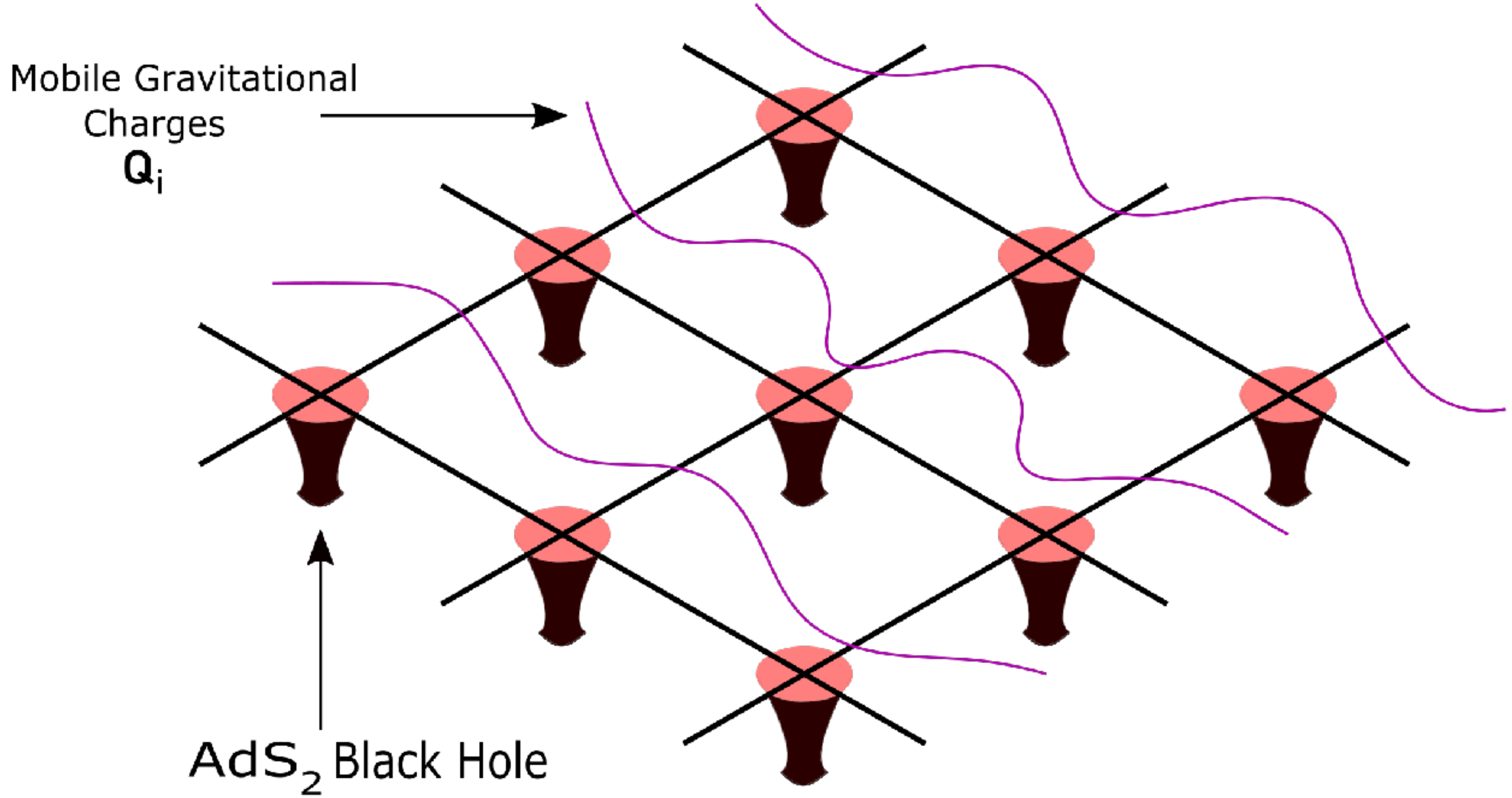
Fragmentation instability



The near-extremal horizon can fragment into multiple throats via instantons [Brill (1992); Maldacena, Michelson, Strominger (1999)]

There are abundant low energy modes at the boundaries of instanton moduli space where two or more throats are separated by sub-Planckian distance

Cartoon of our model



INGREDIENTS

- A NAdS2 throat lattice — the lattice has same dimensionality of the BH horizon. The throats carry $\mathcal{Q}_i$ SL(2,R) gravitational charges.
- Pure gravitational hair in the form of *mobile* SL(2,R) charges

WHAT WE CANNOT MISS

- The $SL(2,R) \otimes SL(2,R) \otimes \cdots \otimes SL(2,R)$ symmetry needs to broken to the diagonal $SL(2,R)$ as if there is one unfragmented horizon
- An overall conserved energy

It's like that old black hole,
No matter how you try,
You set out each day
Never to arrive

Like the mystery in the dark
Oh, it's just another kind of light

I don't expect you to believe me (YET)
But everything is alright

Dr. Dog

The justification of our model is based on phenomenology

The key is the coupling between the lattice SL(2,R) charges and the mobile gravitational hair

The periodic chain model

$$M_i' = -\lambda \left(\boldsymbol{\mathcal{Q}}_{i-1} + \boldsymbol{\mathcal{Q}}_{i+1} - 2\boldsymbol{\mathcal{Q}}_i \right) \cdot \mathbf{Q}_i',$$
$$\mathbf{Q}_i'' = \frac{1}{\sigma^2} \left(\mathbf{Q}_{i-1} + \mathbf{Q}_{i+1} - 2\mathbf{Q}_i \right) + \frac{1}{\lambda^2} \left(\boldsymbol{\mathcal{Q}}_{i-1} + \boldsymbol{\mathcal{Q}}_{i+1} - 2\boldsymbol{\mathcal{Q}}_i \right).$$

M_i -> The AdM mass (holographic em-tensor) of the i-th throat

Each throat is described by JT gravity with a dilaton and appropriate matter em-tensor

The conserved energy

$$\mathcal{E} = \mathcal{E}_{\mathcal{Q}} + \mathcal{E}_{\mathsf{Q}}, \mathcal{E}_{\mathcal{Q}} = \sum_i M_i,$$

$$\mathcal{E}_{\mathsf{Q}} = \frac{\lambda^3}{2} \sum_i \mathbf{Q}'_i \cdot \mathbf{Q}'_i + \frac{\lambda^3}{2\sigma^2} \sum_i (\mathbf{Q}_{i+1} - \mathbf{Q}_i) \cdot (\mathbf{Q}_{i+1} - \mathbf{Q}_i)$$

The total energy is simply the sum of the ADM masses of the AdS2 throats and the energy in the hair charges.

The NAdS2 throat

The action

$$S=\frac{1}{16\pi G}\left[\int \mathrm{d}^2x\sqrt{-g}\Phi\left(R+\frac{2}{l^2}\right)+S_{matter}\right]$$

The equations of motion:

$$R+\frac{2}{l^2}=0$$

metric is always locally AdS2

$$\nabla_\mu\nabla_\nu\Phi-g_{\mu\nu}\nabla^2\Phi+\frac{1}{l^2}g_{\mu\nu}\Phi+T^{matter}_{\mu\nu}=0.$$

matter in bulk reparametrizes time in the boundary up to a SL(2,R)

SOLUTION

The metric

$$ds^2 = -2\frac{l^2}{r^2}\mathrm{d}u\mathrm{d}r - \left(\frac{l^2}{r^2} - M(u)l^2\right)\mathrm{d}u^2. \qquad (1)$$

The ADM mass $M(u)$ is the variable. Alternatively the variable is $t(u)$ the boundary time-reparametrization, i.e. the boundary limit of the uniformization map:

$$t = t(u), \quad \rho = \frac{t'(u)r}{1 - \frac{t''(u)}{t'(u)}r}$$

to pure AdS2

$$ds^2 = -\frac{2}{\rho^2}\mathrm{d}\rho\mathrm{d}t - \frac{1}{\rho^2}\mathrm{d}t^2$$

with $$M(u) = -2\,\mathrm{Sch}(t(u), u)$$

$$\mathrm{Sch}(f(u), u) := \frac{f'''(u)}{f'(u)} - \frac{3}{2}\left(\frac{f''(u)}{f'(u)}\right)^2$$

<u>Solution for the dilaton and matter em-tensor</u>

$\Phi(r, u) = 2/r$ rather simple!

