Hydrodynamic fluctuations

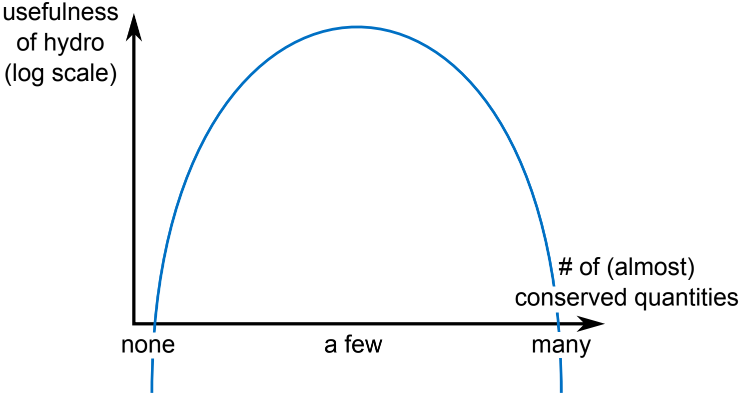
Luca Delacrétaz

University of Chicago

HoloMatter @IFT UAM Madrid

Universality of Hydrodynamics

Any thermalizing system, quantum or classical, is described by hydrodynamics at sufficiently late times



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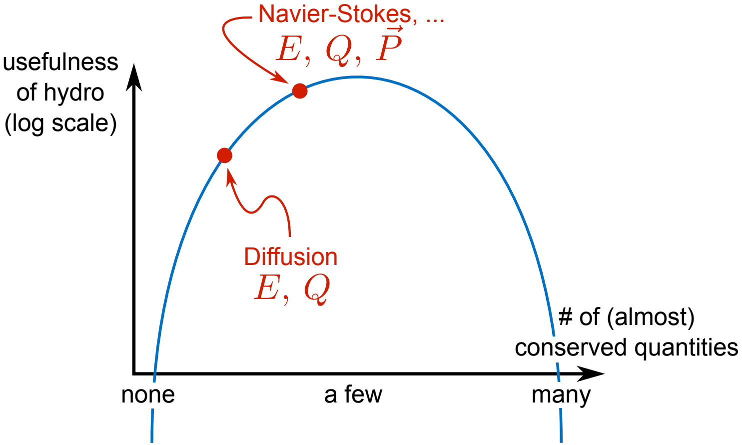

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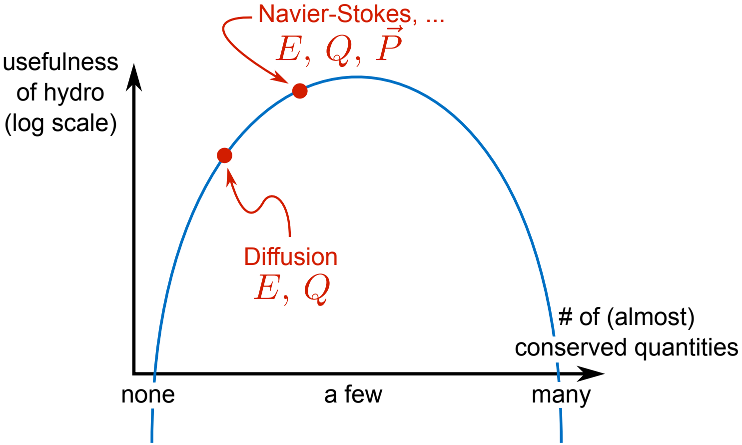

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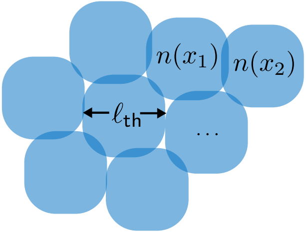
Most excitations 'relax' at finite $T \rightsquigarrow$ thermalization time τ_{th}

Conserved densities related to symmetries decay with rate $\Gamma \sim k^2$

For k small enough these are parametrically slower than generic excitations

Hydrodynamics is the late time $(t \gg \tau_{\mathrm{th}})$ description of these 'coarse grained' quantities



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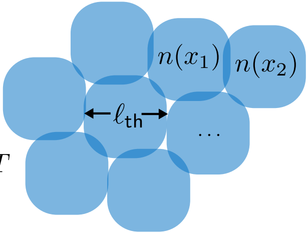


At weak coupling $(g \ll 1)$: $\tau_{\rm th} \sim \tau_{\rm Planck}/g^2$

At strong coupling expect: $\tau_{\rm th} \sim \tau_{\rm Planck} = \hbar/k_B T$

How it works

Theory of a conserved density n (or its potential μ), subject to

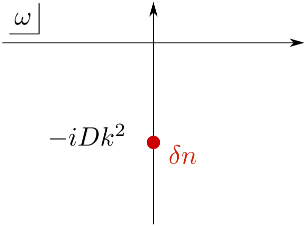
$$\dot{n} + \nabla \cdot j = 0$$

This also involves j. Close the equation with a constitutive relation

$$j_i = -D\partial_i n + \cdots$$

Solving these equations yields a diffusive Greens function

$$G^R_{nn}(\omega, k) = \frac{\chi D k^2}{-i\omega + Dk^2} + \cdots$$



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Two expansions: gradients $\partial + \partial^2 + \cdots$ and fluctuations $\delta n + \delta n^2 + \cdots$

Always controlled (in principle)

Controlled when interactions are *irrelevant*

How it works II

Theory of conserved densities T_{00}, T_{0i} or their potentials $\beta(x)$, $u_\mu(x)$, subject to $\partial_\mu T^{\mu\nu} = 0$.

