Intertwined Orders and Fermionic Spectral Functions in Holography

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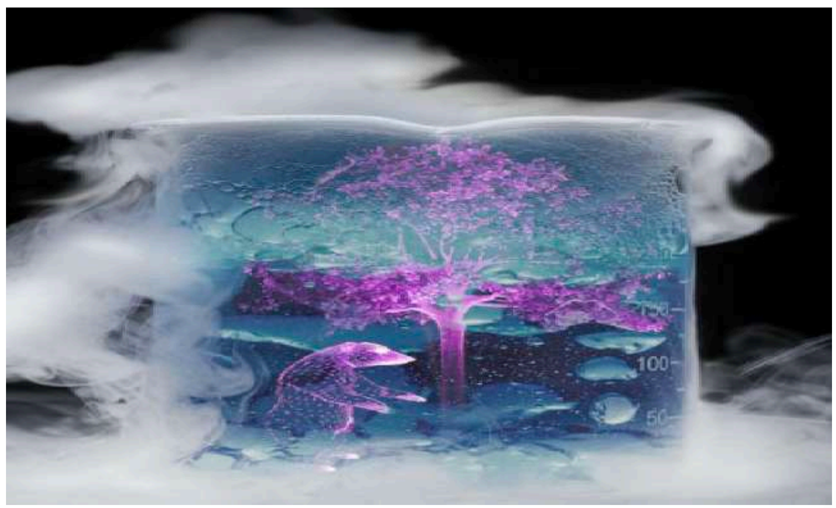


in collaboration with Sera Cremonini (Lehigh U.) and Jie Ren(Zhongshan U.)
Rong-Gen Cai (ITP-CAS), Yong-Qiang Wang (Lanzhou U.)
and Jan Zaanen(Leiden U.)

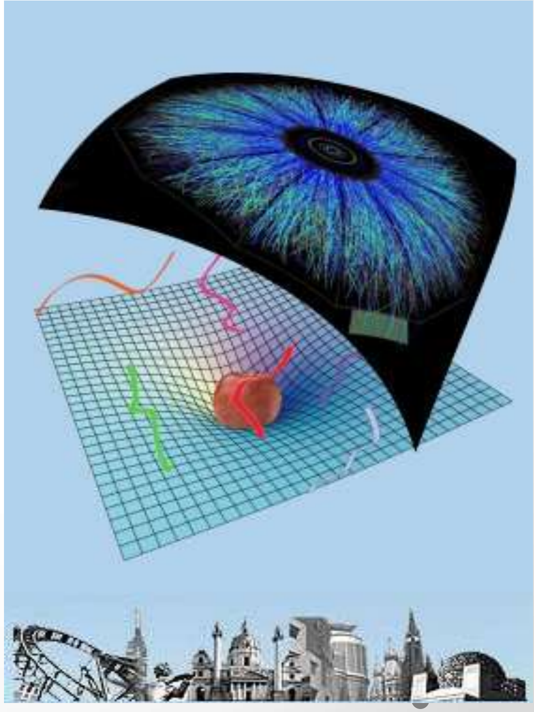
Outline

- Introduction

- Holographic Setup

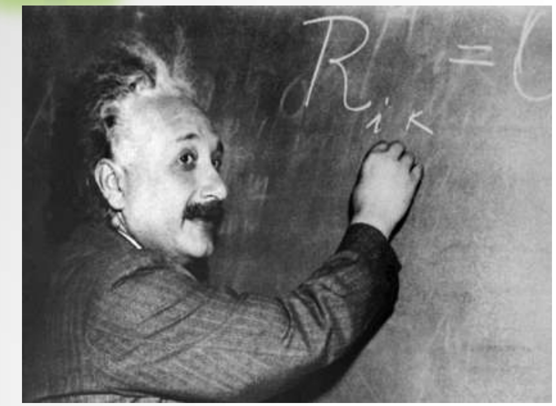
  Spatially modulated black holes with intertwined orders

  Fermi response in holographic striped phases

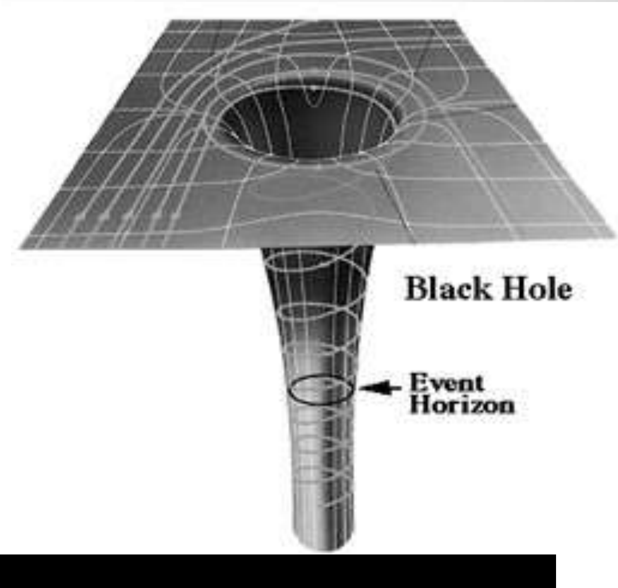


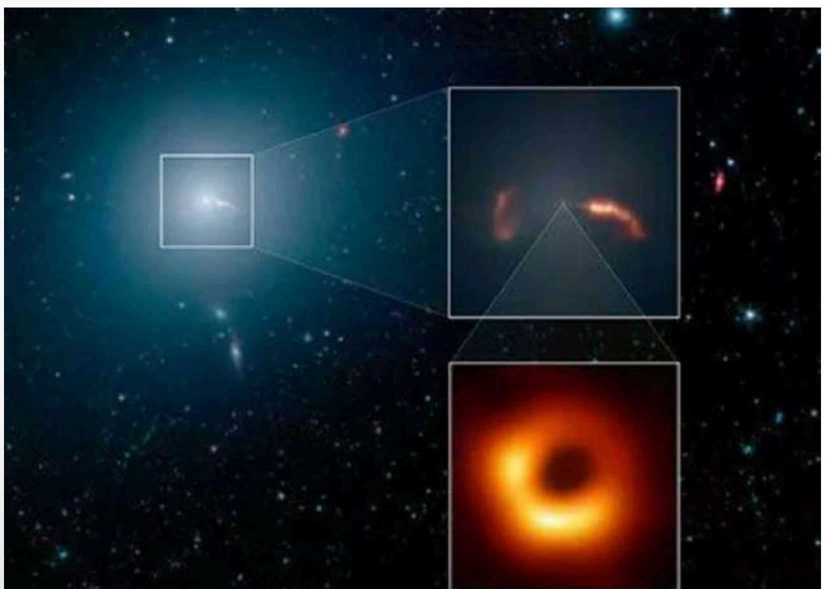
- Conclusion

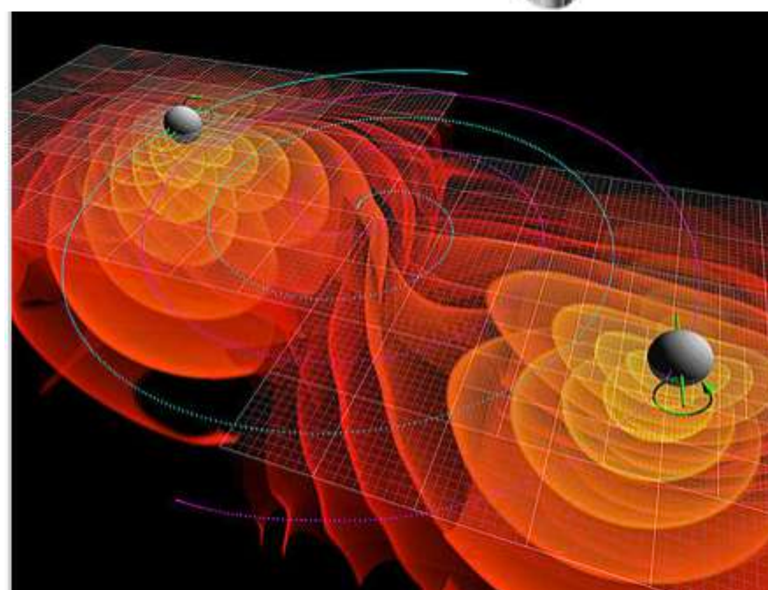
Introduction



General Relativity: Black Hole

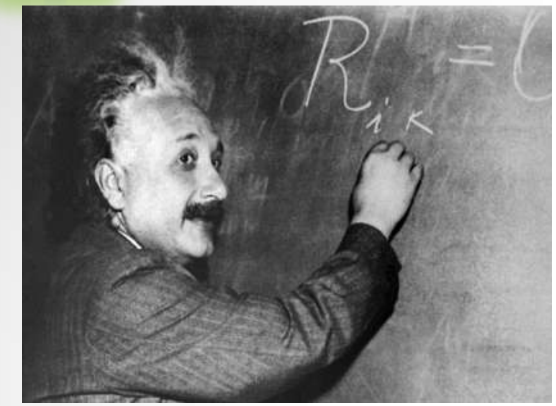






- ETH: black hole shadow from M87
- LIGO: **GW150914**

Introduction



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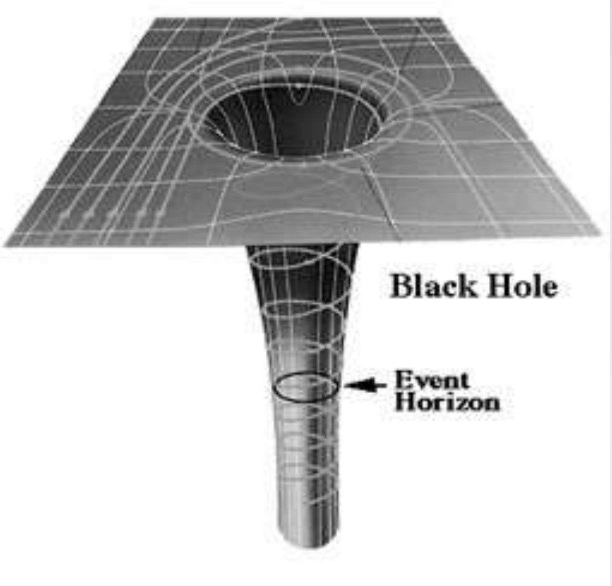
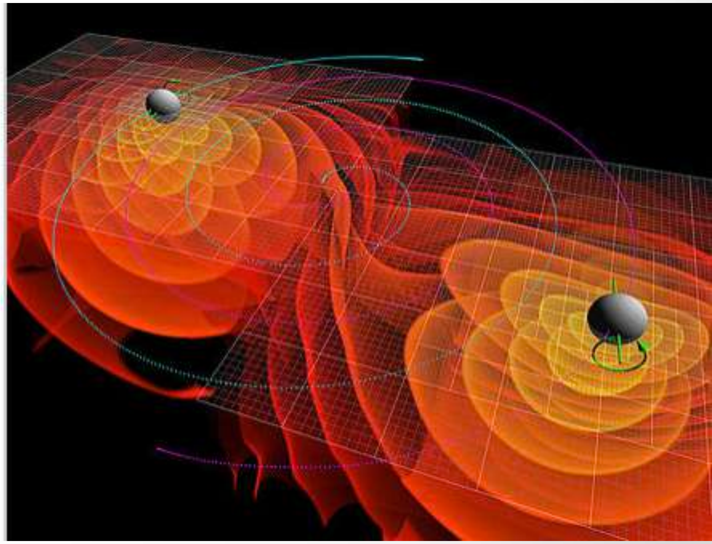


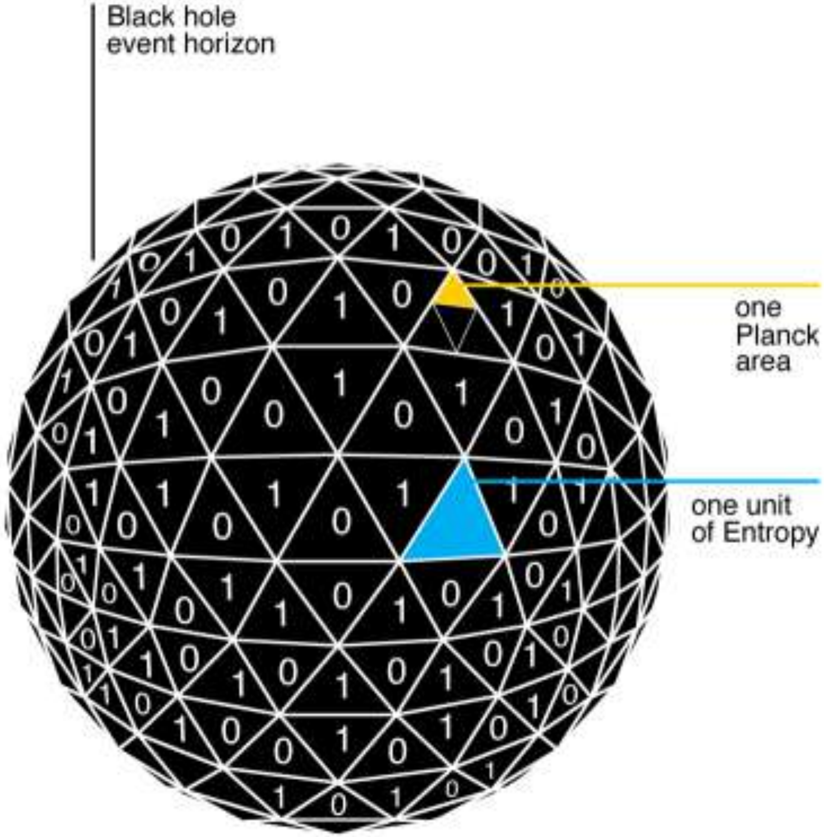
Insights from quantum physics (S. Hawking 1974):
Black holes aren't completely black !

a. Hawking radiation: lose energy and evaporate
b. Have finite temperature and entropy



LIGO: **GW150914**

Black Hole Entropy: a large number of microstates



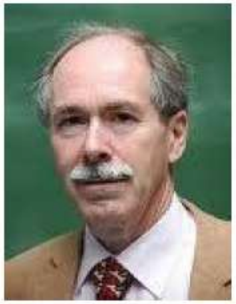
$$S_{BH} = \frac{kc^3}{4\hbar G} A$$

Area Law: The black hole entropy is proportional to the **area of its event horizon** divided by the Planck area $l_P^2 = \frac{\hbar G}{c^3} \sim 2.61223 \times 10^{-70}\ \text{m}^2$

- **Black hole thermodynamics**: a very deep and fundamental relationship between **gravitation**, **thermodynamics** and **quantum theory** !

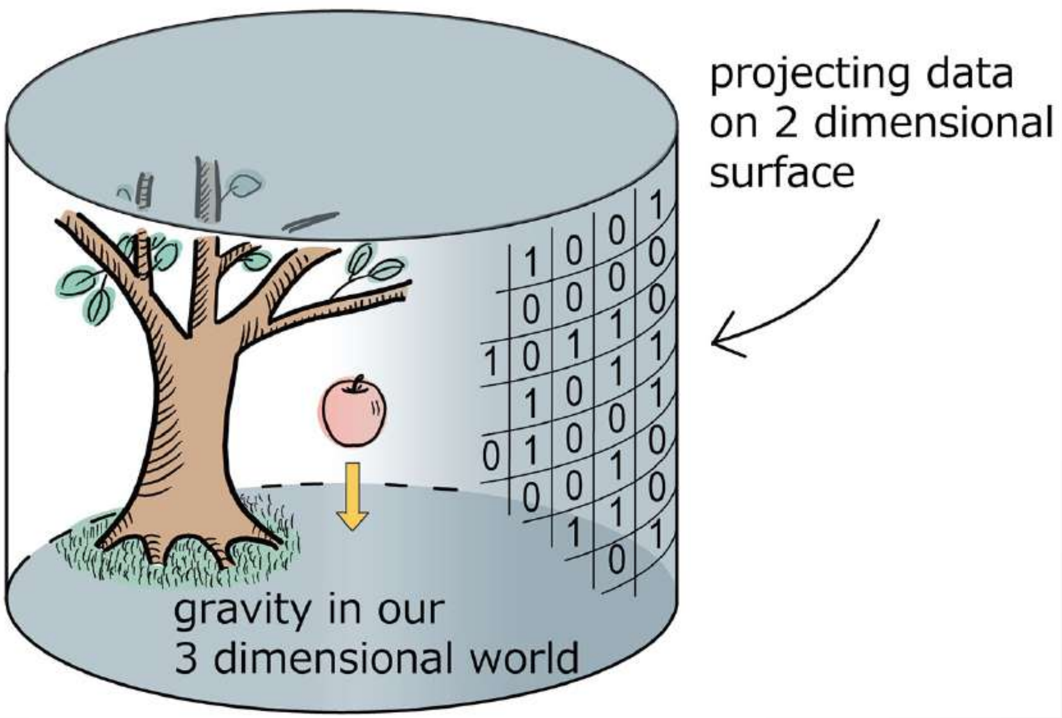
- To **unify GR and quantum mechanism** is one of the greatest challenges of modern Physics ! (Quantum Gravity)

Holographic Principle



A supposed property of quantum gravity:

the description of a volume of space can be thought of as encoded on a lower dimensional boundary to the region.



How the degrees of freedom can be described by the "boundary" theory?

