RELATIONS BETWEEN TRANSPORT & CHAOS IN HOLOGRAPHIC THEORIES

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OVERVIEW

- In some QFTs, there are connections between transport properties and underlying chaotic dynamics.

- First motivated by studying a particular transport process: diffusion of a U(1) charge.

  Blake

  Its relation to underlying chaotic dynamics turns out to not be very robust.

- I will describe subsequent work identifying more robust relations between transport and chaos.

  I will focus on QFTs with a gravity description.

MOTIVATION

- Transport properties characterize the dynamics of a system's conserved charges (e.g. energy, momentum, $U(1)$ charge, etc.)

- Transport properties are experimentally important
  - ✳ Easy to measure
  - ✳ Exhibit universality in interesting materials

- Often governed by properties of underlying quasiparticle degrees of freedom.

- What about in systems with no quasiparticle description?

TRANSPORT PROPERTIES

- Objects of interest: retarded Green's functions of conserved charge densities

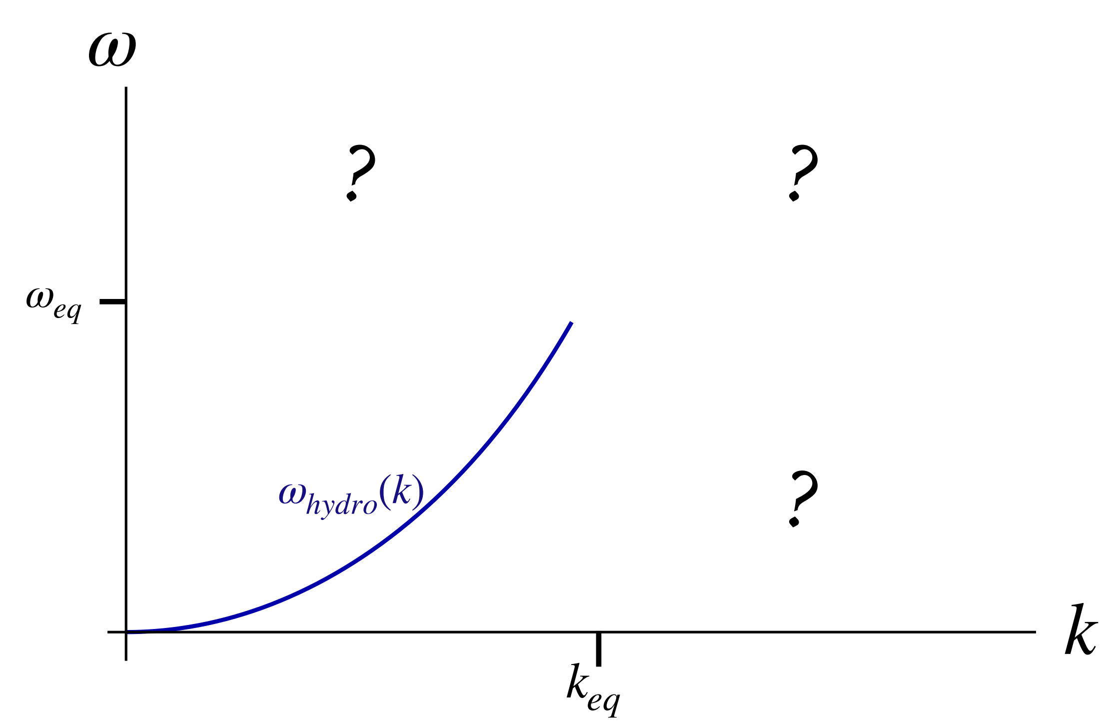
  e.g. $G_{\varepsilon\varepsilon}(\omega, k)$: change in energy density $\langle\varepsilon\rangle$ due to a small source

- Green's function poles $\longleftrightarrow$ dispersion relations $\omega(k)$ of collective modes

- Can identify some general features, even in absence of quasiparticles

  * When the system is in local equilibrium, transport is governed by simple effective theories: **hydrodynamics**.

