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Strongly Correlated Dirac Materials, Electron Hydrodynamics & AdS/CFT

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w. D. Rodriguez-Fernandez,
I. Matthaiakakis, J. Erdmenger (PRB 2018)
w. E. Hankiewicz & C. Tutschku (PRB 2019)
w. R. Thomale, D. di Sante, E. van Loon, T. Wehling (1911.06810)
w. E. Hankiewicz, C. Tutschku & J. Böttcher (2003.03146)

The AdS/CFT correspondence

Maldacena 1997

Strongly Coupled Quantum Theory (without gravity)

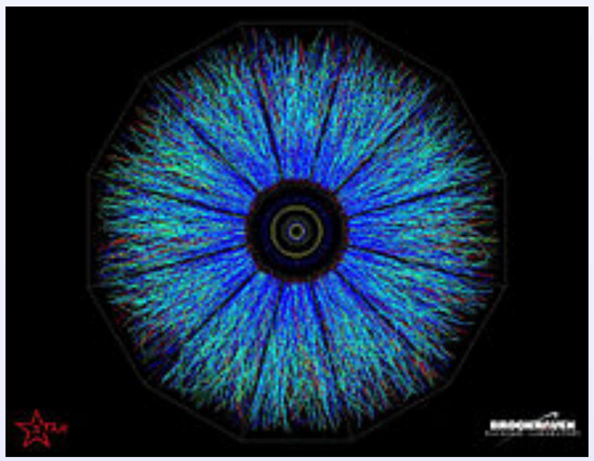
Duality

Gravity in Anti de Sitter Space-Time

What does the AdS/CFT correspondence tell us about universal properties of strongly coupled fluids?

The AdS/CFT correspondence

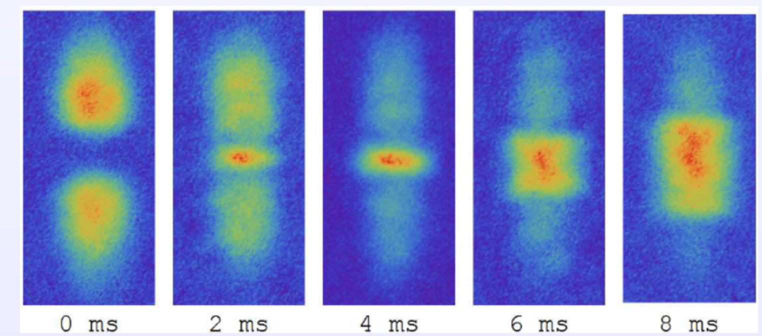
Which systems to test AdS/CFT in experiment?



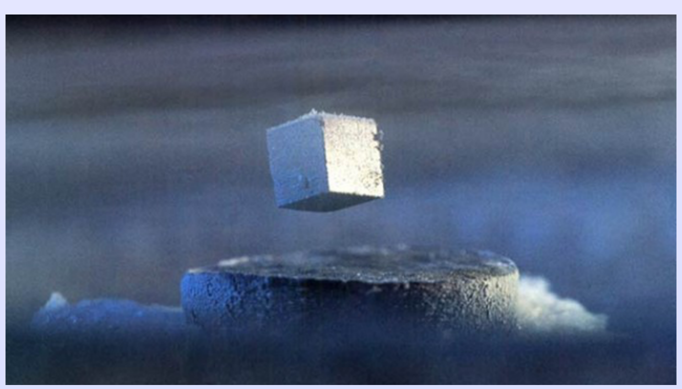
Quark-Gluon
Plasma

Strongly Coupled
Hydrodynamics

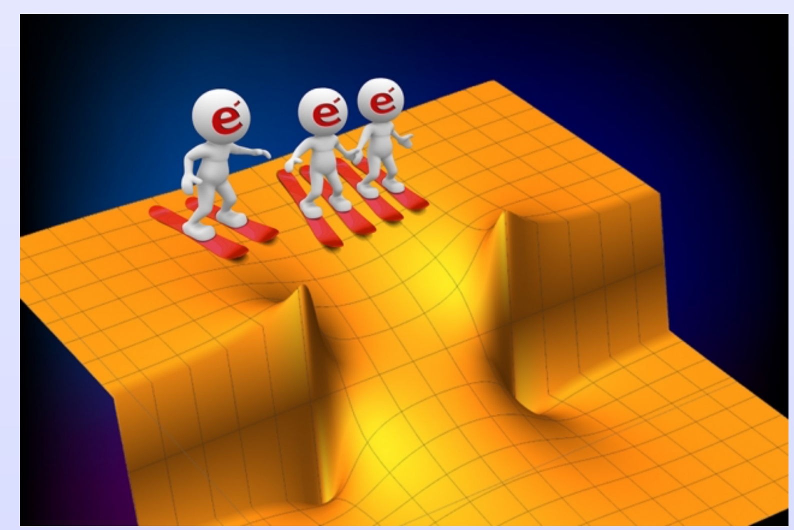
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$



Unitary Fermions



High Tc Superconductors
Quantum Critical Phases



Electronic Fluids

Motivation and Overview

New Proposal for a strongly correlated Dirac material:

Turbulent hydrodynamics in strongly correlated Kagome metals

Domenico Di Sante, Johanna Erdmenger, Martin Greiter, Ioannis Matthaiakakis, Rene Meyer, David Rodriguez Fernandez, Ronny Thomale, Erik van Loon, Tim Wehling

1911.06810

Scandium-substituted Herbertsmithite: New Dirac material with stronger Coulomb coupling between electrons than in Graphene

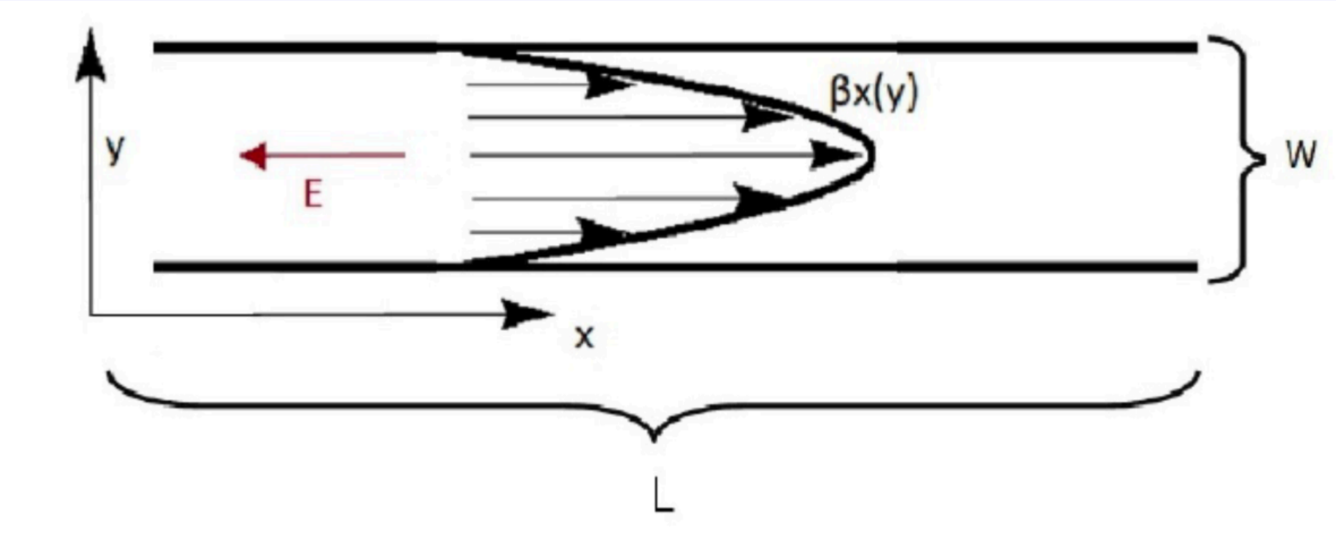
- Enhanced applicability of hydrodynamic regime
- Allows for smaller η/s
  (shear viscosity to entropy density ratio)
- Closer to AdS/CFT regime
- Larger Reynolds numbers, turbulent flow regime

Electron Hydrodynamics in Solids

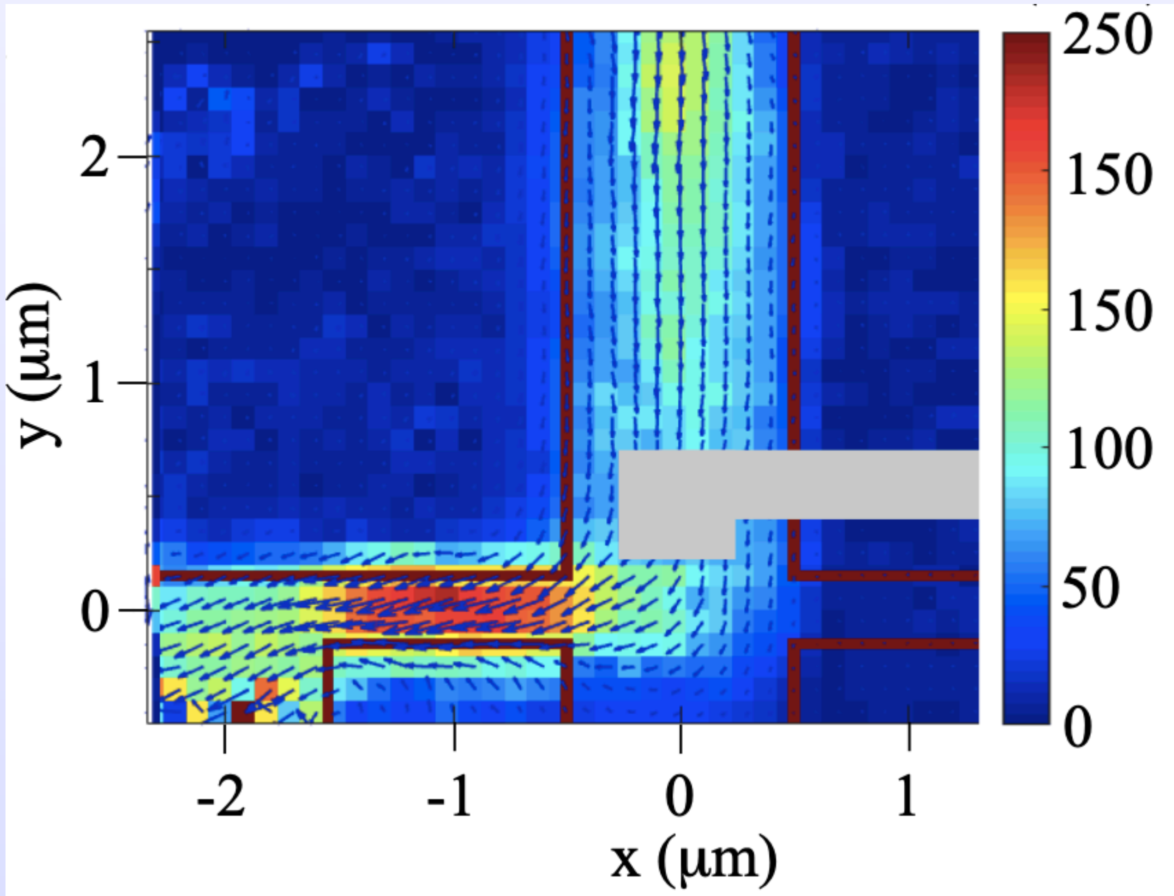
Electrons first Coulomb interact with each other, before loosing energy or momentum to phonons, impurities, or walls.

$$\ell_{ee} < \ell_{imp}, \ell_{phonon}, W$$

→ Hydrodynamic electron flow



Poiseuille flow

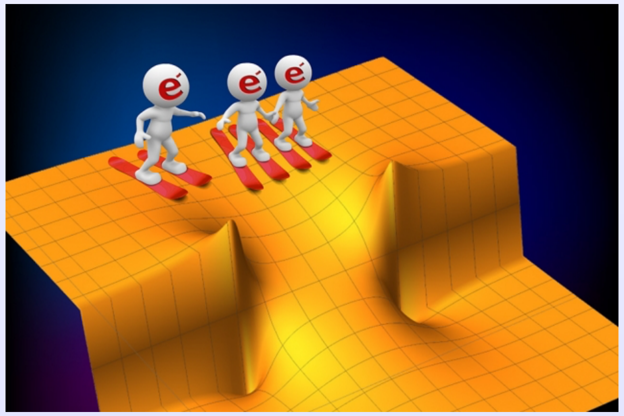


Ku et. al. 1905.10791

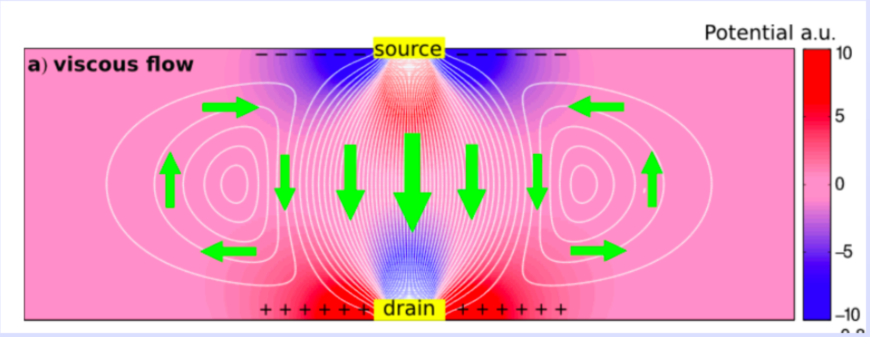
New Transport from Electron Flow

- Decreasing differential resistance dV/dI with increasing current I (non-linear response)
  [Molenkamp+de Jong 1994,95]

- Larger than ballistic conductances
  [Geim et.al. Nat. Phys. 2017]

