Transport in (clean) quantum critical superfluids

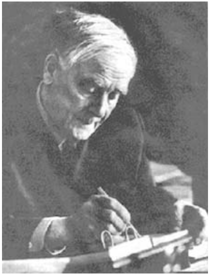
1912.08849, 2005.xxxxx (w/ B. Goutéraux)

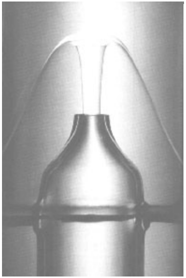
Eric Mefford (CPHT, CNRS, École Polytechnique, IP Paris)

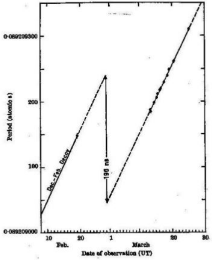
May 5, 2020

[Horizon 2020: grant agreement 758759]

Experimental knowledge of superfluidity



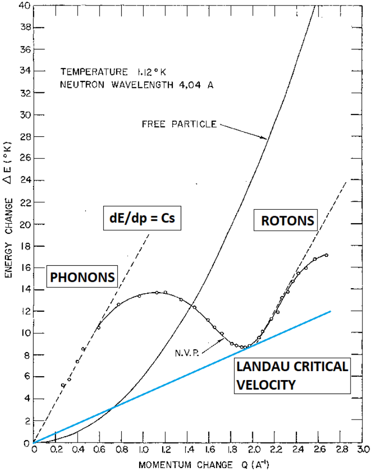




- ^{4}He discovered in 1937 by Kapitsa and by Allen and Misener (26y after superconductivity)
- ^{3}He discovered in 1972 (spin 1/2)
- These are the *only* observations to date
- Predicted in nuclear matter (pulsar glitches)

[Kapitsa], [Radhakrishnan and Manchester],[Ridley and Downs]

low T degrees of freedom



[Henshaw and Woods '60]

- order parameter $\eta = \rho \exp[i\varphi]$
- $\langle\rho\rangle_0 = 0$: unbroken $U(1)$
- $\langle\rho\rangle_0 \neq 0$: SSB $U(1)$ symmetry
- φ is the gapless Goldstone boson (*phonon* of ^{4}He)

Two fluid hydrodynamics (vortex free)

Conservation laws:

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} j_\mu$$
$$\partial_\mu j^\mu = 0$$

Constitutive relations:

$$j^\mu = \rho_n u^\mu + \rho_s v^\mu$$
$$T^{\mu\nu} = (\epsilon_n + P) u^\mu u^\nu + P\eta^{\mu\nu} + \mu\rho_s v^\mu v^\nu$$

Thermodynamic relations:

$$\epsilon_n + P = sT + \mu\rho_n, \qquad \epsilon + P = sT + \mu\rho$$
$$\partial_\mu(su^\mu) = 0, \qquad \rho = \rho_n + \rho_s$$

[Landau], [Tisza], [Carter], [Khalatnikov], [Herzog, Kovtun, Son]

Thermodynamics of φ

- Conserved $U(1)$ charge density ρ: $p[T,\mu] = Ts - \epsilon + \mu\rho$
- $\varphi \sim \varphi + 2\pi \Rightarrow \oint_{C_i} \nabla\varphi \cdot dx = \alpha \Rightarrow p[T,\mu,(\nabla\varphi)^2]$
- For $F_{\mu\nu} = 0$, treat $\mu = u^\nu A_\nu$ and $\xi_\mu = D_\mu\varphi = \partial_\mu\varphi - A_\mu$.

Two new relations:

Josephson relation $\quad u^\mu \xi_\mu = -\mu$

Gibbs relation $\quad dP = sdT + \rho d\mu - \frac{\rho_s}{2\mu} d\left(\xi_\mu\xi^\mu + \mu^2\right)$

- To match hydro: $v^\mu = \frac{1}{\mu}\xi^\mu$.