  * In this regime, transport is dominated by a handful of gapless modes $\omega_{hydro}(k)$



HYDRODYNAMICS

- **Example**: system whose only conserved charge is its total energy

  ∗ Local thermal equilibrium ⟶ state characterized by slowly-varying $\varepsilon(\underline{x}, t)$

  ∗ Dynamics of this variable are constrained by symmetries:

$$\longrightarrow \qquad \partial_t \varepsilon = D\,\nabla^2 \varepsilon + \Gamma\,\nabla^4 \varepsilon + O(\,\nabla^6)$$

$$\text{or} \qquad \omega_{hydro}(k) = -\,iDk^2 - i\Gamma k^4 + O(k^6)$$

  Energy diffuses over long distances.

- The values of the parameters of the effective theory $(D, \Gamma$ etc$)$ depend on the details of the particular system.

  In a Fermi liquid, $D \sim v_F^2 \tau_{qp}$.

CHAOTIC PROPERTIES

- Chaotic dynamics are seemingly something very different from transport.

$$C(t,\underline{x}) = -\left\langle [V(t,\underline{x}), W(0,\underline{0})]^2 \right\rangle_T$$

- In theories with a classical gravity dual, these correlations have the form

$$C(t,\underline{x}) \sim e^{\tau_L^{-1}\left(t - |\underline{x}|/v_B\right)}$$

  - ✶ The timescale is always $\tau_L = (2\pi T)^{-1}$ Shenker, Stanford
  - ✶ But the "butterfly velocity" v_B depends on the particular theory.

Roberts, Stanford, Susskind

  - ✶ In the gravity description, governed by near-horizon physics.

SUMMARY OF RESULTS

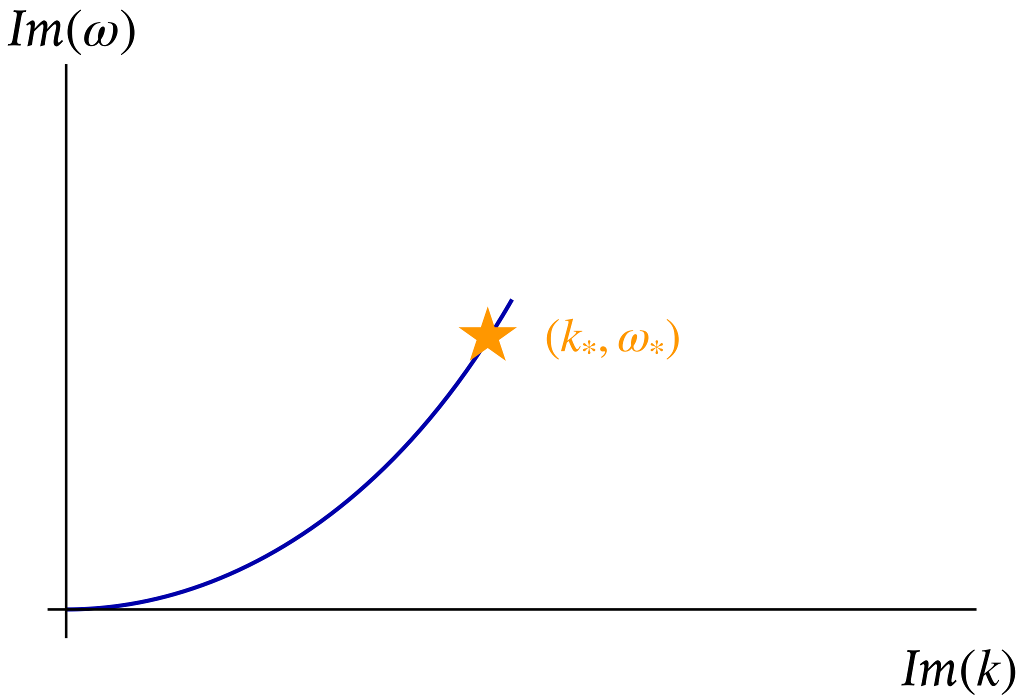
- In QFTs with a gravity dual, the transport properties are constrained by v_B, τ_L