$T_{uu} = f(u), \quad T_{ur} = T_{ru} = T_{rr} = 0$ conserved in the metric (1)

$M'(u) = f(u)$ i.e. $\mathrm{Sch}'(t(u), u) = -\frac{1}{2} f(u)$ matter reparametrizes time up to SL(2,R)

Simplest example:

If $\quad t(u) = \tanh\left(\dfrac{\pi u}{\beta}\right), \quad$ then $\quad M(u) = -2\mathrm{Sch}(t(u), u) = \dfrac{4\pi^2}{\beta^2}$

The Euclidean time has periodicity β and there is no bulk em-tensor

A thermal state with temperature β^{-1}.

Generally states belong to the coset space: $\mathrm{Diff}/SL(2,R)$

SL(2,R) Charges:

$$t(u) = \tanh\left(\frac{\pi\tau(u)}{\beta}\right)$$

$\tau(u)$ maps physical time u to the time of a black hole with temperature β^{-1}

$$\mathcal{Q}_i^0 = \frac{\beta}{2\pi}\left(\frac{\tau_i'''}{\tau_i'^2} - \frac{\tau_i''^2}{\tau_i'^3}\right) - \frac{2\pi}{\beta}\tau_i',$$

$$\mathcal{Q}_i^+ = e^{\frac{2\pi\tau_i}{\beta}}\left(\frac{\beta}{2\pi}\left(\frac{\tau_i'''}{\tau_i'^2} - \frac{\tau_i''^2}{\tau_i'^3}\right) - \frac{\tau_i''}{\tau_i'}\right),$$

$$\mathcal{Q}_i^- = e^{-\frac{2\pi\tau_i}{\beta}}\left(\frac{\beta}{2\pi}\left(\frac{\tau_i'''}{\tau_i'^2} - \frac{\tau_i''^2}{\tau_i'^3}\right) + \frac{\tau_i''}{\tau_i'}\right).$$

Note β disappears if we re-express with $t(u)$

We can set $\beta = 2\pi$

A finite β is good for simulation

How to simuate:

Use these identites with $\beta = 2\pi$:

$$\mathcal{Q}^{0'} = \frac{1}{\tau'}\mathrm{Sch}' = -\frac{1}{2\tau'}M', \quad \mathcal{Q}^{\pm'} = \frac{e^{\pm\tau}}{\tau'}\mathrm{Sch}' = -\frac{e^{\pm\tau}}{2\tau'}M' \qquad \tau' = \frac{1}{2}\left(\mathcal{Q}^{-}e^{\tau} + \mathcal{Q}^{+}e^{-\tau} - 2\mathcal{Q}^{0}\right)$$

Initialize: $\mathcal{Q}^0(u), \quad \mathcal{Q}^+(u), \quad \mathcal{Q}^-(u), \quad \tau(u)$

For given $f(u)$ i.e. null matter evolve via these to find $\tau(u)$. Simulation protocol: (Joshi, Mukhopadhyay, Soloviev 2019) — can work for arbitrary matter also

$$\mathcal{Q}^{0'} = -\frac{1}{2\tau'}f, \quad \mathcal{Q}^{\pm'} = -\frac{e^{\pm\tau}}{2\tau'}f, \quad \tau' = \frac{1}{2}\left(\mathcal{Q}^{-}e^{\tau} + \mathcal{Q}^{+}e^{-\tau} - 2\mathcal{Q}^{0}\right)$$

Back to our model

The inter-throat couplings imply inflow/outflow of null matter in each throat

$$f_i(u) = -\lambda\left(\mathcal{Q}_{i-1} + \mathcal{Q}_{i+1} - 2\mathcal{Q}_i\right)\cdot \mathbf{Q}'_i$$

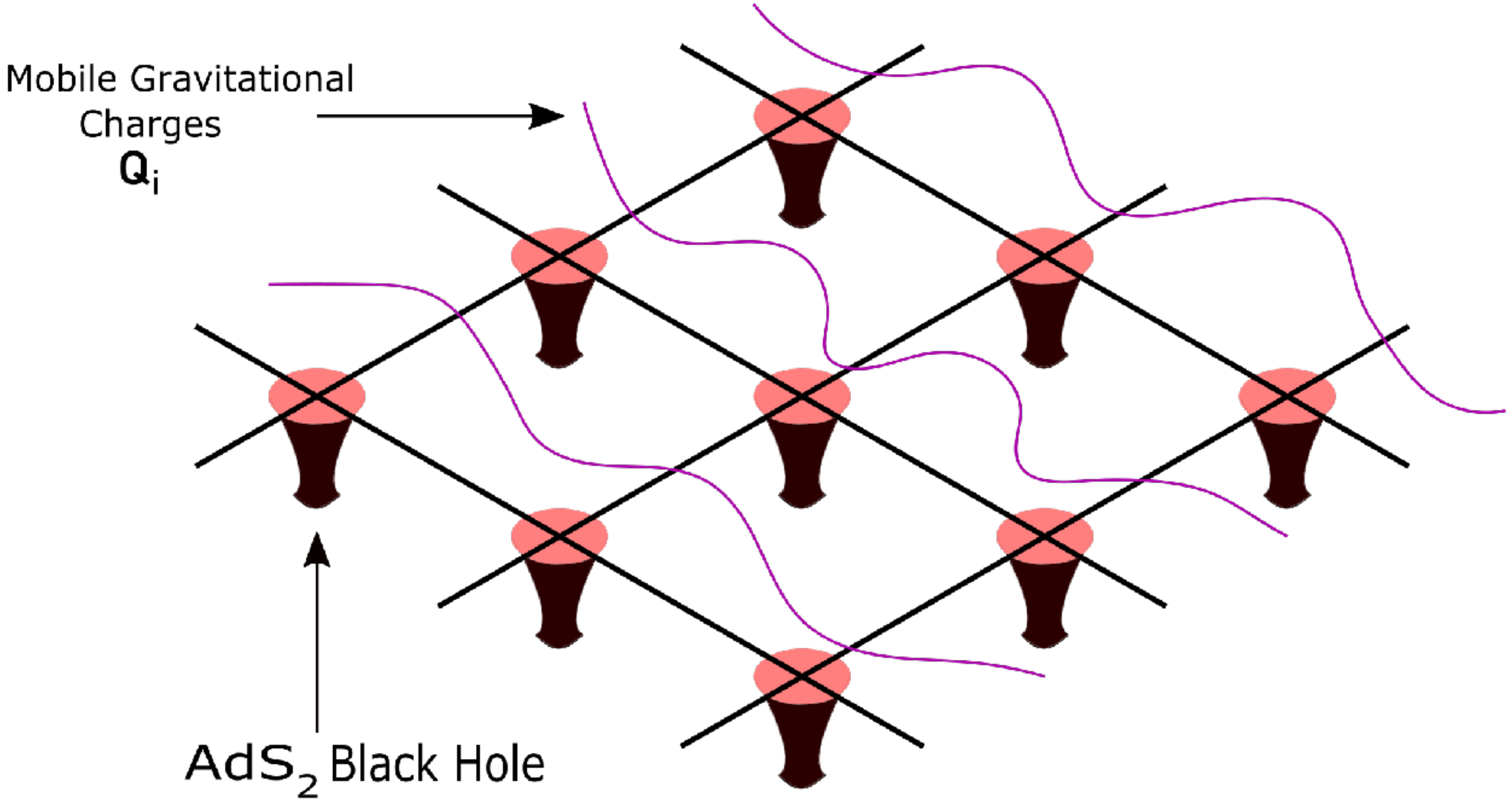
Initialize: $\mathcal{Q}_i^0, \quad \mathcal{Q}_i^+, \quad \mathcal{Q}_i^-, \quad \tau_i(u), \quad \mathbf{Q}_i, \quad \mathbf{Q}_i'$

Simulate:

$$\begin{aligned} M'_i &= -\lambda\left(\mathcal{Q}_{i-1} + \mathcal{Q}_{i+1} - 2\mathcal{Q}_i\right)\cdot \mathbf{Q}'_i, \\ \mathbf{Q}''_i &= \frac{1}{\sigma^2}\left(\mathbf{Q}_{i-1} + \mathbf{Q}_{i+1} - 2\mathbf{Q}_i\right) + \frac{1}{\lambda^2}\left(\mathcal{Q}_{i-1} + \mathcal{Q}_{i+1} - 2\mathcal{Q}_i\right). \end{aligned}$$

Note the gravitational hair satisfy sourced discrete Klein-Gordon equations

For homogeneous SL(2,R) lattice they decouple from the BH interior



Comments

We are assuming large N limit in each throat so we can utilize semiclassical gravity approximation in each of them

The hair $\mathbf{Q}_i$ should in principle be treated quantum-mechanically. However, if they are in a coherent state, we can also treat them classically to a good approximation

The Page curve can be computed in our model, however we will focus here on the mechanisms of information processing

Our model has remarkable resemblances with color glass condensate theory describing saturated gluons in pQCD. (See our paper for explicit details.)