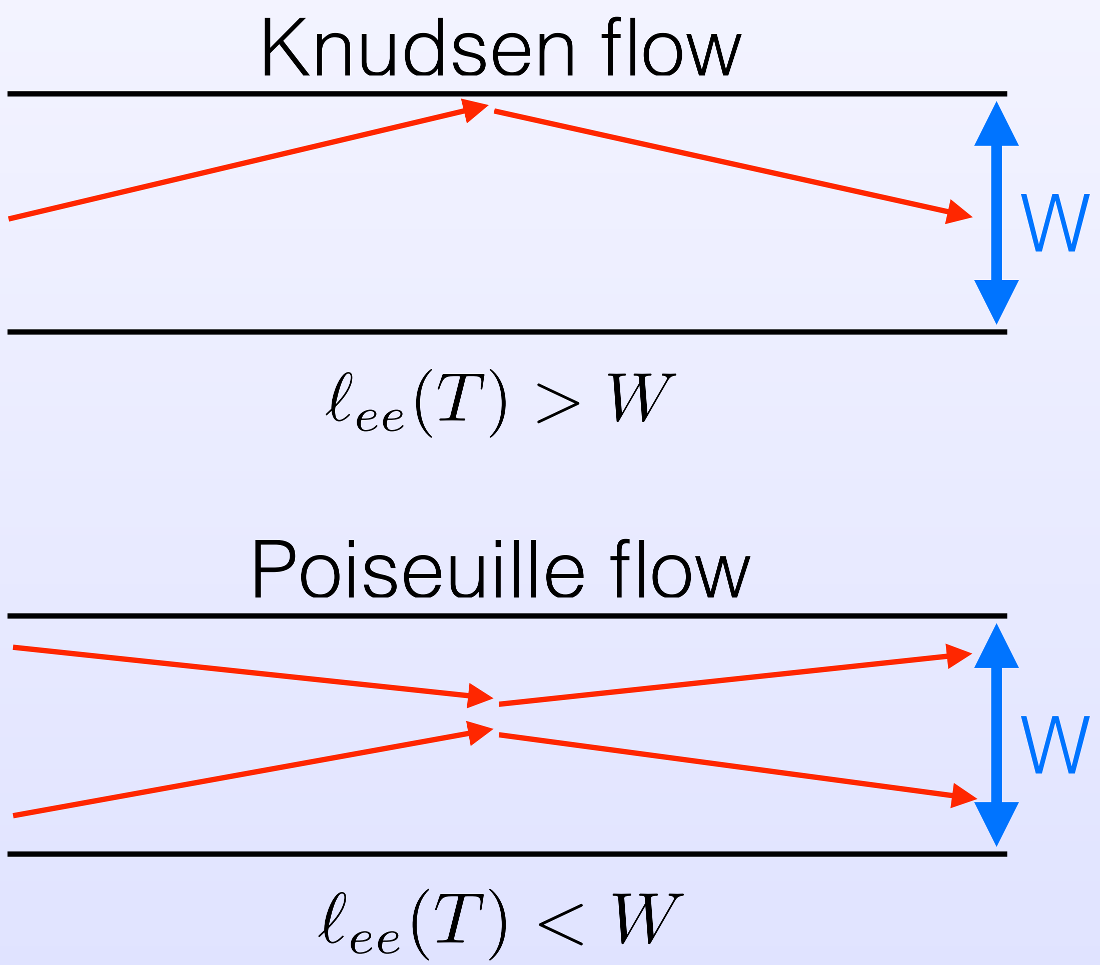


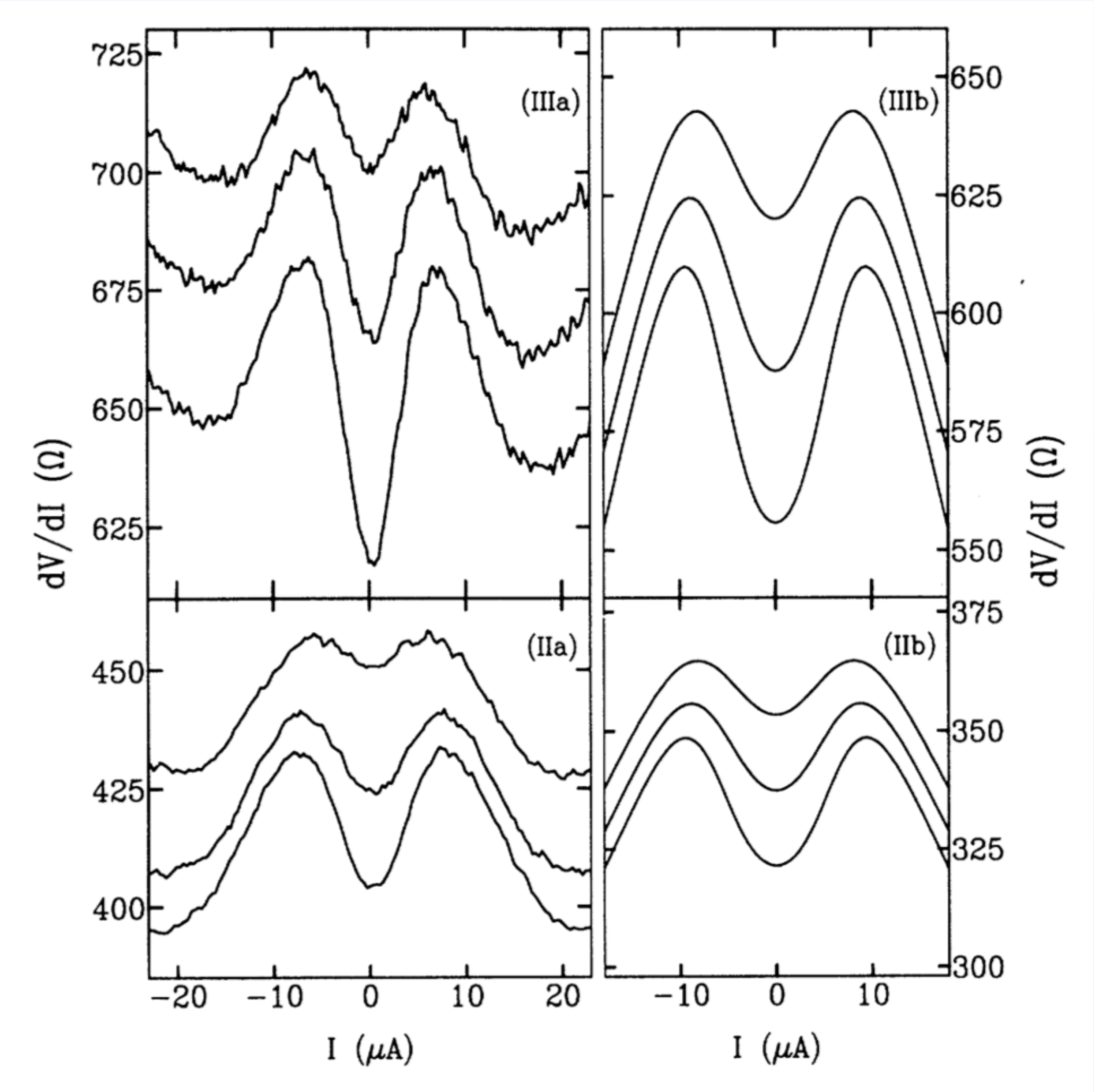
- Negative nonlocal resistance
  [Falkovich et.al. Nat. Phys. 2016]
  [Geim et.al. Science 2016]



Gurzhi Effect

- 2D Electrons in (Al)GaAs Heterostructures



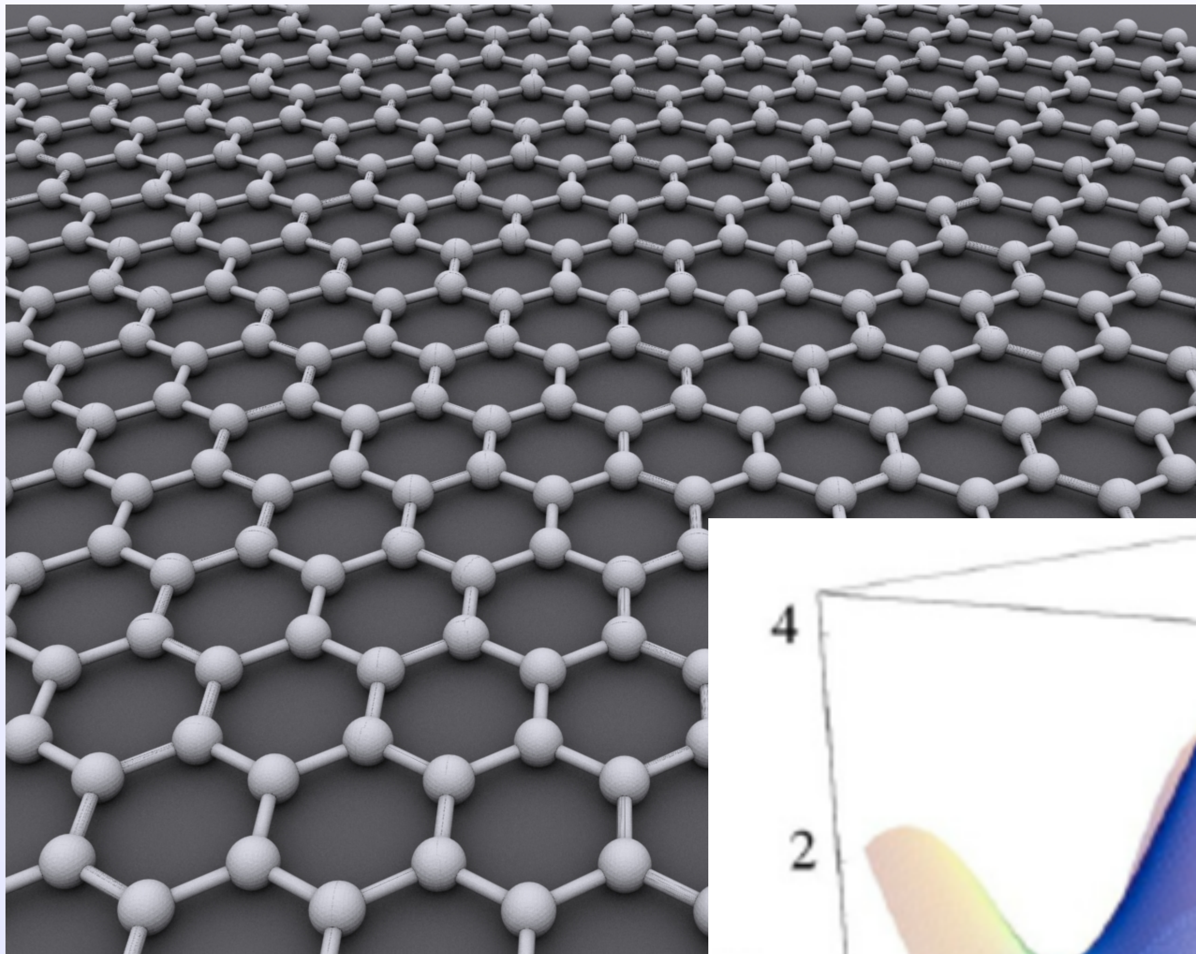


[Gurzhi 1968]

Weakly Coupled!

[Molenkamp+de Jong 1994,95]

Graphene

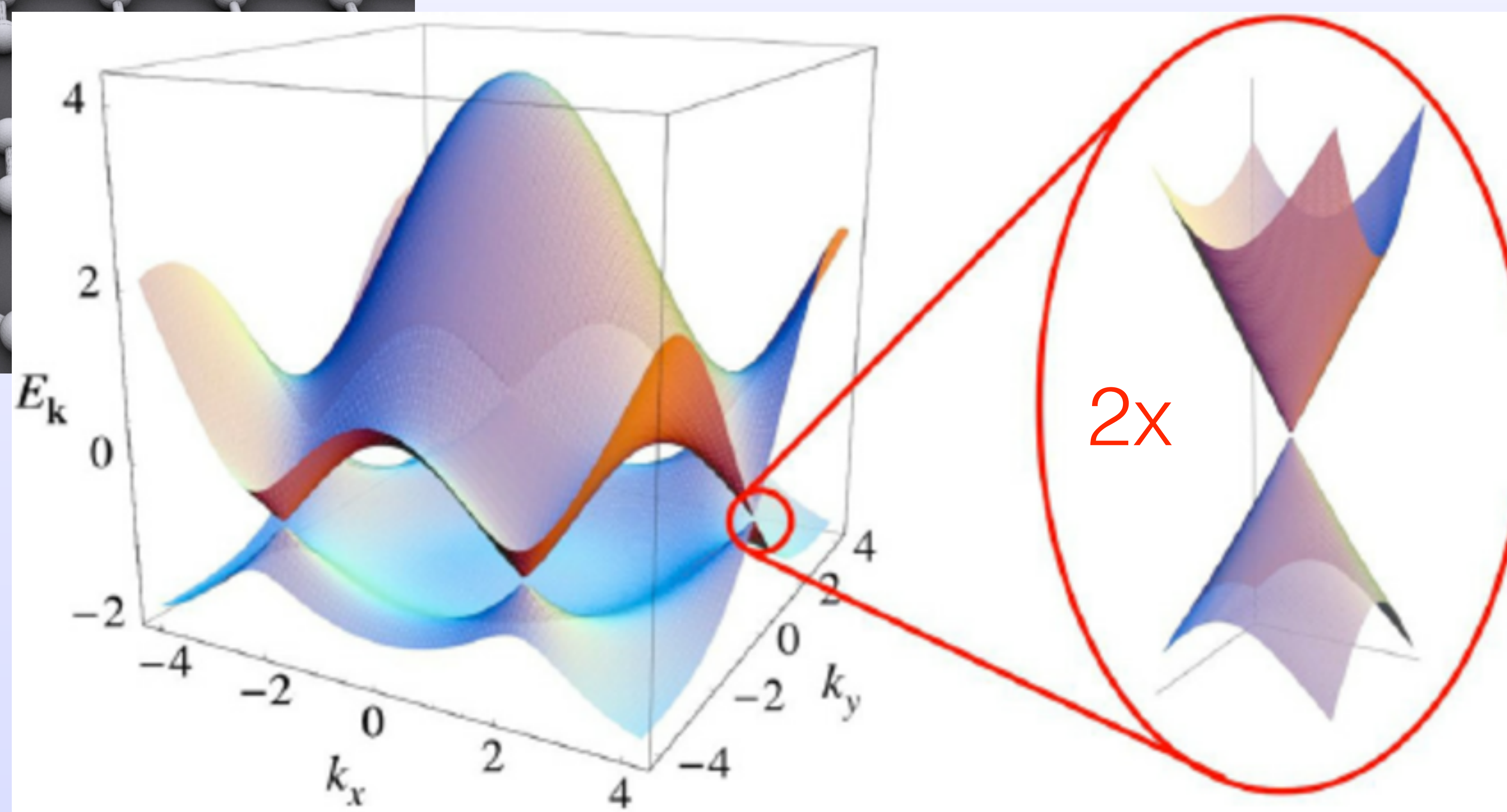


Very clean: $\ell_{ee} < 0.1\ell_{mr}$

Phonons decouple up to high T

$$\nu = \frac{\eta}{mn} \approx 0.1\frac{m^2}{s}$$

$$v_F \approx \frac{c}{300}$$



[Geim, Novozelov Nature 2005, Nobel Prize 2010; Geim etal. Science 2016]