- There is a collective mode transporting energy whose dispersion relation obeys

$$\omega(k_*) = i\tau_L \quad \text{where} \quad k_*^2 = -(v_B\tau_L)^{-2} .$$

Blake, RD, Grozdanov, Liu

see also: Grozdanov, Schalm, Scopelliti

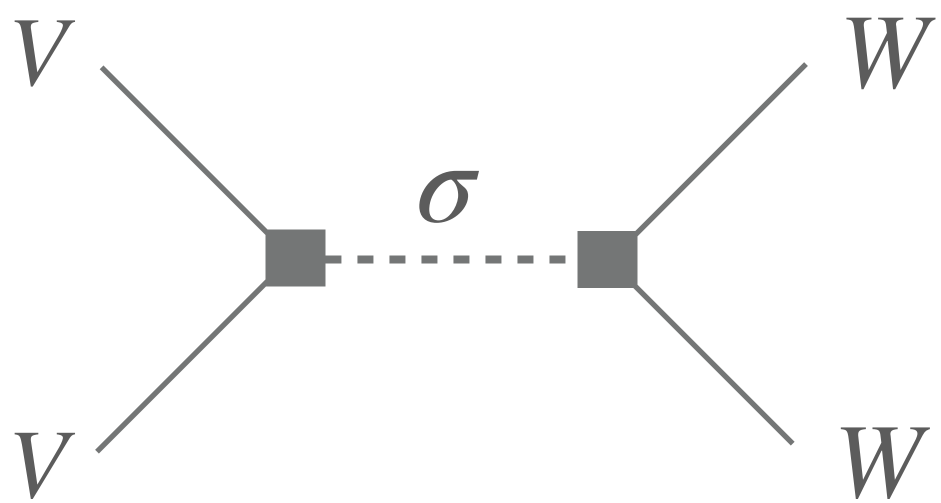


- In the limit of low temperatures, there is diffusive transport of energy with

$$D \sim v_B^2 \tau_L$$

INTERPRETATION

- Consistent with proposal that chaotic behavior has hydrodynamic origin



Blake, Lee, Liu

σ : hydrodynamic mode of energy conservation

- If it is a hydro mode that satisfies $\omega_{hydro}(k_*) = i\tau_L$ for $k_*^2 = -(v_B\tau_L)^{-2}$

  And this mode is approximately diffusive $\omega_{hydro}(k) \sim -iDk^2$ up to $k = k_*$

$$\longrightarrow \quad D \sim v_B^2 \tau_L$$

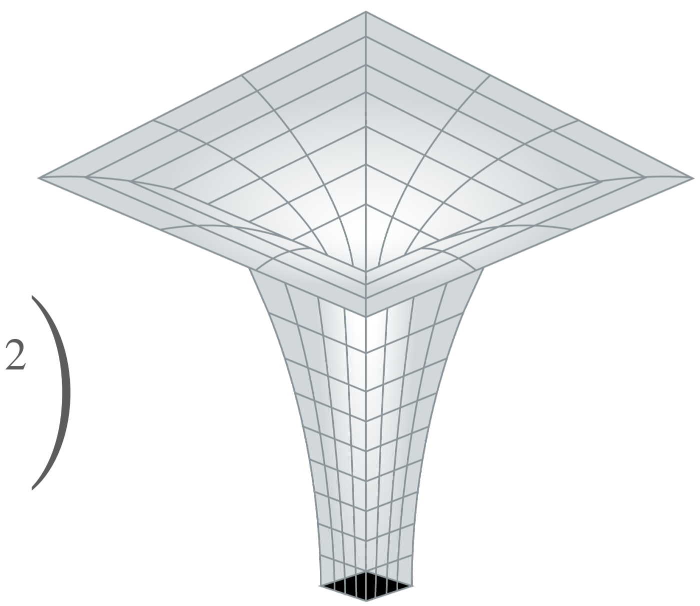
THE GRAVITATIONAL THEORIES

- I will discuss asymptotically AdS_{d+2} black branes supported by matter fields:

$$ds^2 = -f(r)dv^2 + 2dvdr + h(r)d\underline{x}_d^2$$

Arising as classical solutions of

$$S = \int d^{d+2}x\sqrt{-g}\left(R - Z(\phi)F^2 - \frac{1}{2}(\partial\phi)^2 + V(\phi) - Y(\phi)\sum_{i=1}^{d}\left(\partial\chi_i\right)^2\right)$$