The microstates and the arrow of time

What should be the microstates

Solutions with:

(i) time-independent lattice charges $\mathcal{Q}_i$ and thus also constant M_i

(ii) the hair charges $\mathbf{Q}_i$ either (a) locked with the BH interior i.e. $\mathcal{Q}_i$, or (b) decoupled and freely propagating without affecting the BH interior

Clearly, the total mass $M = \mathcal{E}_{\mathcal{Q}}$ of the AdS2 throats and the energy in the hair charges are separately constant.

The general microstate solutions are:

$$\mathscr{Q}_i^0 = Q, \quad \mathscr{Q}_i^{\pm} \text{ random}$$

A disordered ferromagnet

$$\mathrm{Q}_i^0 = q_i(u) + \alpha\, u + \mathscr{K}^0, \qquad \mathrm{Q}_i^{\pm} = -\frac{\sigma^2}{\lambda^2}\mathscr{Q}_i^{\pm} + \mathscr{K}^{\pm}$$

Decoupled

Locked with BH interior

$$q_i'' = \frac{1}{\sigma^2}(q_{i-1} + q_{i+1} - 2q_i)$$

$$\sum_i q_i = 0, \quad \sum_i q_i' = 0.$$

No double-counting: Allowed by the equations of motion

Note we have exhausted the global SL(2,R) symmetry to align the homogeneous component of $\mathcal{Q}_i$ in the zero-direction.

We can set $\tau_i(u = 0) = \tau_{oi}$.

Positivity of energy requires $M_i \geq 0$.

Also, $\tau_i, \tau_i', \tau_i''$ need to be continuous for $-\infty < u < \infty$ and therefore

$$Q \leq -M_i, \quad \mathcal{Q}_i^{\pm} \leq 0, \quad \mathcal{Q}_i^{+} + \mathcal{Q}_i^{-} \geq 2Q$$

We choose this.

or

$$Q \geq M_i, \quad \mathcal{Q}_i^{\pm} \geq 0, \quad \mathcal{Q}_i^{+} + \mathcal{Q}_i^{-} \leq 2Q$$

Remarkably these inequalities imply that $\tau_i' \geq 0$ or $\tau_i' \leq 0$ for all i.

The uniform arrow of time emerges from our model. We choose the future direction.

The hair charges in the solutions discussed are linear superpositions of three terms

$$\mathbf{Q}_i^{loc}: \qquad \mathrm{Q}_i^0 = 0, \qquad \mathrm{Q}_i^{\pm} = -\frac{\sigma^2}{\lambda^2}\mathcal{Q}_i + \mathcal{K}$$

locked with interior

$$\mathbf{Q}_i^{mon}: \qquad \mathrm{Q}_i^0 = \alpha u, \qquad \mathrm{Q}_i^{\pm} = 0$$

$\alpha = \sum_i q_i'$: conserved monopole charge

$$\mathbf{Q}_i^{rad}: \qquad \mathrm{Q}_i^0 = q_i(u), \qquad \mathrm{Q}_i^{\pm} = 0$$

with $\sum_i q_i = 0, \quad \sum_i q_i' = 0.$

We expect $\mathbf{Q}_i^{rad}$ satisfying source-free Klein Gordon equation to decay via interaction with the asymptotic geometry. If present, the microstate is hairy.

In these (hairy) microstate solutions

$$\mathcal{E}_{\mathsf{Q}} = \mathcal{E}_{\mathsf{Q}}^{pot} + \mathcal{E}_{\mathsf{Q}}^{mon} + \mathcal{E}_{\mathsf{Q}}^{rad}, \quad \mathcal{E}_{\mathsf{Q}}^{pot} = -\frac{\sigma^2}{2\lambda}\sum_i \left(\mathcal{Q}_{i+1}^{+} - \mathcal{Q}_i^{+}\right)\left(\mathcal{Q}_{i+1}^{-} - \mathcal{Q}_i^{-}\right),$$
$$\mathcal{E}_{\mathsf{Q}}^{mon} = \frac{1}{2}\lambda^3\alpha^2, \quad \mathcal{E}_{\mathsf{Q}}^{rad} = \frac{\lambda^3}{2}\sum_i q_i'^2 + \frac{\lambda^3}{2\sigma^2}\sum_i \left(q_{i+1} - q_i\right)^2 .$$

Positivity of energy (on average) requires $\lambda > 0$

Hairless microstate ensemble:

Fix total mass M, Q and α.

Allocate M_i, $\mathcal{Q}_i^{\pm}$ subject to inequalities discussed.

Adding hair on top:

Each microstate solution supports hair oscillations $\mathbf{Q}_i^{rad}$ that can propagate freely over the lattice without affecting it

Shock and Verify

Phenomenological viability

Dr. Dog asked you to trust our model. But now it is time to verify if everything is all right, i.e. whether these actually hold

(i) If shockwaves fall into one or more of our throats in a microstate solution, the full system must relax into another (hairy) microstate solution

(ii) The injected energy must almost fully go into increasing the total black hole mass

(i) During the transition time, the total BH mass $M = \mathscr{E}_{\mathscr{Q}}$ and the hair energy $\mathscr{E}_{\text{Q}}$ should be separately conserved to a very good approximation

These work out if $\alpha = \sum_i q_i' > 0$ (it is easy to see α is unaffected by shocks).

In presence of shocks, the equations of motion are:

$$M_i' = -\lambda\,(\boldsymbol{\mathcal{Q}}_{i-1} + \boldsymbol{\mathcal{Q}}_{i+1} - 2\boldsymbol{\mathcal{Q}}_i) \cdot \mathbf{Q}_i' + \sum_A e_{i,A}\delta(u - u_{i,A}),$$