This also involves the 'currents' T_{ij}

To close equations we need constitutive relations

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P\eta_{\mu\nu} - \zeta\eta_{\mu\nu}\partial \cdot u - \eta\partial_{(\mu}u_{\nu)} + O(\partial^2)\,,$$

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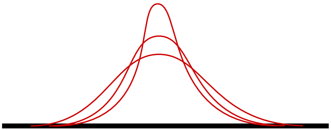
Solving around equilibrium $u_\mu(x) = \delta^0_\mu + \delta u_\mu$ and $\beta(x) = \beta + \delta\beta$ gives

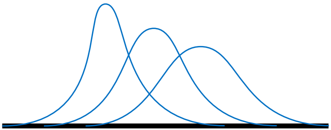
$$G^R_{T_{0i}T_{0j}}(\omega,k) \simeq \frac{s}{\beta}\left[\,\frac{k_ik_j}{k^2}\underbrace{\frac{\omega^2}{c_s^2k^2 - \omega^2 - i\Gamma k^2\omega}}_{\text{sound}} + \left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\underbrace{\frac{Dk^2}{-i\omega + Dk^2}}_{\text{diffusion}}\,\right]$$

Ballistic v. Diffusive

$$\omega = -iDk^2 + \cdots$$



$$\omega = ck - \frac{i}{2}\Gamma k^2 + \cdots$$



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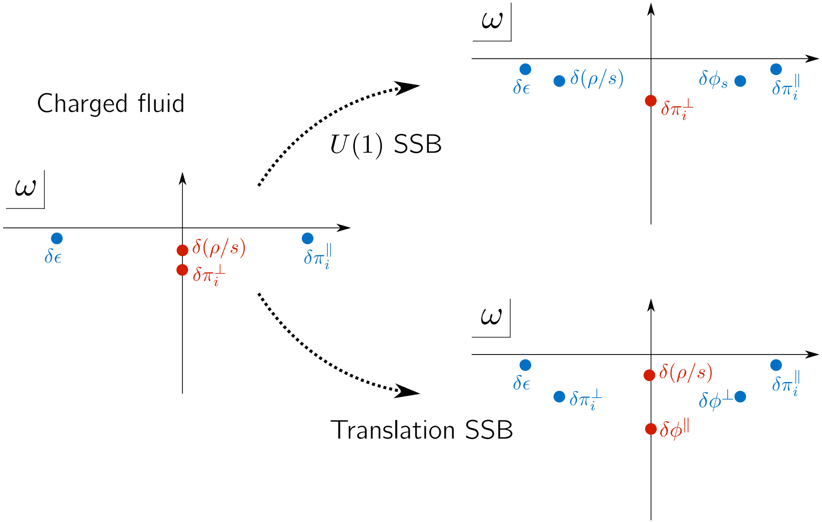

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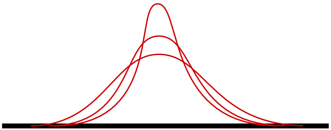

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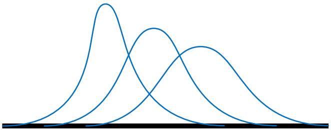

Other possibilities, e.g.:

- $\omega = ck \sin\theta - iDk^2$ (smectic, MHD)
- $\omega = \pm k^2 - ik^2$ (nematic)
- $\omega = \pm k^2 - ik^4$ (spin waves in a ferromagnet)
- $\ldots$

Ballistic v. Diffusive

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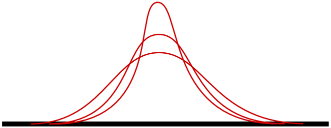

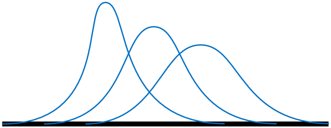

When are fluctuations big? Diffusive:

$$0 = \dot{n} + \nabla[D\nabla n] + \cdots$$

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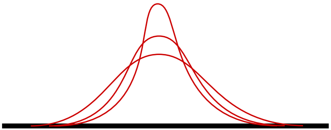

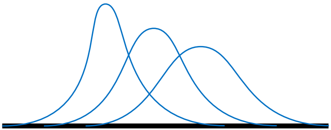

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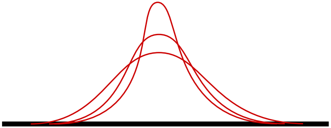

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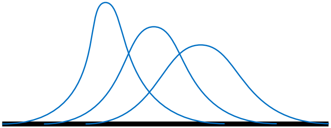

When are fluctuations big? Diffusive:

$$
\begin{aligned}
0 &= \dot{n} + \nabla[D(n)\nabla n] + \cdots \\
&= \left(\partial_t + D\nabla^2 + D'\delta n \nabla^2 + \cdots\right)\delta n
\end{aligned}
$$

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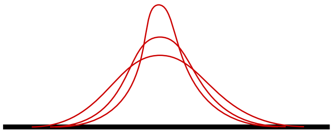

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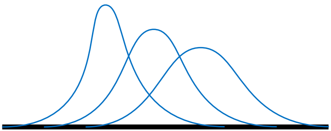