- Matter fields induce an RG flow from the UV CFT :

$$\phi(r) \neq 0 \qquad F_{vr}(r) \neq 0 \qquad \chi_i = mx^i$$

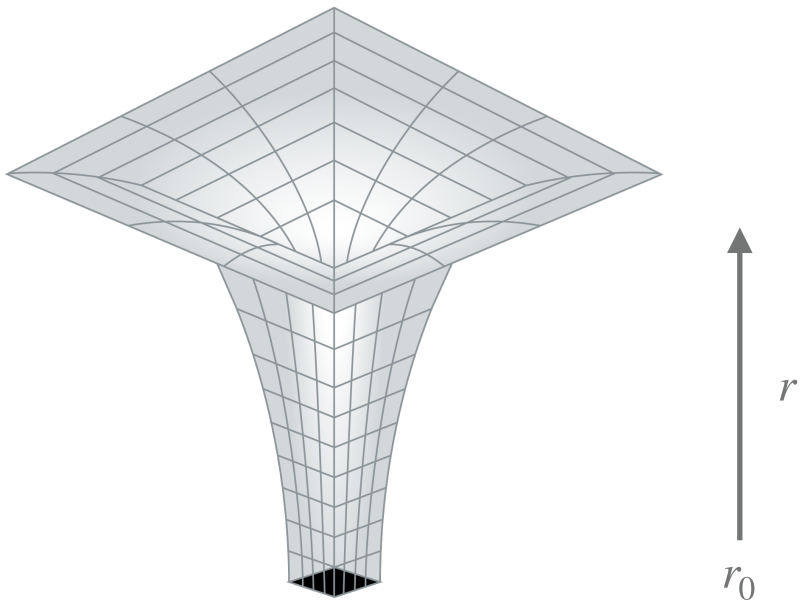
- Broad family of solutions with different symmetries and hydrodynamics.

GREEN'S FUNCTIONS FROM GRAVITY

- Simplest case: scalar operator. $\partial_a(\sqrt{-g}\partial^a\varphi) - m^2\sqrt{-g}\varphi = 0$

- Each Fourier mode has two independent solutions:

$$\varphi_{norm}(r,\omega,k) \quad \text{and} \quad \varphi_{non-norm}(r,\omega,k)$$



- Find the linear combination that is regular at the horizon $r = r_0$:

$$\varphi_{ingoing} = a(\omega,k)\varphi_{norm} + b(\omega,k)\varphi_{non-norm}$$

- The QFT retarded Green's function is $G(\omega,k) = \dfrac{b(\omega,k)}{a(\omega,k)}$

GREEN'S FUNCTIONS FROM GRAVITY

- Green's functions of conserved charges (e.g. $G_{\varepsilon\varepsilon}$) depend in detail on the metric and matter field profiles throughout the spacetime.

  i.e. on many specific details of the particular QFT state.

- But there are two situations when only near-horizon dynamics is important

  $\ast$ $\omega \to 0$, $k \to 0$ limit where radial evolution is simple: $\dfrac{d}{dr}\big(\ldots\varphi'(r)\big) = 0$

  $\ast$ Points in Fourier space (ω_*, k_*) where the ingoing solution is not unique

⟶ features of the Green's functions that are insensitive to many details of the state

HORIZON CONSTRAINTS ON THE SPECTRUM

- Identifying points (ω_*, k_*) where the ingoing solution is not unique

⟶ exact constraints on the spectrum $\omega(k)$ of collective modes

- **Example**: probe scalar field

Blake, RD, Vegh ; see also Grozdanov et al

  - ✳ Ansatz: solution that is regular at the horizon $\varphi(r) = \sum_{n=0}^{\infty} \varphi_n (r - r_0)^n$

  - ✳ Solve iteratively for $\varphi_{n>0}$:

$$2h(r_0)(2\pi T - i\omega)\varphi_1 = \left(k^2 + m^2 h(r_0) + i\omega \frac{dh'(r_0)}{2}\right)\varphi_0 \qquad \text{etc.}$$

  - ✳ At (ω_*, k_*) both solutions are regular at the horizon

$$\omega_* = -i2\pi T, \qquad k_*^2 = -\left(m^2 h(r_0) + d\pi T h'(r_0)\right)$$

HORIZON CONSTRAINTS ON THE SPECTRUM

- Moving infinitesimally away from (ω_*, k_*) yields one regular solution:

$$\begin{aligned} \omega &= \omega_* + i\delta\omega \\ k &= k_* + i\delta k \end{aligned} \longrightarrow \frac{\varphi_1}{\varphi_0} = \frac{1}{4h(r_0)}\left(4ik_*\frac{\delta k}{\delta\omega} - dh'(r_0)\right)$$

But this regular solution depends on the arbitrary slope $\delta k/\delta\omega$.