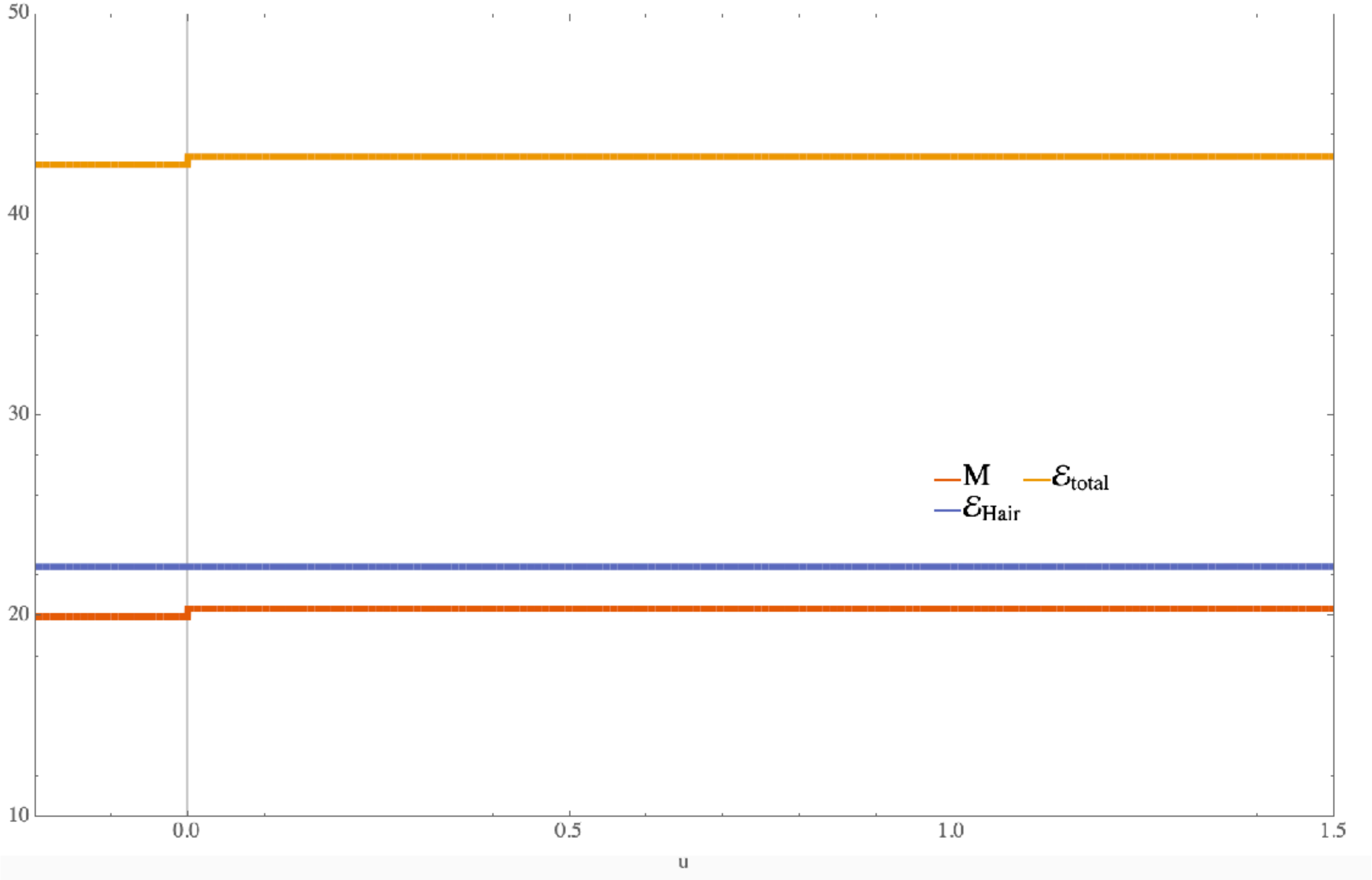
$$\mathbf{Q}_i'' = \frac{1}{\sigma^2}\,(\mathbf{Q}_{i-1} + \mathbf{Q}_{i+1} - 2\mathbf{Q}_i) + \frac{1}{\lambda^2}\,(\boldsymbol{\mathcal{Q}}_{i-1} + \boldsymbol{\mathcal{Q}}_{i+1} - 2\boldsymbol{\mathcal{Q}}_i)$$

The null matter inside the throats has additional shockwaves falling along null geodesics

$$T_{(i)uu}(r,u) = f_i(u), \quad T_{(i)ur}(r,u) = T_{(i)rr}(r,u) = 0.$$

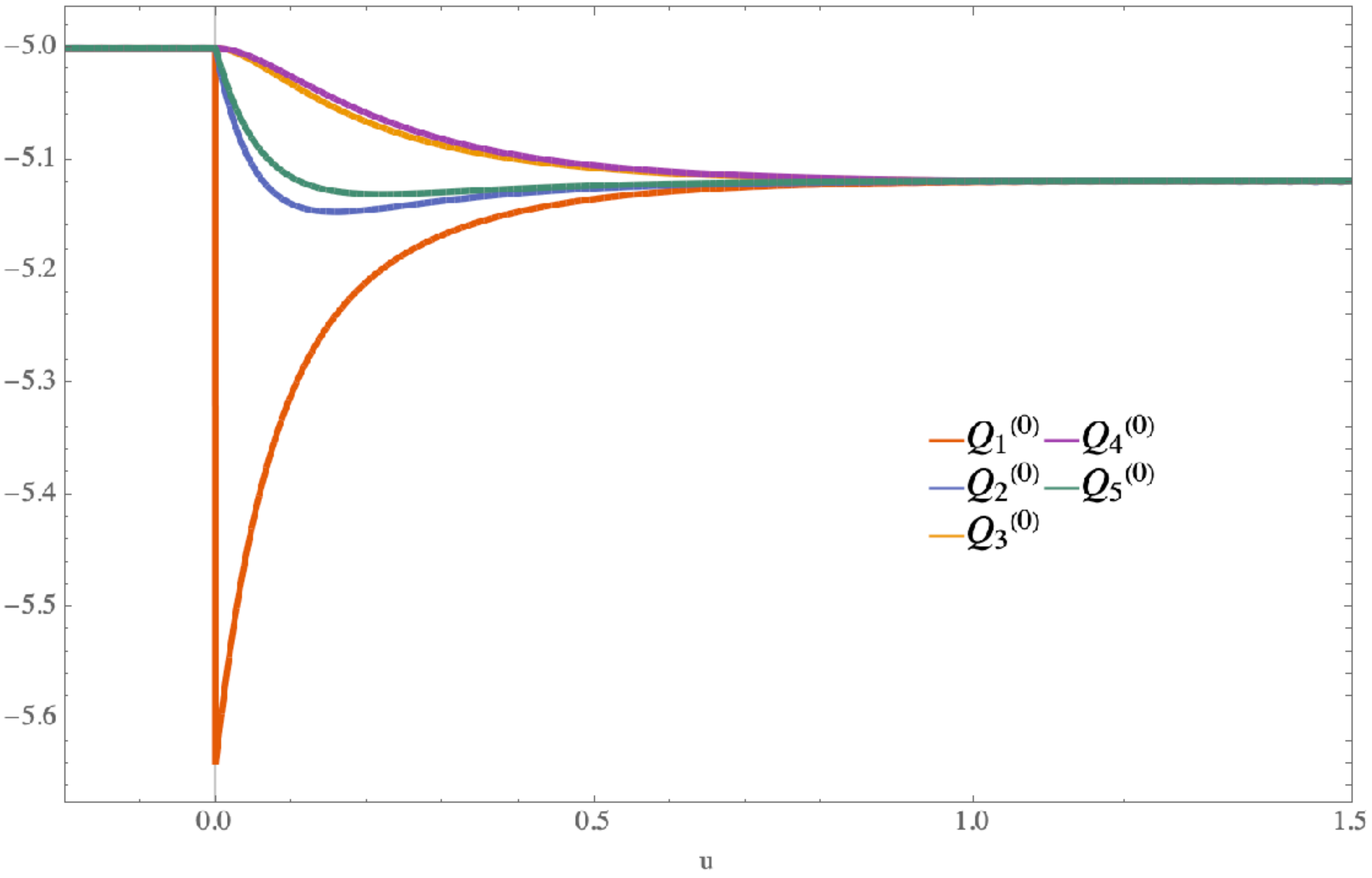
$$f_i(u) = -\lambda\,(\boldsymbol{\mathcal{Q}}_{i-1} + \boldsymbol{\mathcal{Q}}_{i+1} - 2\boldsymbol{\mathcal{Q}}_i) \cdot \mathbf{Q}_i' + \sum_A e_{i,A}\delta(u - u_{i,A}),$$