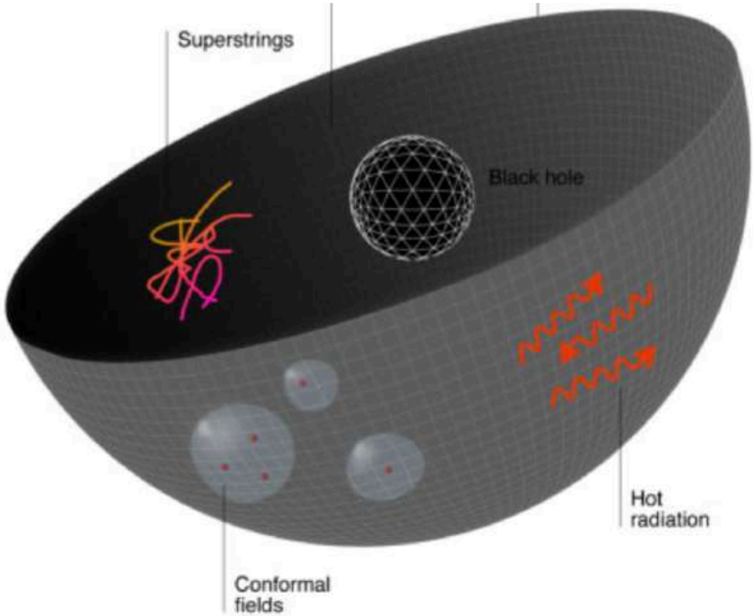
Gauge/Gravity duality: the first example of holographic principle (J. Maldacena, 1997)

Gauge Theory
Quantum field theory

↔

Gravity theory
in high dimensions

conjecture from low energy limit of string theory



Holography:

Quantum field theory
d dimensional spacetime

↔

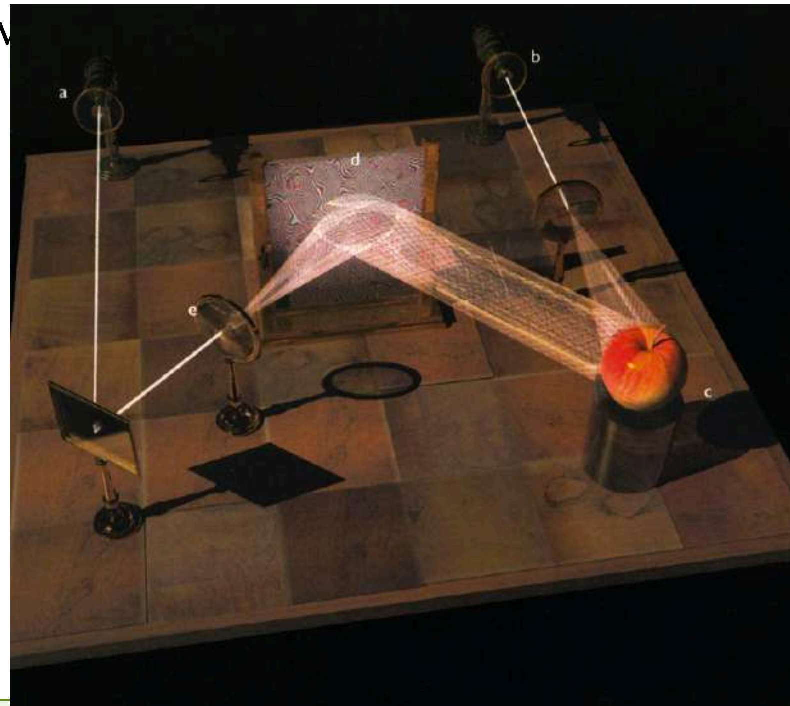
Gravitational theory
d+1 dimensional spacetime

J. Maldacena, arXiv:hep-th/9711200; E. Witten, arXiv:hep-th/9802150; S.S.Gubser et al, arXiv:hep-th/9802109

Gauge/Gravity duality: the prime example of holography
(J. Maldacena, 1997)

Gauge Theory Quantum field theory	⟷	Gravity theory in high dimensions

conjecture from low



Holography:

Quantum field theory **d dimensional** spacetime	⟷	Gravitational theory **d+1 dimensional** spacetime

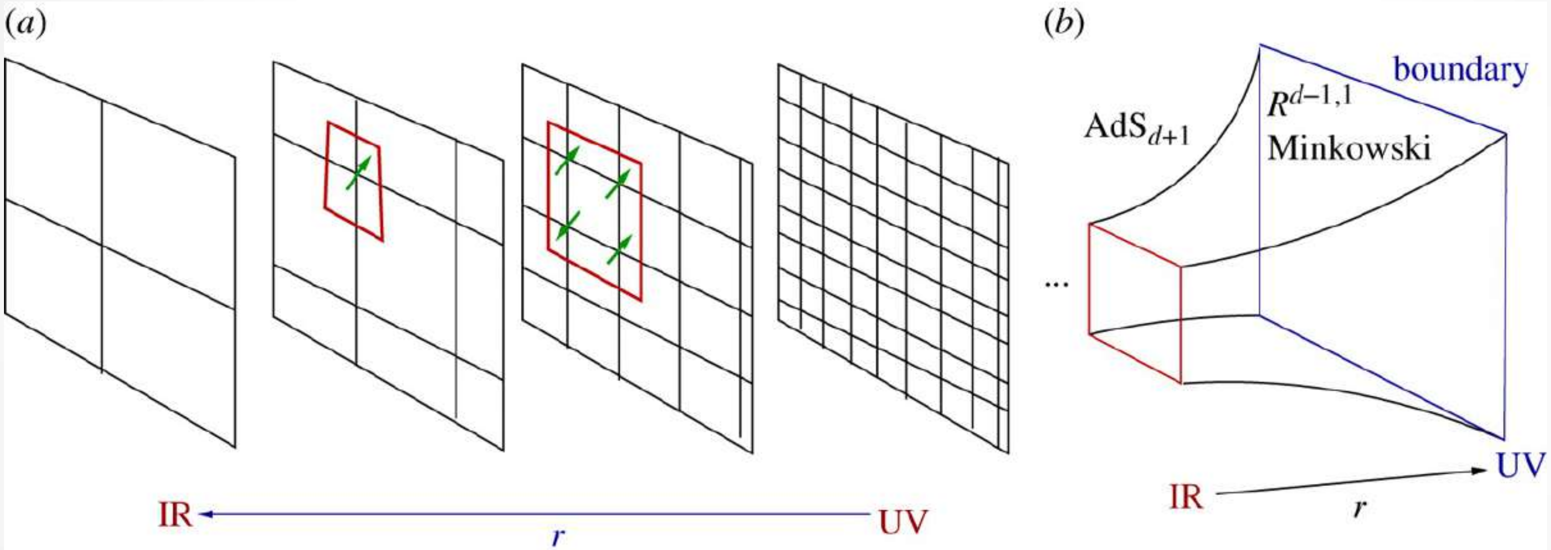
J. Maldacena, arXiv:hep-th/9711200; E. Witten, arXiv:hep-th/9802150; S.S.Gubser et al, arXiv:hep-th/9802109

General Relativity = Renormalization Group

The basic example: AdS=CFT

(dimensions: d+1 d)

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\boldsymbol{x}^2) + R^2\frac{dr^2}{r^2}$$



(a): A series of block spin transformations labeled by a parameter r.

(b): AdS space, which organizes the field theory information in the same way.

Excitations with different wavelength get put in different place in the bulk picture.

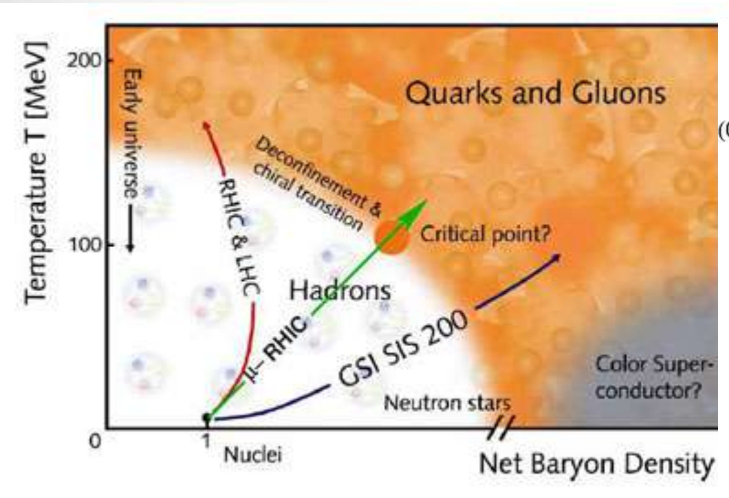
(arXiv: 1101.0597[hep-th])

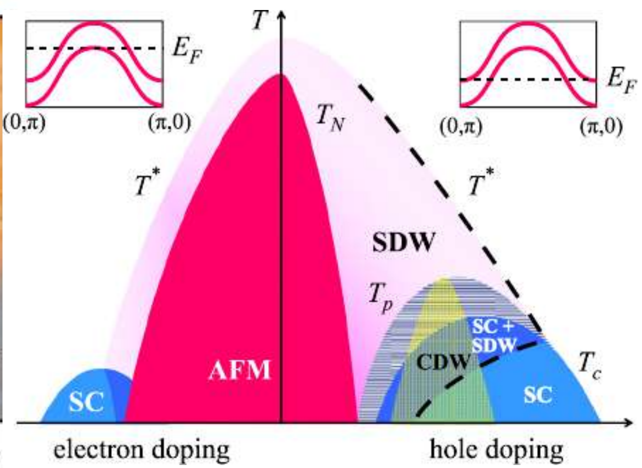
Holography as a Theoretical Laboratory

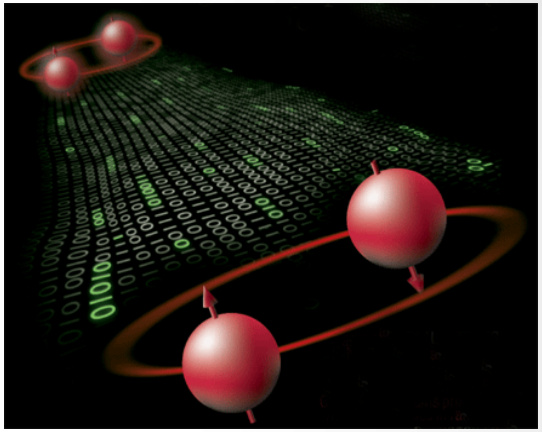
Quantum field theory
at strong coupling

⟷

Theory of gravitation
at weak coupling







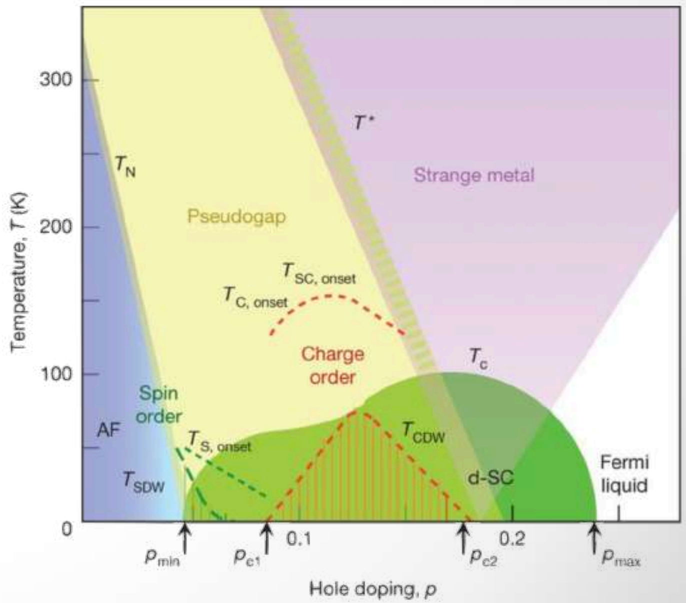
Applied holography:

QGP and QCD (drag force, jet quenching, confinement/deconfinement,…),
Condensed matter (quantum criticality, strange metal, superconductivity,…),
Quantum Entanglement, Non-equilibrium dynamics…

Challenges for Strongly Coupled Quantum Phases of Matter

- Breakdown of Fermi-liquid theory, quantum matter without quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
- Segmented Fermi surfaces (`Fermi arcs')
- Anomalous transport
- Planckian dissipation
- Long-range entanglement
- …

(Jan's talk on Wednesday)



Keimer et al, Nature (2015)

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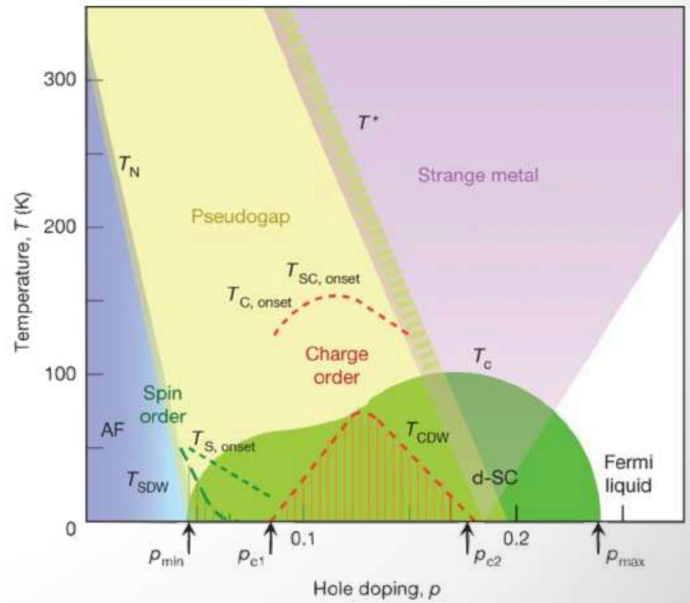


Keimer et al, Nature (2015)

Holography as a Theoretical Laboratory

Strategy:
Study **solvable models that may be in the same universality class** as strongly correlated phases of interest

Goal:
Draw qualitative and quantitative lessons – universal features?
Shed light on **basic mechanisms** underlying the dynamics

Solvable often implies working with overly **simplified bottom-up toy models**

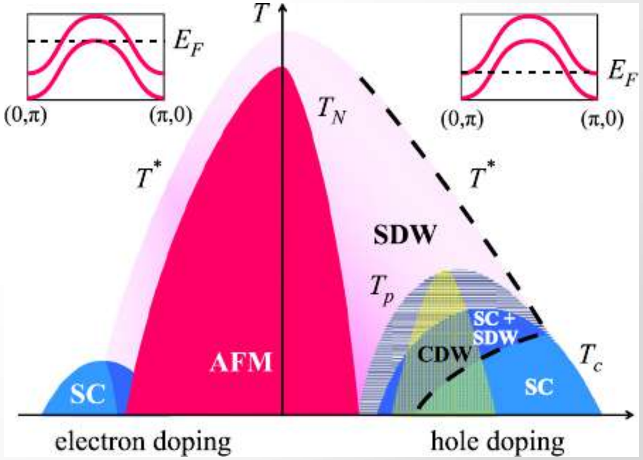
Spatially Modulated Black Holes

Construct stationary black hole solutions in which spatially translational symmetry is broken (**spontaneously** vs. explicitly).

Motivation:

Stationary solutions of the Einstein equation, especially black holes, are the most fundamental of all gravitational objects. The search for new stationary solutions will help to understand general relativity more broadly and deeply.

This novel black hole solution provides a dual description of quantum phases, where various orders appear to be intertwined.
These spontaneous orders are believed to play an important role in the rich phase diagram of strongly correlated systems.



A number of striped quantum phases are **generated spontaneously** in strongly correlated electron systems.

E. Berg, E. Fradkin, S.A.Kivelson, arXiv:0810.1564 [cond-mat.supr-con]

Pair Density Wave (PDW)

- SC condensate is **spatially modulated** but has a **zero average**

$$\langle O \rangle \sim \Delta_Q \cos(Q\, x)$$

- charge density **oscillates twice** the frequency of the condensate

$$\rho \sim \rho_0 + \rho_{2Q} \cos(2Q\, x) \quad \longleftrightarrow \quad \text{CDW}$$

Co-exiting SC and CDW phase (SC+CDW)

- The condensate has a **nonzero uniform** component

$$\langle O \rangle \sim \Delta_0 + \Delta_Q \cos(Q\, x)$$

- CDW oscillates at the **same frequency** as the condensate

$$\rho \sim \rho_0 + \rho_Q \cos(Q\, x)$$

The holographic model

3+1 D bottom-up model in the bulk:

$$S_0 = \frac{1}{2\kappa_N^2} \int d^4x \sqrt{-g} \left[\mathcal{R} - 2\Lambda + \mathcal{L}_m + \mathcal{L}_{cs} \right] ,$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \mathcal{F}(\chi)(\partial_\mu \theta - q A_\mu)^2 - \frac{Z(\chi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\chi) ,$$

$$\mathcal{L}_{cs} = -\vartheta(\chi) \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma} .$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Field content:

- Gravity+negative cosmological constant $\Lambda < 0$
- Two real scalars χ and θ
- One U(1) gauge fields A_μ :**charge density** of field theory

R.G. Cai, **LL**, Y.Q.Wang J.Zaanen, PRL, 2017

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

"Stuckelberg term" (Josephson action)
→ allows for more general couplings

Break U(1) symmetry spontaneously

Holographic superconductor

[0906.1214,0907.3610, 0912.0480,1510.00020,…]

The holographic model

3+1 D bottom-up model in the bulk:

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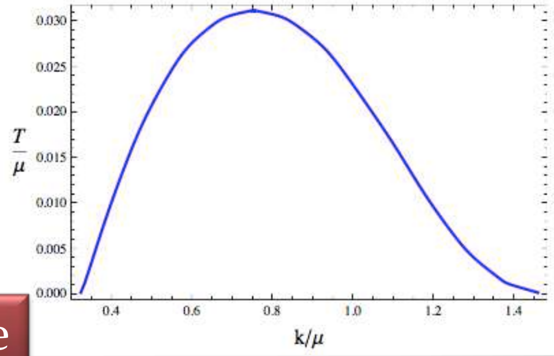
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\langle O\rangle \sim \cos(k\,x)$



Crucial coupling **for seeding spatially modulated instabilities**
n=0 → leading unstable mode is not striped

$$\vartheta(\chi) = \frac{n}{2}\chi + \cdots$$

bell curve

Broken Phase: Spatially Modulated Black Holes

$$ds^2 = \frac{L^2}{z^2}\left[-H(z)U_1 dt^2 + \frac{U_2}{H(z)}dz^2 + U_3(dx + z^2 U_5 dz)^2 + U_4(dy + (1-z)U_6 dt)^2\right]$$

$$A = (1-z)\,\phi\,dt + A_y\,dy, \quad \chi = z\,\psi,$$

where the nine functions (U1, U2, U3, U4, U5, U6, φ, Ay, ψ) depend on both z and x.

$$U_1(0,x) = U_2(0,x) = U_3(0,x) = U_4(0,x) = 1\,,$$

$$\psi(0,x) = A_y(0,x) = U_5(0,x) = U_6(0,x) = 0, \quad \phi(0,x) = \mu$$

spontaneously symmetry breaking

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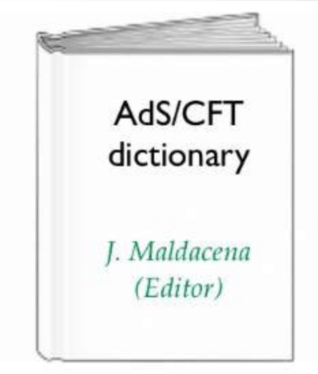
spontaneously symmetry breaking

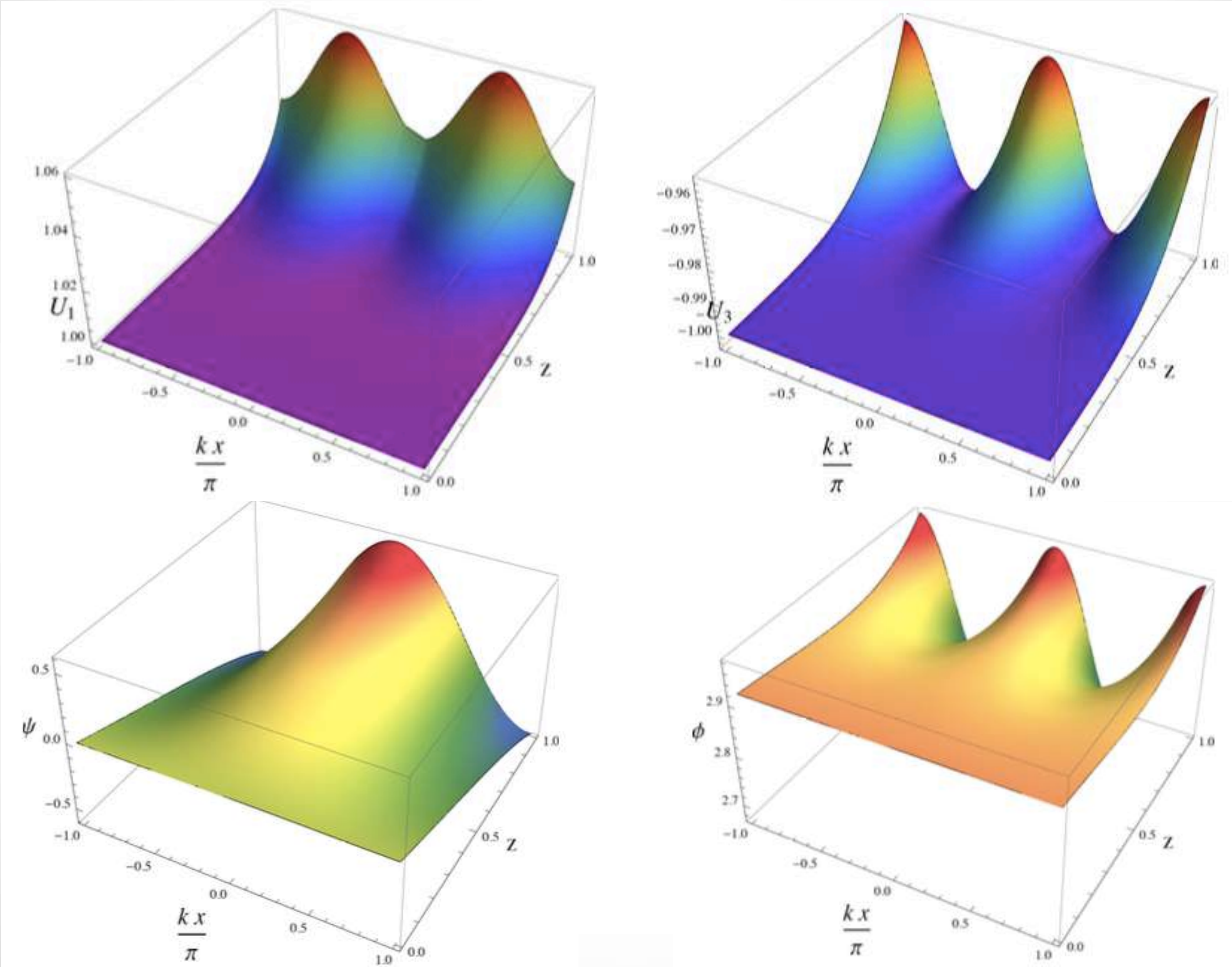
Physical quantities can be read off from the boundary data at z=0.