- Can obtain an arbitrary combination of φ_{norm} and $\varphi_{non-norm}$ by tuning $\delta k/\delta\omega$:

$$\varphi_{ingoing}(\omega_* + i\delta\omega, k_* + i\delta k) = C\left(1 - v_z\frac{\delta k}{\delta\omega}\right)\varphi_{norm} + \left(1 - v_p\frac{\delta k}{\delta\omega}\right)\varphi_{non-norm}$$

$$\longrightarrow G(\omega_* + i\delta\omega, k_* + i\delta k) = C\frac{\delta\omega - v_z\delta k}{\delta\omega - v_p\delta k}$$

$\longrightarrow$ there must be a pole with dispersion relation obeying $\omega(k_*) = \omega_*$

HORIZON CONSTRAINTS ON THE SPECTRUM

- This constraint on $\omega(k)$ is obtained only from the near-horizon dynamics.

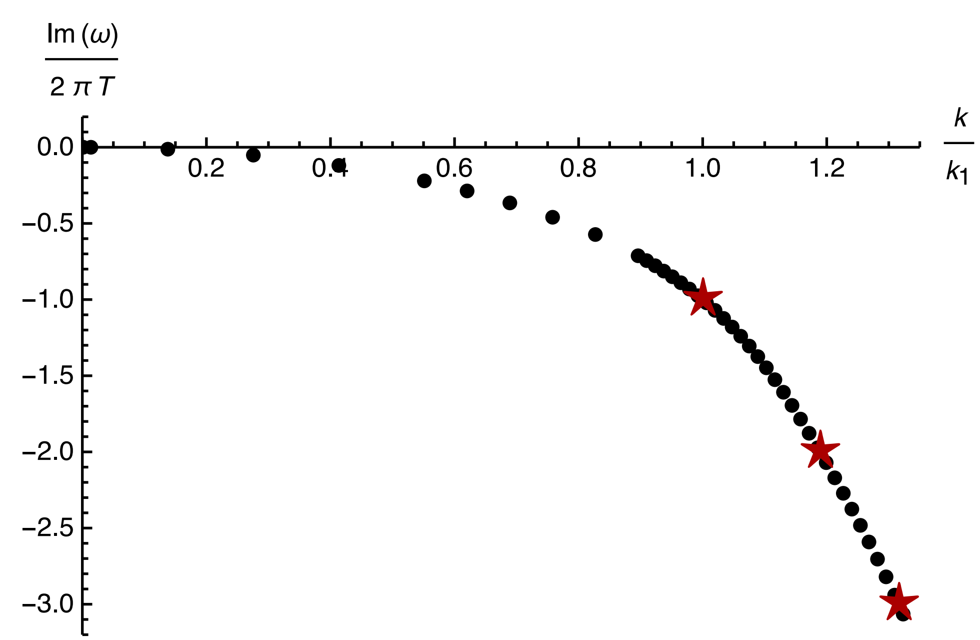
  ⟶ a part of the spectrum that is independent of the rest of the spacetime

- A more thorough analysis yields infinitely many constraints of this kind

  e.g. a collective mode of the momentum density operator (Schwarzschild-AdS$_4$)

  ● exact (numerical) dispersion relation $\omega(k)$

  ★ near-horizon constraints



- This argument can be generalized to any type of field.

e.g. Ceplak, Ramdial, Vegh

CONSTRAINTS ON ENERGY DENSITY MODES

- For $G_{\varepsilon\varepsilon}(\omega, k)$, the equations are seemingly more complicated.