Results for a single shock



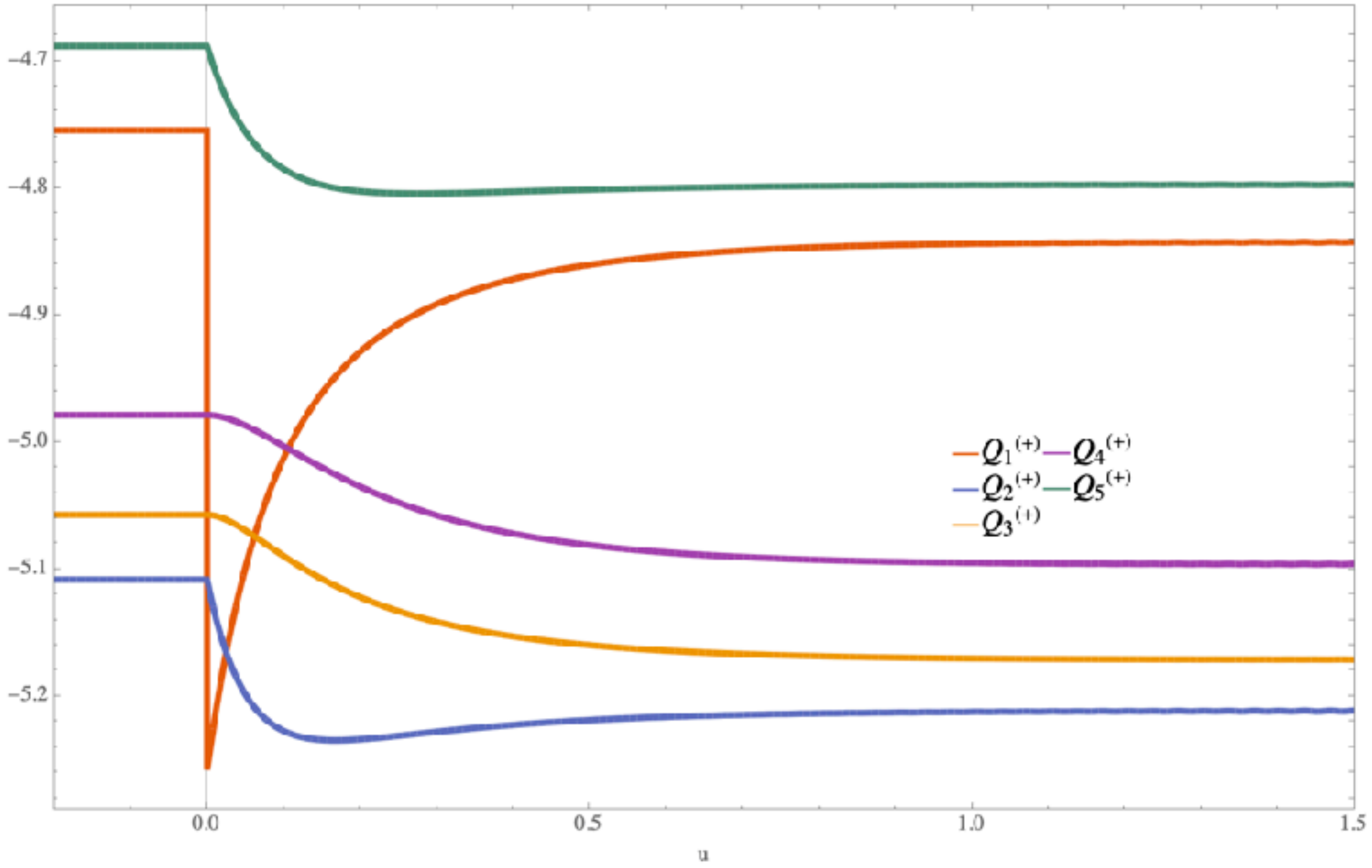
We shock a 5-site periodic chain at site 1. The initial microstate is randomly chosen.

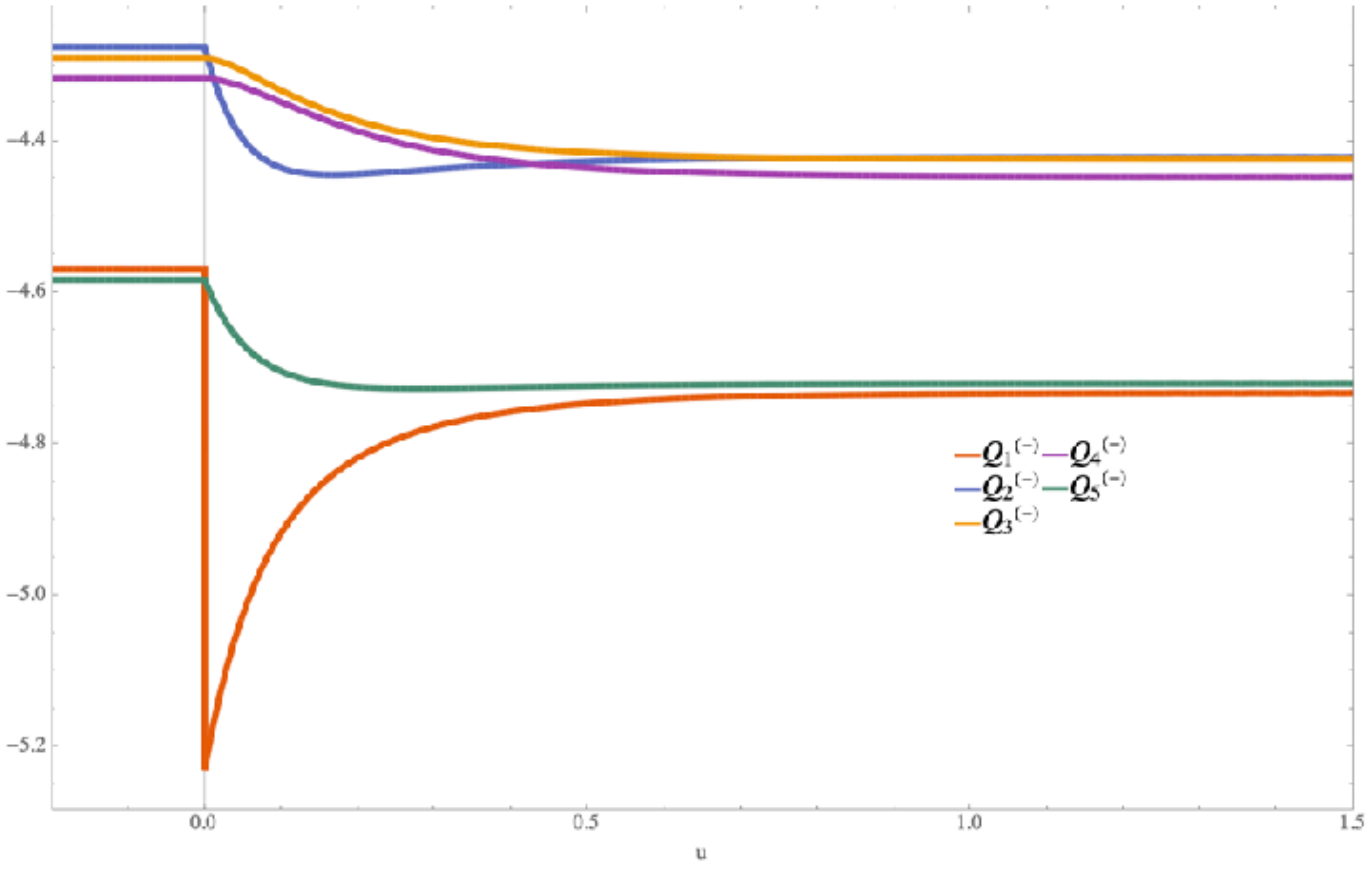
The total mass is approximately conserved even during transition to final microstate. The ratio of oscillations to total mass goes to zero in continuum limit.



Crucially the $\mathcal{Q}_i^0$ relax to the same value as must happen in the microstate.

The conservation of the monopole charge implies that the final homogenous component should also be in the zero-direction



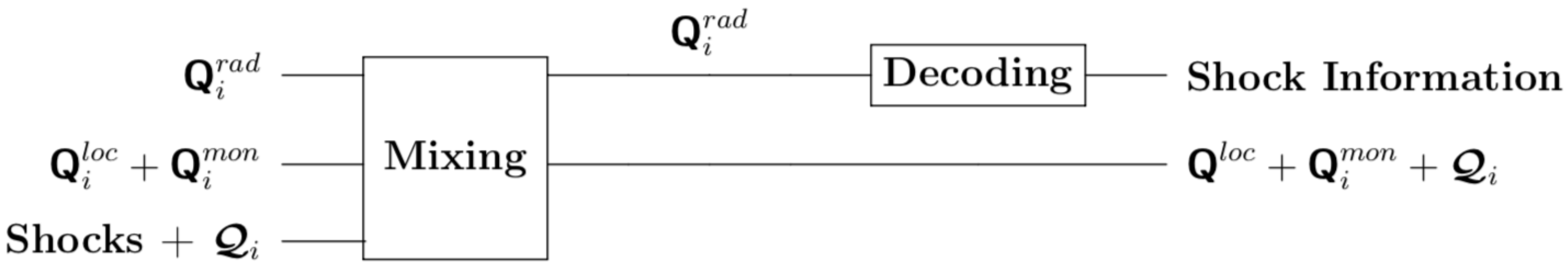


Also $\mathcal{Q}_i^{\pm}$ relax to constants as they should in a (final) microstate.