condensate	$\psi(z,x) = \langle O_\chi(x)\rangle\, z + \mathcal{O}(z^2)$
charge density	$A_t(z,x) = \mu - \rho(x) z + \mathcal{O}(z^2)$
current density	$A_y(z,x) = 0 + j_y(x) z + \mathcal{O}(z^2)$



Numerical Method: pseudo-spectral collocation+DeTurck+Newton-Raphson

Full solutions: novel black hole with scalar, charge and current hairs



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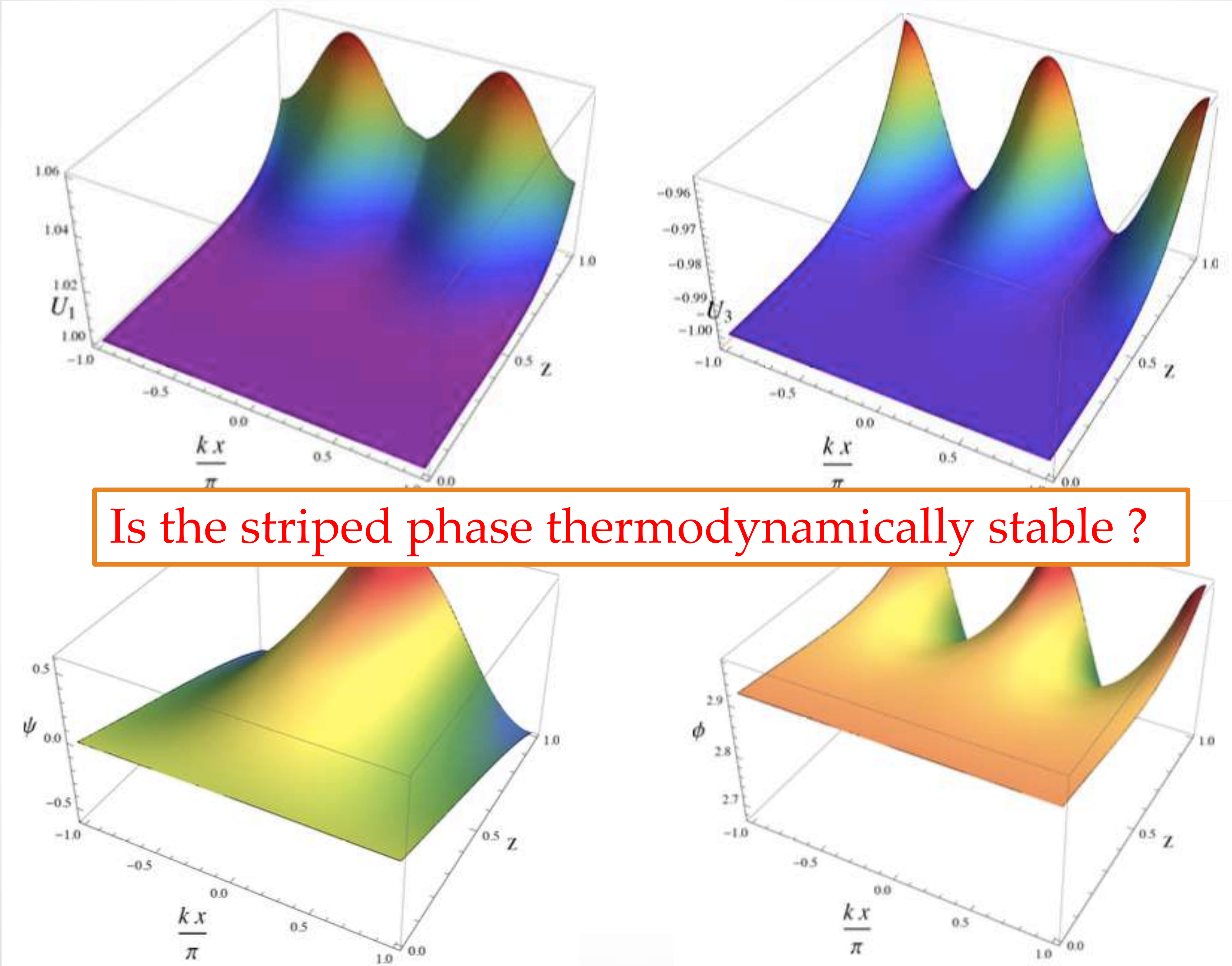
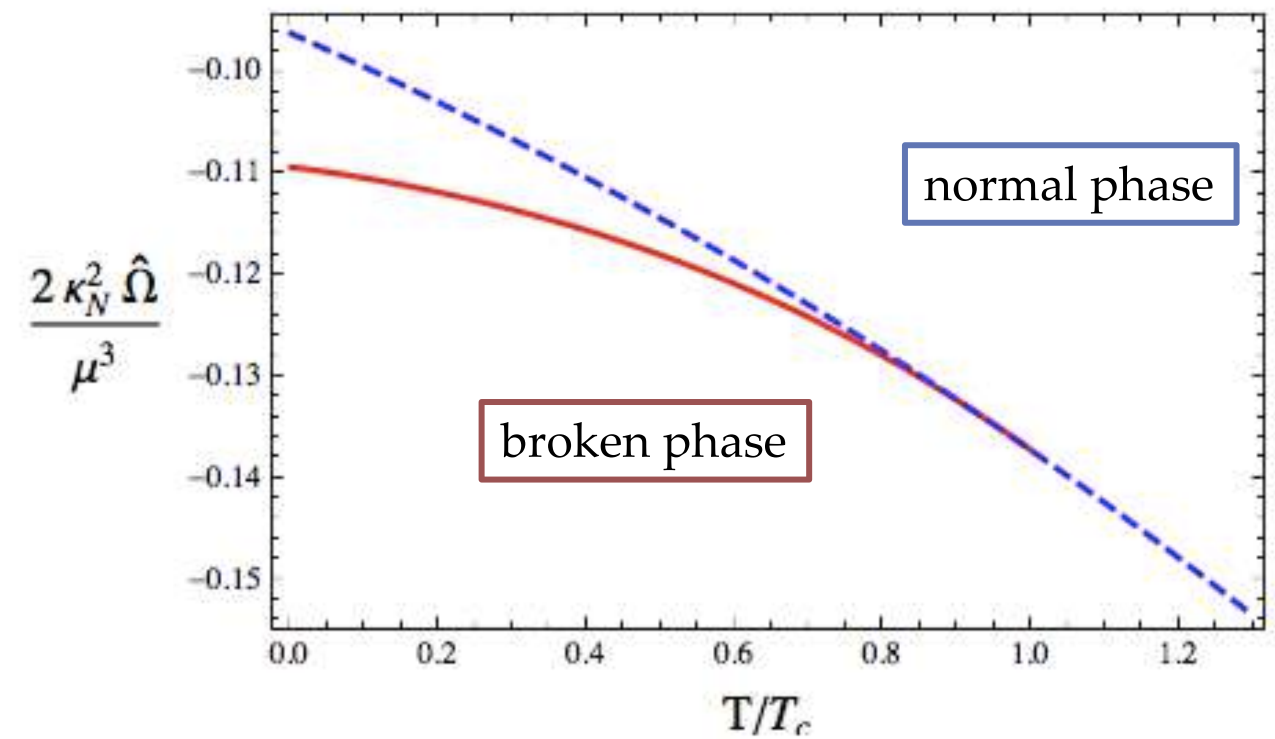
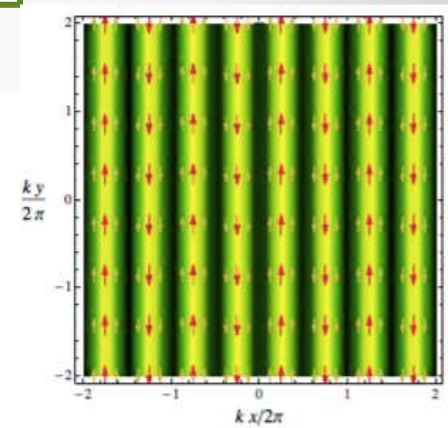


Is the striped phase thermodynamically stable ?

Thermodynamics: the striped phase has a lower free energy!



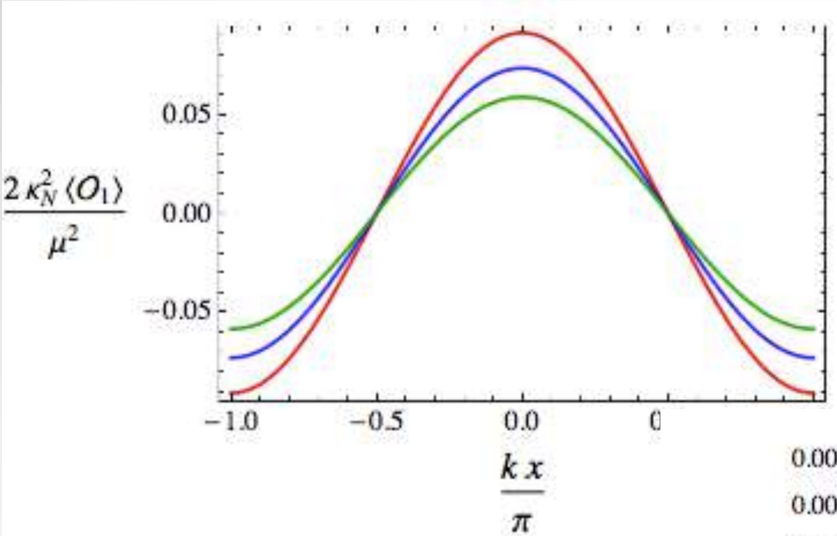


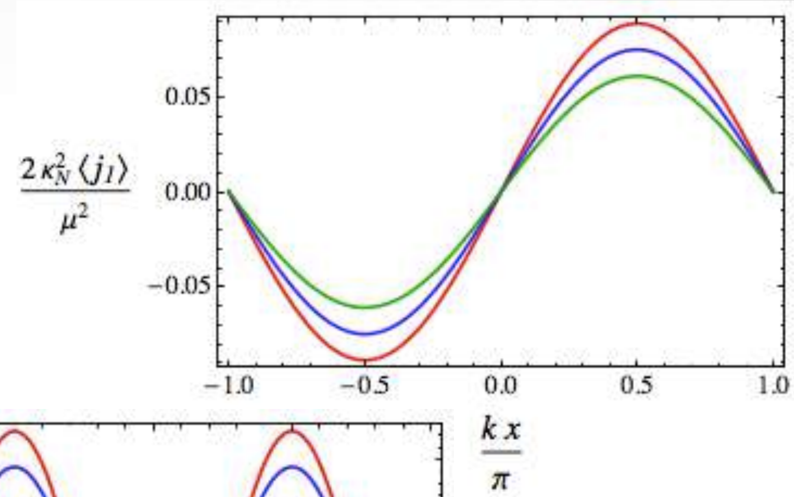
$T_c \approx 0.1284\mu$

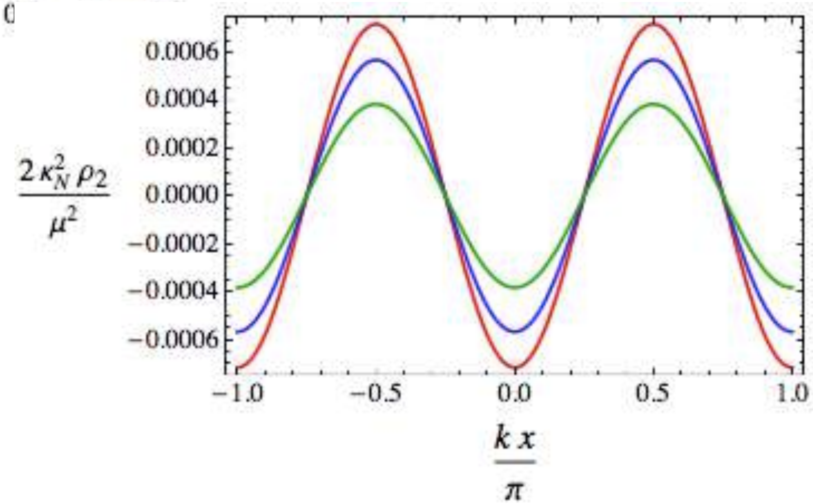
$$Z(\chi) = \frac{1}{\cosh(\sqrt{3}\chi)}, \qquad V(\chi) = 1 - \cosh(\sqrt{2}\chi),$$

$$\mathcal{F}(\chi) = \cosh(\chi) - 1, \quad \vartheta(\chi) = \frac{1}{4\sqrt{3}}\tanh(\sqrt{3}\chi).$$

Second order phase transition

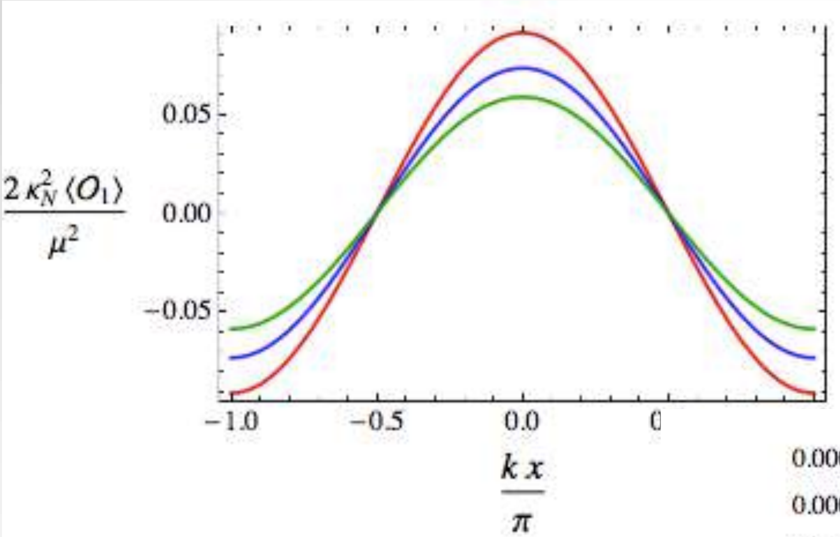


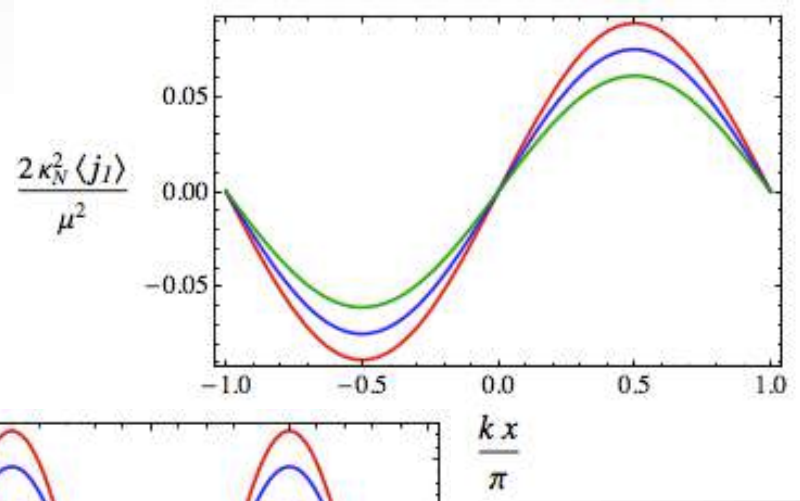


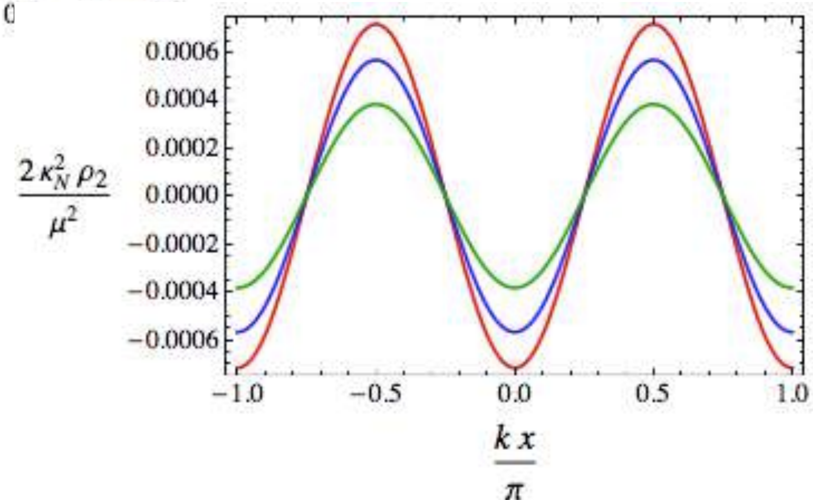


Interesting features:

- The SC condensate is spatially modulated in such a way that its uniform component is zero.
- The charge density oscillates at twice the frequency of the current and condensate.
- The current density wave and condensate modulation are precisely out of phase.







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(Pair density wave !)