  δg_{vv} couples to other metric perturbations and to matter field perturbations

  e.g. $$\left(-i\omega d h'(r_0) + 2k^2\right)\delta g_{vv}(r_0) - 2\left(2\pi T + i\omega\right)\left(\omega\delta g_{x^i x^i}(r_0) + 2k\delta g_{vx}(r_0)\right) = 4h(r_0)\left(\delta T_{vv}(r_0) - T_{vr}(r_0)\delta g_{vv}(r_0)\right)$$

- But the constraint is very simple, and **independent of matter field profiles**

$$\omega(k_*) = +\, i2\pi T \qquad k_*^2 = -\, d\pi T h'(r_0)$$

$\longrightarrow$

$$\omega(k_*) = +\, i\tau_L^{-1} \qquad k_*^2 = -\, (v_B\tau_L)^{-2}$$

- Robust to some further generalizations

  e.g. higher-derivative gravity, magnetic fields & anomalies

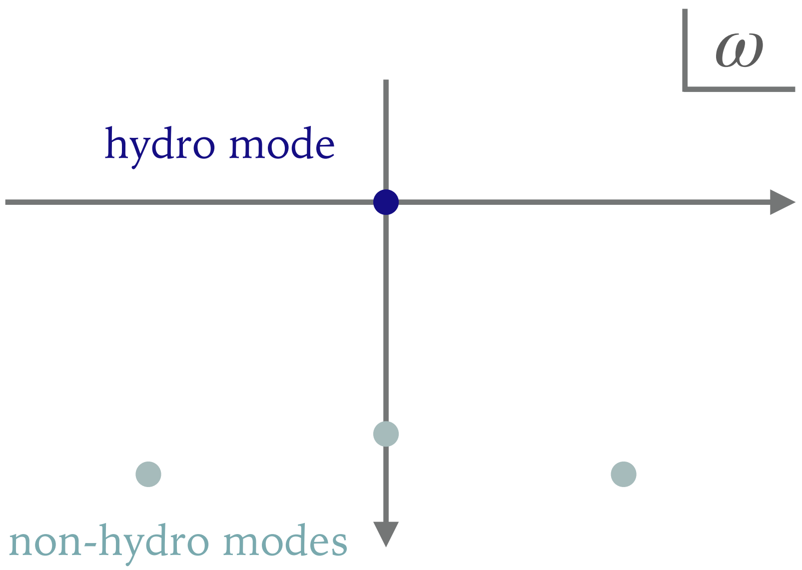
Grozdanov ;
Abbasi, Tabatabaei

RELATION TO DIFFUSIVITY

- It is reasonable to expect one of the hydro modes to obey the universal constraint.

  In a few cases, this has been verified.

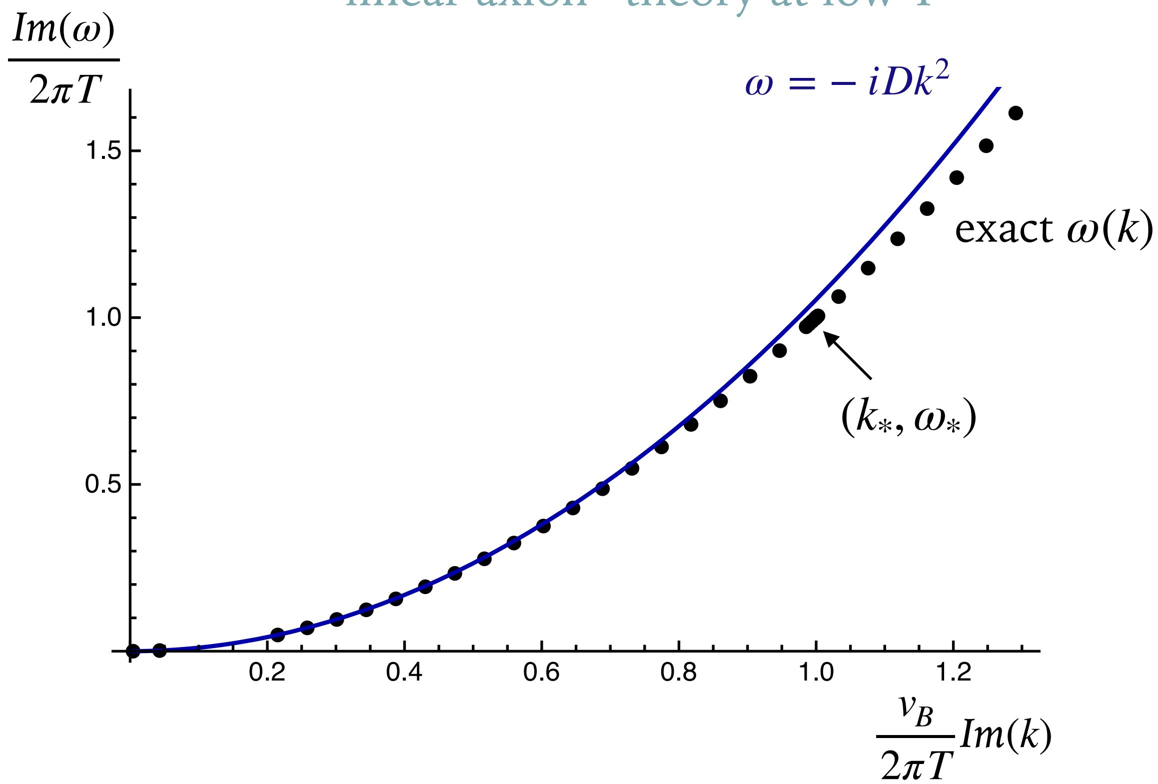
  Grozdanov, Schalm, Scopelliti ;
  Blake, RD, Grozdanov, Liu