[Landau], [Tisza], [Carter], [Khalatnikov], [Herzog, Kovtun, Son]

Fluctuations

Consider the thermodynamic equilibrium state with no relative velocity, i.e. let $\xi_i = 0$. Work in fluid rest frame.

$$\delta T^{00} = \delta\epsilon \qquad \delta j^0 = \delta\rho$$

$$\delta T^{0i} = (\mu\rho_n + sT)\delta u^i + \rho_s \delta\xi^i \qquad \delta j^i = \rho_n \delta u^i + \frac{\rho_s}{\mu}\delta\xi^i$$

$$\delta T^{ij} = \delta P\, \delta^{ij} \qquad \delta\xi_0 = -\delta\mu$$

The hydro equations imply

$$-\rho\delta F_{i0} = (\mu\rho_n + sT)\partial_0\delta u_i + \rho_s\partial_0\delta\xi_i + s\partial_i\delta T + (\rho_n + \rho_s)\partial_i\delta\mu$$

So that as $T \to 0$,

$$\Rightarrow \boxed{\rho_n^{(0)}\partial_0(\mu\delta u_i - \delta\xi_i) = 0}$$

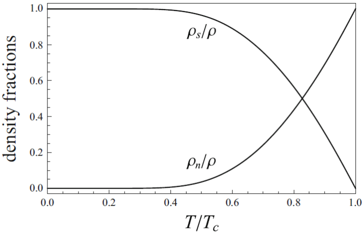
[Leggett]

Back to ^{4}He

$$f(\epsilon_p) = [1 - e^{c_s p/T}]^{-1}$$

$$P = T \int \frac{d^3p}{(2\pi)^3} \ln f(\epsilon_p) = \frac{\pi^2 T^4}{90 c_s^3}$$

$$s = dP/dT = \frac{2\pi^2 T^3}{45 c_s^3}$$



In the rest frame of the superfluid, any momentum is from $\rho_n(\vec{v}_n - \vec{v}_s)$. If this comes from fluctuations,

$$\rho_n = \lim_{\delta\vec{v}\to 0} \frac{1}{|\delta\vec{v}|^2} \int \frac{d^3p}{(2\pi)^3} (\vec{p}\cdot\delta\vec{v})^2 \frac{\partial f}{\partial_{\epsilon_p}}|_{\epsilon_p = c_s p} = \frac{2\pi^2 T^4}{45 c_s^5} = \boxed{\frac{sT}{c_s^2}}$$

[Schmitt]

Other examples: $\rho_n = \frac{sT}{\mu}\frac{1-c_s^2}{c_s^2}$

$$\mathcal{L} = |\partial\eta|^2 - m^2|\eta|^2 - \lambda|\eta|^4$$

$$c_s^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2}$$

$$P = \frac{(\mu^2 - m^2)^2}{4\lambda} + \frac{\pi^2 T^4}{90 c_s^3}\left[1 + O(\nabla\varphi)^2\right]$$

$$s = \frac{2\pi^2 T^3}{45 c_s^3}\left[1 + O(\nabla\varphi)^2\right]$$

$$\rho_n = \frac{\mu\pi^2 T^4}{45 c_s^5}\frac{12\mu^2 - m^2}{(3\mu^2 - m^2)^2}$$

[Schmitt]

$$\mathcal{L}_{eff} = P(\sqrt{D_\mu\varphi D^\mu\varphi})$$

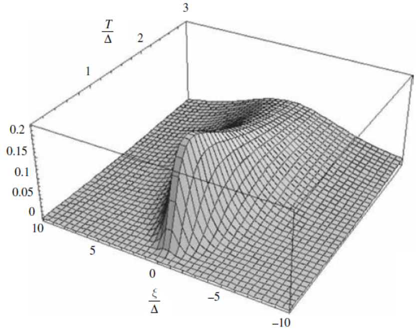
$$P(\mu) \sim \mu^{d+1}$$

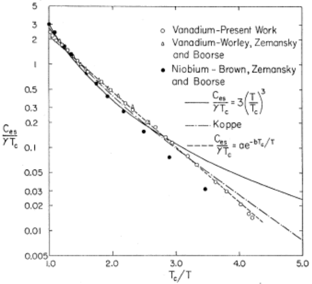
$$\rho_n \sim \frac{sT}{\mu}\frac{1 - c_s^2}{c_s^2}$$