Optical Conductivity

The conductivity matrix is

$$\begin{pmatrix} J^x \\ J^y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Map to gravity side

$$\frac{1}{2\kappa_N^2} \begin{pmatrix} a_x^v \\ a_y^v \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} i\omega a_x^s \\ i\omega a_y^s \end{pmatrix}$$

where $a_x = a_x^s + a_x^v(x)\, z + \mathcal{O}(z^2), \quad a_y = a_y^s + a_y^v(x)\, z + \mathcal{O}(z^2)$ at AdS boundary

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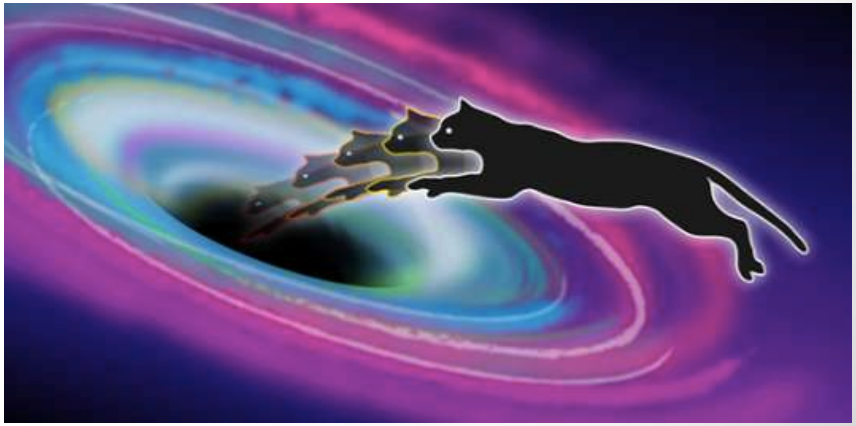
Impose in-going condition near the black hole horizon



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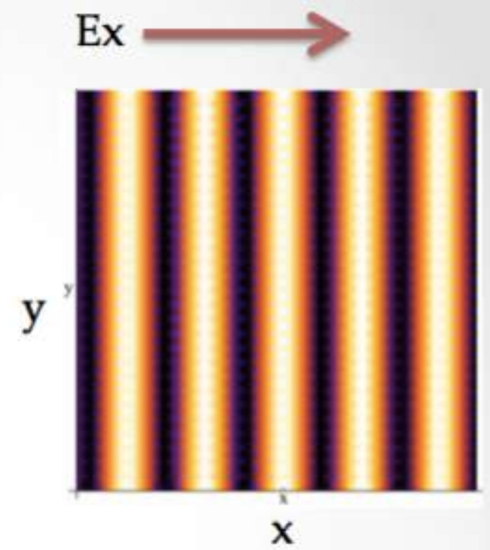
Retarded **two-point Green's function**

Optical Conductivity: perpendicular to stripes



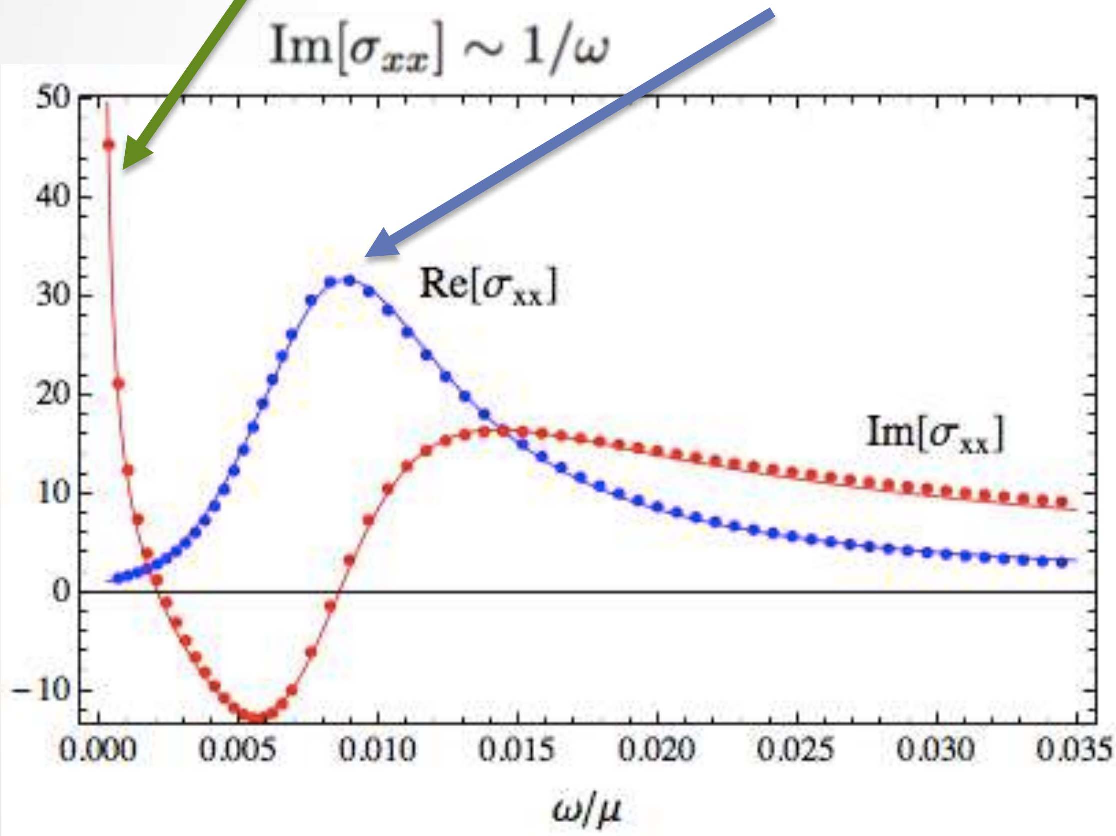
Ionic lattice

$$\mu(x) = \mu\,[1 + \mathbf{A}\cos(p\,x)]$$

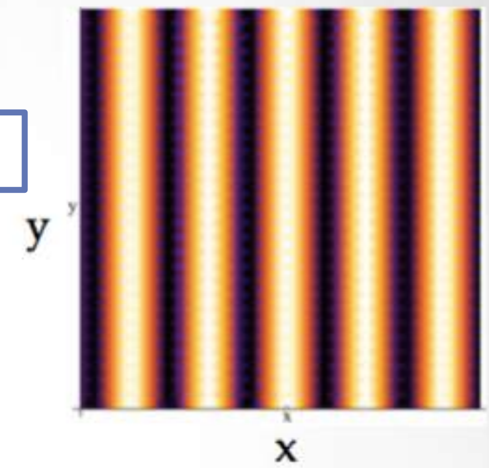
Optical Conductivity: perpendicular to stripes

Superconducting mode

phonon collective mode



Ex



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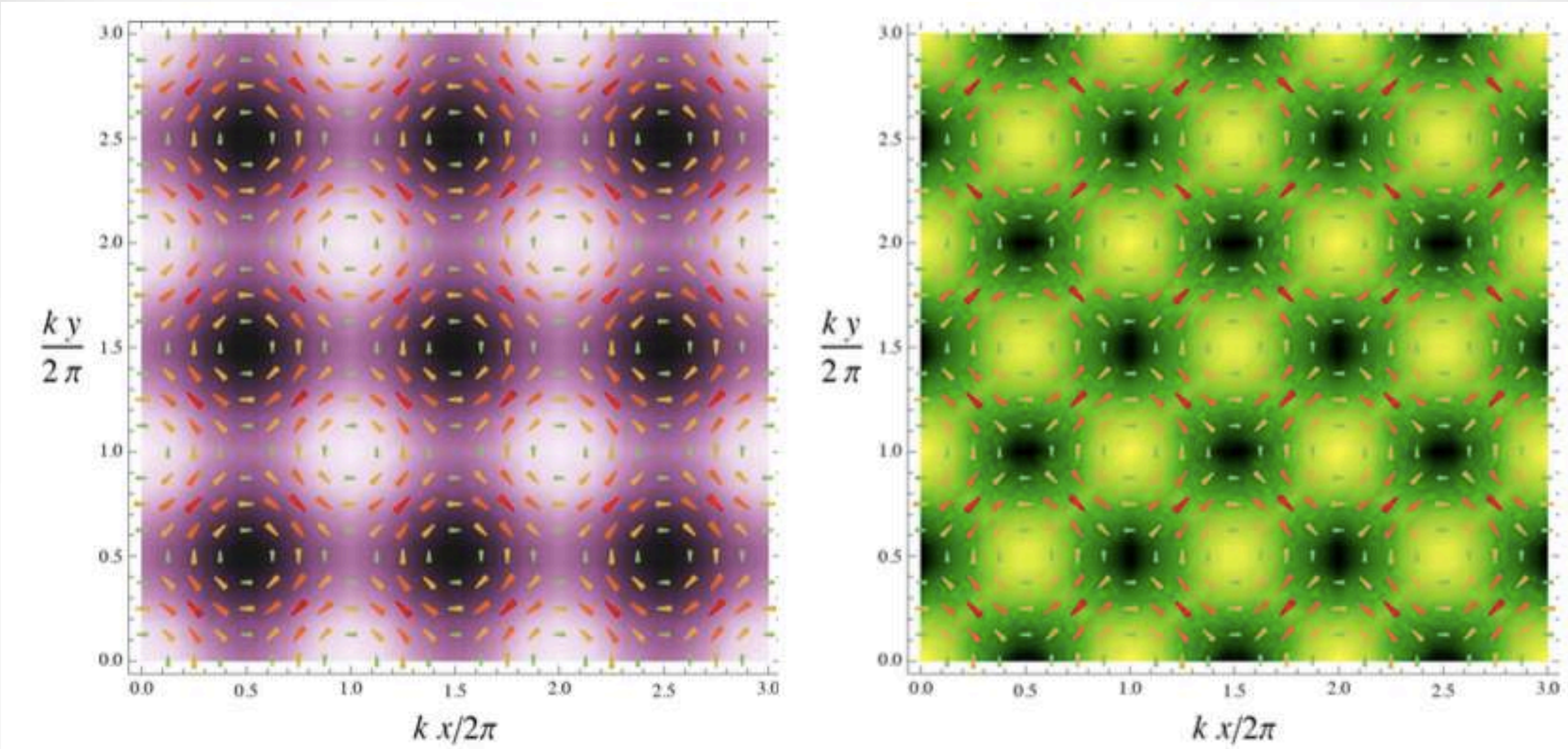
Lorentz formula

$$\sigma(\omega) = i\frac{\rho_s}{\omega} + \sum_{j=1}^{N} \frac{S_j^2\,\tau_j}{1 - i\omega\tau_j(1 - \omega_j^2/\omega^2)}$$

Fully crystallized phase: 2D PDW

Parity breaking $\mathcal{L}_{cs} \sim \chi \, \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$



The rules of striped order repeat themselves in the tetragonally ("checkerboard") ordered case.

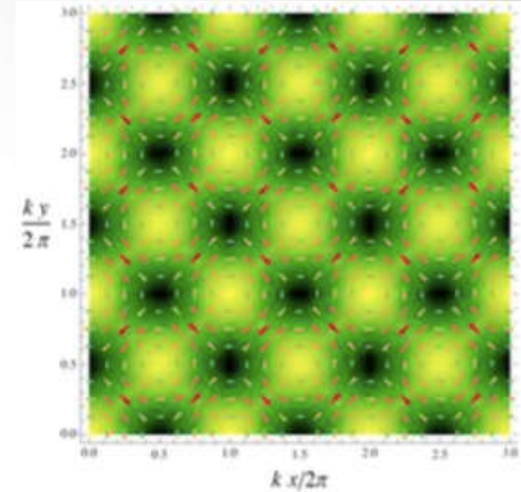
The charge order is now accompanied by spontaneous staggered current patterns similar to the "d-density wave" of condensed matter physics.

Density-wave states of nonzero angular momentum

Chetan Nayak
Phys. Rev. B **62**, 4880 – Published 15 August 2000

Holographic PDW :

holographic quantum phase with **intertwined orders**

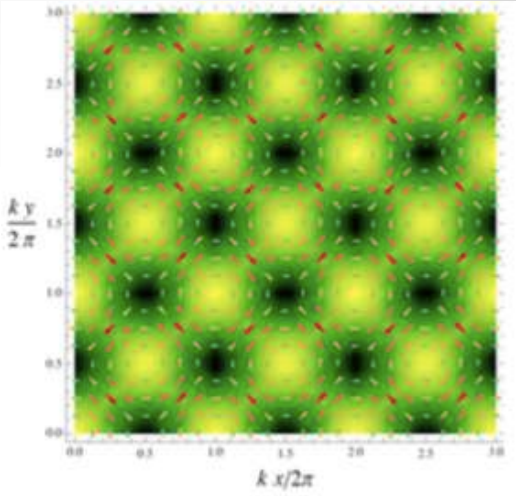


- Superconducting order
- Charge density wave
- Parity breaking order
- Current density wave (d-density wave)

a symphony of quantum matter

Holographic PDW :

holographic quantum phase with **intertwined orders**



Superconducting order

Charge density wave

a symphony of quantum matter

Science Contents News Careers Journals

Magnetic field–induced pair density wave state in the cuprate vortex halo

S. D. Edkins[1,2,3], A. Kostin[1], K. Fujita[1,4], A. P. Mackenzie[3,5], H. Eisaki[6], S. Uchida[7], Subir Sachdev[8], Michael J. Lawler[1,9], E.-...
+ See all authors and affiliations

Science 07 Jun 2019:
Vol. 364, Issue 6444, pp. 976-980
DOI: 10.1126/science.aat1773

The holographic model without parity breaking

3+1 D bottom-up model in the bulk:

$$S = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\left[\mathcal{R} - 2\Lambda + \mathcal{L}_m\right]$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Two U(1) vector fields

Two real scalars

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Field content:

- Gravity+negative cosmological constant $\Lambda < 0$
- Two real scalars χ and θ
- Two U(1) vector fields A_μ and B_μ with different physical interpretations
  - A_μ :**charge density** of field theory
  - B_μ : **spectator field** or proxy for "spin" density or **second species of charge carriers**

S.Cremonini., **LL**, J. Ren (1612.04385, 1705.05390)

The holographic model

3+1 D bottom-up model in the bulk:

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$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

"Stuckelberg term"
→ allows for more general couplings

Break U(1) symmetry spontaneously

[0906.1214,0907.3610, 0912.0480,1510.00020,…]

S.Cremonini., **LL**, J. Ren (1612.04385, 1705.05390)

The holographic model

3+1 D bottom-up model in the bulk:

$$Z_A(\chi) = 1 + \frac{a}{2}\chi^2 + \cdots, \quad Z_B(\chi) = 1 + \frac{b}{2}\chi^2 + \cdots$$

$$S = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2\Lambda + \mathcal{L}_m]$$

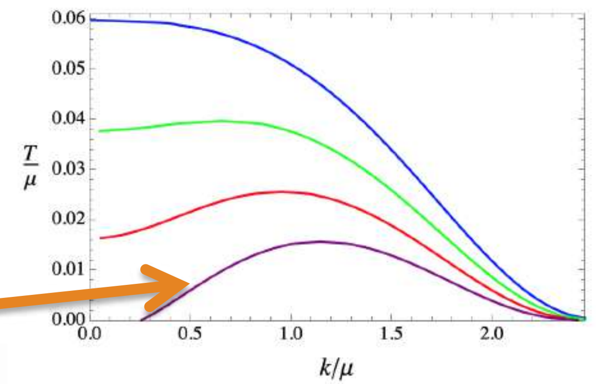
$$Z_{AB}(\chi) = c\chi + \cdots$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\langle O\rangle \sim \cos(k\,x)$$

Crucial coupling **for seeding spatially modulated instabilities**
c=0 → leading unstable mode is not striped



bell curve

S.Cremonini., **LL**, J. Ren (1612.04385, 1705.05390)

Different quantum phases can be described holographically whthin this model.

$$\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2$$

order parameters for different quantum phases

	Δ_0	Δ_Q	ρ_Q	ρ_{2Q}
PDW	0	✓	0	✓
CDW	0	0	✓	✓
CDW′	0	0	0	✓
SC	✓	0	0	0
SC+CDW	✓	✓	✓	✓

In the holographic theory:

- q_A≠ 0, q_B=0 (PDW)
- q_A= 0, q_B= 0 (CDW)
- q_A≠ 0 (SC)
- q_A≠ 0, q_B≠ 0 (SC+CDW)

together with other appropriate model parameters which can be fixed analytically.

PHYSICAL REVIEW B

covering condensed matter and materials physics

Highlights Recent Accepted Authors Referees

Editors' Suggestion

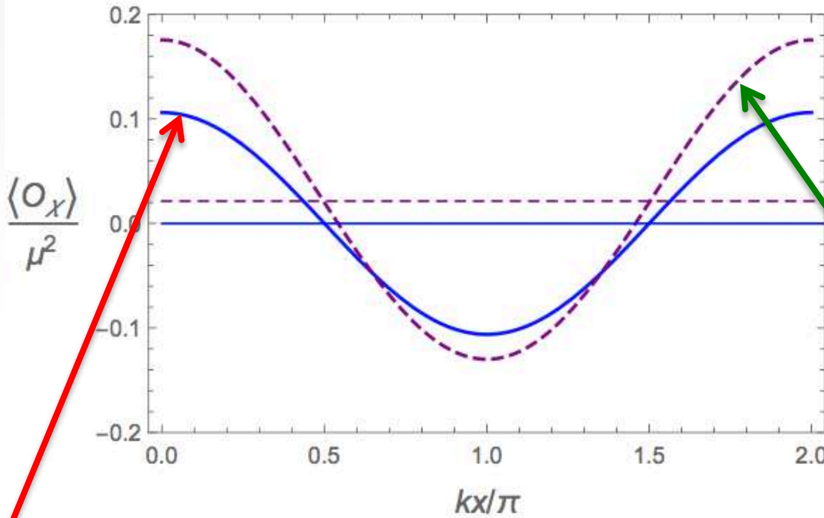
Theory of the striped superconductor

Erez Berg, Eduardo Fradkin, and Steven A. Kivelson

Phys. Rev. B **79**, 064515 – Published 12 February 2009

Translational symmetry is broken Spontaneously in PDW, CDW and SC+CDW

condensate and charge density

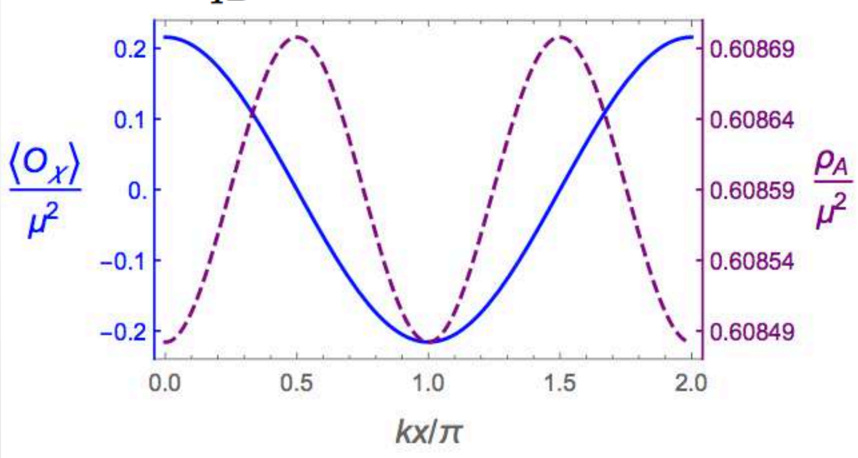


The two horizontal lines denote the average values of the condensate.

