Axion hot dark matter bound, reliably

ESRs Webinar - 11/01/2021

Gioacchino Piazza

IJCLab, Pôle Théorie, CNRS node

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860881-HIDDeN

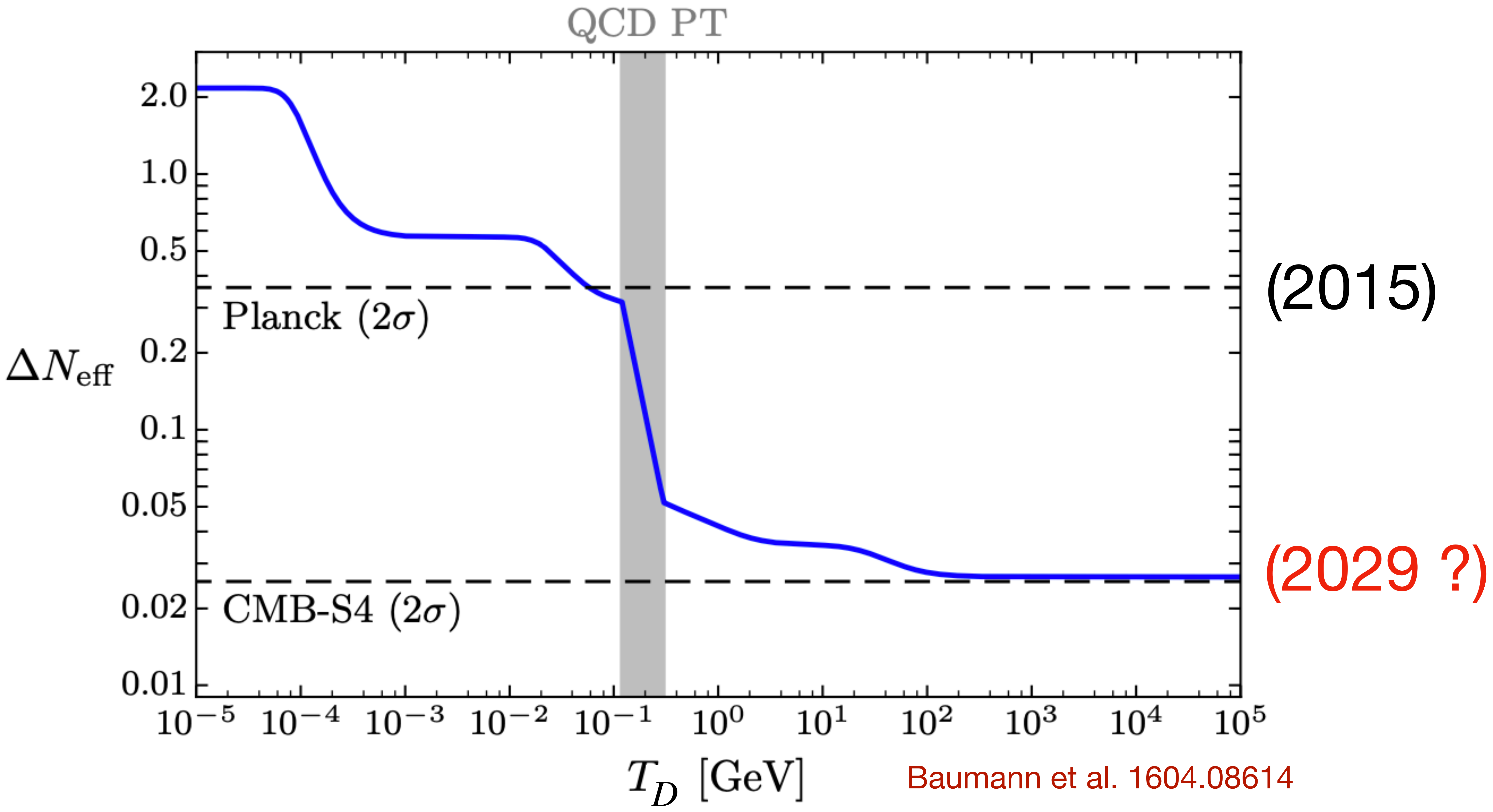
A possible discovery channel for the axion!

Axion contributes to the Number of extra relativistic species

- Axions once in equilibrium with SM bath contribute to the radiation density of the Universe