The dynamics is pseudorandom (necessary for Harlow-Hayden scenario)

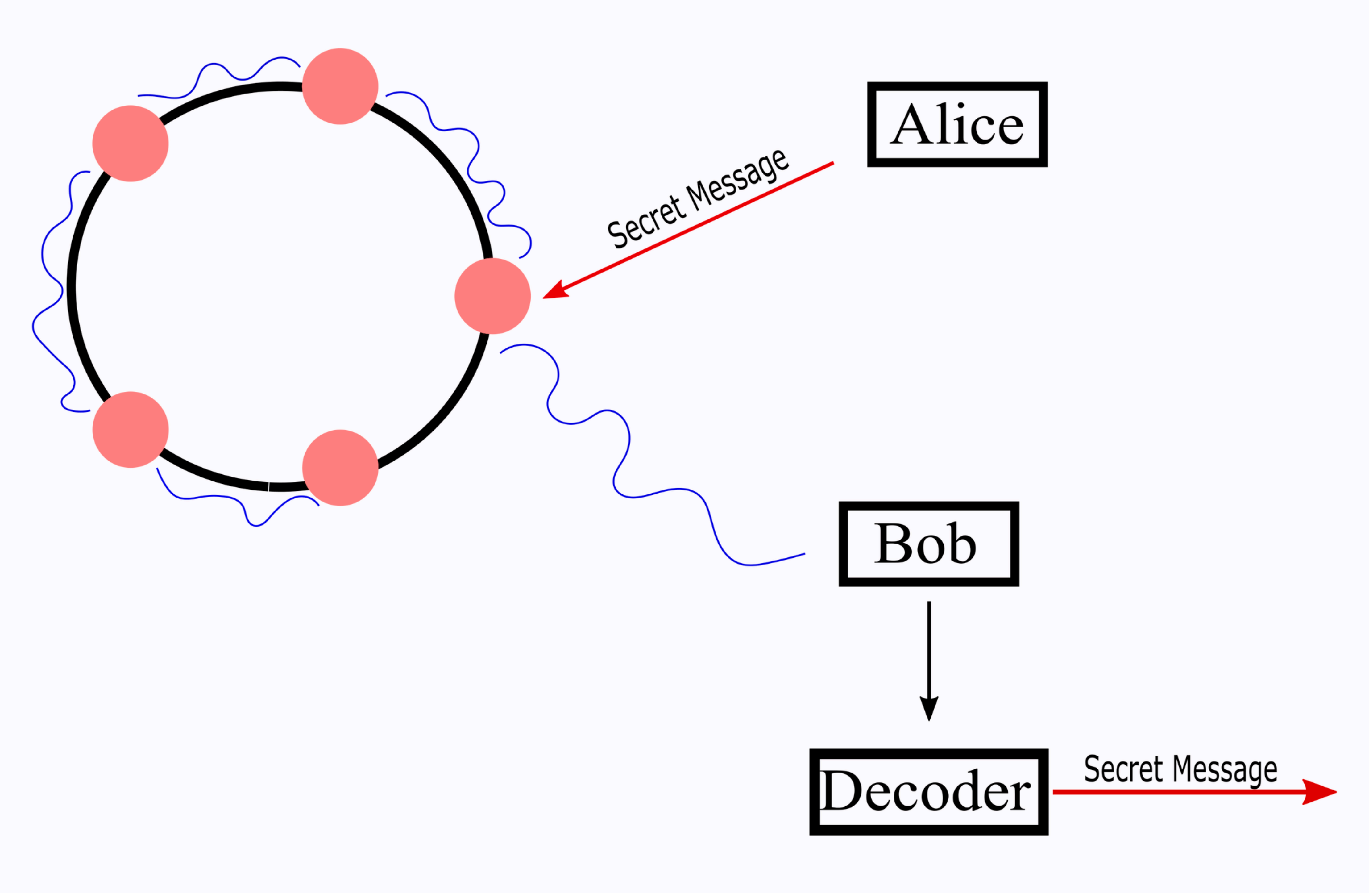
The final microstate is hairy with decoupled q_i oscillations

The Hayden-Preskill protocol



Alice throws in her secret information into our BH in the form of the positions and time-ordering of shocks.

Bob can decode the classical information from $\mathbf{Q}_i^{rad}$ as soon as it decouples from the final microstate

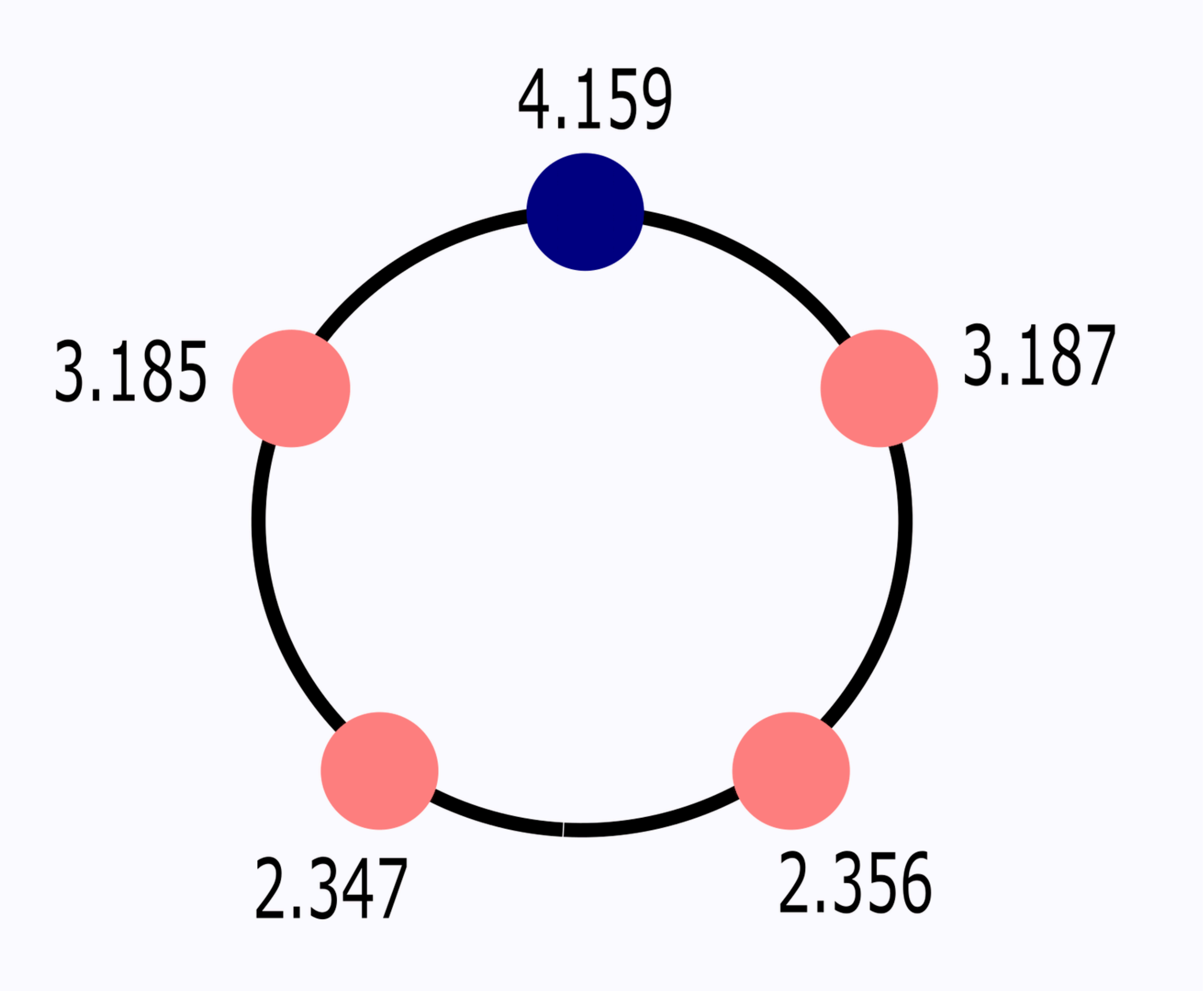


In a 5-site chain $\mathbf{Q}_i^{rad}$ has two positive frequency normal modes.