[Delacrétaz, Hofman, Mathys],

[Son]

Other examples: BCS

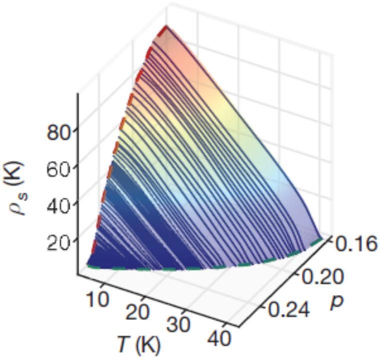


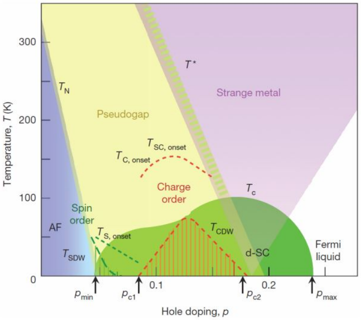


$$\rho_n \sim sT$$

[Altland & Simons], [Tinkham], [Corak et al.], [BCS], [Hinken]

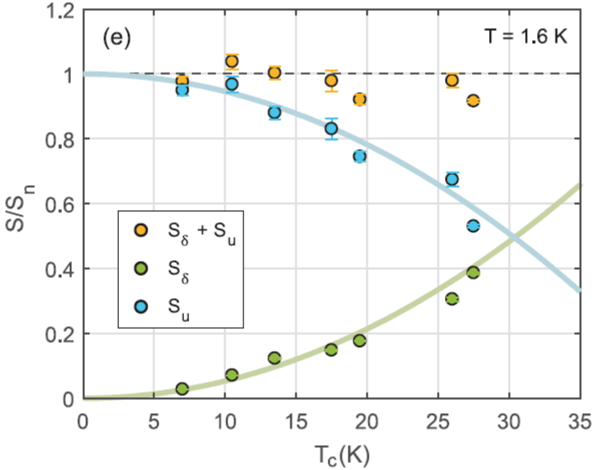
Is $\lim_{T\to 0} \rho_n = 0$ always?





[Bozovic et al. '16]: very overdoped $La_{2-x}Sr_xCuO_4$ has anomalously small $\rho_s(T=0)$ and $\rho_s \sim \rho_s^{(0)} + \# T$

Is $\lim_{T\to 0} \rho_n = 0$ always?



[Mahmood et al. '18]: ac conductivity measurements on very overdoped $La_{2-x}Sr_xCuO_4$ show $\rho_n(T=0) \neq 0$

Hydro: consistent sources and linear response

$$\boxed{\rho_n^{(0)}\partial_0(\mu\delta u_i - \delta\xi_i) = 0}$$

If $\rho_n^{(0)} \neq 0$, then fluctuations in u^i and ξ^i are not independent!

$$\delta T^{0i} = (\mu\rho_n + sT)\delta u^i + \rho_s\delta\xi^i \qquad (1)$$

$$\delta\xi^i = \mu(\delta u^i + [\frac{\delta\xi^i}{\mu} - \delta u^i]) \qquad (2)$$

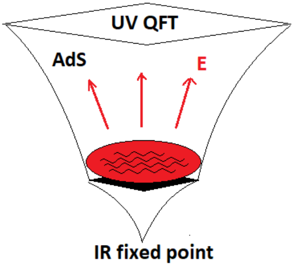
$$\chi_{P^iP^i} = h \Rightarrow s_{\xi^i} = [\frac{\xi^i}{\mu} - u^i] \qquad (3)$$