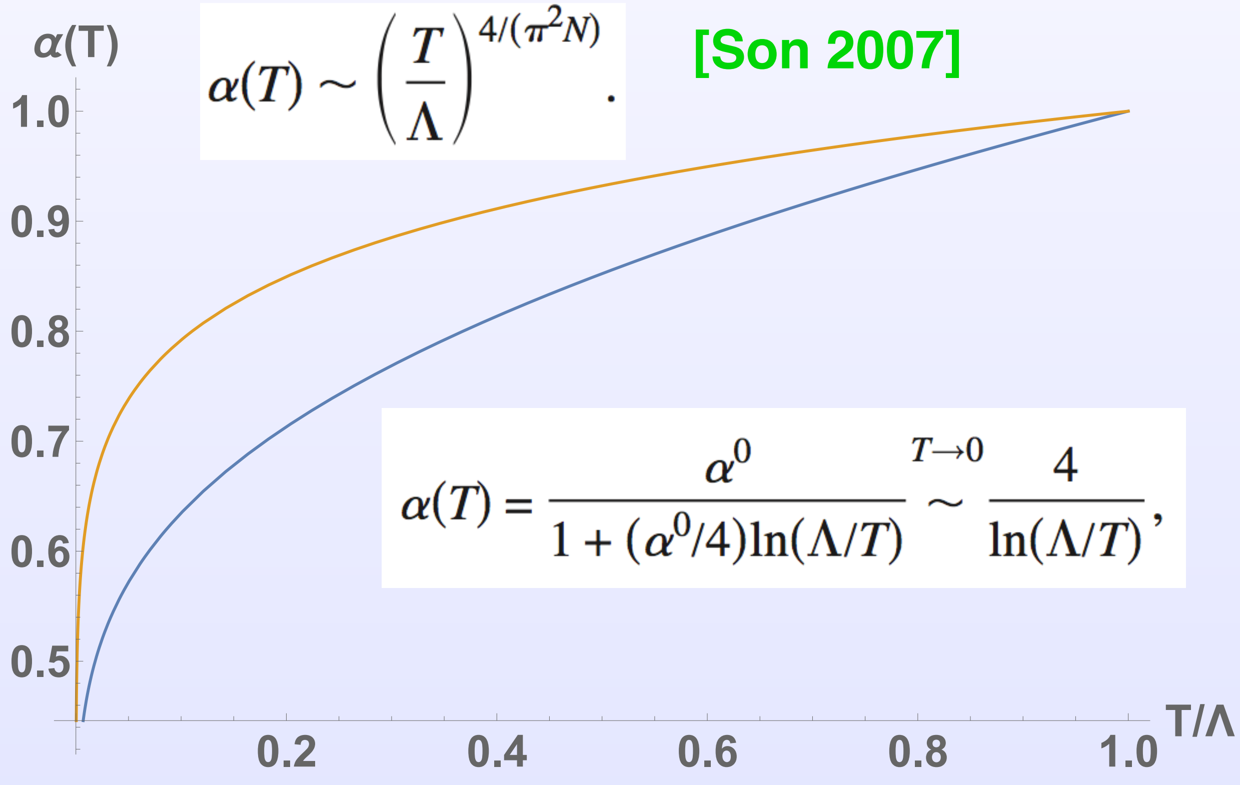
Typical Values

$$\nu \sim 10^{-6}\, m^2/s$$

$$\nu \sim 10^{-3}\, m^2/s$$

Hydrodynamic Fluid in Graphene

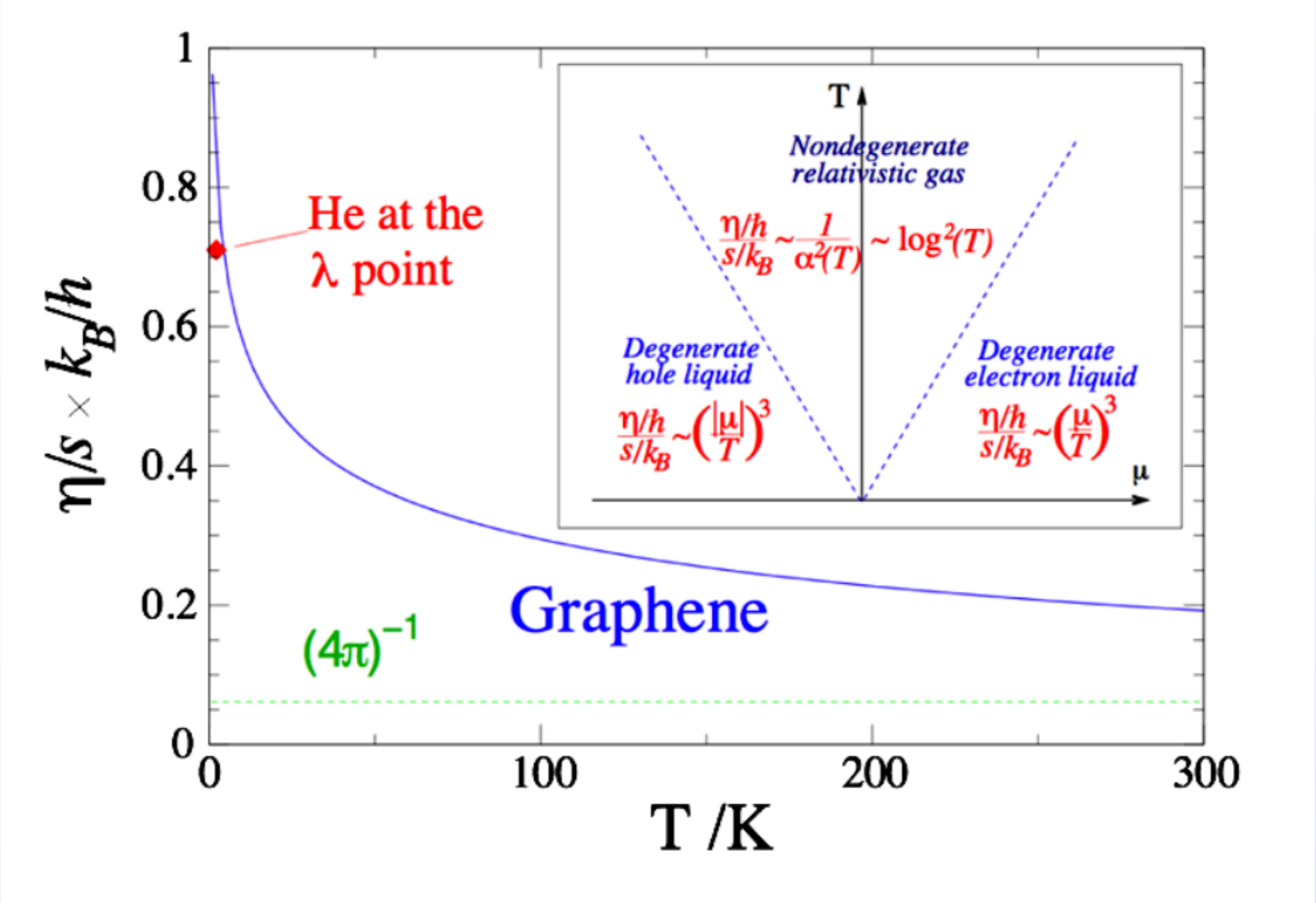
- Graphene in the nonperturbative regime



$$\alpha(T) \sim \left(\frac{T}{\Lambda}\right)^{4/(\pi^2 N)}.$$

[Son 2007]

$$\alpha(T) = \frac{\alpha^0}{1 + (\alpha^0/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)},$$



$$\eta/s = \frac{\hbar}{k_B}\frac{C_\eta \pi}{9\zeta(3)}\frac{1}{\alpha^2(T)} \simeq 0.008\,15 \times \left(\log\frac{T_\Lambda}{T}\right)^2.$$

[Fritz, Schmalian, Sachdev etal PRB, PRL 2008,09]

Hydrodynamics

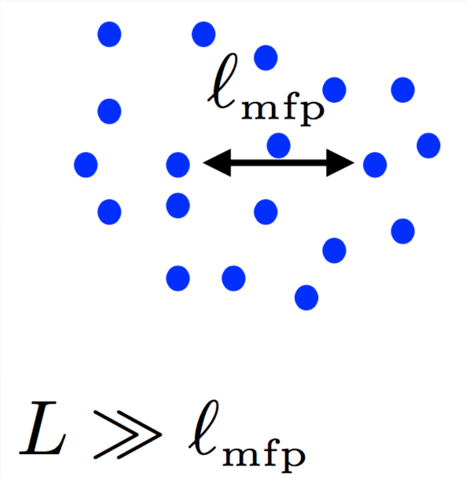
Hydrodynamics: Long wavelength, low frequency perturbations of a fluid away from global equilibrium

Theory of transport of (approx.) conserved quantities (energy, momentum, charges)

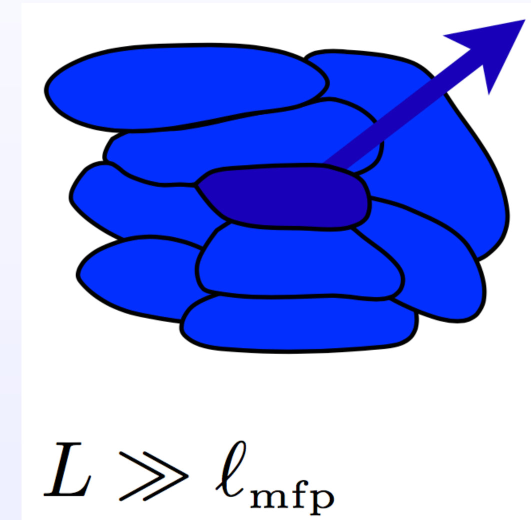
Black hole horizons show hydrodynamic response

[T. Damour 1970s, AdS/CFT]

Charged Relativistic Fluid



$\epsilon(x^\mu)$	Energy density	$\leftrightarrow T(x^\mu)$
$\rho(x^\mu)$	Charge density	$\leftrightarrow \mu(x^\mu)$
$u^\nu(x^\mu)$	Velocity field $(u_\mu u^\mu = -1)$	



Theory of Transport of (approx.) conserved quantities:

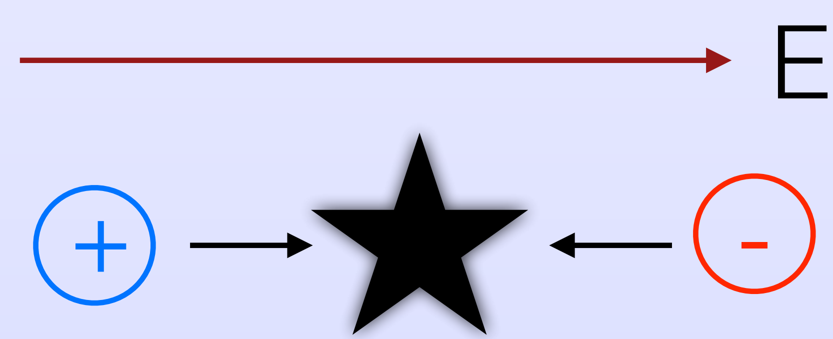
$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho - \frac{T^{0i}\delta_i^\nu}{\tau_{\mathrm{imp}}}$$

$$\nabla_\rho J^\rho = 0$$

Relevant transport coefficients:

η: Shear viscosity

σ: Quantum Critical conductivity

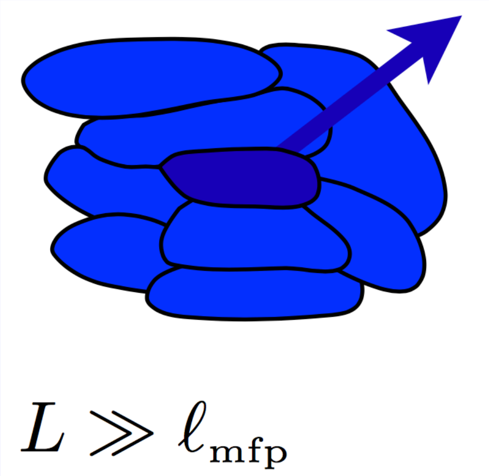


Bulk viscosity can be neglected
for conformal fluids or incompressible flows ($\partial_\mu v^\mu = 0$)

Hydrodynamics as EFT

Expand $T_{\mu\nu}$ and J_μ in

$$\ell_{mfp}\frac{\partial}{\partial x^\mu} = \frac{\ell_{mfp}}{L}\frac{\partial}{\partial \xi^\mu}$$