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We need to know how charge fluctuations scale: scaling $\omega \sim k^2$

$$\langle n(x,t) n\rangle \propto \frac{e^{-x^2/4Dt}}{t^{d/2}} \quad \Rightarrow \quad \delta n \sim \omega^{d/4} \sim k^{d/2}$$

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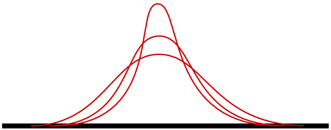

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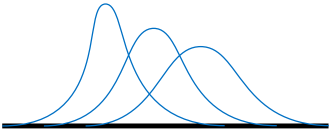

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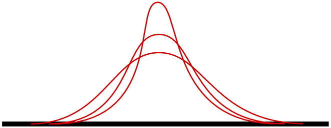

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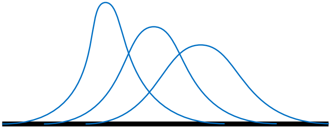

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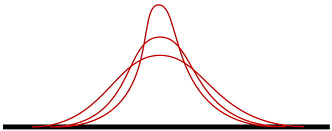

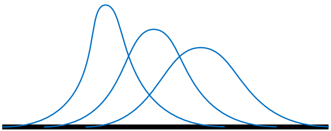

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$$0 = \dot{n} + c(n)\nabla n + D(n)\nabla^2 n + \cdots$$
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In the reference frame of the pulse $x' = x - ct$ we again scale $\omega' \sim k^2$

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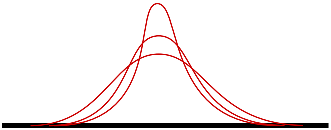

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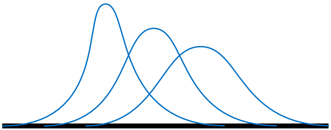

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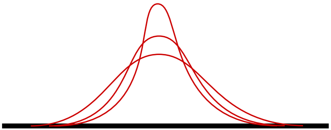

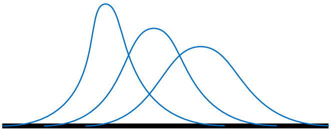

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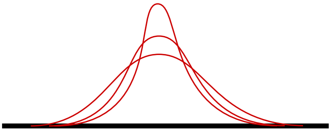

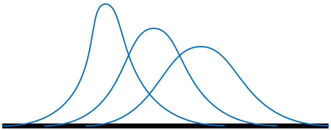

When are fluctuations big? Ballistic: when $d \leq d_c = 2$

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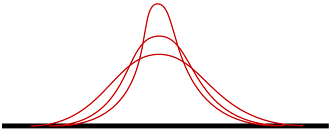

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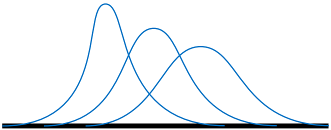

Bottomline:

	Diffusive modes	**Ballistic modes**
Weak fluctuations	$d > 0$	$d > 2$
Strong fluctuations	never	$d < 2$

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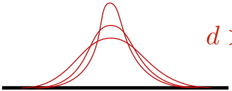
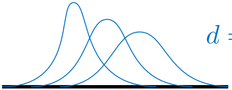


$d > 0$



$d = 1$

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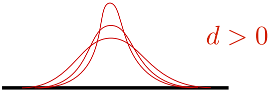
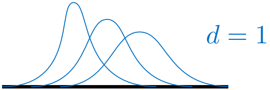
2. Weak fluctuations
3. Strong fluctuations





"Breakdown of diffusion on the edge"
2002.08365
with Paolo Glorioso

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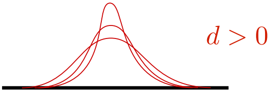
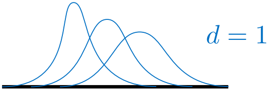
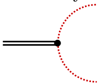




Irrelevant interactions

We found that diffusive fluctuations $\delta n \sim k^{d/2}$ are irrelevant in $d > 0$

$$j_i \quad = \quad D\partial_i n \quad + \quad D' n \partial_i n \quad + \quad \cdots$$

$=$  $+$  $+ \cdots$

Studied within theory of hydrodynamic fluctuations

Martin Siggia Rose ’73, Forster Nelson Stephen ’77

(long history: Zwanzig ’61 Mori ’65 Kawasaki ’68 Alder Wainwright ’70 Ernst Hauge van Leeuwen ’70 Pomeau Résibois ’75 ...)

$$j_i \, = \, D\partial_i n + D' n \partial_i n + \xi_i + \cdots \ , \qquad \langle \xi_i(x,t)\xi_j \rangle = 2\chi D T \delta^d(x)\delta(t)\delta_{ij}$$

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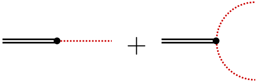
$=$  $+ \cdots$

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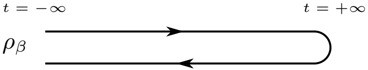
Modern approach: path integral on a Schwinger-Keldysh contour

Kamenev '11, Grozdanov Polonyi '13, Crossley Glorioso Liu '15, Haehl Loganayagam Rangamani '15, Jensen Pinzani-Fokeeva Yarom '17

Roughly: $n \sim \phi_{\text{top}} + \phi_{\text{bottom}}$

$\xi \sim \phi_{\text{top}} - \phi_{\text{bottom}}$



Irrelevant interactions

We found that diffusive fluctuations $\delta n \sim k^{d/2}$ are irrelevant in $d > 0$