- In some cases, $\omega_{hydro}(k) \approx = -iDk^2$ is an excellent approximation up to $k = k_*$

$$\longrightarrow \qquad D \approx v_B^2 \tau_L$$

"linear axion" theory at low T



THERMAL DIFFUSIVITY

- dc conductivities are sensitive only to the near-horizon part of the spacetime

  Thermal conductivity κ is independent of the matter field profiles

$$\kappa \equiv \kappa - \frac{\alpha^2 T}{\sigma} = 4\pi \left. \frac{f'(r)h(r)^{d-2}}{\frac{d}{dr}\left(f'(r)h(r)^{d/2-1}\right)} \right|_{r=r_0}$$

Blake, RD, Sachdev

- At low T (near IR fixed point), heat capacity c is set by the horizon area.

  Also independent of matter field profiles.

- In this limit, there is a collective mode of energy density with diffusivity

$$D = \frac{\kappa}{c}$$

LOW TEMPERATURE THERMAL DIFFUSIVITY

- Quantitative relations between diffusive transport and chaos
  - For a large class of theories with $AdS_2 x R^d$ IR fixed points

as $T \to 0$

$$D = v_B^2 \tau_L$$

Blake, Donos

  - Generic IR fixed point has symmetry $t \to \Lambda^z t, \quad \underline{x} \to \Lambda \underline{x}$

as $T \to 0$

$$D = \frac{z}{2(z-1)} v_B^2 \tau_L$$

Blake, RD, Sachdev

  - When $z = 1$, diffusive approximation breaks down at $\omega \ll \tau_L^{-1}$.

RD, Gentle, Goutéraux

SUMMARY

- In some QFTs, there are connections between transport properties and underlying chaotic dynamics.

- Near-horizon dynamics yield exact constraints on the dispersion relations of collective modes.

  There is a universal constraint for collective modes of energy density

$$\omega(k_*) = + i\tau_L^{-1} \qquad\qquad k_*^2 = -\,(v_B\tau_L)^{-2}$$

- At low temperatures, there is a diffusive mode carrying energy with diffusivity

$$D \sim v_B^2\tau_L$$

OPEN QUESTIONS

- Hydrodynamics and chaos
  - ✳ Effective action of holographic theories
  - ✳ Generalisations outside holography

    e.g. Gu, Qi, Stanford ; Patel, Sachdev ; Gu, Lucas, Qi ; Grozdanov, Schalm, Scopelliti ; …
  - ✳ Regime of validity of (diffusive) hydro in holographic theories?

    e.g. Hartman, Hartnoll, Mahajan ; Lucas ; Withers ; Grozdanov et al ; ….
- Exact "pole-skipping" constraints from near-horizon dynamics
  - ✳ Necessary conditions in a QFT?
  - ✳ Robustness of universal constraint at $\omega = +i2\pi T$
  - ✳ Constraints on other transport coefficients?

THANK YOU!