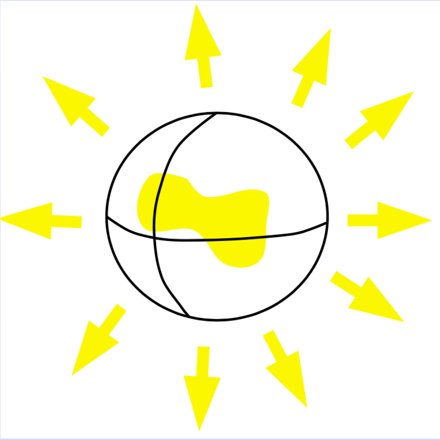


$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + T^{\mu\nu}_{(1)} + \ldots \quad J^\mu = \rho(T,\mu)u^\mu + J^\mu_{(1)} + \ldots$$

$$T^{\mu\nu}_{ideal} = \epsilon(T,\mu)u^\mu u^\nu + p(T,\mu)\underbrace{(u^\mu u^\nu + g^{\mu\nu})}_{\equiv\Delta^{\mu\nu}}$$

$$T^{\mu\nu}_{(1)} = \eta\,\underbrace{\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho - g_{\rho\sigma}(\nabla u)\right)}_{shear} - \zeta\Delta^{\mu\nu}(\nabla u)$$

$$J^\mu_{(1)} = \sigma\left(E^\mu - T\Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right)\right)$$



Local version of the 2nd law: $\partial_\mu J^\mu_s \geq 0$

Hydrodynamics as EFT

$$T^{\mu\nu}_{(1)} = \eta \underbrace{\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho - g_{\rho\sigma}(\nabla u)\right)}_{shear} - \zeta \Delta^{\mu\nu}(\nabla u)$$

$$J^{\mu}_{(1)} = \sigma\left(E^\mu - T\Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right)\right)$$

Linear Response:
(e.g. in 2+1D)

$$G_R^{ij,kl}(\omega,0) = \langle T^{ij}(-\omega)T^{kl}(\omega)\rangle_R$$

$$G_R^{i,j}(\omega,0) = \langle J^{i}(-\omega)J^{j}(\omega)\rangle_R$$

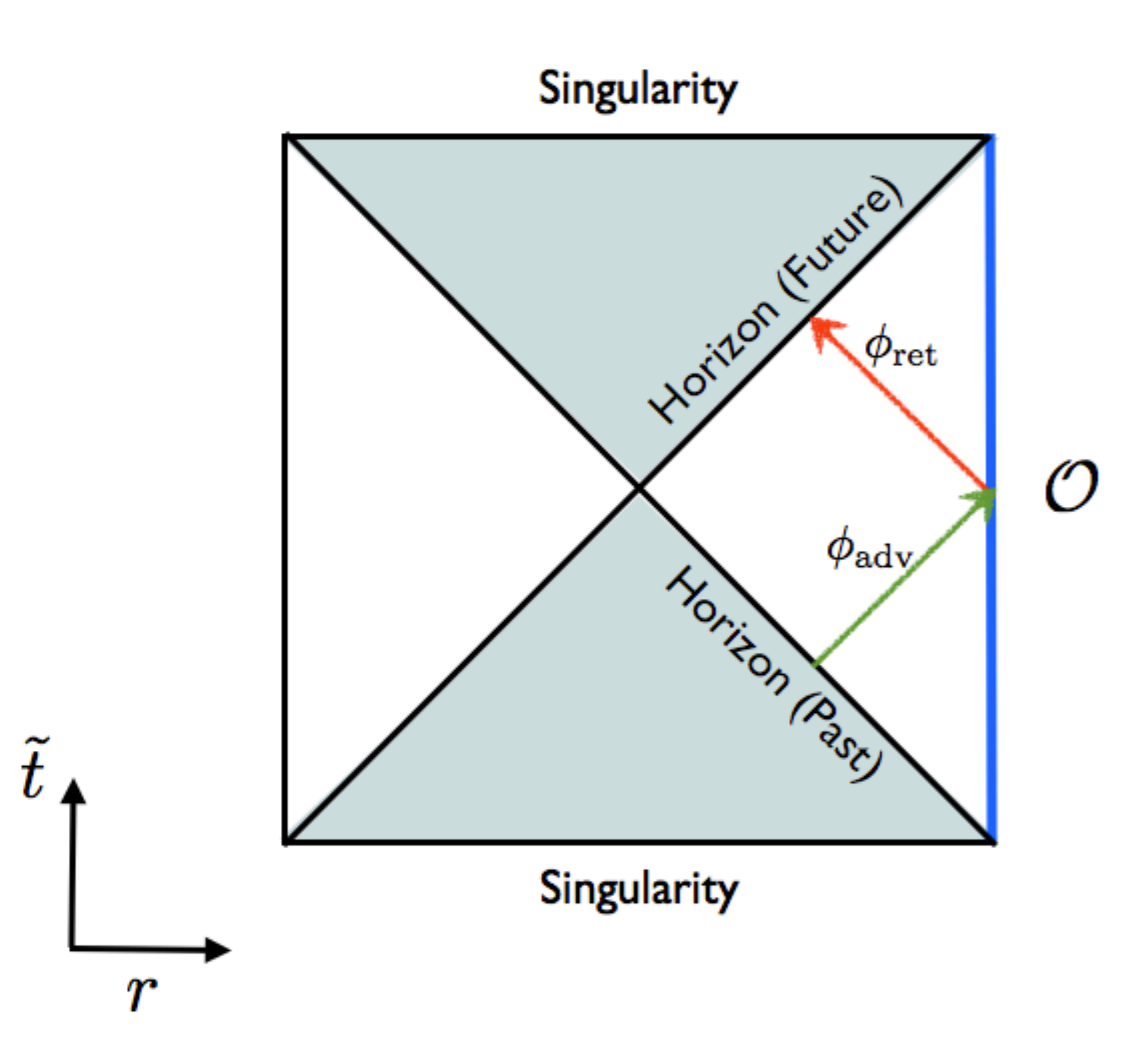
$$\eta = \lim_{\omega\to 0}\frac{1}{8\omega}(\delta_{ik}\delta_{jl} - \epsilon_{ik}\epsilon_{jl})\,\mathrm{Im}\,G_R^{ij,kl}(\omega,0)\,,$$

$$\sigma = \lim_{\omega\to 0}\frac{1}{2\omega}\delta_{ij}\,\mathrm{Im}\,G_R^{i,j}(\omega,0)\,,$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{4\omega}\delta_{ij}\delta_{kl}\,\mathrm{Im}\,G_R^{ij,kl}(\omega,0)\,,$$

η, σ, ζ calculated from microscopic or other effective models (kinetic theory, AdS/CFT)

Hydrodynamics from Black Holes



$$\eta = \lim_{\omega\to 0} \frac{1}{8\omega}(\delta_{ik}\delta_{jl} - \epsilon_{ik}\epsilon_{jl})\,\mathrm{Im}\,G_R^{ij,kl}(\omega,0)$$

$$G_R^{ij,kl} = \frac{\langle \delta T^{ij}\rangle_R}{\delta h_{(0),kl}}$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

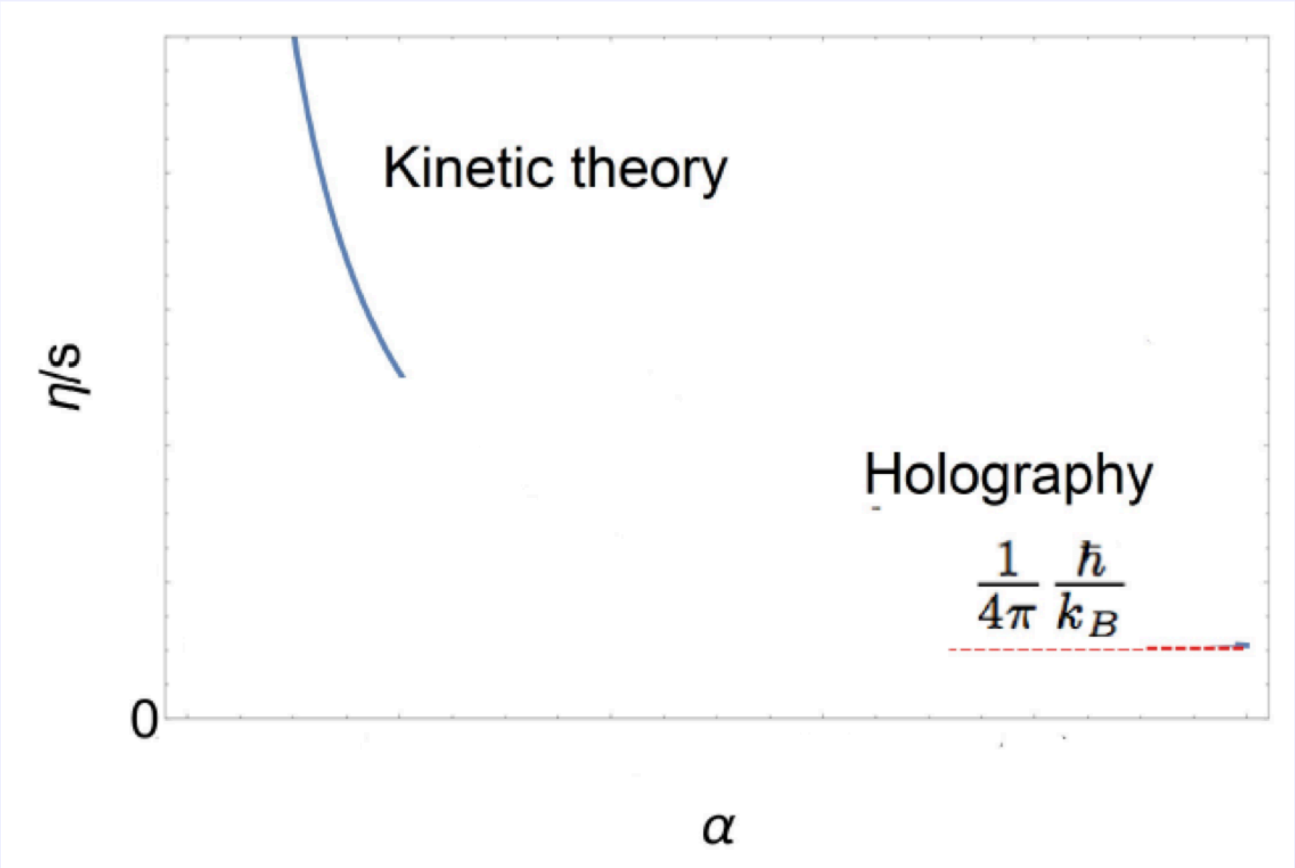
[Kovtun, Son, Starinets ’04]

- Universal in AdS/CFT at large N and coupling
- Same in 2+1D and 3+1D, Rotational invariance
- Corrections can be systematically included

Improving the hydrodynamic approximation to get closer to AdS/CFT

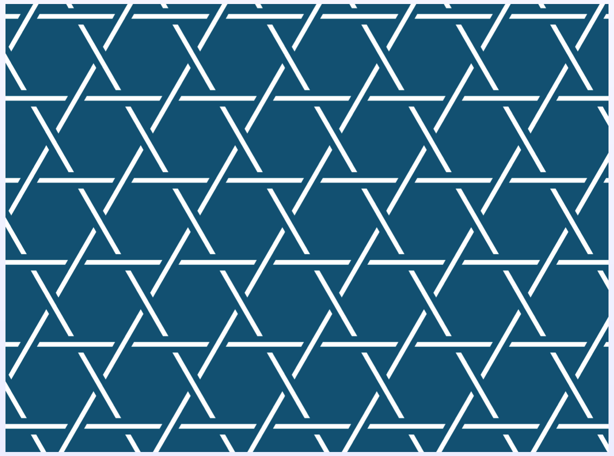
Electron-electron scattering length: $\ell_{ee} \sim \frac{1}{\alpha_{eff}^2}$

Effective fine structure constant: $\alpha_{eff} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_F}$



Kagome Lattices

Kagome: Basket weaving pattern from Japan

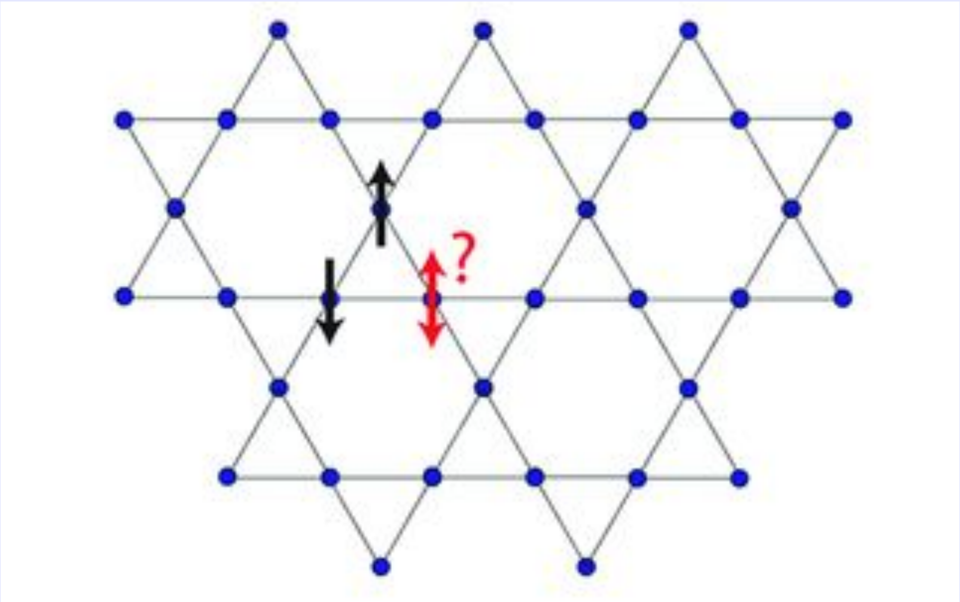


[Wikipedia]