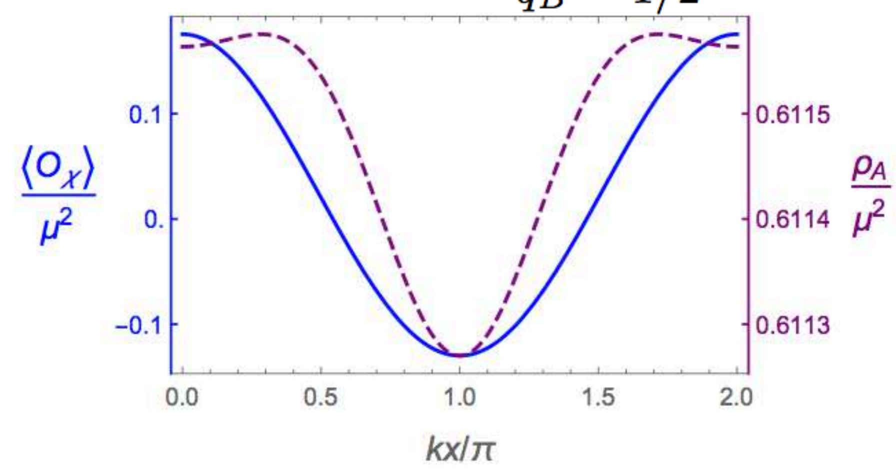
PDW

SC+CDW

$q_B = 0$



$q_B = 1/2$

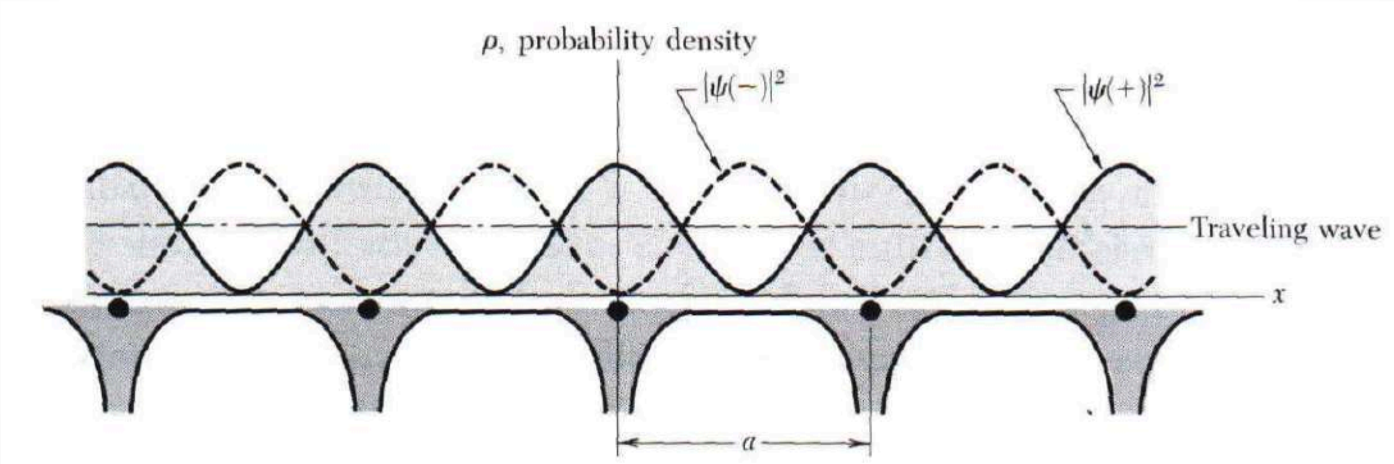


The period of charge density is one half of that of the condensate.

The condensate and charge density share the same period.

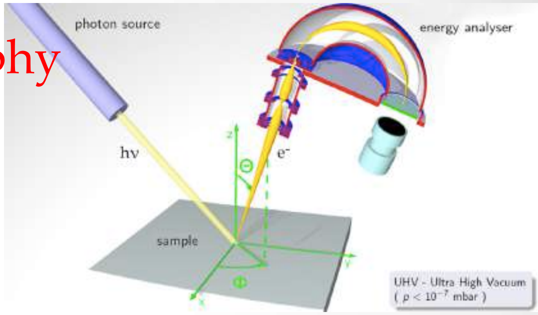
Next:

Examine **fermionic response** in these spontaneously generated striped phases
(including the effects of explicit breaking of translations)



Holographic Fermions in Striped Phases

ARPES

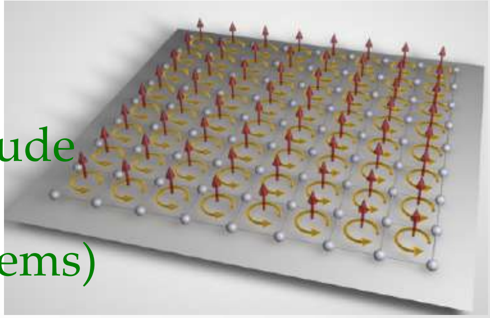


◆There are a lot of work on fermionic response in holography

Single fermion spectral function computations

- Cubrovic, Zaanen, Schalm, Science 325 (2009) 439
- Faulkner, Liu, McGreevy, Vegh, Science 329 (2010) 1043
- See e.g. Iqbal, Liu and Mezei, arXiv:1110.3814 for a review

Most studies focus on cases with **translational invariance or homogeneous** lattices



◆To make contact with real materials, it is important to include effects **of periodic structure (stripes/lattices)**
(also, rich striped phases in strongly correlated electron systems)

A.Bagrov, N. Kaplis, A. Krikun, K. Schalm, J. Zaanen, arXiv:1608.03738

Holographic Fermions in Striped Phases

- Very few holographic investigations on fermions in **inhomogeneous systems**
  - Y. Liu, K. Schalm, Y. W. Sun, J. Zaanen [1205.5277]
    Small periodic modulation of chemical potential perturbatively
  - Y. Ling, C. Niu, J. P. Wu, Z. Y. Xian, H. B. Zhang [1304.2128]
    Include backreaction from lattice
- Interesting features identified: **anisotropic FS** and **appearance of a gap**
- In these studies the lattice is introduced by hand and is irrelevant in the IR

◆The bulk model provides a framework with a periodic structure that is IR relevant (crystalline structure generated spontaneously)

◆Our main interest:

the role of spontaneous vs. explicit translational symmetry breaking on fermionic spectral function.
Note: breaking of U(1) doesn't play a role in what I discuss today (future work)

Gravity setup:

❖Place a probe fermion in the **spontaneously generated striped** background, and then turn on an **explicit lattice** in the UV to break translations explicitly.

❖Solve Dirac equation

$$\left[\Gamma^{\underline{a}} e_{\underline{a}}^{\mu} \left(\partial_\mu + \frac{1}{4}(\omega_{\underline{ab}})_\mu \Gamma^{\underline{ab}} - iqA_\mu\right) - m\right] \psi = 0$$

numerically. Note that the background geometry has **periodic modulation**, so solutions will reflect this periodicity. (consider m=0)

S. Cremonini, **LL**, J. Ren, arXiv:1807.11730

Translational symmetry breaking: spontaneous vs. explicit

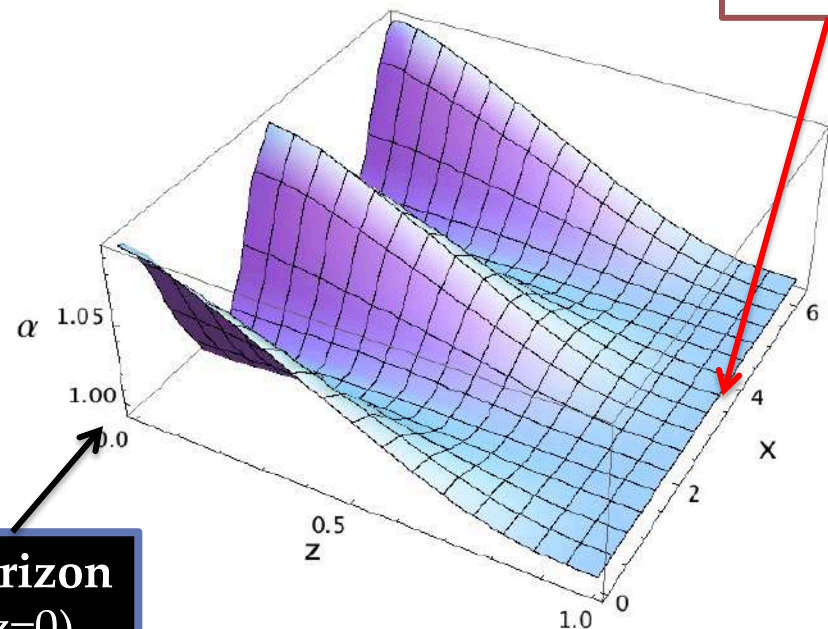
$$ds^2 = \frac{r_h^2}{L^2(1-z^2)^2}\left[-F(z)Q_{tt}dt^2 + \frac{4z^2L^4Q_{zz}}{r_h^2F(z)}dz^2 + Q_{xx}(dx - 2z(1-z^2)^2Q_{xz}dz)^2 + Q_{yy}dy^2\right]$$

$$A_t = \mu\, z^2\alpha, \quad B_t = z^2\beta, \quad \chi = (1-z^2)\phi$$

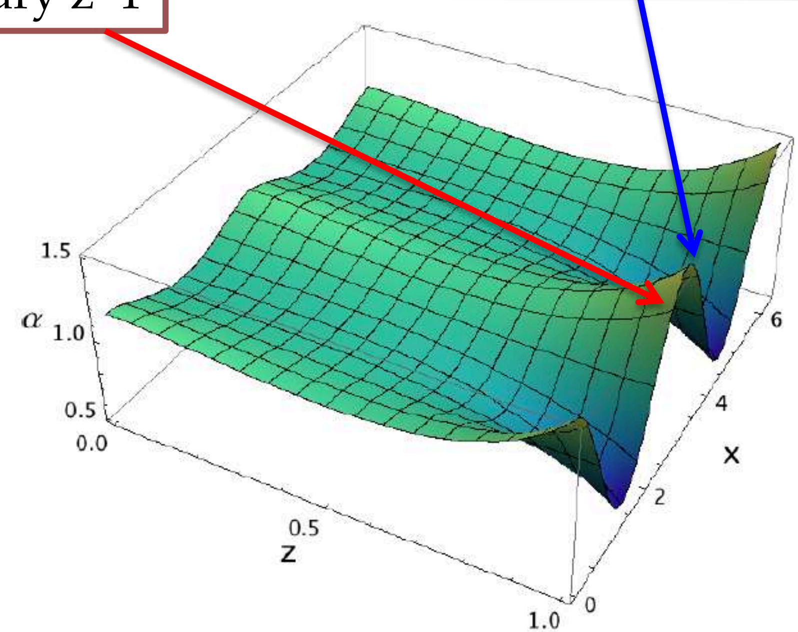
$$\mu(x) = A_t(1,x) = \mu[1 + a_0\cos(p\,x)]$$

Typical profile for gauge field

AdS boundary z=1



Horizon
(z=0)



PDW order
(spontaneously breaking)

PDW +ionic lattice
(explicit breaking in UV, spontanous in IR)

Spectral function and criteria for Fermi surface

- Periodicity of spatial modulation sets the size of **Umklapp** vector K
- Solutions will reflect periodicity of background:
  Bloch expansion

$$k_x \in [-\frac{K}{2}, \frac{K}{2}]$$

$$\Psi_\alpha = \int \frac{d\omega dk_x dk_y}{2\pi} \sum_{n=0,\pm1,\pm2,\cdots} \mathcal{F}_\alpha^{(n)}(z,\omega,k_x,k_y) e^{-i\omega t + i(k_x+nK)x + ik_y y}$$

n: Brillouin zone
K: Umklapp vector

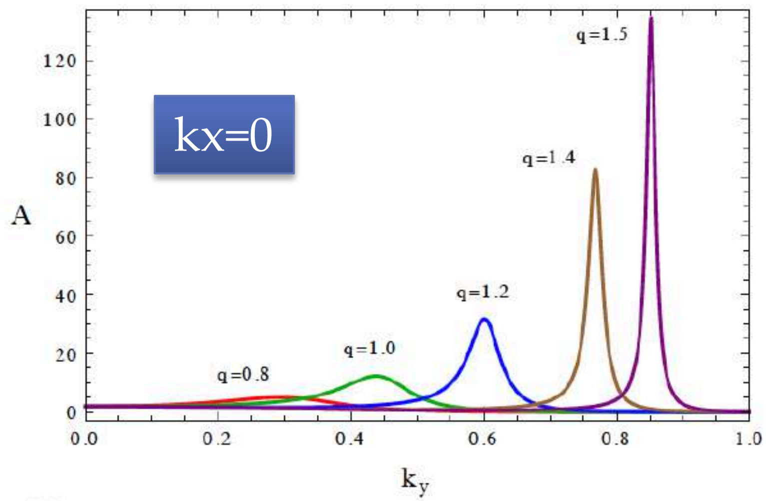
Spectral function

$$A(\omega,k_x,k_y) = \sum_{n=0,\pm1,\pm2,\cdots} \mathrm{Tr}\ \mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega,k_x,k_y)]$$

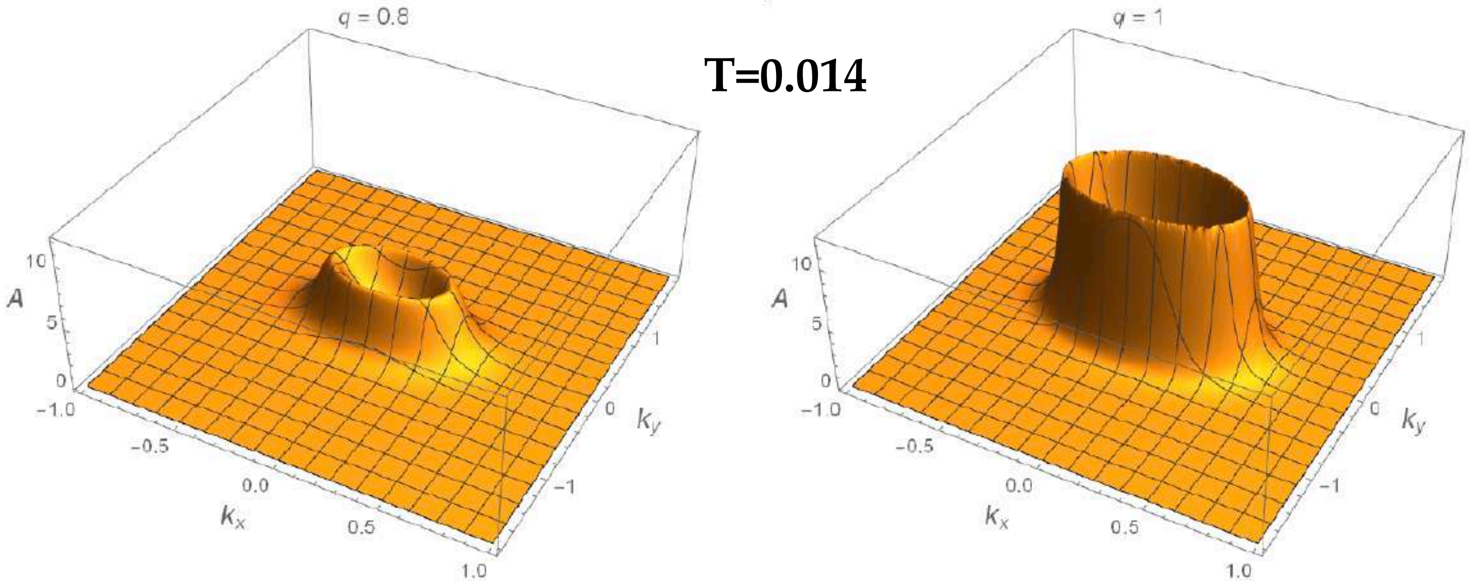
- At zero T Fermi surface: pole in spectral function as w=0 (with respect to the chemical potential).
- **Finite T criteria** to identify Fermi surface: **width, frequency** and **magnitude** criteria introduced by C. H-Horeau and S. Gubser [1411.5384].

Fermi surface develops when the fermionic charge q is large enough

Peaks in the spectral function become sharper and sharper as q is increased.



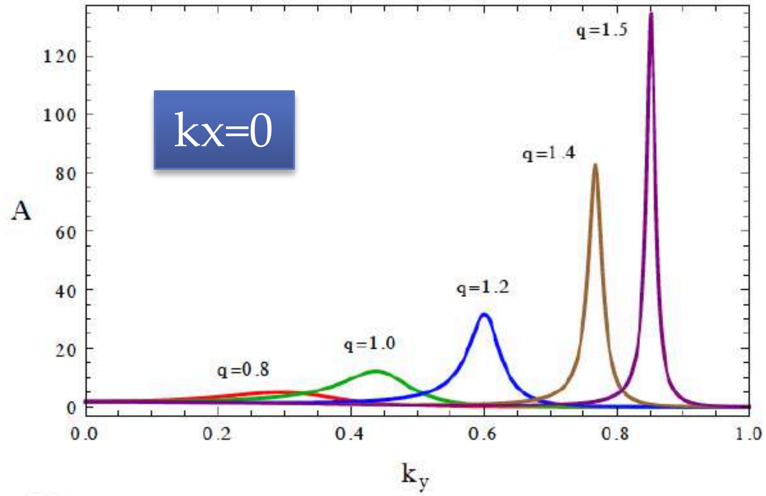
T=0.014



$\omega = 10^{-6}$

Fermi surface develops when the fermionic charge q is large enough

Not a Fermi surface



Peaks in the spectral function become sharper and sharper as q is increased.

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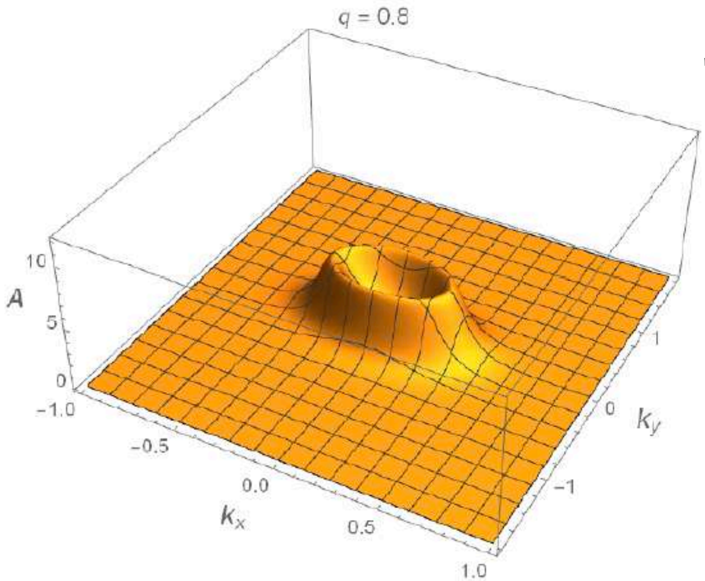