We assume for simplicity that $\mathbf{Q}_i^{rad} = 0$ in the initial microstate (indeed it decays so a realistic assumption)

We decode the positions and time-ordering of the shocks (with not too large or too small energies which can be different) from the phase differences of the normal mode oscillations

Decoding a single shock

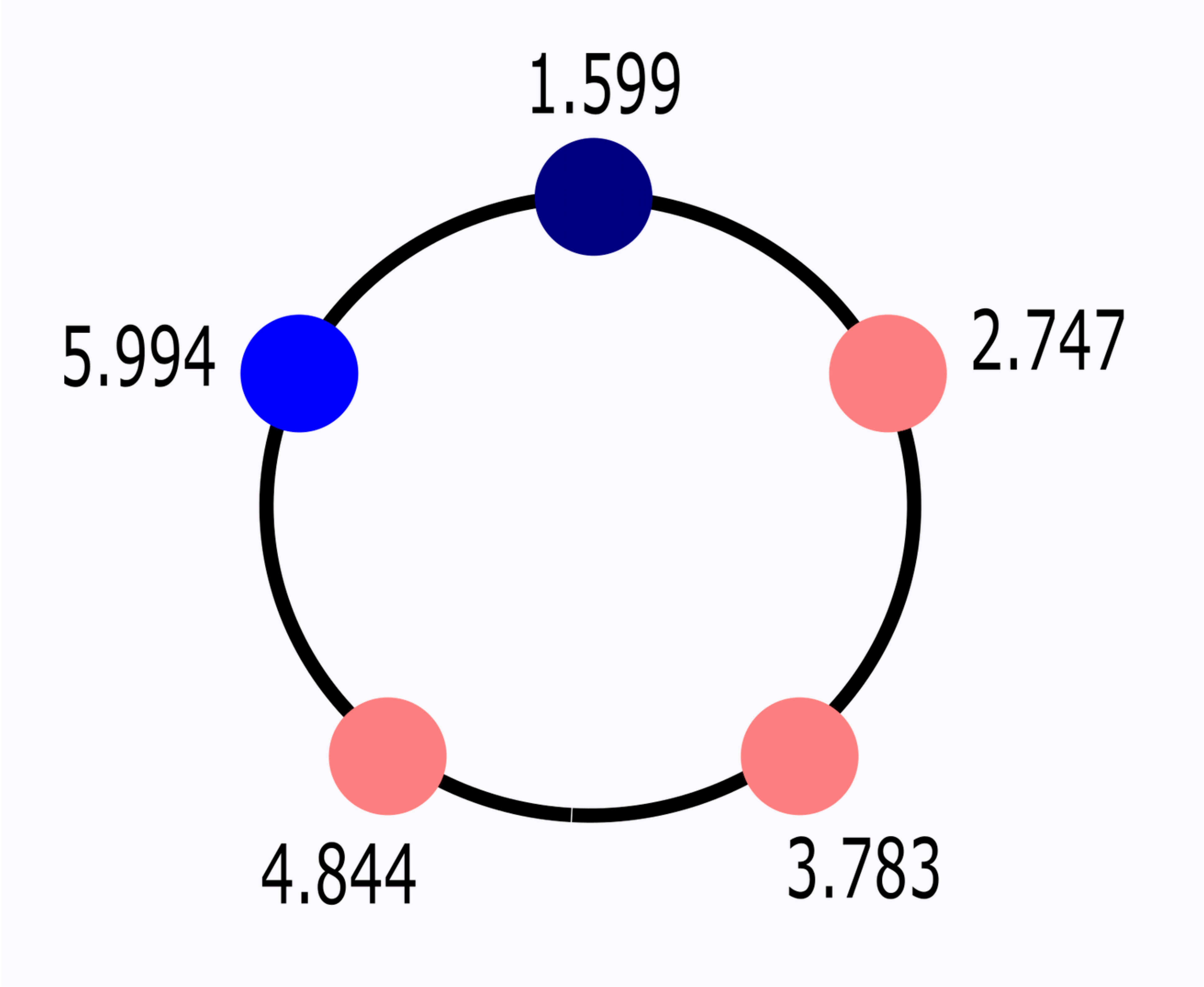


The symmetry in phase differences reveals which site was shocked

A highly non-trivial result because we start from highly asymmetric random initial conditions.

Also not all features of $\mathbf{Q}_i^{rad}$ has this symmetry

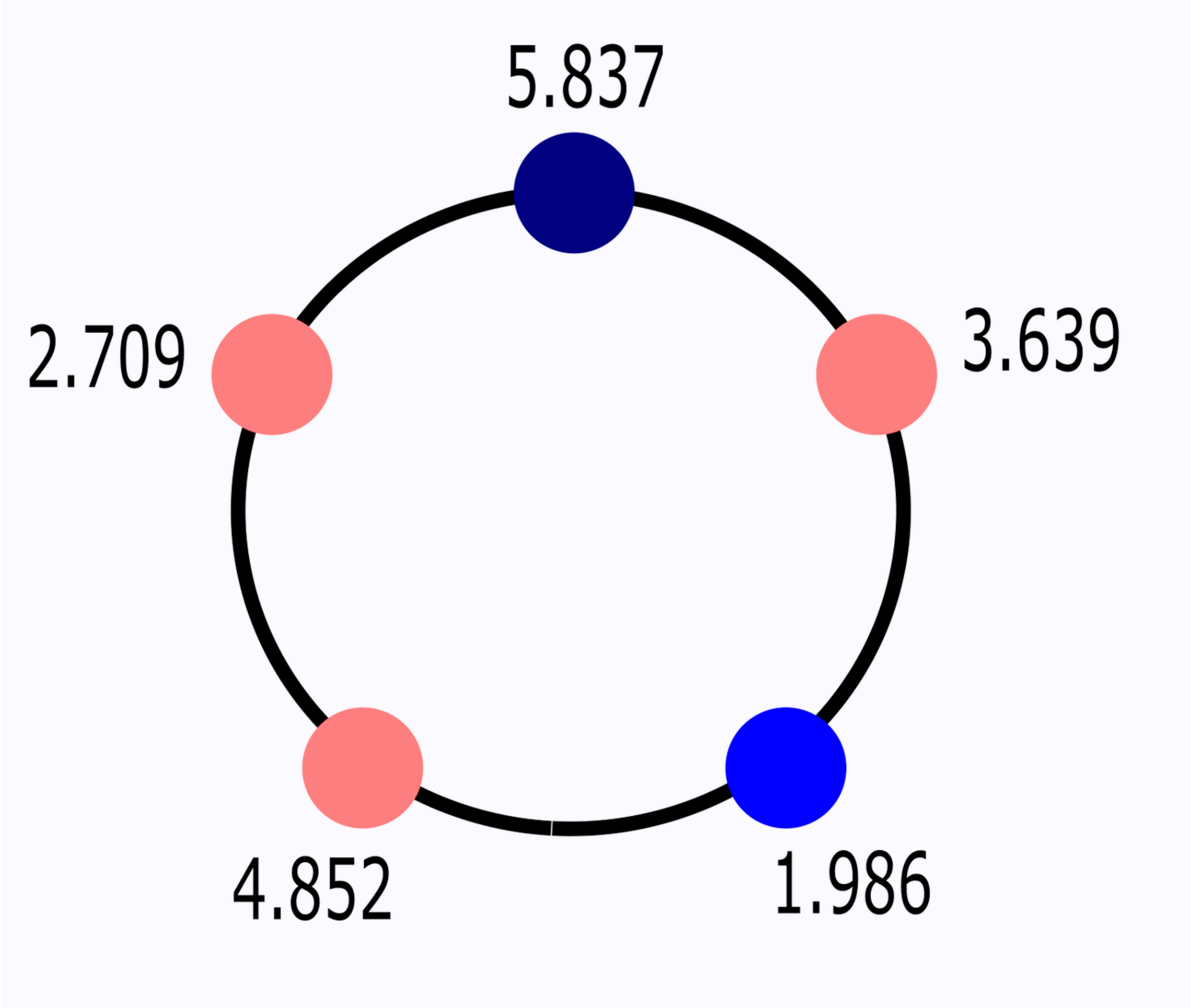
Decoding two shocks: 1



The maximum and minimum phase differences are the positions of the shocks

The minimum phase difference site was shocked first if the shocked sites are nearest neighbors