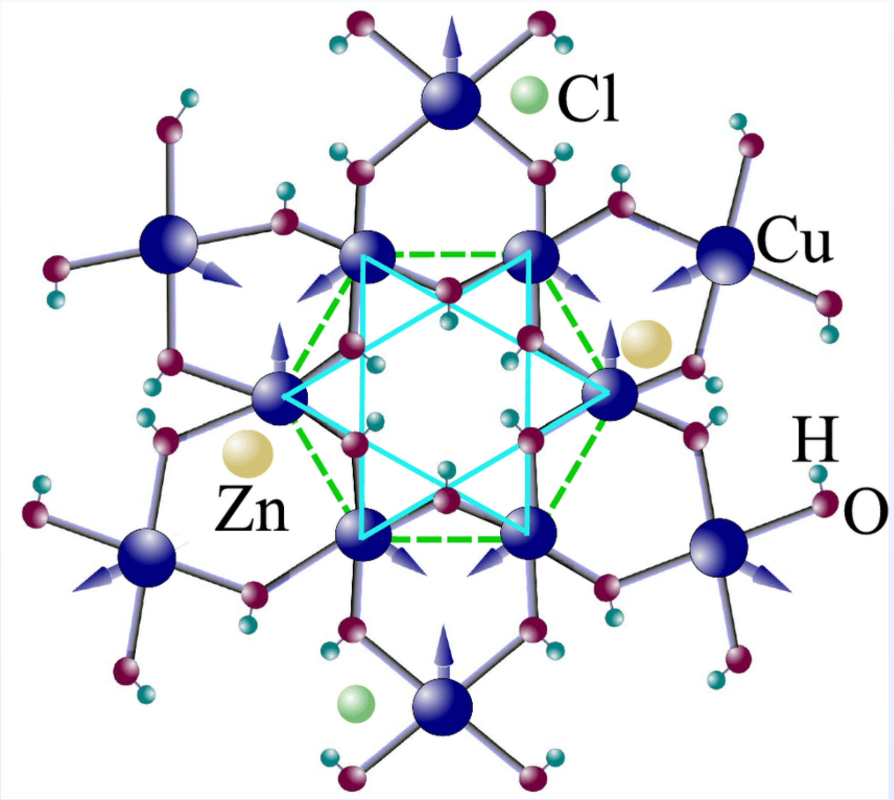
籠 (kago) Basket

目 (me) Eye

Kagome lattice: Frustration prevents ordering phenomena

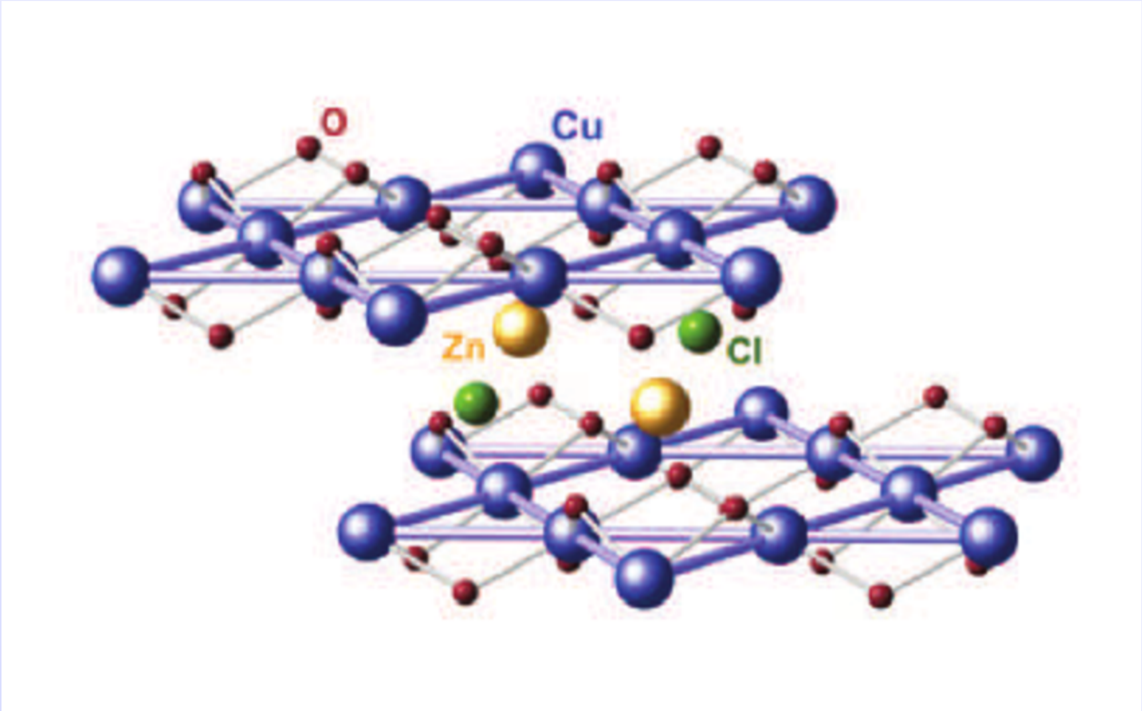


Kagome Materials



Herbertsmithite

[Wikipedia]



[J. Mat. Sciences]

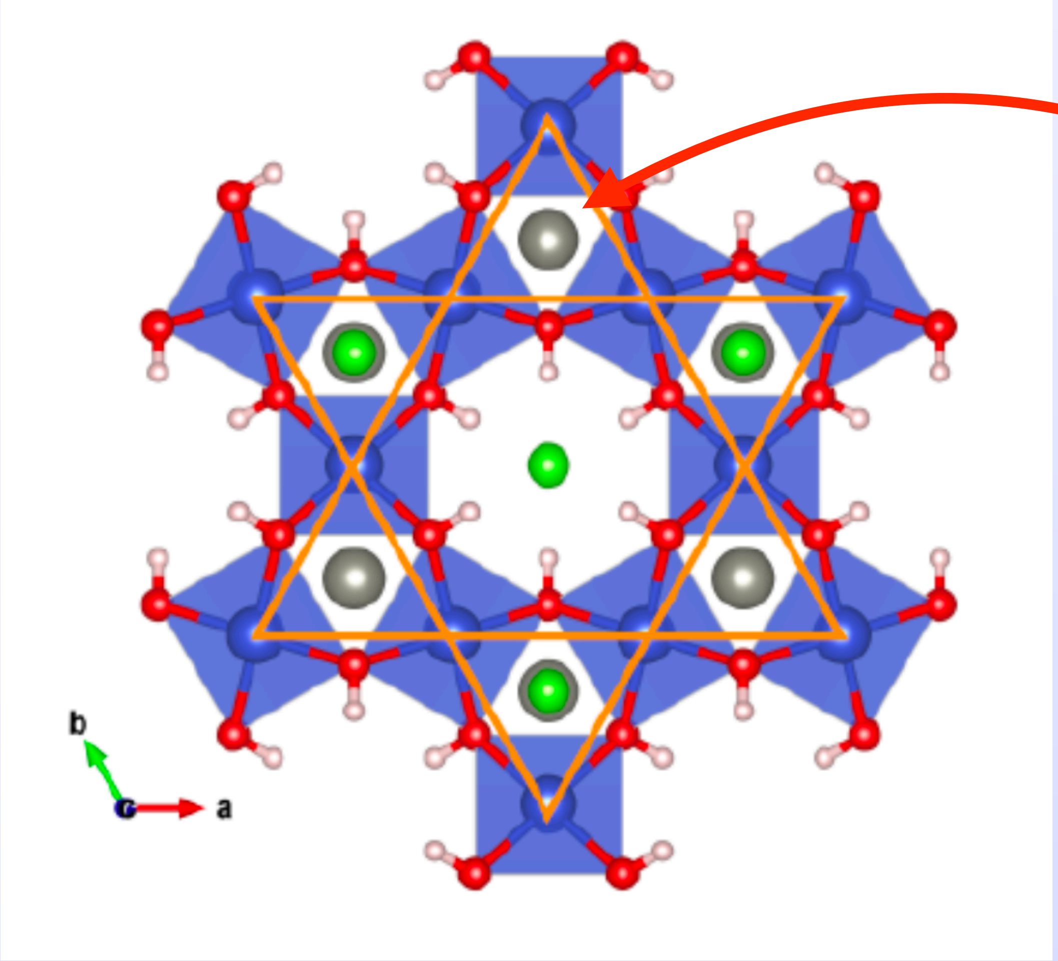
$$ZnCu_3(OH)_6Cl_2$$

Scandium-Herbertsmithite

Herbertsmithite: Zn^{2+} Fermi surface below Dirac point

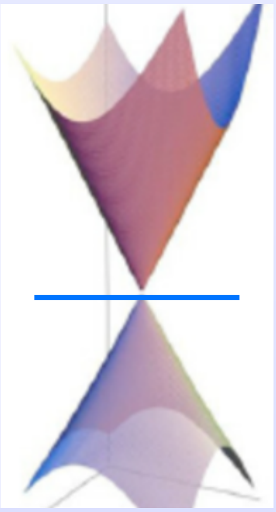
Sc-Herbertsmithite: Sc^{3+} Fermi surface at Dirac point

Same low energy band structure as Graphene



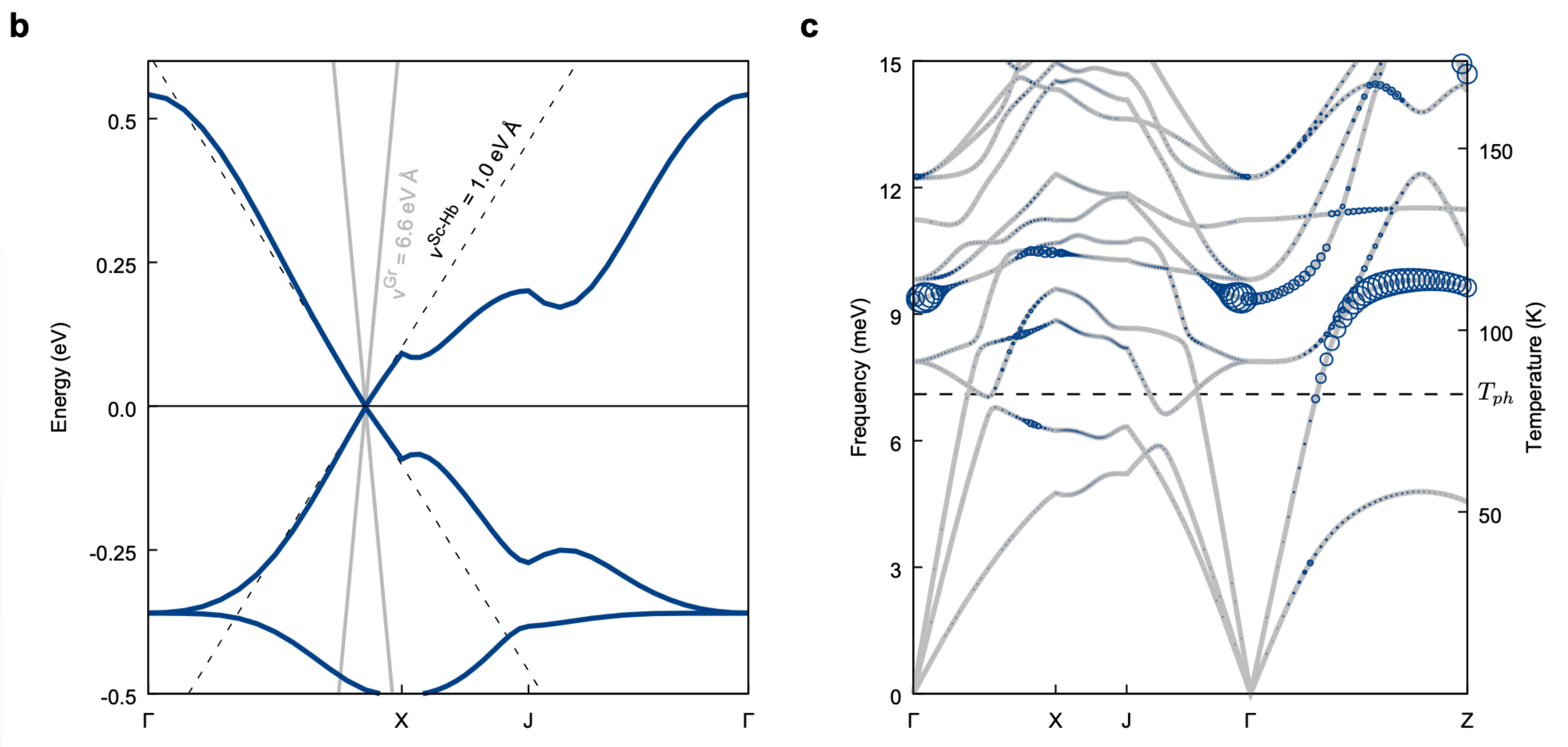
Sc^{3+}

2x



I.L. Mazin et. al. Nat. Comm. (2014)

Scandium-Herbertsmithite



Electronic Band Structure

Phonon Dispersion

$v_{F,Sc} \approx 0.15 v_{F,Gr}$

$T_{phon} \sim 80K$

J.E., I.M, R.M., D.R-F., R. Thomale, D. Di Sante, E. van Loon, T. Wehling 1911.06810

Scandium-Herbertsmithite

Table I. **Dirac fluid parameters.** The Fermi velocity v_F, the relative dielectric constant ϵ_r and the fine-structure constant α for electrodynamics in vacuum, (hBN encapsulated) graphene [25, 26] and stochiometric Scandium substituted Herbertsmithite. In graphite [27] the low-energy dispersion at the K point of the Brillouin zone is quadratic and not linear as in graphene.

	v_F (eVÅ)	ϵ_r	$\alpha = e^2/\epsilon_0\epsilon_r\hbar v_F$
ED in vacuum	2×10^3	1	1/137
hBN/graphene/hBN	6.6	2.2 – 4.0	0.5 – 1.0
graphite	–	2.5	–
Sc-Herbertsmithite	1.0	5.0	2.9

$$\alpha_{eff} = \frac{e^2}{\epsilon_0 \epsilon_r \hbar v_F}$$

Increase in fine structure constant driven by smaller Fermi velocity

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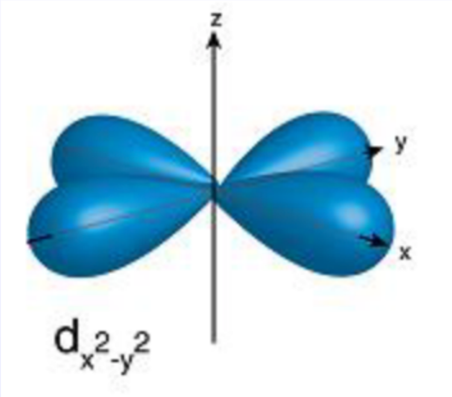
Scandium-Herbertsmithite