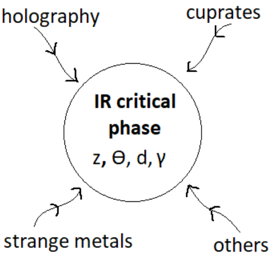
$$\chi_{P^i\xi^i} = \chi_{\xi^iP^i} = \mu \quad \text{Onsager relations} \qquad (4)$$

$$s_{\xi^i} = 0 \Rightarrow \boxed{\delta\xi^i - \mu\delta u^i = 0}$$

Holography

- hydrodynamic regime of certain strongly coupled QFT_{d+1} dual to near-horizon physics of black holes in AdS_{d+2} [Maldacena], [Bhattacharyya, Hubeny, Minwalla, Rangamani], [Horowitz, Rangamani], ...
- The radial direction parametrizes an RG flow [Susskind and Witten; Balasubramanian and Krauss; de Boer, Verlinde and Verlinde;...]
- Often the IR fixed point is quantum critical [Sachdev,...]
- The hydrodynamic regime of quantum critical systems are often universal [Sachdev]

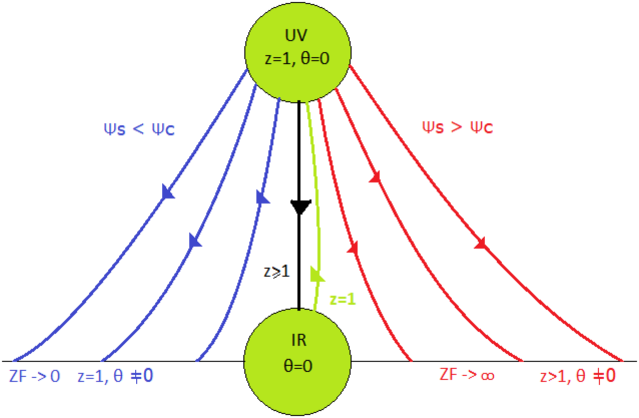




A generic holographic superconductor: quartic V

[Gubser], [Hartnoll, Herzog, Horowitz], [Gubser and Nellore], [Horowitz and Roberts], [Adams, Crampton, Sonner, Withers],

$$S = \int d^{d+2}x\sqrt{-g}\left[R - \frac{Z_F(\psi)}{4}F^2 - |D\eta|^2 - \frac{1}{2}(\partial\psi)^2 - V(\psi,|\eta|)\right]$$



Calculation of ρ_n

- Ansatz: $ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}_d^2$, $A = A_t(r)dt$, $\eta = \eta^* = \eta(r)$, $\psi = \psi(r)$
- Perturb $\delta a_x = a_x(r)e^{-i\omega t}$, $\quad \delta g_{tx} = g_{tx}(r)e^{-i\omega t}$
- $\sigma(\omega) = \frac{1}{i\omega}\frac{a_x^{(1)}}{a_x^{(0)}} = \frac{i}{\omega}\left[\frac{\rho_n^2}{\mu\rho_n + sT} + \frac{\rho - \rho_n}{\mu}\right]$ so can solve the $\omega \to 0$ equation to find ρ_n.

$$\frac{1}{C^{d/2-1}}\frac{d}{dr}\left[C^{d/2-1}Z_F\sqrt{\frac{D}{B}}a_x'\right] - \sqrt{\frac{B}{D}}\left(2q^2D\eta^2 + \frac{Z_F^2(A_t')^2}{B}\right)a_x = 0$$

$$\frac{d}{dr}\left[\frac{g_{tx}}{C}\right] + \frac{Z_F}{C}a_xA_t' = 0 \qquad \Rightarrow \qquad \boxed{\delta u^x - \frac{\xi^x}{\mu} = 0}$$

Calculation of ρ_n

Using $\sqrt{\frac{C^d}{BD}}\left[C\left(\frac{D}{C}\right)' - Z_F A_t A_t'\right] = -sT$
We find

$$\frac{d}{dr}\left[C^{d/2-1}\sqrt{\frac{D}{B}}Z_F A_t^2\left(1+\frac{sT}{A_t R}\right)\left(\frac{a_x}{A_t}\right)' + sT\frac{D}{C}\left(\frac{a_x}{A_t}\right)\right] = -(sT)\frac{2q^2\eta^2 C^{d-1} Z_F A_t^2}{R^2}\left(\frac{a_x}{A_t}\right)'$$

- $R(r)$ is the electric flux. At the UV boundary, $R \to \rho$ and at the horizon $R \to \rho_{in}$.
- When $\eta = 0$, $\frac{a_x^{(1)}}{a_x^{(0)}} = \frac{\rho^2}{sT+\mu\rho} \sim \frac{\rho}{\mu}\sum_{n=0}^{\infty}\left(\frac{-sT}{\mu\rho}\right)^n$ [Davison, Goutéraux, Hartnoll]
- When $\eta \neq 0$, a_x can be solved perturbatively in sT.