Decoding two shocks: 2



The maximum and minimum phase differences are the positions of the shocks

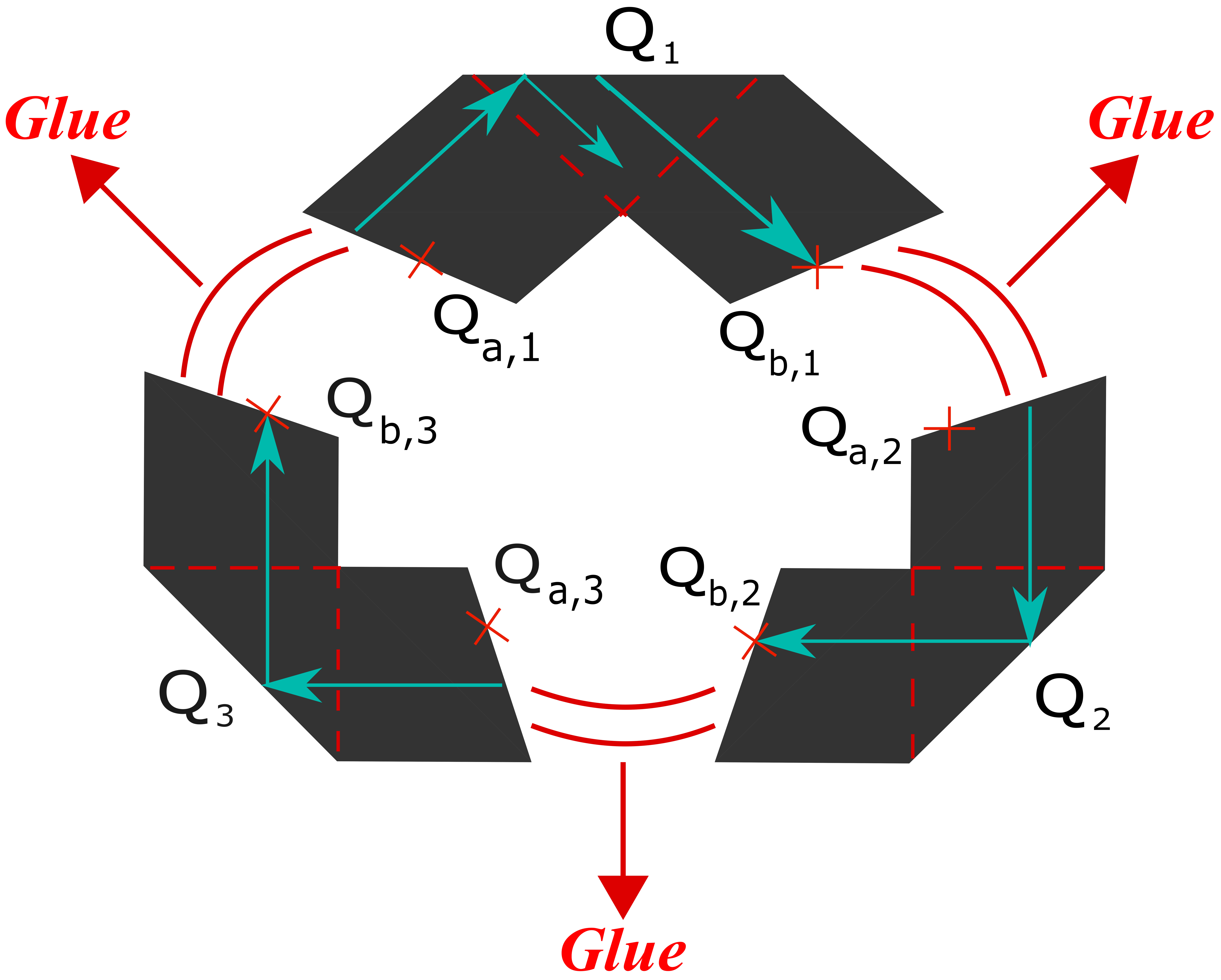
The minimum phase difference site was shocked <u>later</u> if the shocked sites are <u>not</u> nearest neighbors

Although not easy to generalize to multiple number of sites and/or multiple shocks, we expect a similar protocol to exist generically that will scale with the complexity of the inputs and not with that of the BH microstate.

If we throw in qubits, we need to study the open quantum system of the hair charges.

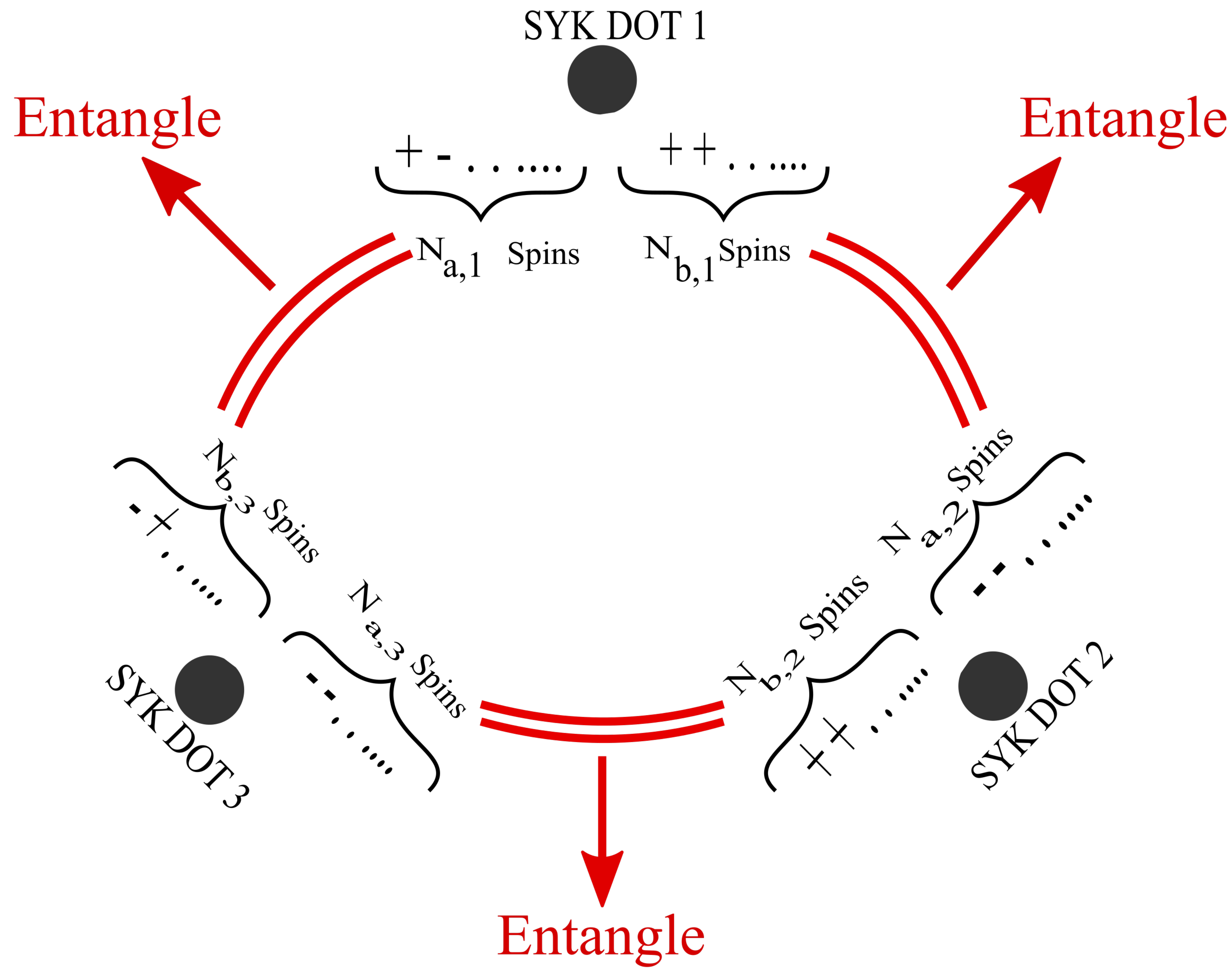
It will also be wonderful to study the Hawking radiation — Bob can of course genuinely access the hair decoupled from the BH interior.

Fragmented Holography



A more general model which preserves fast scrambling and information mirroring will involve wormhole networks of AdS2 throats.

This construction has similarities but also differences with replica wormholes



A possible dual is a MPS network of SYK-spin states (defined by Kourkoulou and Maldacena)

Simulation of MPS networks in real time is feasible. We can perhaps simulate this too.

Conclusions and Outlook

- We have constructed a phenomenological model of an old black hole that captures its semi-classical dynamics (relaxation, absorption of energy into mass, etc) and also its information-processing features
- We need to answer some basic questions now: (i) can we reproduce thermodynamic properties, such as the entropy? (ii) how does the parameters of the network (entanglement between sites) affect thermodynamics?
- Is this useful for something else? A general framework for semi-local non-Fermi liquids? Strange metals? Instanton liquid of QCD?
- Fragmented holography in some form may capture (non-supersymmetric) holographic physics at thermal scales and is likely to lead to *universal* predictions too

A blackbuck in our IIT Madras campus.

THANK YOU FOR YOUR ATTENTION