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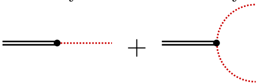
$=$  $+ \cdots$

For this talk, a simplified treatment of hydro fluctuations will be enough to illustrate concepts Ernst Hauge van Leeuwen '70, Kovtun Yaffe '03

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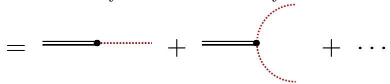

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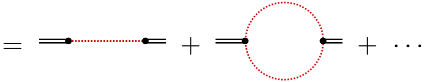
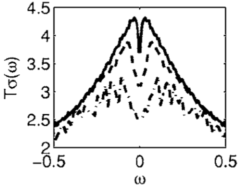
$$\sigma(\omega) = \frac{1}{2T}\langle jj\rangle(\omega, k=0) = \chi D + \#|\omega|^{d/2} + \#\omega + \cdots$$





Mukerjee Oganesyan Huse '05

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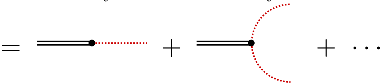

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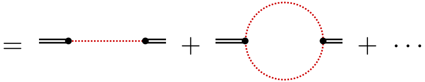
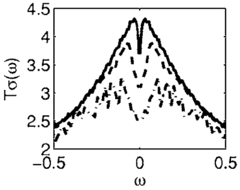




Mukerjee Oganesyan Huse '05

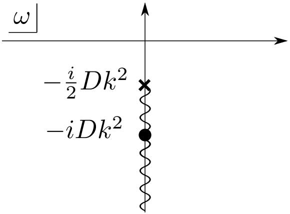
Long-time tails

- Discovered in molecular dynamics numerics Alder Wainwright '70
- Seen in holography Caron-Huot Saremi '09
- Recent interest for RHIC

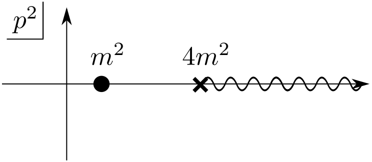
Analytic structure

These calculations can also be performed at finite k Chen-Lin LVD Hartnoll '18

$$G^R_{nn}(\omega,k) = \frac{\chi D k^2 + \cdots}{-i\omega + Dk^2 + \Sigma k^2}\,, \qquad \Sigma(\omega,k) = (\# i\omega + \# k^2)\left[k^2 - \frac{2i\omega}{D}\right]^{\frac{d-2}{2}}$$



Relativistic massive particle $G(p^2)$



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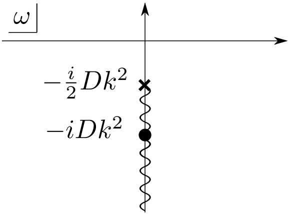

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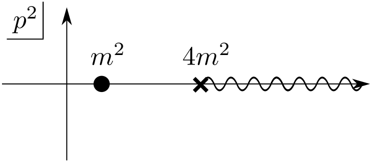
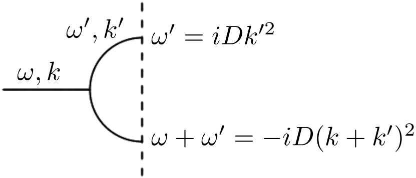


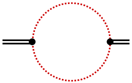
Two-'diffuson' threshold:



Real time explanation

$\langle \mathcal{O}\mathcal{O} \rangle(t,k) =$

$+$



$+ \quad \cdots$

$$\sim \quad g(t,k) \quad + \quad k^d g(t, \tfrac{k}{2})^2 \quad + \quad \cdots$$

$$\sim \quad e^{-Dk^2 t} \quad + \quad k^d e^{-Dk^2 t/2} \quad + \quad \cdots$$

For $\tau_{\mathrm{th}} \lesssim t \lesssim \dfrac{1}{Dk^2}$, the first term dominates

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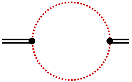
$\langle \mathcal{O}\mathcal{O} \rangle(t,k) =$  $+$  $+ \quad \cdots$

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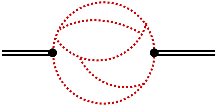
The n-diffuson contributions take the form

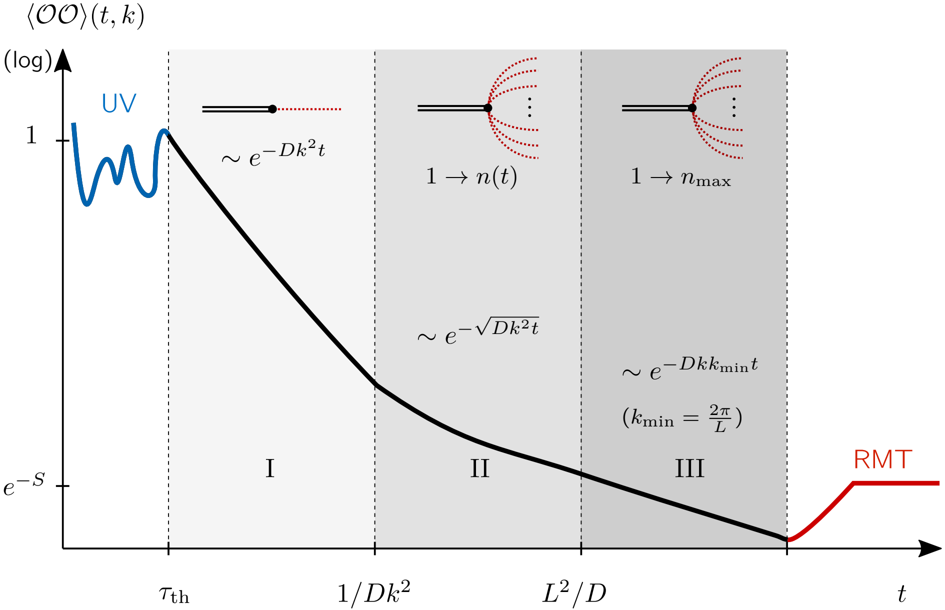
 $\sim \quad n!\,(k\ell_{\rm th})^{dn}\, e^{-Dk^2 t/n}$