Analytic solution when $\eta \neq 0$

$$\frac{d}{dr}\left[C^{d/2-1}\sqrt{\frac{D}{B}}Z_F A_t^2\left(1+\frac{sT}{A_t R}\right)(\mathcal{A})' + sT\frac{D}{C}(\mathcal{A})\right] = -(sT)\frac{2q^2\eta^2 C^{d-1}Z_F A_t^2}{R^2}(\mathcal{A})'$$

$$\mathcal{A} = \frac{\mu}{a_x^{(0)}}\frac{a_x}{A_t} = \mathcal{A}_0 + (sT)\mathcal{A}_1 + (sT)^2\mathcal{A}_2 + \dots$$

$$\mathcal{A}_0 = 1$$

$$\mathcal{A}_1 = -\int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C} + c_1\right]dr'$$

$$\begin{aligned}\mathcal{A}_2 = &\int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C} + c_2\right]dr' \int_0^{r'} \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C} + c_1\right]d\tilde{r} \\ &+ \int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^3 R}\left[\frac{D}{C} + c_1\right]dr' \\ &+ \int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}dr' \int_{r_h}^{r'} \sqrt{\frac{B}{D}}\frac{2q^2\eta^2 C^{d/2}}{R^2}\left[\frac{D}{C} + c_1\right]d\tilde{r}\end{aligned}$$

What is c_1?

- $\lim_{r\to r_h} A_t \mathcal{A}_i = \text{constant} + c_i(\text{divergent})$ (σ_Q [Davison, Goutéraux])
- Divergences in $\mathcal{A}_{i-1}$ can be cancelled by c_i, i.e. $c_i \propto c_1$

$$\frac{d}{dr}\mathcal{A}|_{r\to 0} = \frac{\mu}{a_x^{(0)}}\frac{\mu a_x^{(1)} - \rho a_x^{(0)}}{\mu^2} = \lim_{\omega\to 0}\omega\ Im[\sigma(\omega)] - \frac{\rho}{\mu}$$

$$= -\frac{sT}{\mu^2}[1+c_1] + \frac{(sT)^2}{\mu^3\rho}[1+c_1] - \frac{(sT)^2}{\mu^2}I[c_1] + \dots$$

$$I[c_1] = \int_{r_h}^{0} dr \frac{B}{\sqrt{g}}\frac{2q^2\eta^2 D}{Z_F^2(A_t')^2/B}[\frac{D}{C} + c_1] + \dots$$

- Competing $U(1)$'s: $\lim_{r_h\to\infty} I \to \infty,\ c_1 = -\frac{D_0}{C_0}(r_h) = -c_{ir}^2$ or $\lim_{r_h\to\infty} I \to \text{constant},\ c_1 = 0$

$\psi = 0$

When $\psi = 0$, the flux vanishes in the IR. All charge is in the condensate. The precise IR phase depends on the relevance of A_t. [Gubser and Nellore, Horowitz and Roberts]

$$ds^2 = -L_t^2 \left(\frac{L}{r}\right)^{2z} f dt^2 + \tilde{L}^2 \frac{dr^2}{r^2 f} + \left(\frac{L}{r}\right)^2 d\vec{x}_d^2, \quad f(r) = 1 - \left(\frac{r}{r_h}\right)^{d+z}$$

$z = 1$

- emergent scale invariance in IR
- finite $c_{ir} = \frac{L_t}{L_x}$
- $s \sim T^d$
- $A_t \sim c_A L_t \left(\frac{r}{L}\right)^{\tilde{\Delta}_{A_0}(q,\eta)-1}$