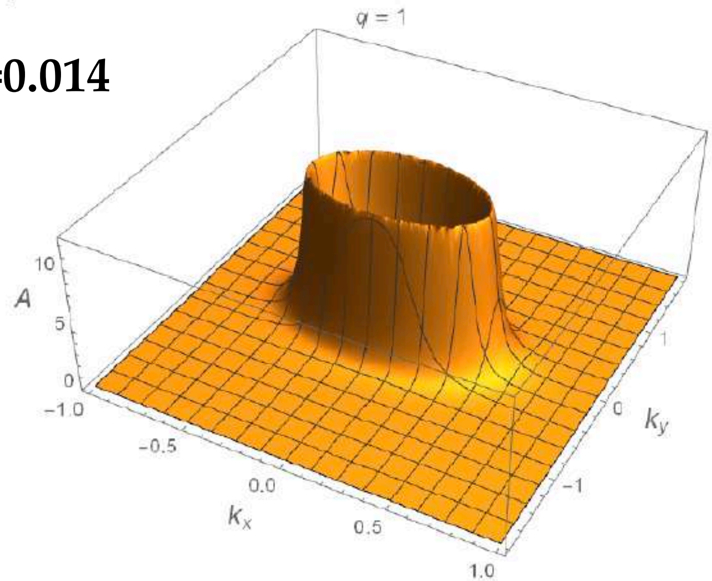

$\omega = 10^{-6}$

Fermi surface develops when the fermionic charge q is large enough

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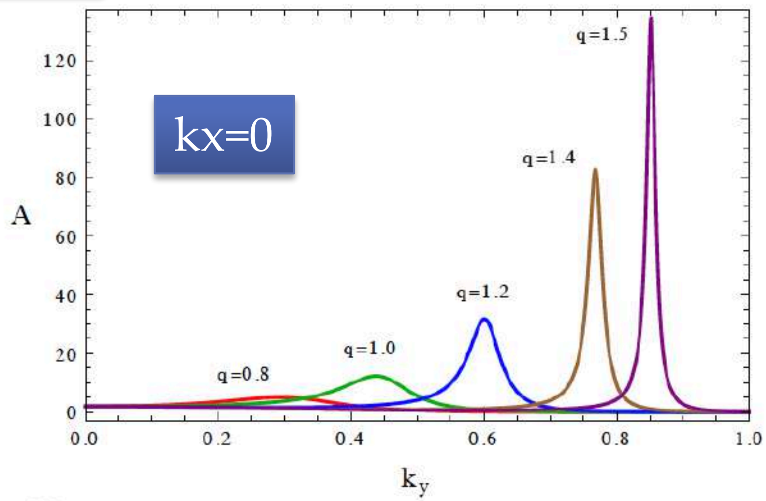
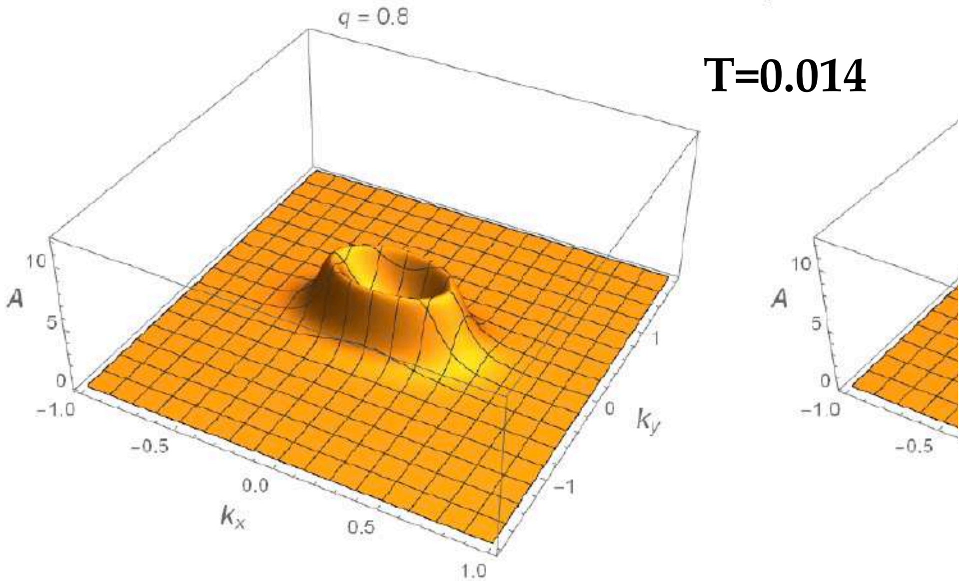
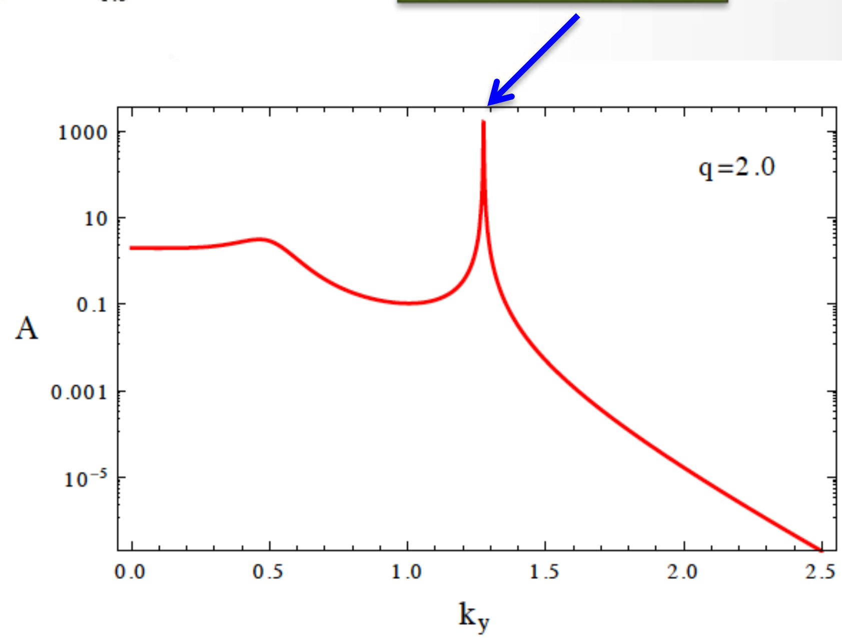


Peaks in the spectral function become sharper and sharper as q is increased.

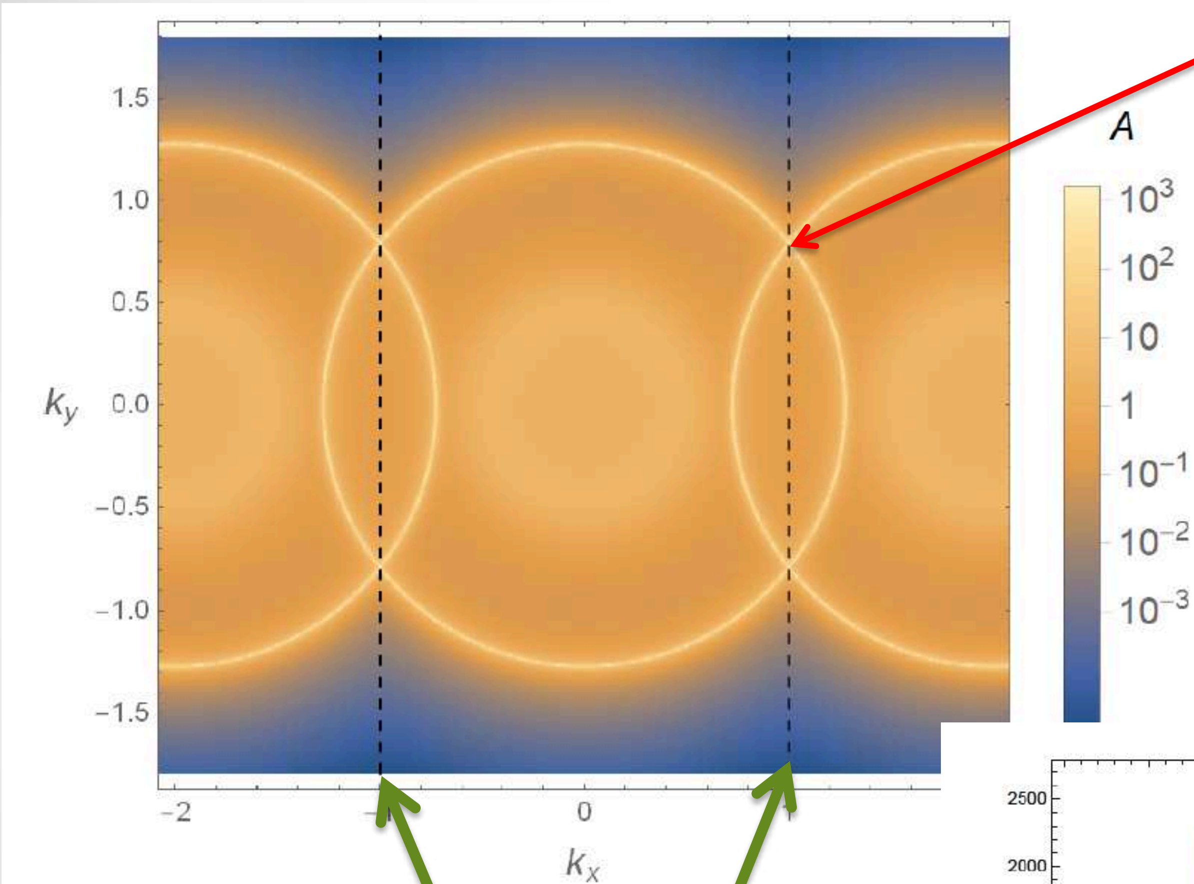


T=0.014

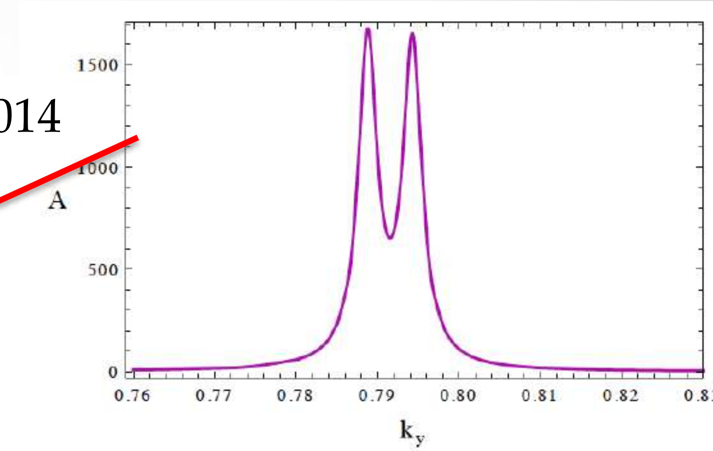
Fermi surface



Spontaneous Case



T=0.014

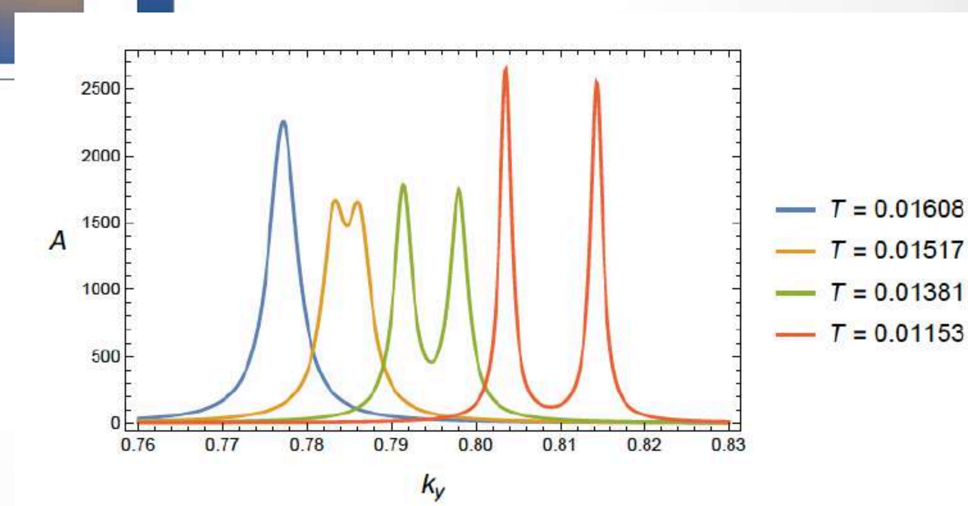


Gap opens up below Tc and increases as T is lowered

$q = 2, \omega = 10^{-6}$

T=0.014

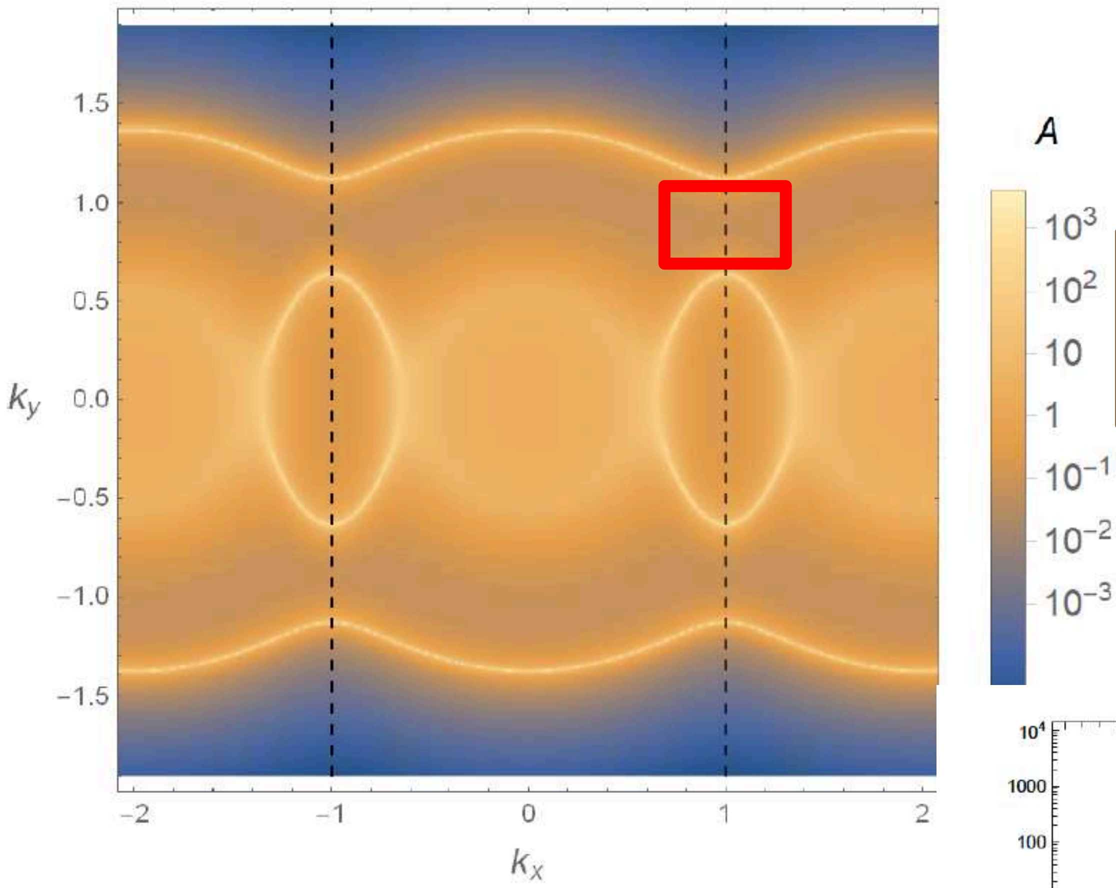
Brillouin zone boundary



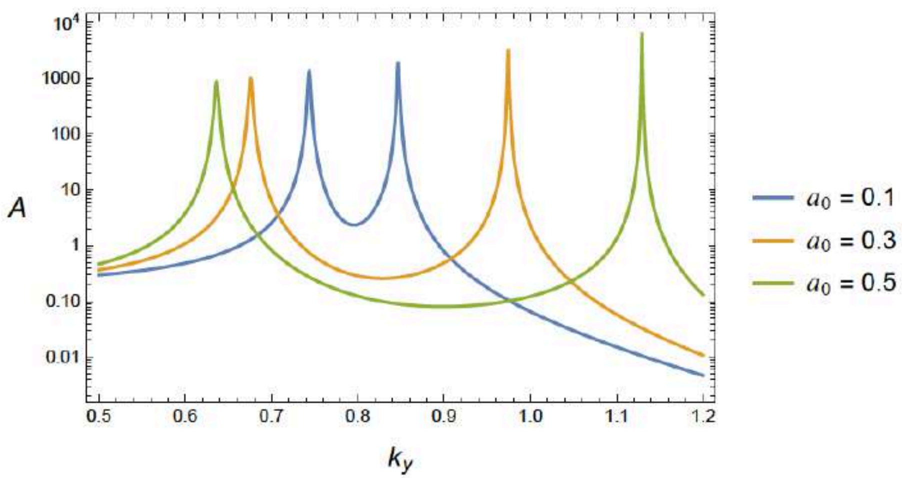
Add Ionic Lattice

$$\mu(x) = A_t(1,x) = \mu[1 + a_0 \cos(p\,x)]$$

$$a_0 = 0.5,\ p = 2.$$



More pronounced anisotropy and larger gap with lattice amplitude increased

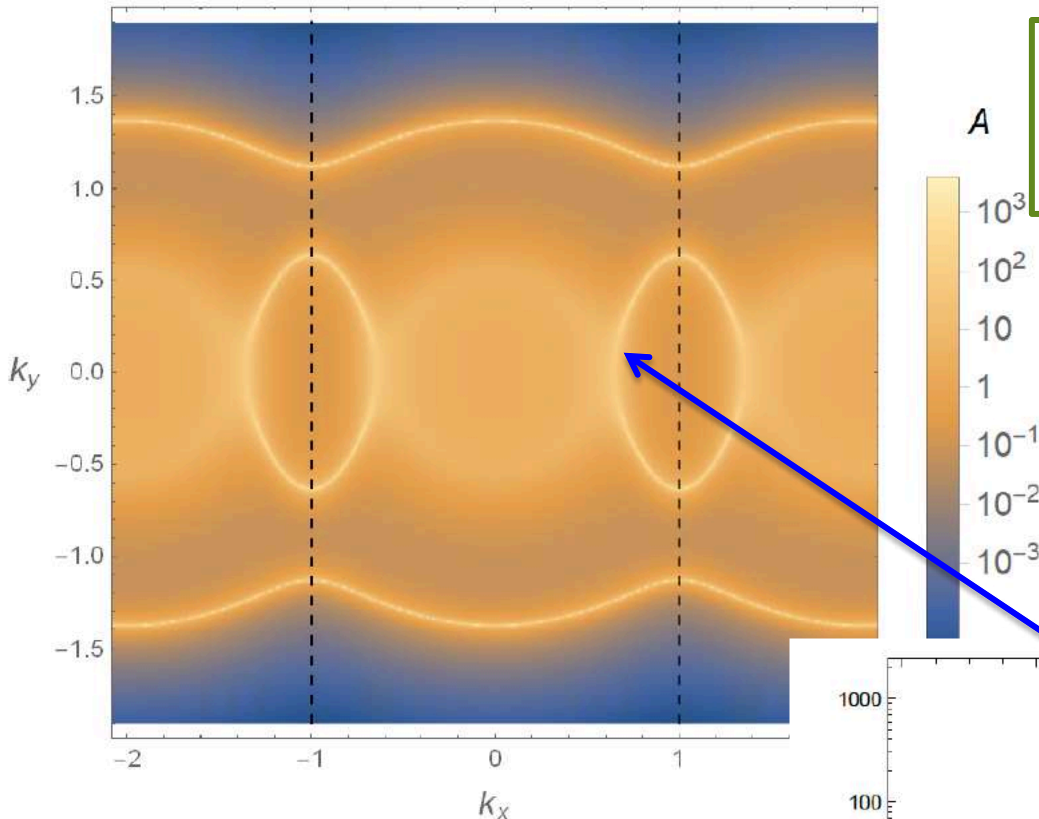


$q = 2, \omega = 10^{-6}.$

T=0.014

More interesting feature:

Part of Fermi surface gradually dissolves with strong inhomogeneity effect

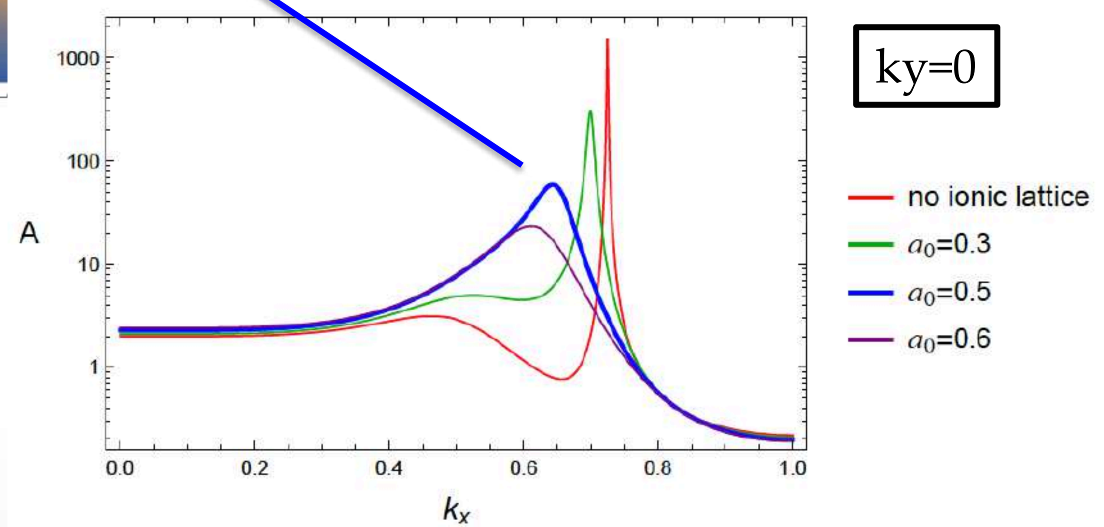


Peaks in spectral function are suppressed with large lattice strength along symmetry breaking direction

Fermi surface gradually dissolves leaving behind detached segments

$q = 2, \omega = 10^{-6}$,

T=0.014



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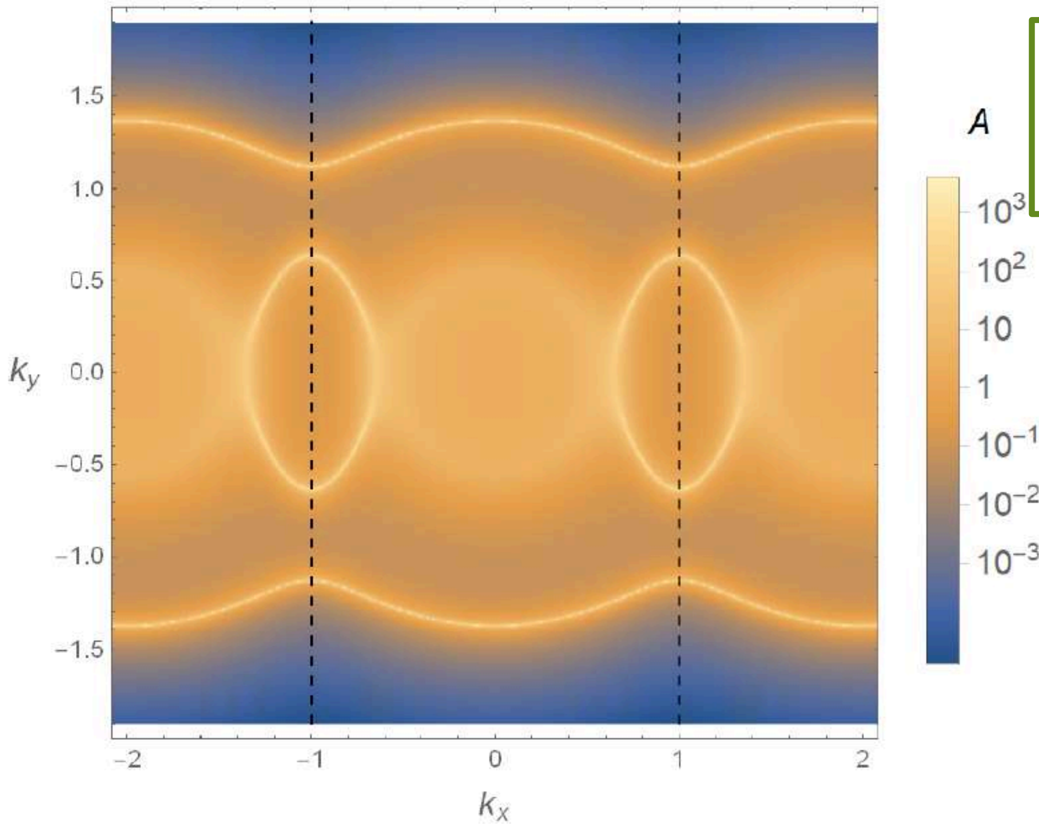
Fermi surface gradually dissolves with strong inhomogeneity effect



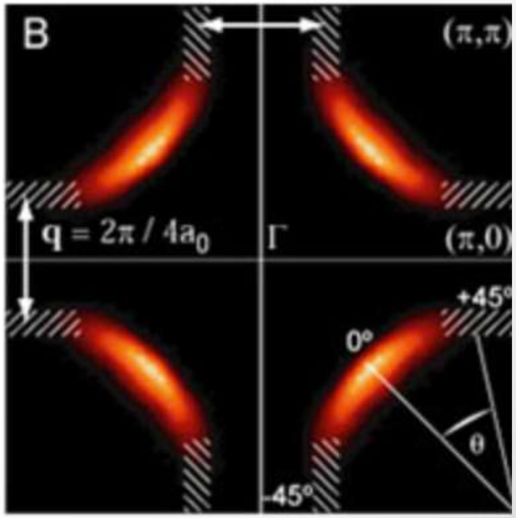
$q = 2, \omega = 10^{-6}$,
T=0.014

Peaks in spectral function are suppressed with large lattice strength along symmetry breaking direction

↓

Fermi surface gradually dissolves leaving behind detached segments

Segmentation of Fermi surface is reminiscent of 'Fermi arcs' in high Tc superconductors (pseudo-gap)



[K. M. Shen et al. Science **307**, 901 (2005)]

Key Question:

Is the **spontaneously generated** order required to see the segmented Fermi surface?

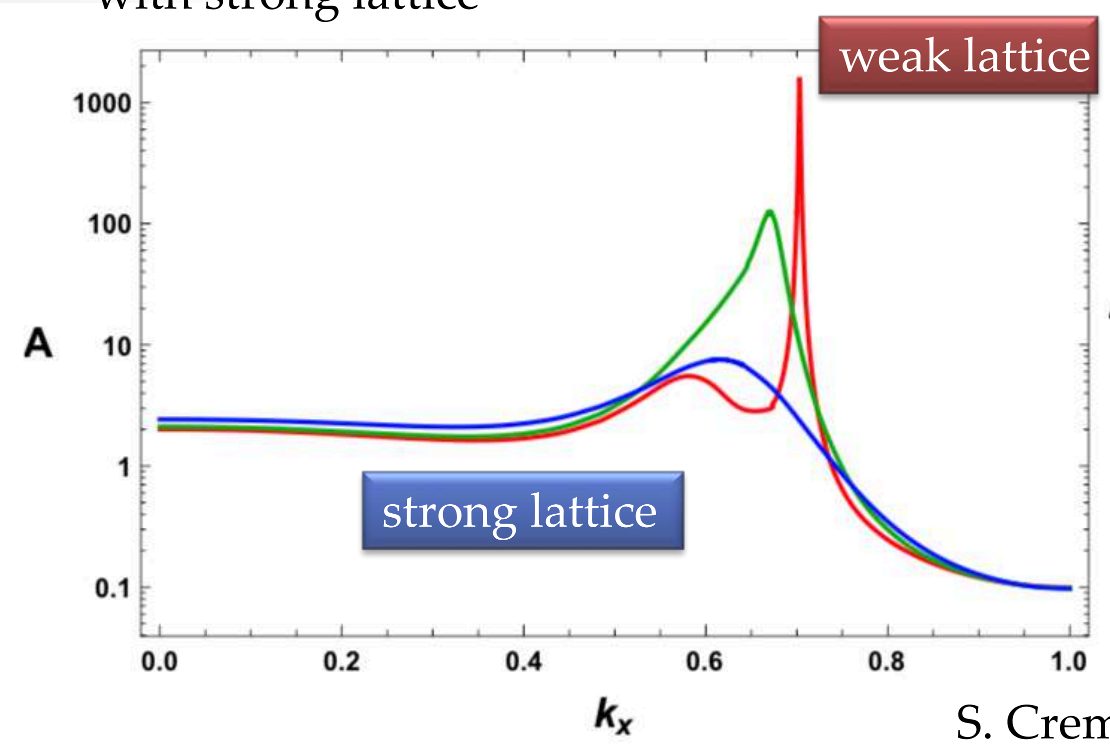
No! Present even with explicit lattice.

Turn off both striped and superconducting orders

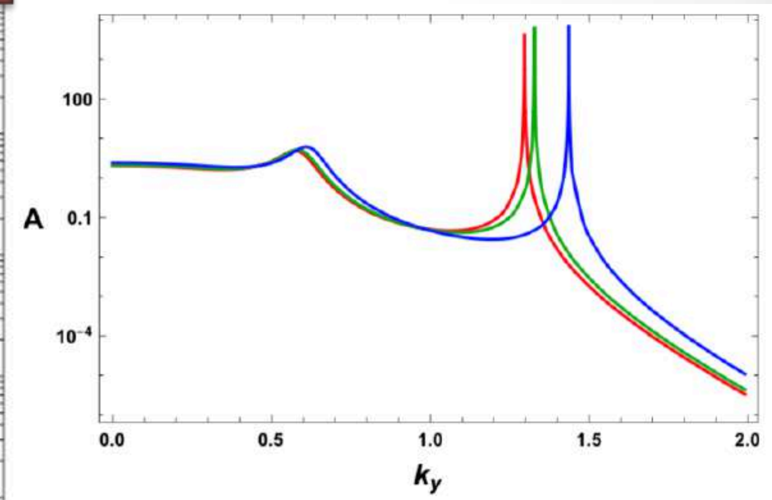
Simple **Einstein-Maxwell model** with an **explicit ionic lattice** provided by a spatially modulated chemical potential

$$\mathcal{L}_m = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \mu(x) = A_t(z=1,x) = \mu[1 + a_0\cos(p_I\, x)]$$

Spectral density suppression observed with strong lattice



No spontaneously broken translations



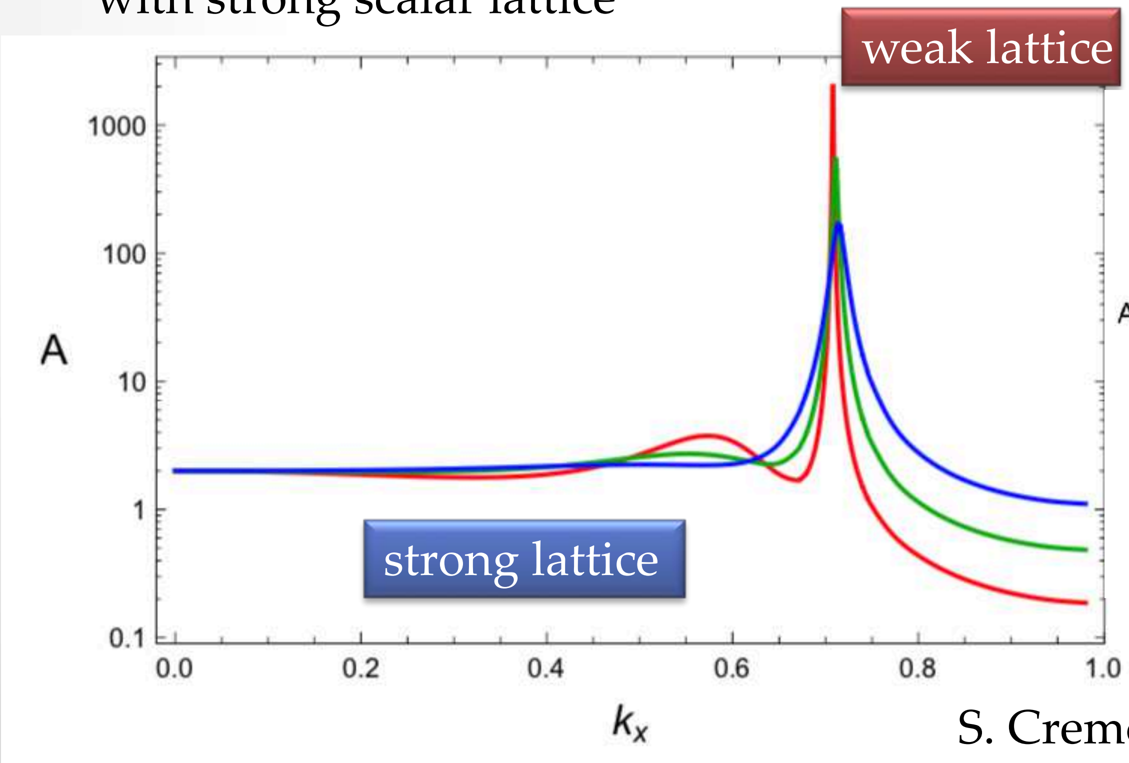
S. Cremonini, **LL**, J. Ren, 1906.02753

Turn off both striped and superconducting orders

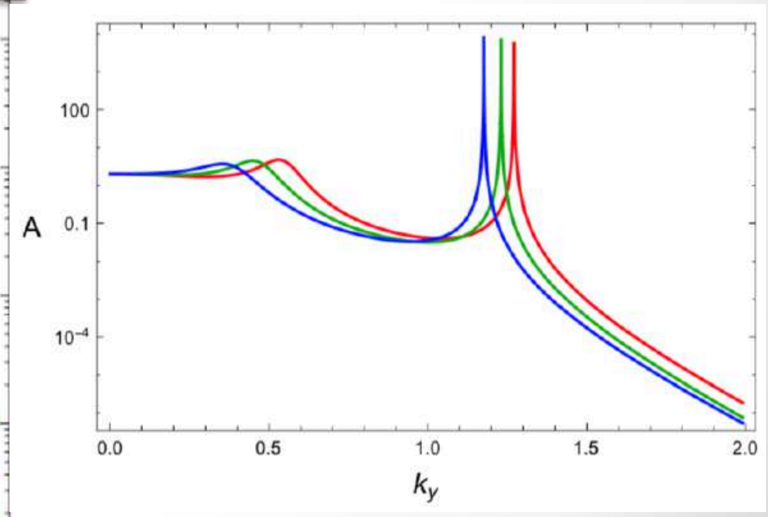
Einstein-Maxwell-scalar model with an **explicit scalar lattice** provided by the source of the scalar operator

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\chi) \qquad \phi_s(x) = A_0\cos(p_S\, x)$$

Spectral density suppression observed with strong scalar lattice



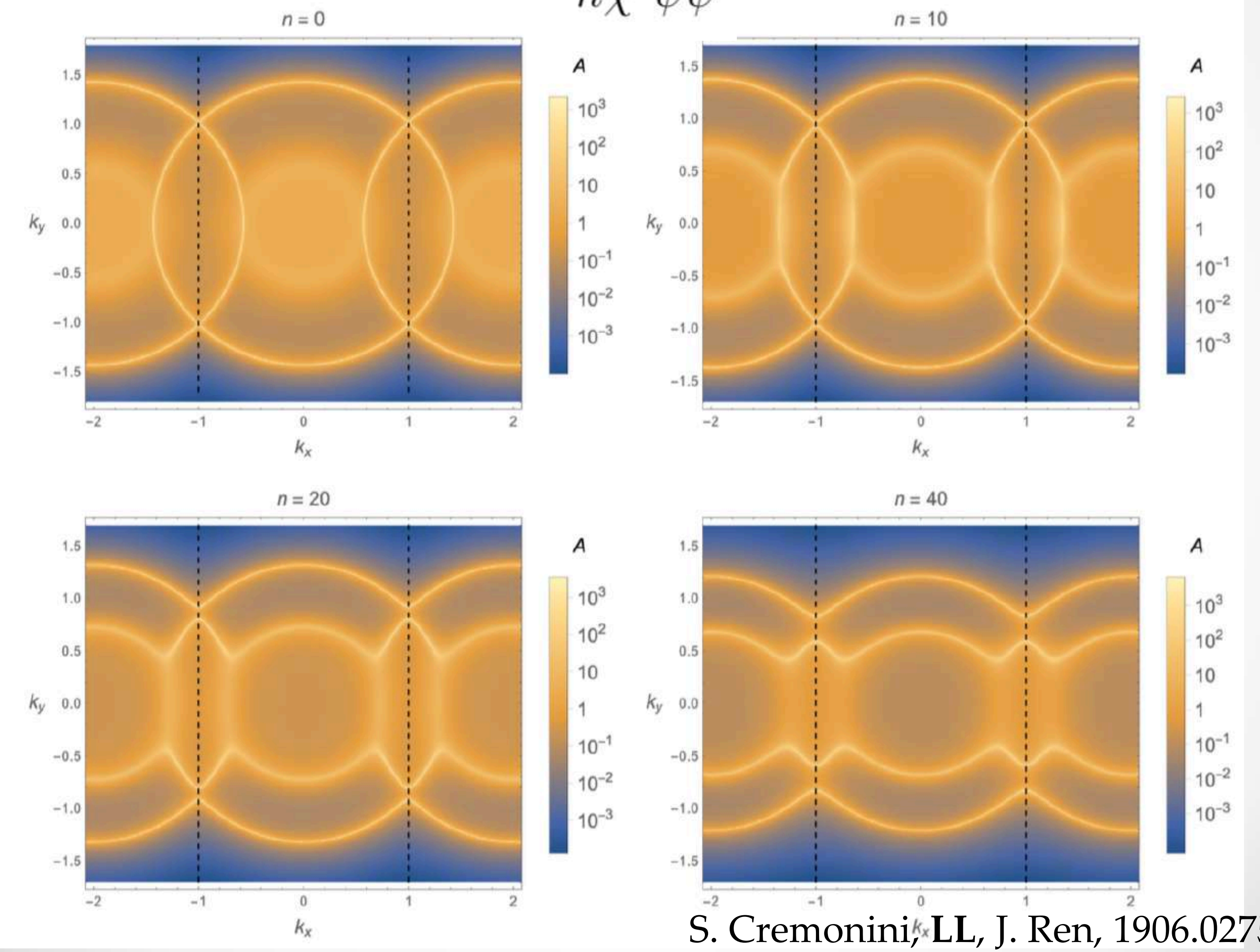
No spontaneously broken translations



S. Cremonini, **LL**, J. Ren, 1906.02753

CDW case (q_A=q_B=0)

To amplify the effect of the spontaneously generated modulations on the fermion, we add a new term $n\chi^2\bar{\psi}\psi$



S. Cremonini, **LL**, J. Ren, 1906.02753

To sum up

While the fine structure of the Fermi surface is sensitive to the details of the theory, Fermi surfaces will be **generically suppressed** when the **inhomogeneity effect is strong** enough.

The real **origin** of the spectral weight suppression is **still unclear**.