At late times $\dfrac{1}{Dk^2} \lesssim t$, the term that dominates have $n(t) \simeq \sqrt{\dfrac{Dk^2 t}{d \log \frac{1}{k\ell_{\rm th}}}}$

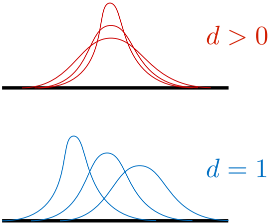
Plugging back gives $\langle \mathcal{O}\mathcal{O} \rangle(t,k) \sim e^{-\sqrt{Dk^2 t}}$! LVD, online soon

(Convergence? Borel summable? Heller Spalinski '15 Grozdanov Kovtun Starinets Tadić '19)

A richer story at finite k



Contents

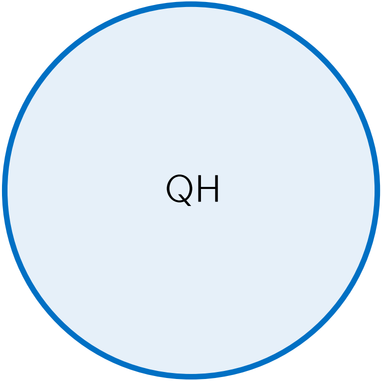


Proposal in a Nutshell

The edge of a QH droplet supports gapless excitations

Theorists like $T = 0$, but thermalization is crucial Polchinski Kane Fisher '94

What is the hydrodynamic description of the edge?



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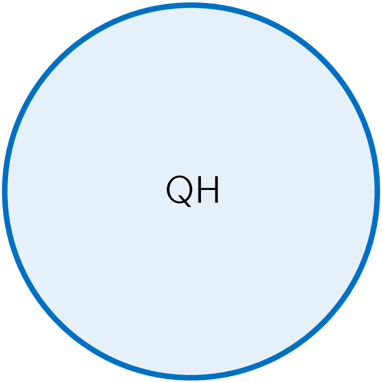
No translation invariance assumed, only charge conservation



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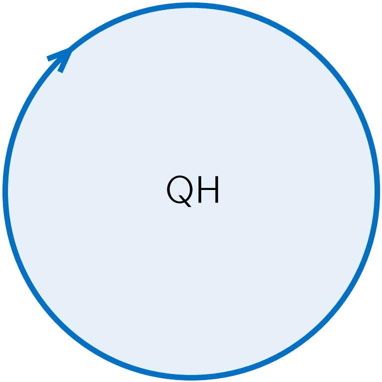
The collective excitation is a chiral ballistic mode

Kane Fisher '95

$$\omega = ck - iDk^2 + \cdots$$



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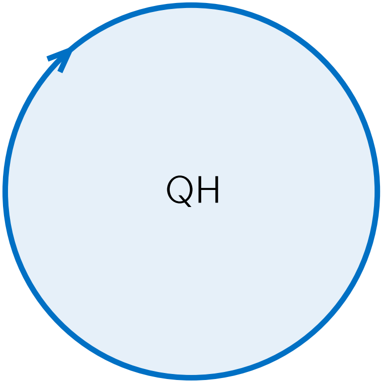

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Breakdown of diffusion drives system to KPZ universality class $z = 3/2$

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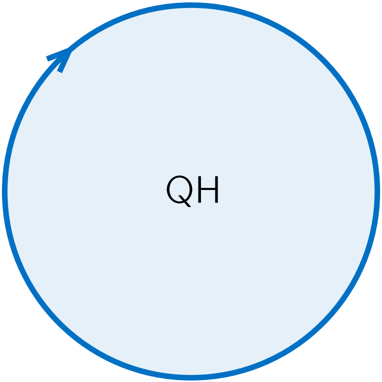
Universal prediction for low-frequency transport on the edge:

$$\sigma(\omega) \sim \frac{1}{\omega^{1/3}} \qquad (\omega \ll 1/\tau_{\text{th}})$$



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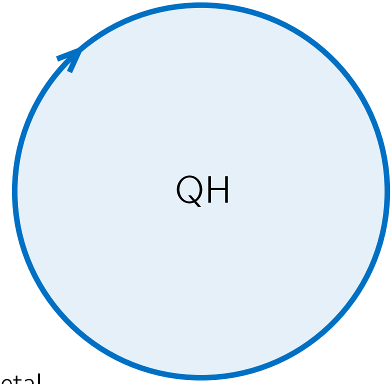
- Heat diffusion also breaks down
- Higher dimension: surface chiral metal



3 Key Aspects

- A hydrodynamic theory describes a condensed matter system, but no translation invariance (no long-lived momentum)