- Kagome lattice formed by $\mathrm{CuO_4}$ plaquettes

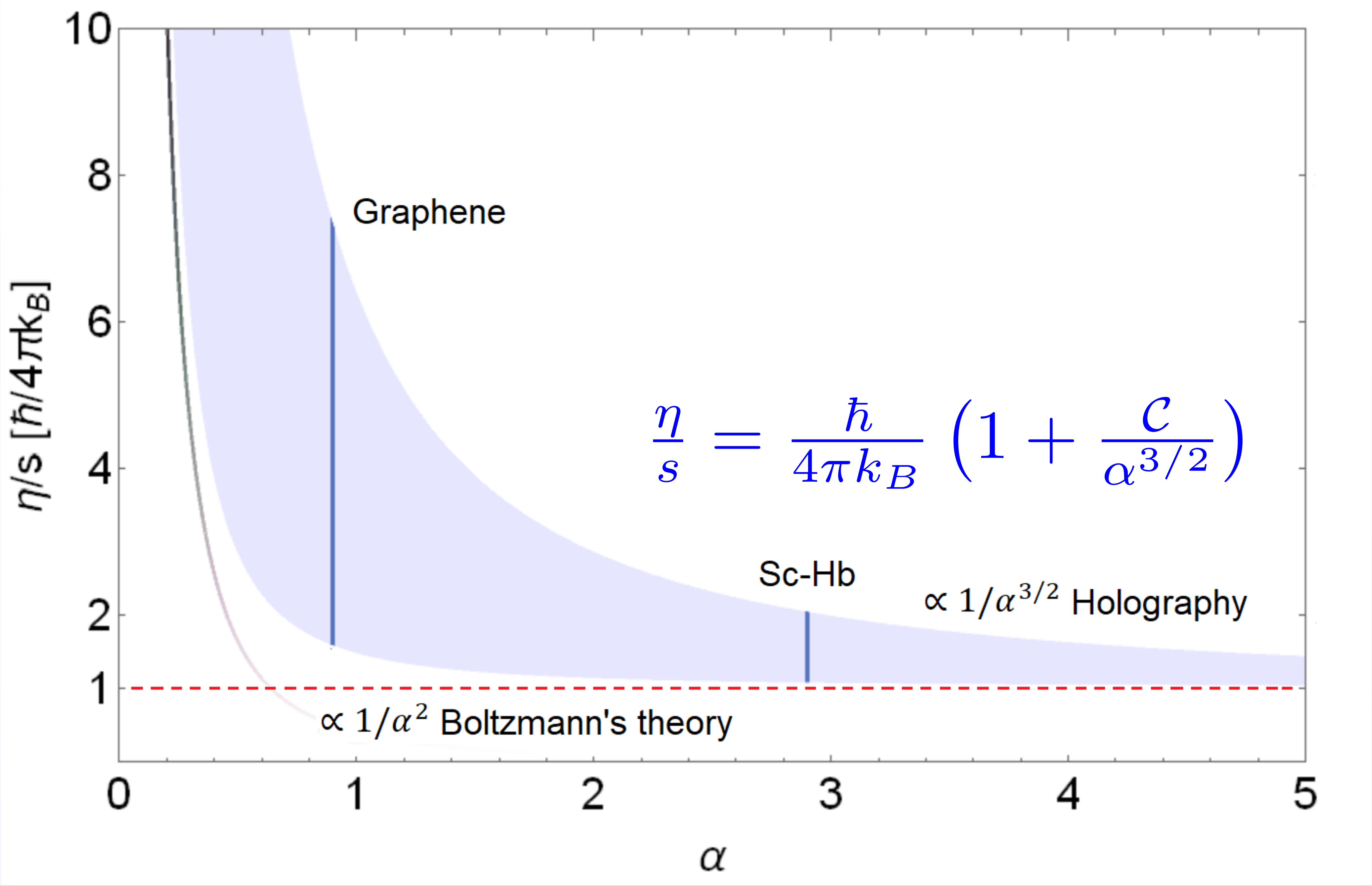


- Cu $d_{x^2-y^2}$ orbitals form Dirac points
- Fermi level at Dirac point (4/3 filling)
- Orbital hybridization leads to larger Coulomb coupling

$$\alpha_{Sc-Hb} = 2.9 \qquad (\alpha_{Gr} = 0.9)$$

- Enhanced hydrodynamic behavior: $\ell_{ee,Sc} = \frac{1}{6}\ell_{ee,Gr}$
- Optical phonons activated above $T_{phon} \sim 80K$
- Candidate to test universal predictions from AdS/CFT

Estimate of Shear Viscosity over Entropy Density



Parametrize leading R^4 correction by $\mathcal{C} = 5 \cdot 10^{-4} \ldots 2$

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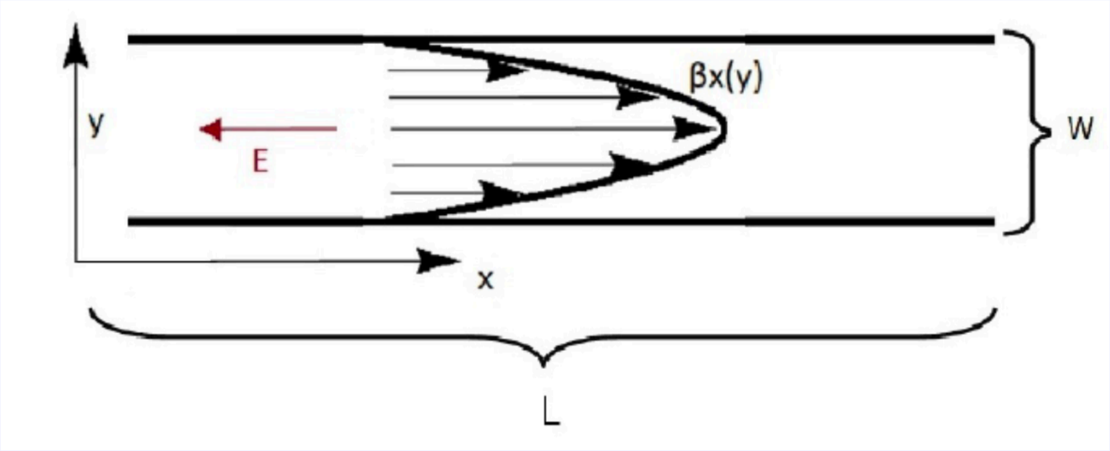
Reynolds number and Turbulence

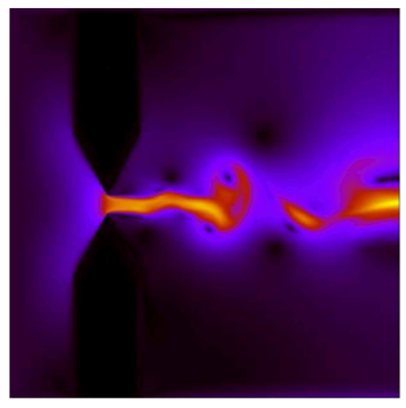
- **Low viscosity fluids easily become turbulent**



$$Re = \left(\frac{\eta}{s}\frac{k_{\mathrm{B}}}{\hbar}\right)^{-1} \frac{k_{\mathrm{B}}T}{\hbar v_{\mathrm{F}}} \frac{u_{\mathrm{typ}}(\eta/s)}{v_{\mathrm{F}}} W,$$

Poiseuille flow

Enhancement of factor 100 in Sc-Hb compared to Graphene



[Mendoza et.al. PRL 2016]

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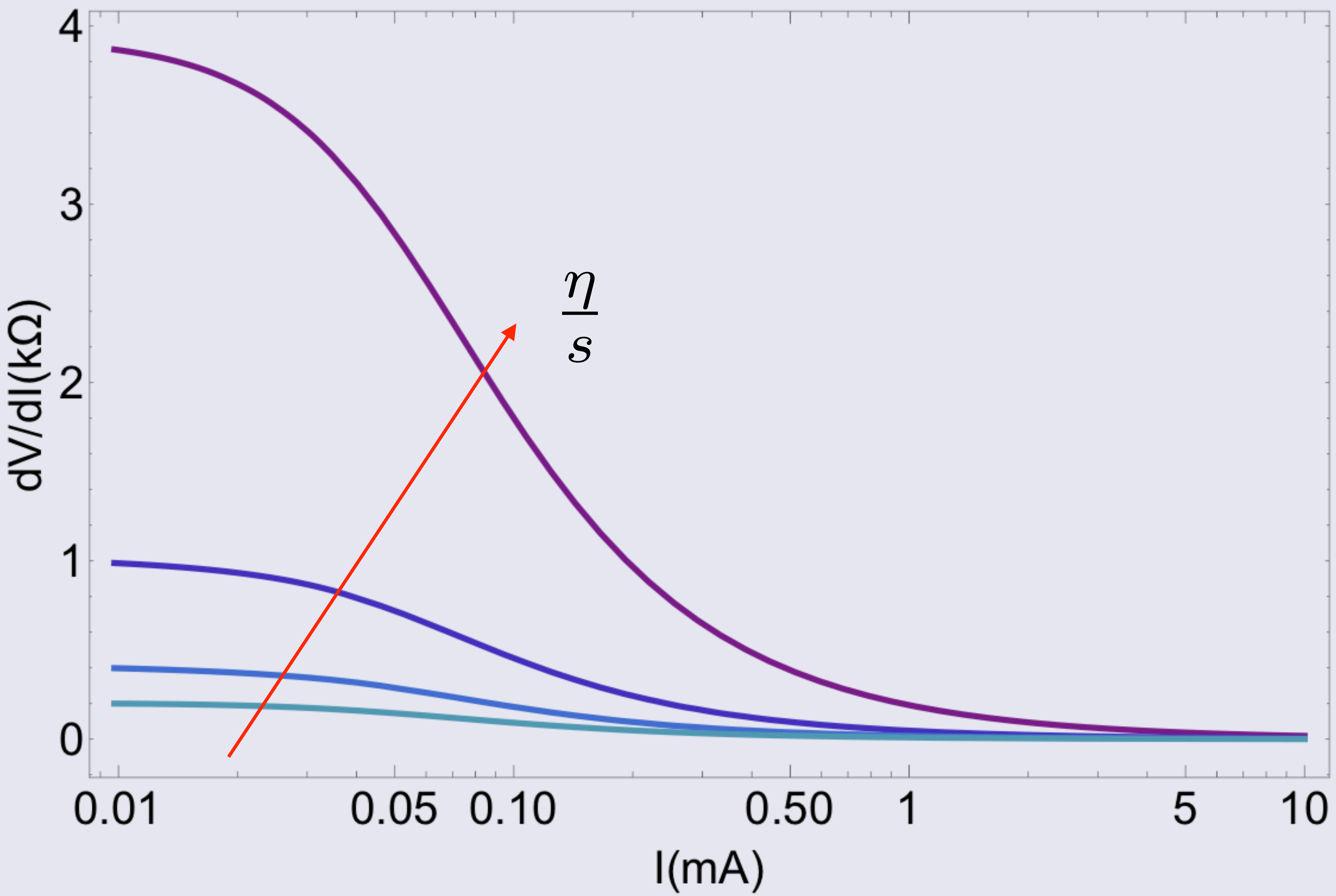
Differential Wire Resistance



R(I) increases as η/s increases

J.E., I. Matthaiakakis, R.M., D. Rodríguez-Fernández (PRB 2018)

Holographic Poiseuille Flows

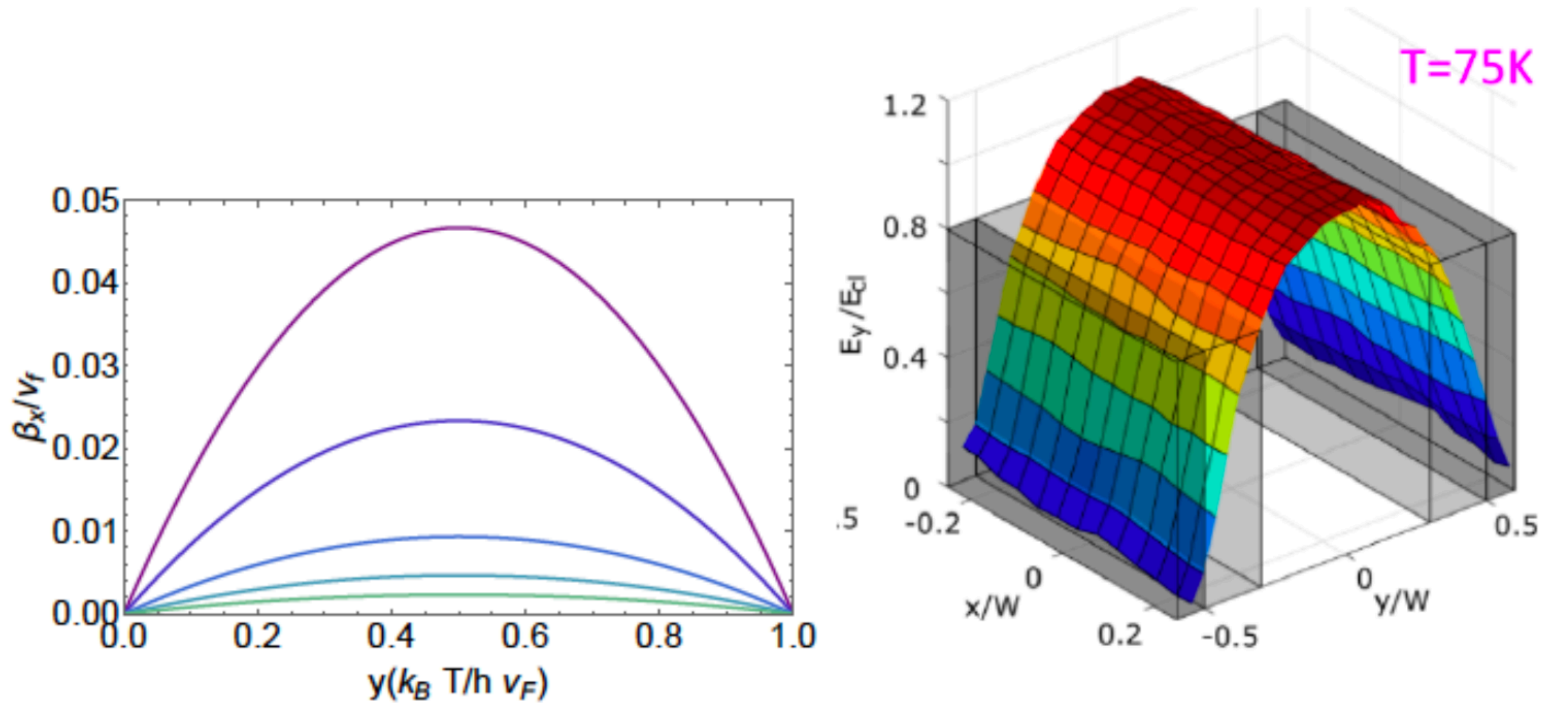
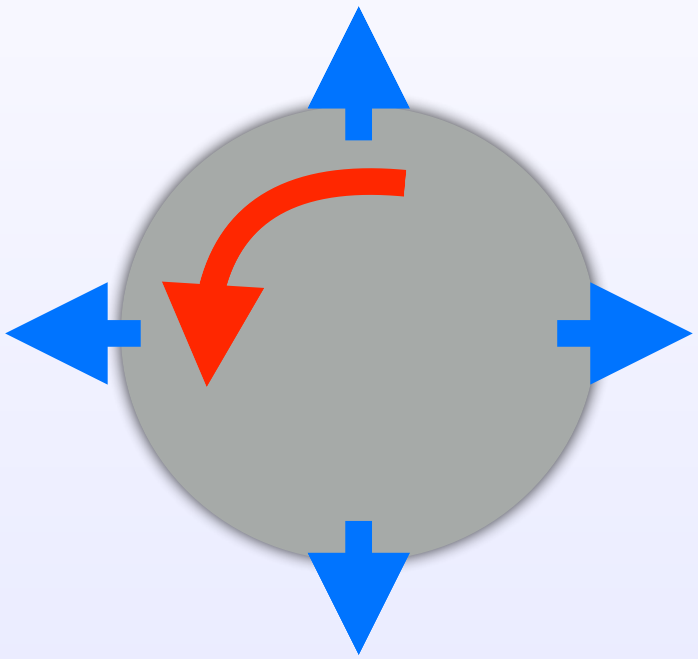