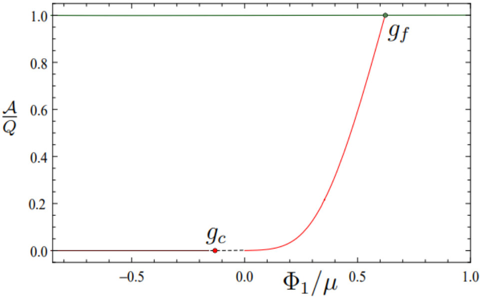
$z > 1$

- Lifshitz symmetry $[t] = z, [x] = 1$
- $c_{ir} = \frac{L_t}{L_x} r_h^{z-1} \sim T^{1-1/z}$
- $s \sim T^{d/z}$
- $A_t \sim L_t \sqrt{2 - 2/z} \left(\frac{r}{L}\right)^{-z}$

$\psi \neq 0$

When $\psi \neq 0$, the flux can be constant in the IR ($Z_F \to \infty$) or vanish ($Z_F \to 0$) [Adams, Crampton, Sonner, Withers],[Charmousis], [Goutéraux and Kiritsis], [Huijse, Sachdev, and Swingle]

$$ds^2 = \left(\frac{r}{L}\right)^{2\frac{\theta}{d}} \left[-L_t^2 \left(\frac{L}{r}\right)^{2z} f dt^2 + \tilde{L}^2 \frac{dr^2}{r^2 f} + \left(\frac{L}{r}\right)^2 d\vec{x}_d^2 \right]$$



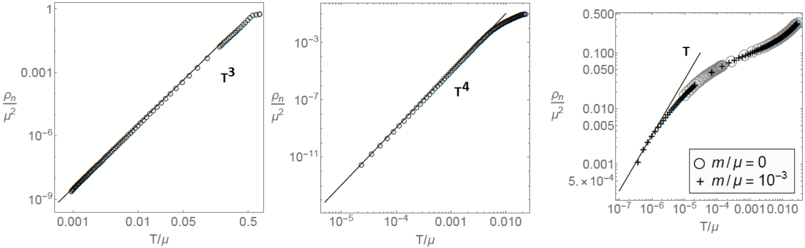
Notably, η is irrelevant. A_t is irrelevant for $Z_F \to 0$ giving $z = 1$.

$z < d + 2 - \theta$

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{\rho}{\mu} - \frac{sT\rho_n}{\mu(\mu\rho_n + sT)}\right]$$

$$\lim_{T\to 0} I \sim \lim_{T\to 0} T^{1-\frac{d+2-\theta}{z}} \to \infty \Rightarrow c_1 = -c_{ir}^2$$

$$\lim_{\omega\to 0} \omega\, Im[\sigma(\omega)] = \frac{\rho}{\mu} - \frac{sT}{\mu^2}(1 - c_{ir}^2) + \ldots \Rightarrow \boxed{\rho_n = sT\frac{1-c_{ir}^2}{c_{ir}^2}} \sim T^{\frac{d+2-\theta}{z}-1}$$



z = 1, Θ = 0

z = 1, Θ = -1

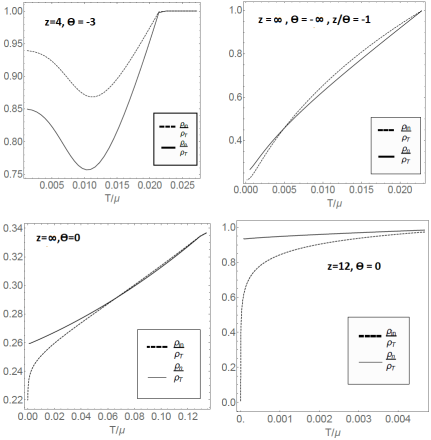
z=2, Θ=0

$$z > d + 2 - \theta$$

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{\rho}{\mu} - \frac{sT\rho_n}{\mu(\mu\rho_n + sT)}\right]$$