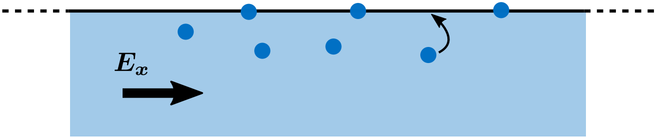
- Anomaly: $\dot{n} + \partial_x j_x \propto E_x$

- Large hydrodynamic fluctuations

The anomaly

Single conserved charge like before, but with an anomaly

$$\dot{n} + \partial_x j_x = \nu E_x$$



Account for anomalies in constitutive relations

Son Surowka '09

$$j_x = \nu\mu - \chi D \partial_x \mu + \cdots$$

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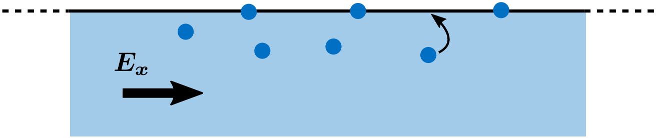

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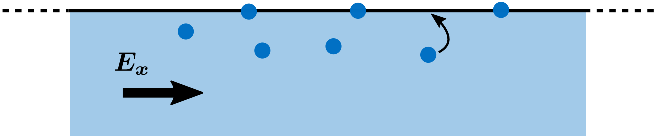

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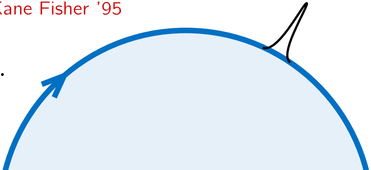

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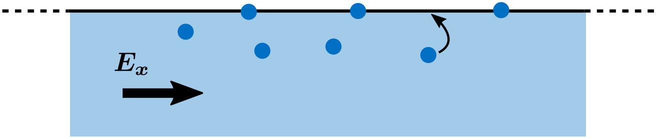

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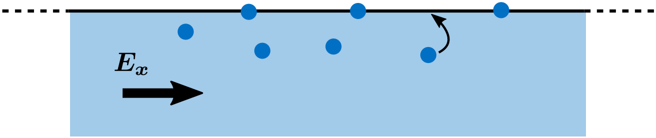

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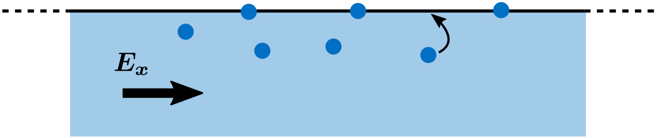

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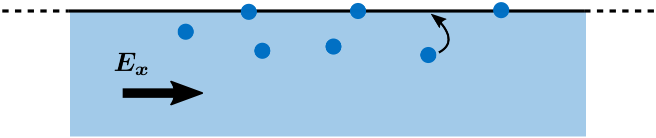

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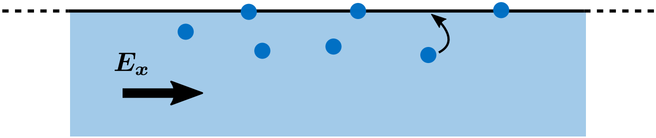

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Breakdown of diffusion

'Anomalous diffusion' equation: (recall $\delta n \sim k^{d/2} = k^{1/2}$)

$$0 = \dot{n} + c\partial_x n - \partial_x(D\partial_x n) + c' n \partial_x n + \cdots$$

$$\sim k^{3/2} \qquad \sim k^{5/2} \qquad \sim k^2$$

$\rightsquigarrow$ breakdown of diffusion!

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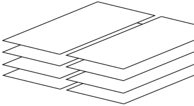
$\rightsquigarrow$ breakdown of diffusion!

What to do?

- Dim reg: expand from upper critical dimension $d_c = 2$.
  The theory at $d_c = 2$ describes chiral surface metals
- Exact solution for $d = 1$?



Burger's equation Forster Nelson Stephen '77, KPZ Kardar Parisi Zhang '86, 1d Navier-Stokes Narayan Ramaswamy '02

KPZ universality on the edge

'Anomalous diffusion' equation:

$$0 = \dot{n} + c\partial_x n - \partial_x(D\partial_x n) + c' n\partial_x n + \cdots$$

Follow chiral front: $x' = x - ct$, so $\partial_{t'} = \partial_t + c\partial_x$

$$0 = \partial_{t'} n - \partial_x(D\partial_x n) + c' n\partial_x n + \cdots$$

Map to KPZ equation $n \leftrightarrow \partial_x h$

Kardar Parisi Zhang '86

$$0 = \partial_{t'} h - D\partial_x^2 h + c'(\partial_x h)^2 + \cdots$$

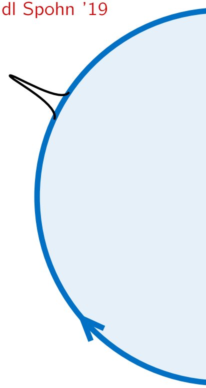
(the noise term also maps appropriately, as it must by fluctuation-dissipation)

Edge is in Burger's-KPZ universality, with $z = 3/2$!