F. Balm et al, 1909.09394

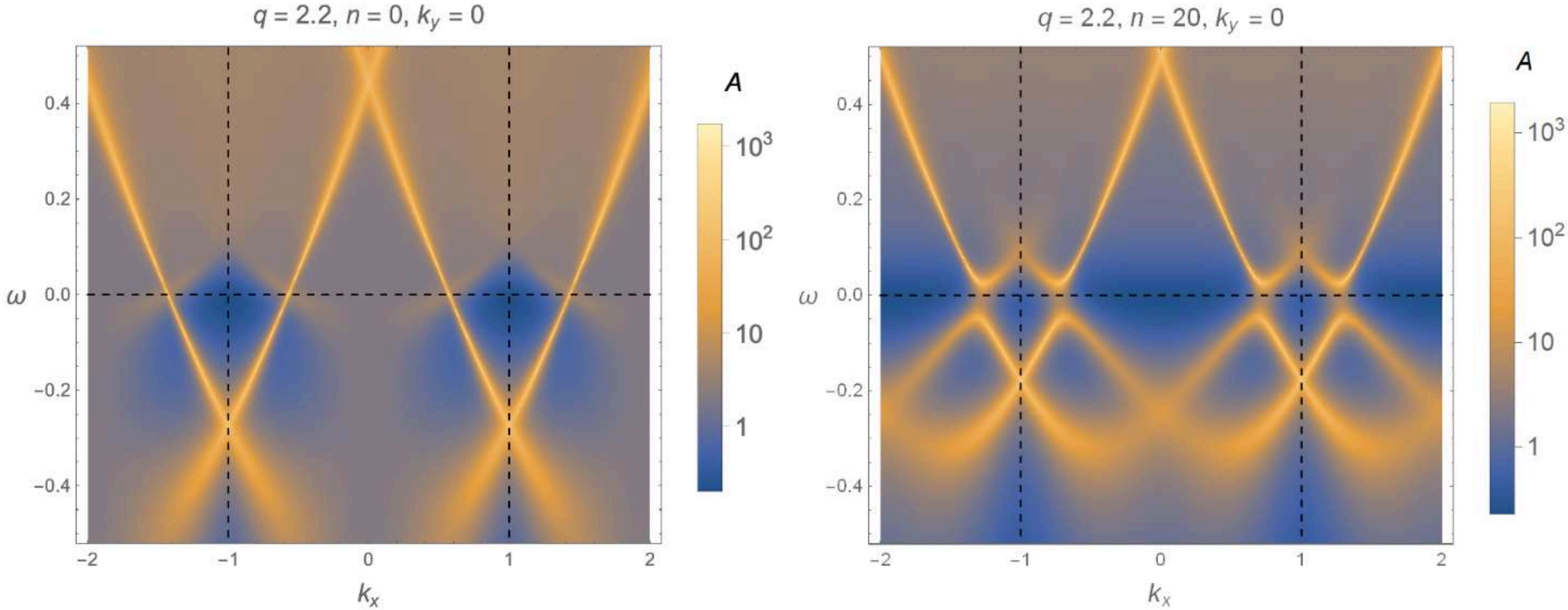
a collision of the Fermi surface pole with zeros of the fermionic Green's function

A. Iliasov, et al, 1910.01542

due to the anisotropic features of the holographic horizon

A possible explanation

Increasing the lattice amplitude might lift the energy band above the Fermi level, due to the strong Umklapp eigenvalue repulsion.



The intersections between the horizontal dashed line ($\omega = 0$) and the brightest points correspond to the location of the Fermi surface.

There is an "energy gap" opening near $\omega = 0$ due to increasing the lattice strength.

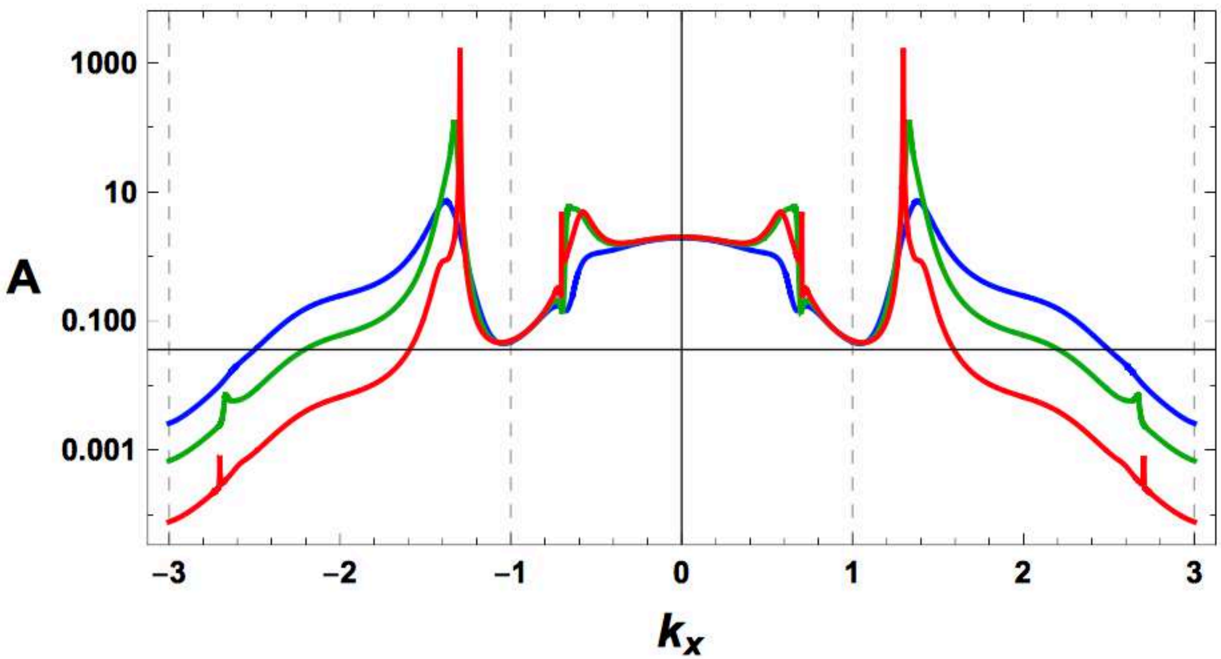
(Thanks A.Krikun and K.Schalm for raising this point with us)

Representation of the spectral density without folding

extended zone scheme

$$A(\omega, k_x = k_0 + nK, k_y) = \mathrm{Tr}\,\mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega, k_0, k_y)]$$

with $k_0 \in [-\frac{K}{2}, \frac{K}{2}]$ and n denoting the momentum level or Brillouin zone.



The spectral density is **not periodic** in the extended zone.

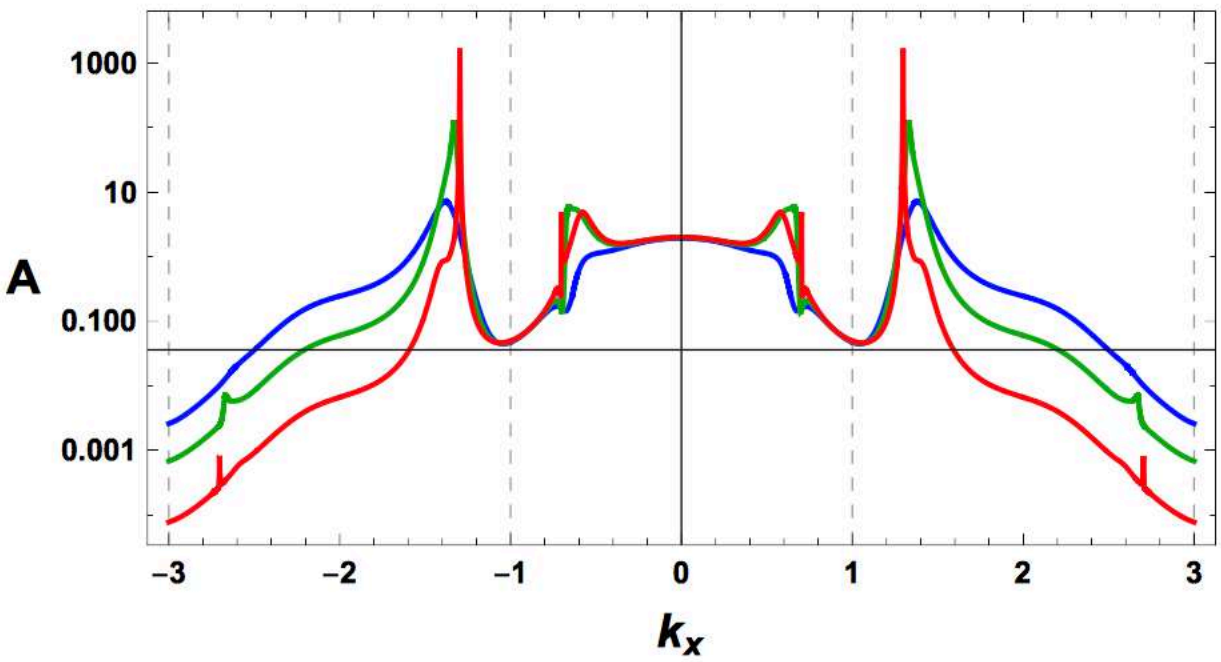
When the peaks appearing in each Brillouin zone are sufficiently sharp, they differ from each other by the Umklapp wave vector K.

Representation of the spectral density without folding

extended zone scheme

$$A(\omega, k_x = k_0 + nK, k_y) = \mathrm{Tr}\,\mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega, k_0, k_y)]$$

with $k_0 \in [-\frac{K}{2}, \frac{K}{2}]$ and n denoting the momentum level or Brillouin zone.

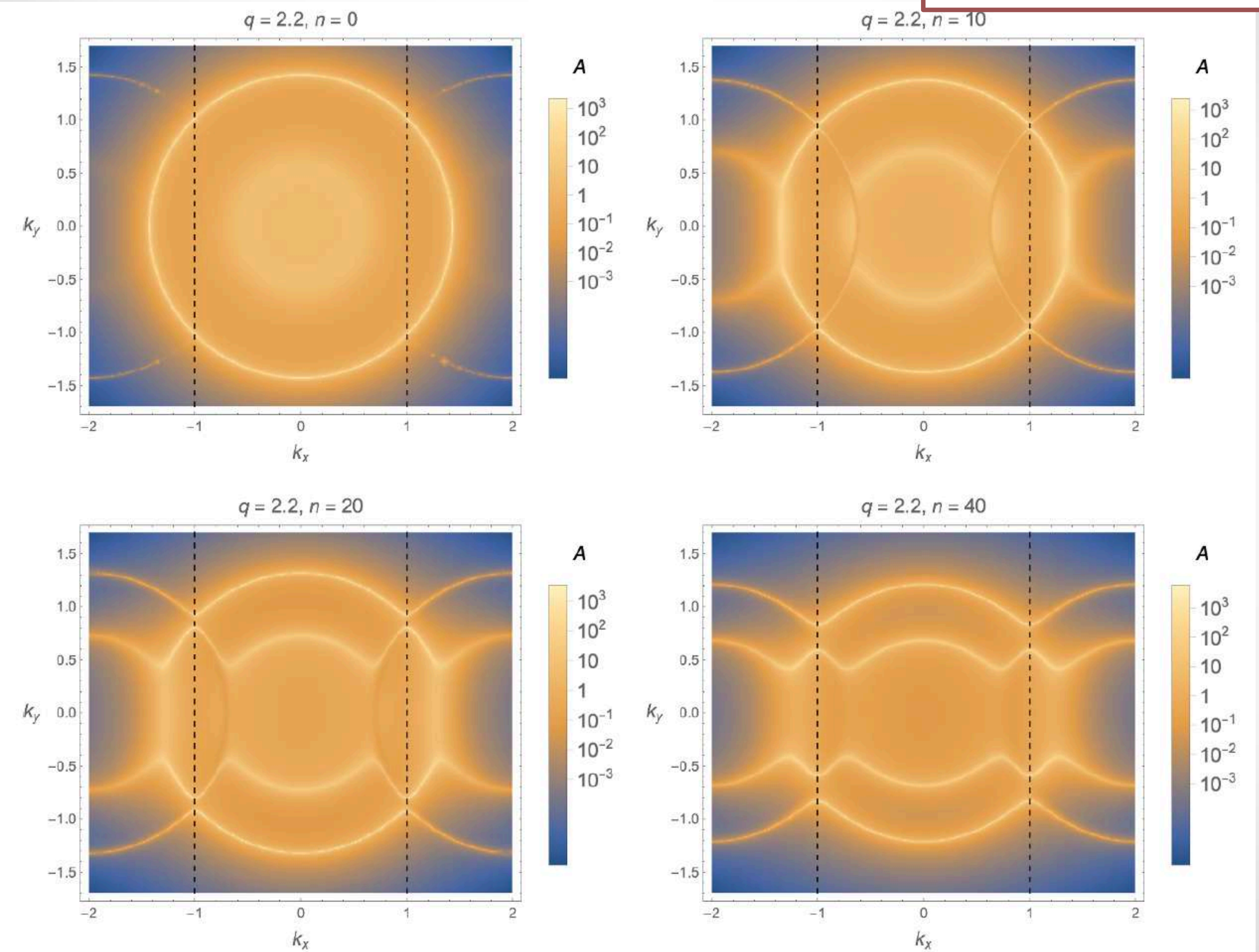


Jan Zaanen

The **non-periodicity** is a generic feature and a direct probe of the **non-Fermi liquid** nature of electron matter, and it would be very interesting to test it experimentally.

CDW case (q_A=q_B=0)

extended zone scheme



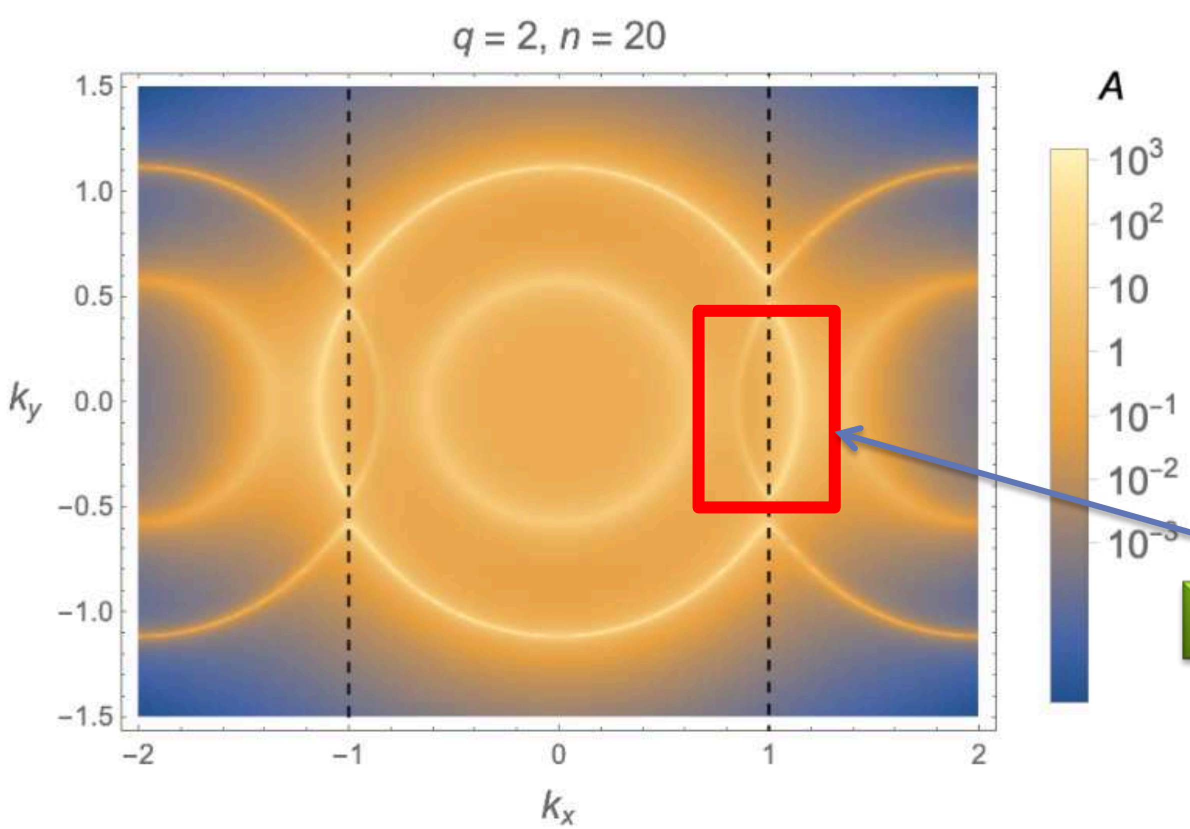
S. Cremonini, **LL**, J. Ren, 1906.02753

Representation of the spectral density without folding

extended zone scheme

$$A(\omega, k_x = k_0 + nK, k_y) = \mathrm{Tr}\,\mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega, k_0, k_y)]$$

with $k_0 \in [-\frac{K}{2}, \frac{K}{2}]$ and n denoting the momentum level or Brillouin zone.

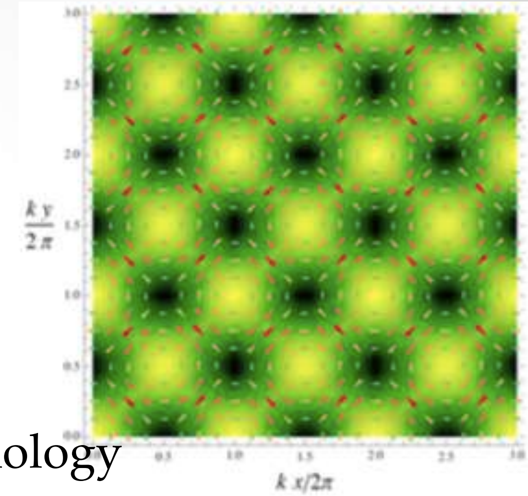


Fermi Arc or pocket?

There is an asymmetry across the Brillouin zone boundary.

III

Conclusion

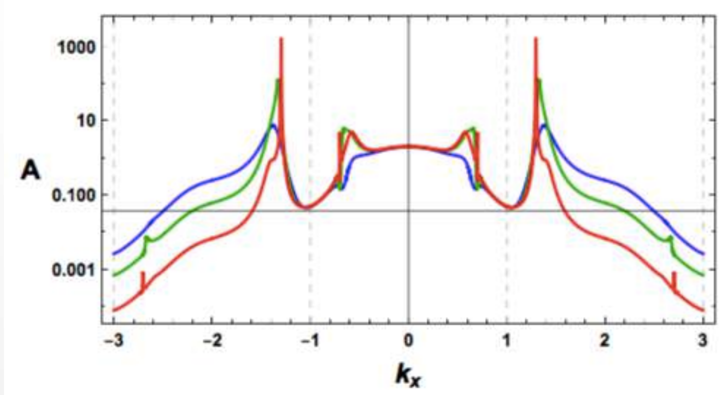


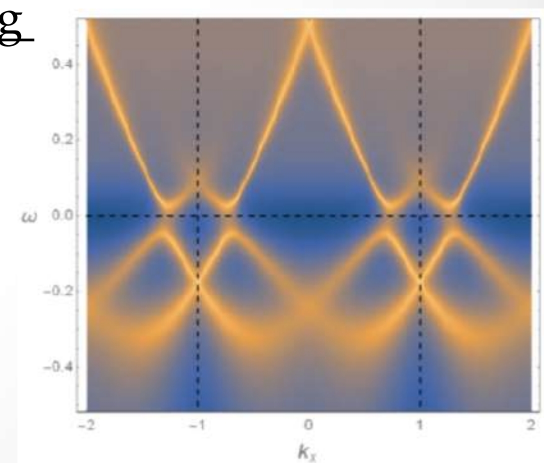
Holography as a Theoretical Laboratory:

(a). Periodic stripes/lattices display interesting phenomenology which is relevant to experimental observations

(b). Umklapp effects can only be seen in spatially dependent backgrounds (homogeneous lattices don't have it)

(c) Disappearance of the Fermi surface seems to be a <u>generic feature of strong translational symmetry breaking</u>





Open questions:

- The nature of the ground state at T=0 for the striped geometry?
- Transport properties and Dispersion relation (excitation from electrons or holes)?
- **Segmented pieces** are left over->related to Fermi arcs? Generic result of strong inhomogeneity? compare to experiments?
- Fermi surface for the fully crystallised solutions ?
- ……

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At the present stage the outcomes of the holographic exercise presented in the above offer no more than a rough cartoon. However, the cartoon is suggestive with regard to generalities.

Map the bulk theory to the real word system?

HoloTube

The Applied Holography Webinars Network

Thank you !