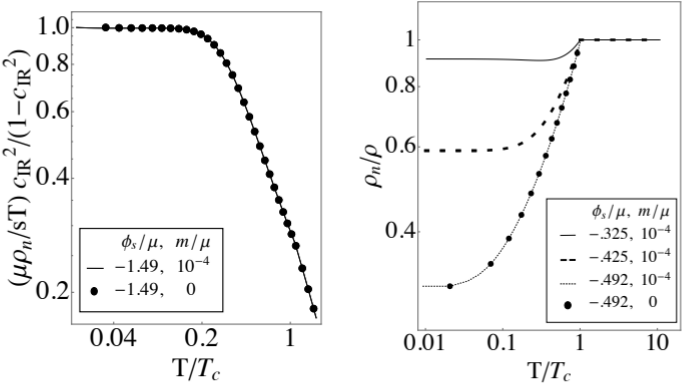
$$\lim_{T\to 0} I \sim \text{constant} \Rightarrow c_1 = 0$$

$$\lim_{\omega\to 0} \omega\, Im[\sigma(\omega)] = \frac{\rho}{\mu} - \frac{sT}{\mu^2} + \frac{(sT)^2}{\mu^3\rho_n^{(0)}} + \ldots \Rightarrow \rho_n = \rho_n^{(0)} + \ldots$$

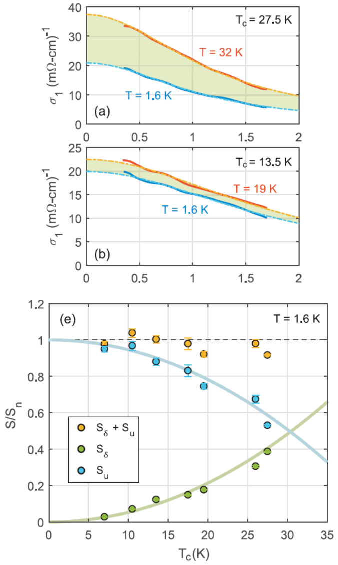


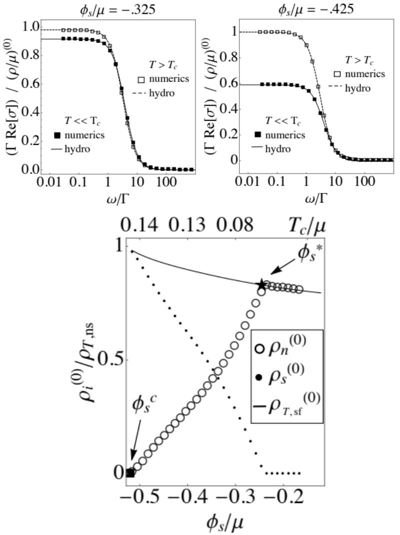
relevance to cuprates

- superfluidity on top of quantum critical groundstate $\Rightarrow \rho_n^{(0)}$
- For $\psi \neq 0$ $\partial_r \psi|_{r=0}$ acts like doping
- Real systems are impure. Break translations via axions [Andrade and Withers] $\Rightarrow \sigma(\omega) = \frac{\rho_n^2}{\mu\rho_n + sT}\frac{1}{\Gamma - i\omega} + \frac{\rho_s}{\mu}\frac{i}{\omega}$

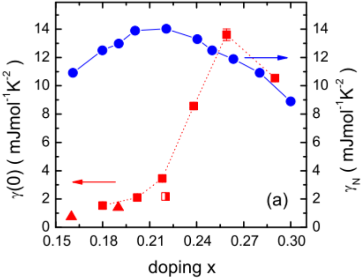


relevance to cuprates

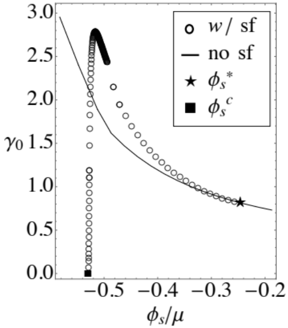




$s \sim \frac{\gamma_0}{2} T^2$



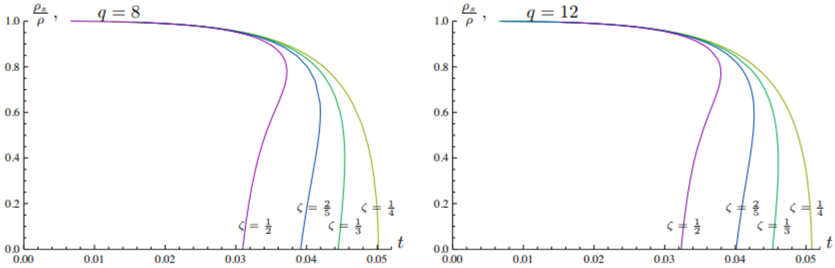
[Wen et al.], [Wang et al.]