Figure: Left figure:Top curve, $\eta/s = \hbar/4\pi k_B$ (Holography). Right figure: Experimental observation of the Poiseuille flow in graphene (fig. taken from J. Sulpizio *et al* [1905.11662]

Faster flow at stronger coupling (smaller viscosity over entropy density ratio)

J.E., I. Matthaiakakis, R.M., D. Rodríguez-Fernández (PRB 2018)

Hall viscosity in Channel Flows



$$\dot{P}^i + \partial_j T^{ji} = F^{i\mu} J_\mu - \Gamma P^i$$

$$T_{ij}^{\mathrm{Hall}} = \eta_{\mathrm{H}} (\epsilon_{ik} v_{kj} + \epsilon_{jk} v_{ki})$$

$$F^y_{visc} = 2\eta_H \partial_y^2 v_x(y)$$

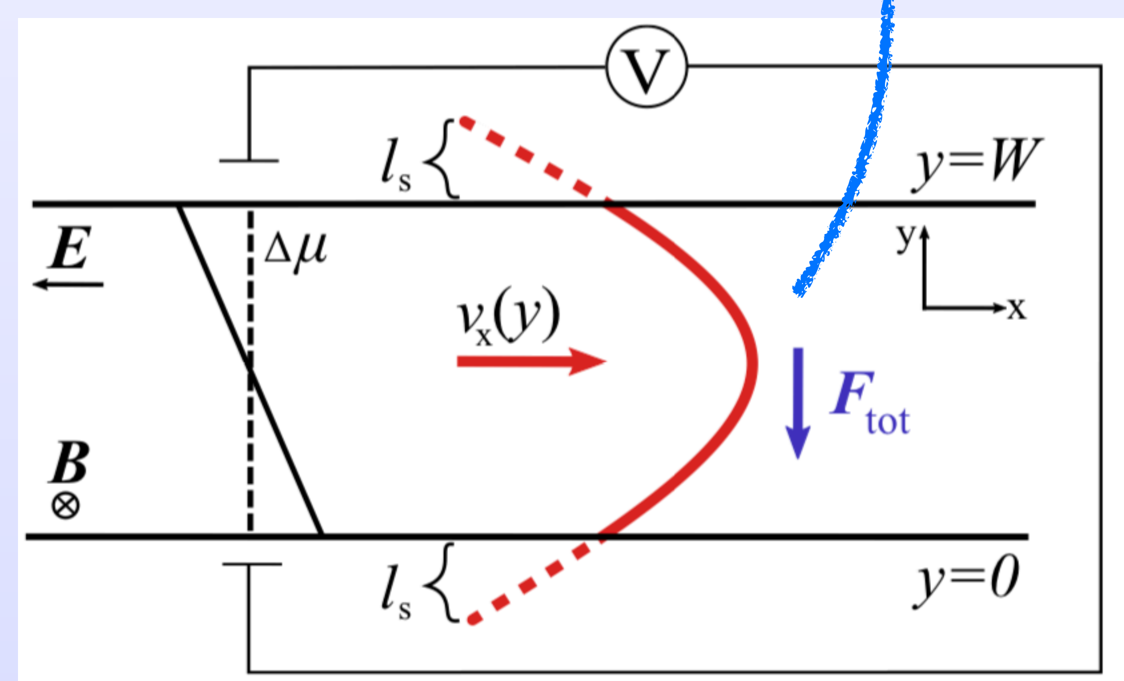
Poiseuille Flow

[Avron, Seiler, Zograf 1995]

Incompressible Flow: $\partial_\mu v^\mu = 0$

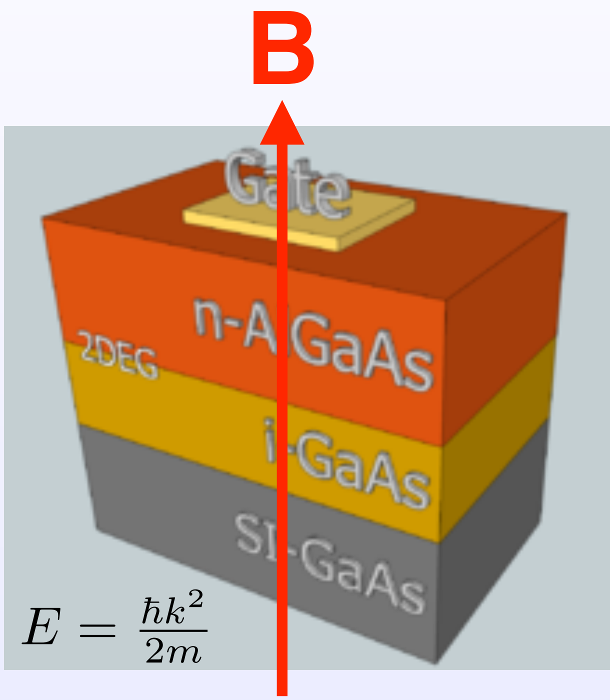
$$(\epsilon + P)\dot{v}^x + \eta \partial_y^2 v^x = E^x \rho - \frac{(\epsilon + P) v^x}{\tau_{\mathrm{imp}}}$$

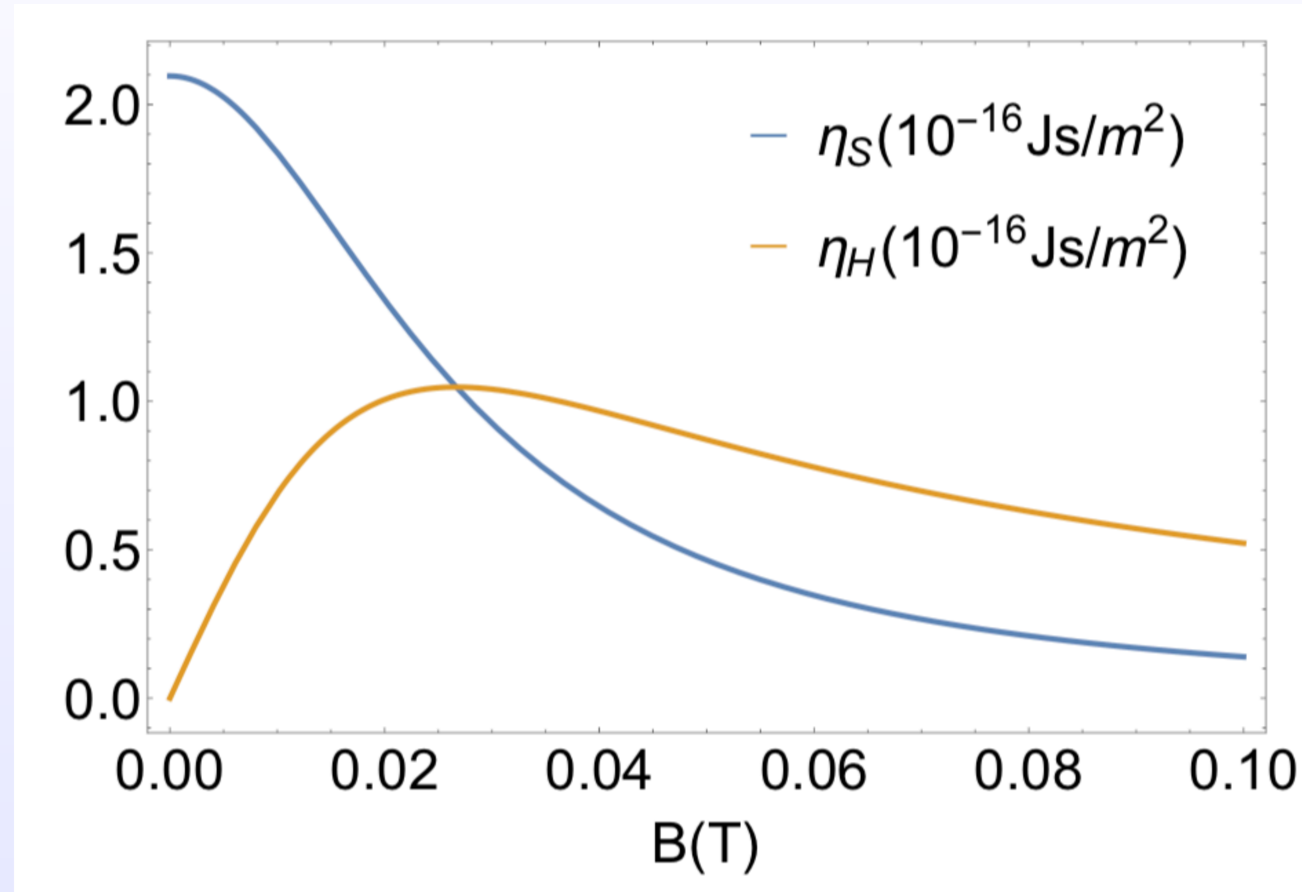
$$2\eta_H \partial_y^2 v^x = \partial_y P = \rho \partial_y V + s \partial_y T$$

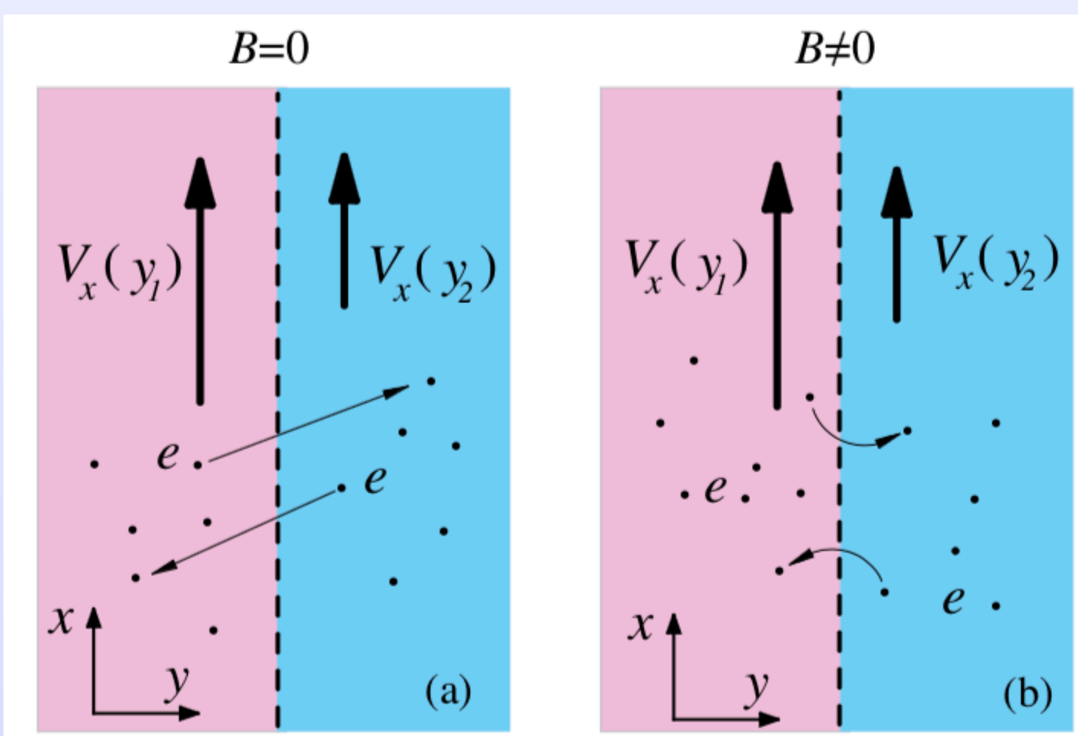


J.E., Hankiewicz, Matthaiakakis, R.M., Rodríguez-Fernández, Tutschku PRB 2019

Hall Viscosity in a 2DEG







$$\eta_{xx} = \frac{\eta}{1+(2\omega_c\tau_2)^2}, \quad \eta_{xy} = \frac{2\omega_c\tau_2\eta}{1+(2\omega_c\tau_2)^2}$$