Collective mode disperses as

$$\omega = ck - i\mathcal{D}k^z + \cdots \qquad \text{with} \quad \mathcal{D} = \sqrt{\frac{T}{\chi^3}\frac{|\nu|}{2\pi}|\chi'|} \quad \text{and} \quad z = \frac{3}{2}$$

similar dispersion relations observed in 1d hydro Narayan Ramaswamy '02 Spohn '14
but these are not robust vs disorder Das Damle Dhar Huse Kulkarni Mendl Spohn '19



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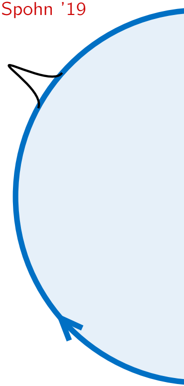

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KPZ UNIVERSALITY ON THE EDGE

Collective mode disperses as

$$\omega = ck - i\mathcal{D}k^z + \cdots \qquad \text{with} \quad \mathcal{D} = \sqrt{\frac{T}{\chi^3}\frac{|\nu|}{2\pi}|\chi'|} \quad \text{and} \quad z = \frac{3}{2}$$

similar dispersion relations observed in 1d hydro Narayan Ramaswamy '02 Spohn '14
but these are not robust vs disorder Das Damle Dhar Huse Kulkarni Mendl Spohn '19

Transport:

KPZ scaling function controls transport

$$G_{nn}(\omega, k) = \frac{\chi T}{\omega} g_{\mathrm{KPZ}}\left(\frac{\omega - ck}{\mathcal{D}k^z}\right) + \cdots$$

and gives

$$\sigma(\omega) = \lim_{k\to 0} \frac{\omega}{k^2} \operatorname{Im} G^R_{nn}(\omega, k) = \# \frac{\chi \mathcal{D}^{4/3}}{\omega^{1/3}} + \cdots$$

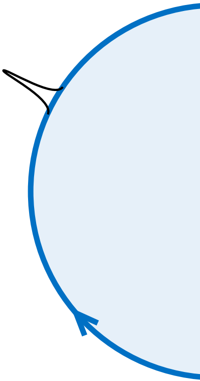
(g_{KPZ} known to high precision, with $\# \simeq 0.417816..$ Prähofer and Spohn '04)

Neglecting thermoelectric effects first:

Charge propagates chirally with velocity σ_{xy}/χ

Heat propagates chirally with velocity κ_{xy}/c_V

$\rightsquigarrow$ 2 decoupled KPZ fronts



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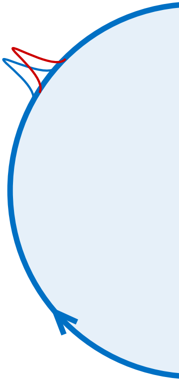

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$\rightsquigarrow$ 2 decoupled KPZ fronts

Interactions between modes are kinematically disfavored because of their different velocities.

$$\omega = ck - i\mathcal{D}k^{3/2} + \cdots \qquad \kappa(\omega) \sim \frac{1}{\omega^{1/3}}$$

Heat

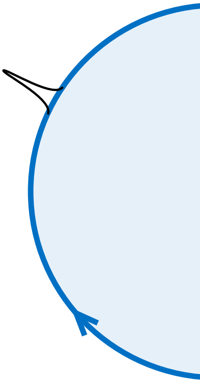
Special case: $\kappa_{xy} = 0$ (e.g. for $\nu = 2/3$)

The chiral velocity of heat vanishes.

Linearized hydro says it should diffuse Kane Fisher '97

$$G^R_{hh}(\omega, k) \simeq \frac{c_V D}{-i\omega + Dk^2}$$

Does this prediction survive hydrodynamic fluctuations?



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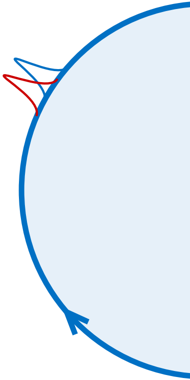

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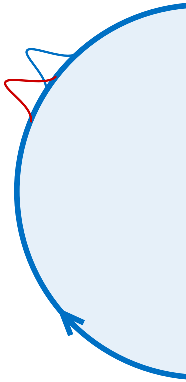

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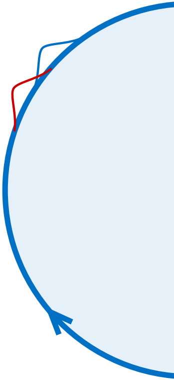

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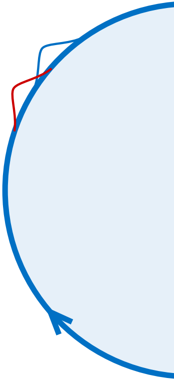
$$\dot{h} - D\partial_x^2 h + \lambda n \partial_x n = 0$$

Open problem in stochastic physics Dhar '08, Spohn '14

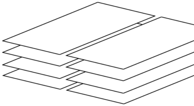
'Mode-coupling' approximation predicts $z = 5/3$, which gives

$$\omega \sim -i\mathcal{D}k^{5/3} \qquad \sigma(\omega) \sim \frac{1}{\omega^{2/5}}$$



Experiments

Singular edge transport: $\sigma(\omega) \sim \frac{1}{\omega^{1/3}}$



Anomalous damping of edge modes:

$$\omega \simeq ck - i\mathcal{D}k^{3/2} \qquad \text{with} \qquad \mathcal{D} = \sqrt{\frac{\chi'^2 T}{\chi^3}\frac{|\nu|}{2\pi}}$$

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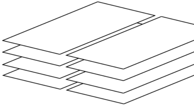