$\gamma_0 \to 0$ signals rapid depletion of normal charge carriers/horizon flux to the condensate (QCP)

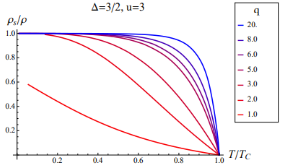
other holographic results: quadratic potential

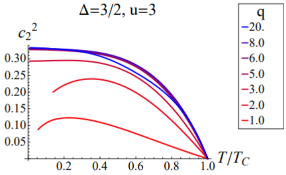
- HHH: $\mathcal{L} = R - 2\Lambda - \frac{1}{4}F^2 - |D\eta|^2 + 2|\psi|^2$
- Horowitz and Roberts showed
  $ds^2 = -L_t^2 \left(\frac{L}{r}\right)^2 dt^2 + \tilde{L}^2 \frac{dr^2}{r^2(\ln[r/L])} + \left(\frac{L}{r}\right)^2 [L_x^2 dx^2 + L_y^2 dy^2]$
- $\lim_{T\to 0} l \to \infty$ and $\lim_{T\to 0} c_{ir} \neq 0$ so $\rho_n = sT\frac{1-c_{ir}^2}{c_{ir}^2}$
- Sonner and Withers showed this model obeys two-fluid hydro

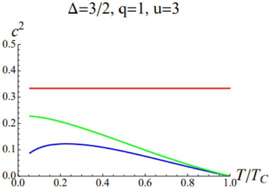


other holographic results: sound modes [Herzog and Yarom]

- $c_2^2 = \left(\frac{s}{\rho}\right)^2 \frac{\rho_s}{(sT+\mu\rho_n)(\partial[s/\rho]/\partial T)_\mu} \sim \begin{cases} \frac{z}{d} c_{ir}^2 \sim T^{2-2/z} & \rho_n^{(0)} = 0 \\ sT & \rho_n^{(0)} \neq 0 \end{cases}$
- $c_4^2 = \frac{\rho_s}{\mu(\partial\rho/\partial\mu)_s} \sim \frac{1}{d}\left[1 - \frac{\rho_n}{\rho}\right]$







Summary and future directions

- holographic superfluidity is an irrelevant deformation of an underlying quantum critical groundstate
- ρ_n can be determined from properties of the groundstate
- When condensate effects are more relevant than finite charge effects, $\rho_n^{(0)} = 0$
- Subleading T dependence either from I or from sT
- while related, ρ_{in} does not determine ρ_n, analagous to BCS superconductors
- hydrodynamics predicts $D_\perp = \eta/(\mu\rho_n + sT)$.
- isotropic holographic theories give $4\pi T D_\perp = c_{ir}^2$ for $\rho_n^{(0)} = 0$
- for $\rho_n^{(0)} \neq 0$ we find $4\pi T D_\perp = sT/\mu\rho_n^{(0)}$
- (suprising?) a universal $4\pi T D_\perp \sim c_2^2$ [Hartnoll], [Blake]
- superfluid EFT: $\omega \sim k \Rightarrow$ finite $c_{ir} \Rightarrow \rho_n \sim T^{d+1}$. What about Lifshitz? ($\omega \sim k^z$)
- finite superfluid velocities?

Thank you!

Other choices of action

We can modify $|\mathcal{D}\eta|^2 \to M_F(\psi)|\mathcal{D}\eta|^2$ to make the condensate more or less relevant compared to particle-hole breaking. For good choices of M_F, η stays irrelevant so the IR does not change.