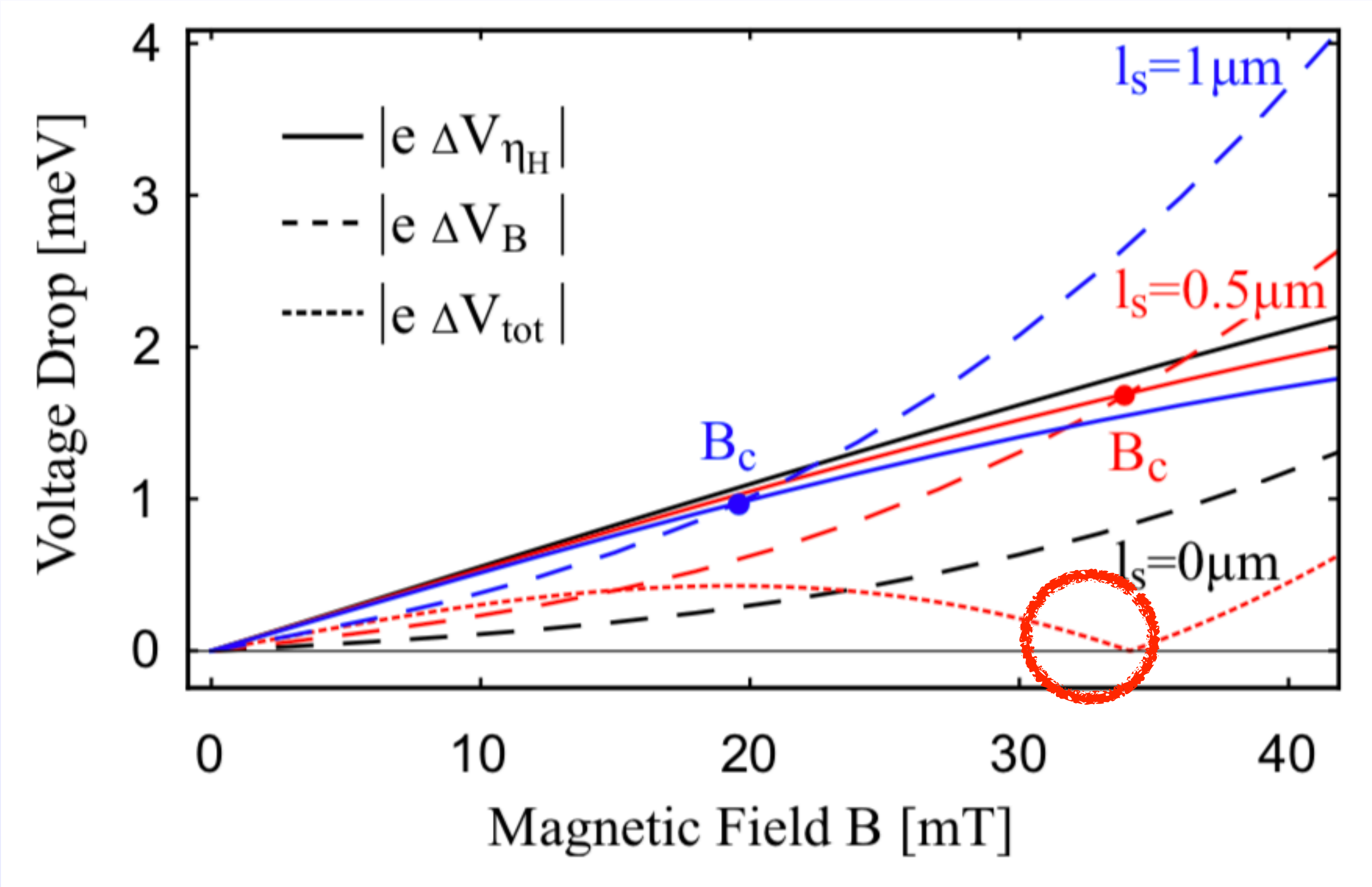
$$\frac{1}{\tau_2(T)} = \frac{1}{\tau_{2,ee}(T)} + \frac{1}{\tau_{2,0}}$$

$$\omega_c = eB/mc$$

Weakly Coupled!

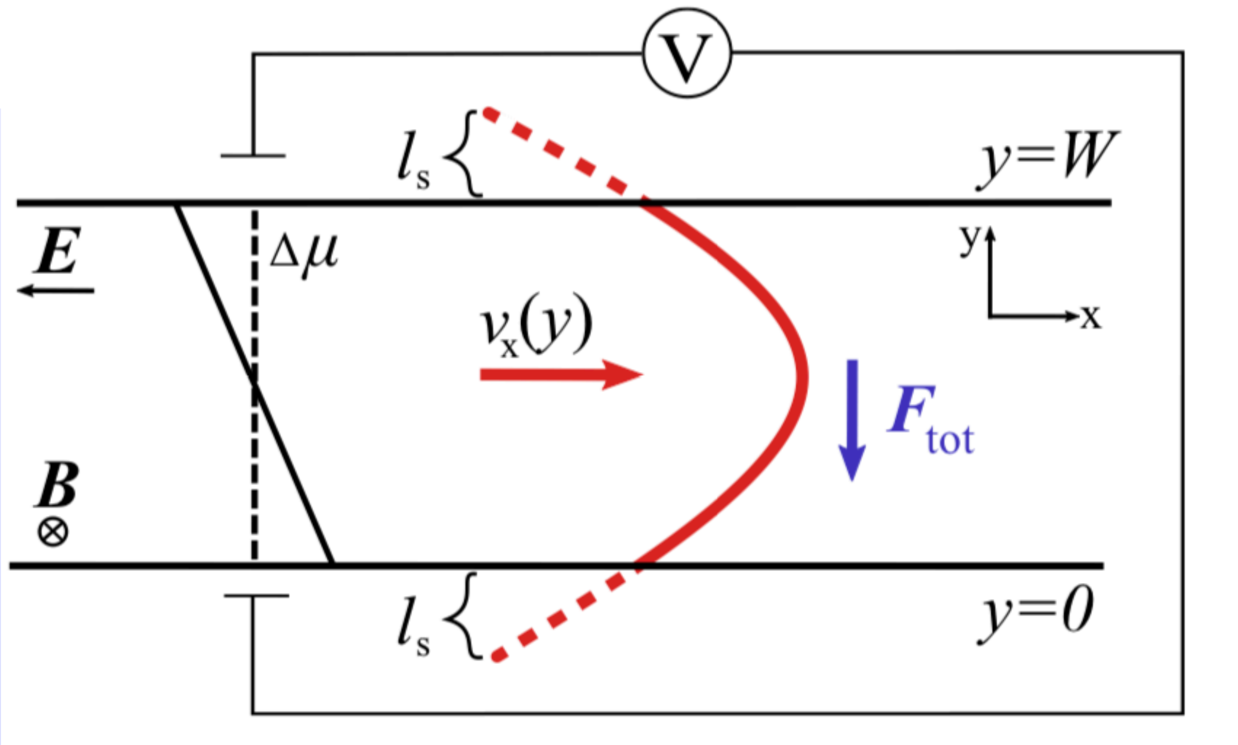
[Alekseev PRL 2016]

Transverse Hall Response



$$f^i_{\eta_H} = \eta_H \epsilon^{ij} \Delta v^j$$

$$\vec{\nabla} P = en\vec{\nabla} V + s\vec{\nabla} T$$



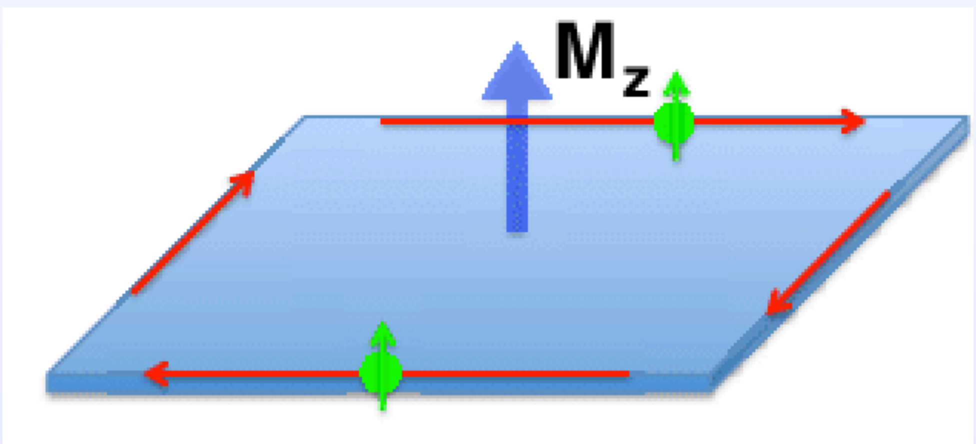
J.E., Hankiewicz, Matthaiakakis, R.M., Rodríguez-Fernández, Tutschku PRB 2019

Nonrelativistic UV terms and nondissipative response

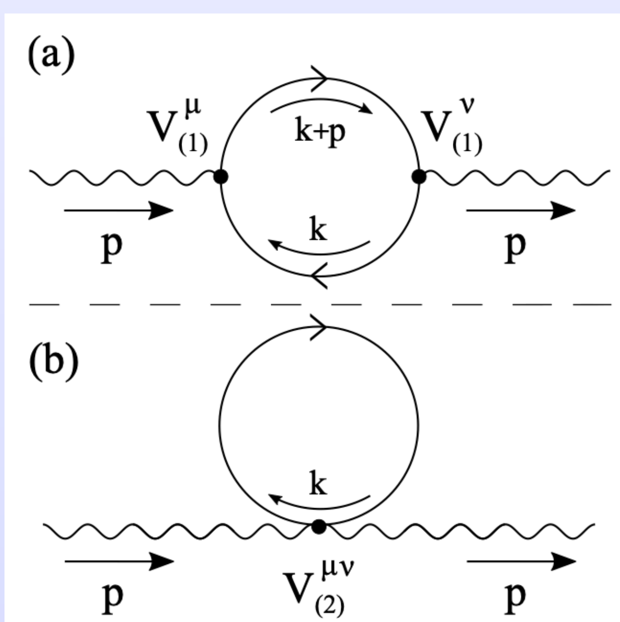
Quantum Anomalous Hall Effect Model

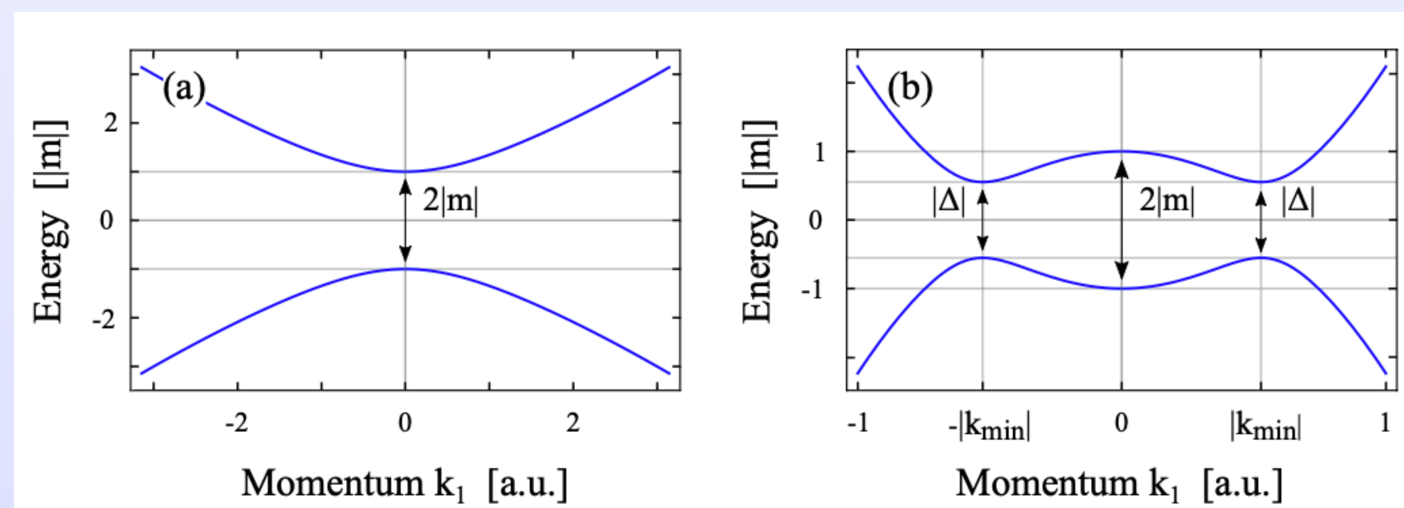
$$\mathcal{H}(\mathbf{k}) = \left(m - B|\mathbf{k}|^2\right)\sigma_3 - \cancel{D|\mathbf{k}|^2\sigma_0} + A\left(k_1\sigma_1 - k_2\sigma_2\right)$$

[Bernevig, Hughes, Zhang Science 2006]



(d) Quantum Anomalous Hall effect





$$\sigma_{xy}(0) = \frac{e^2}{2h}\left[\mathrm{sgn}(m) + \mathrm{sgn}(B)\right] = \frac{e^2}{h}\,\mathcal{C}_{\mathrm{QAH}}$$

C. Tutschku, J. Böttcher, R.M., E. Hankiewicz 2003.03146

Conclusions and Outlook

- Materials with large Coulomb coupling good to test AdS/CFT

- Scandium-substituted Herbertsmithite:
  Effective Coulomb coupling 3 times larger than Graphene
  Smaller shear viscosity to entropy density ratio
  More robustly in the electron hydrodynamics regime
  Turbulent flow regime seems to be at the doorstep

- Strongly coupled holographic fluids may show distinct responses from weakly coupled ones in suitable geometries (e.g. Poiseuille flow)
- Electron hydrodynamics provides a window to non dissipative anomaly-induced transport - mind the UV!

- Interdisciplinary synergy between String theory and AdS/CFT, Condensed Matter Physics, QFT, and effective theories