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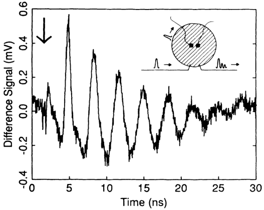
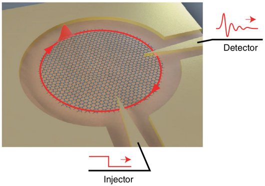


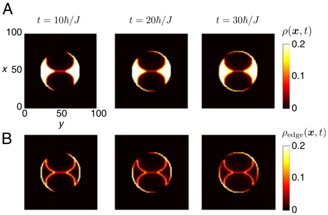
Ashoori Stormer Pfeiffer Baldwin West '92

GaAs



Kumada Glattli et al '14

Graphene



Goldman Spielman et al '13

Cold atoms

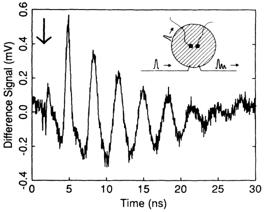
Experimental investigation of the damping of low-frequency edge magnetoplasmons in GaAs-$Al_xGa_{1-x}As$ heterostructures

V. I. Talyanskii,* M. Y. Simmons, J. E. F. Frost, M. Pepper, D. A. Ritchie, A. C. Churchill, and G. A. C. Jones
Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 OHE, United Kingdom
(Received 17 February 1994)

A detailed experimental study of damping and velocity of low-frequency edge magnetoplasmons in GaAs-$Al_xGa_{1-x}As$ heterostructures is presented. The damping is observed to be frequency dependent at filling factors close to integer values. The magnitude of the damping increases with frequency, the dependence being somewhere between linear and quadratic. This finding indicates that the damping of low-frequency edge magnetoplasmons cannot be described by the effective relaxation time. The experimental results are discussed in terms of existing models of low-frequency edge magnetoplasmons.

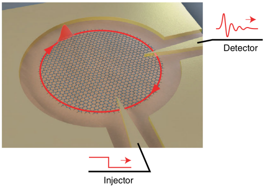
Anomalous damping of edge modes:

$$\omega \simeq ck - i\mathcal{D}k^{3/2} \qquad \text{with} \qquad \mathcal{D} = \sqrt{\frac{\chi'^2 T}{\chi^3}\frac{|\nu|}{2\pi}}$$



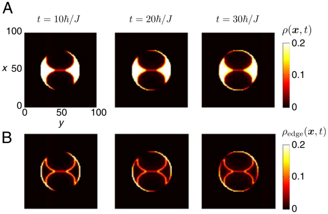
Ashoori Stormer Pfeiffer Baldwin West '92

GaAs



Kumada Glattli et al '14

Graphene



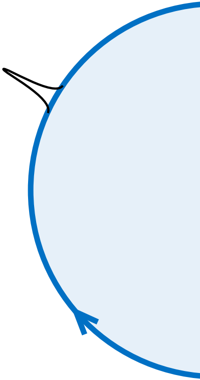
Goldman Spielman et al '13

Cold atoms

Neutral heat mode is similar, except if $\kappa_{xy} = 0$, like for $\nu = 2/3$

$$\omega \sim -i\mathcal{D}k^{5/3}$$

'Upstream' heat transport



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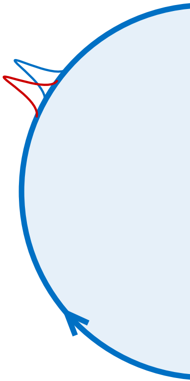

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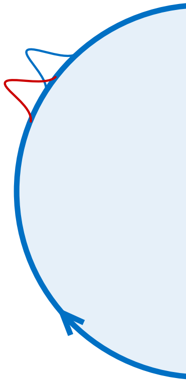

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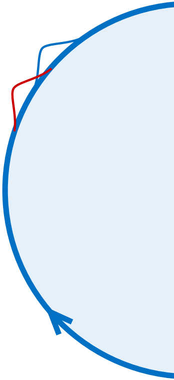

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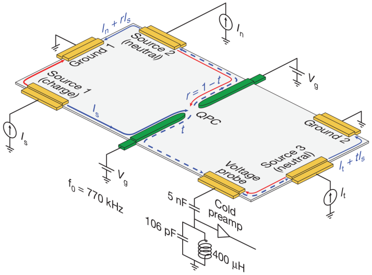
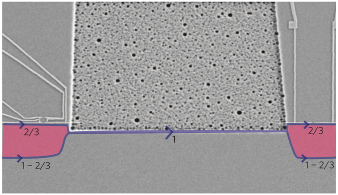
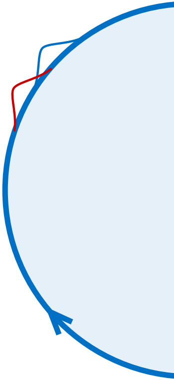


Bid Mahalu et al '**10**

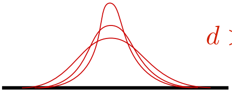


Venkatachalam Hart Yacoby et al '**12**



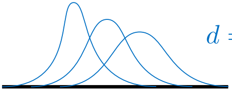
1. Brief Intro to Hydro

2. Weak fluctuations



$d > 0$

3. Strong fluctuations



$d = 1$

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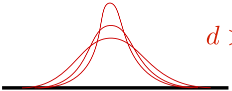

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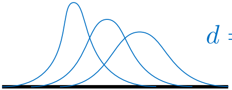

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Thanks!

LVD@uchicago.edu