- T_D depends on the strength of the axion interactions set by f_a

- Full range of allowed $\Delta N_{\rm eff}$ will be covered by CMB-S4



Baumann et al. 1604.08614

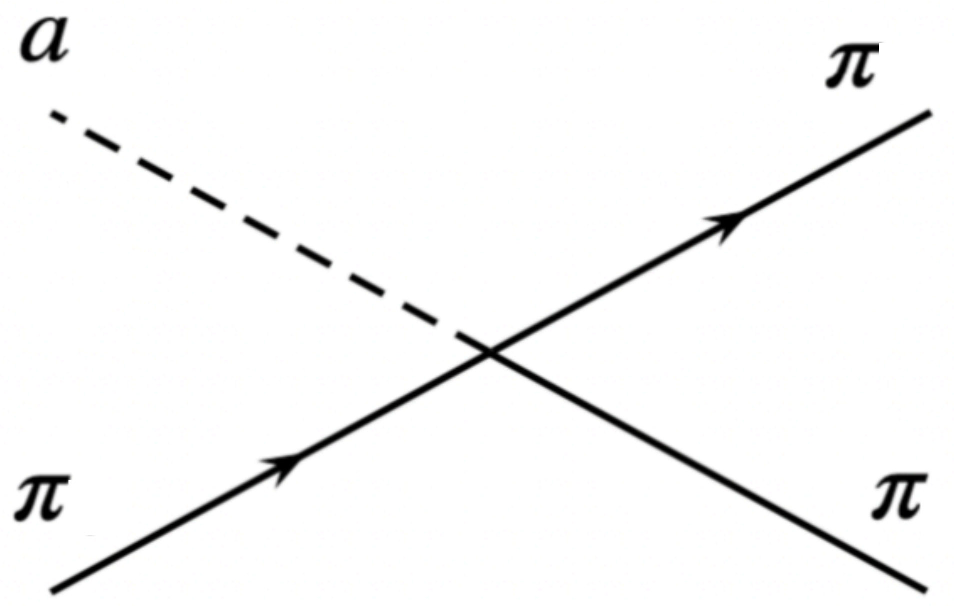
A possible discovery channel for the axion!

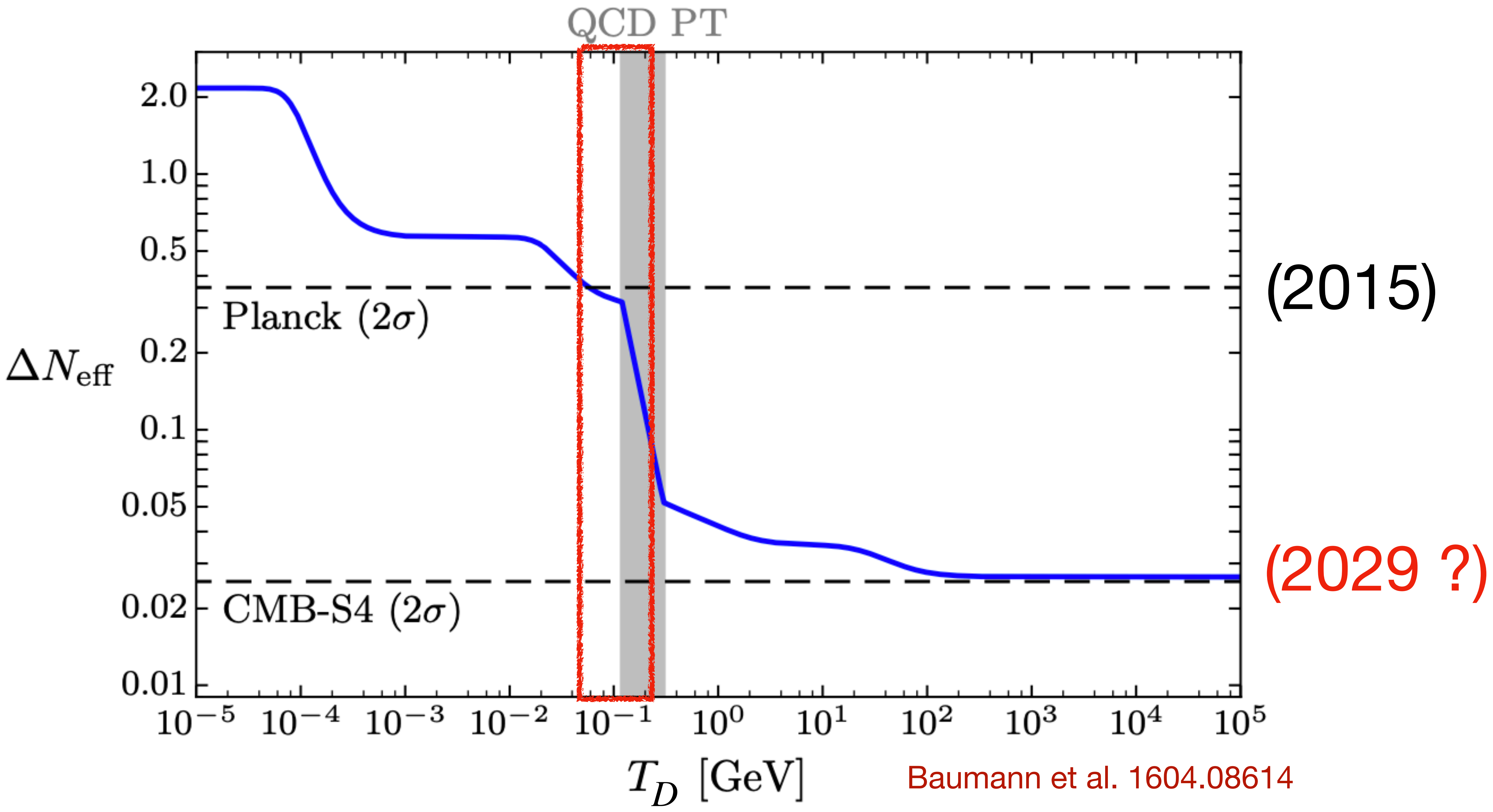
Axion contributes to the Number of extra relativistic species

- Axions once in equilibrium with SM bath contribute to the radiation density of the Universe

$$T_D \sim [50 \text{ MeV}, 200 \text{ MeV}]$$

- Below 200 MeV the main thermalization channel is





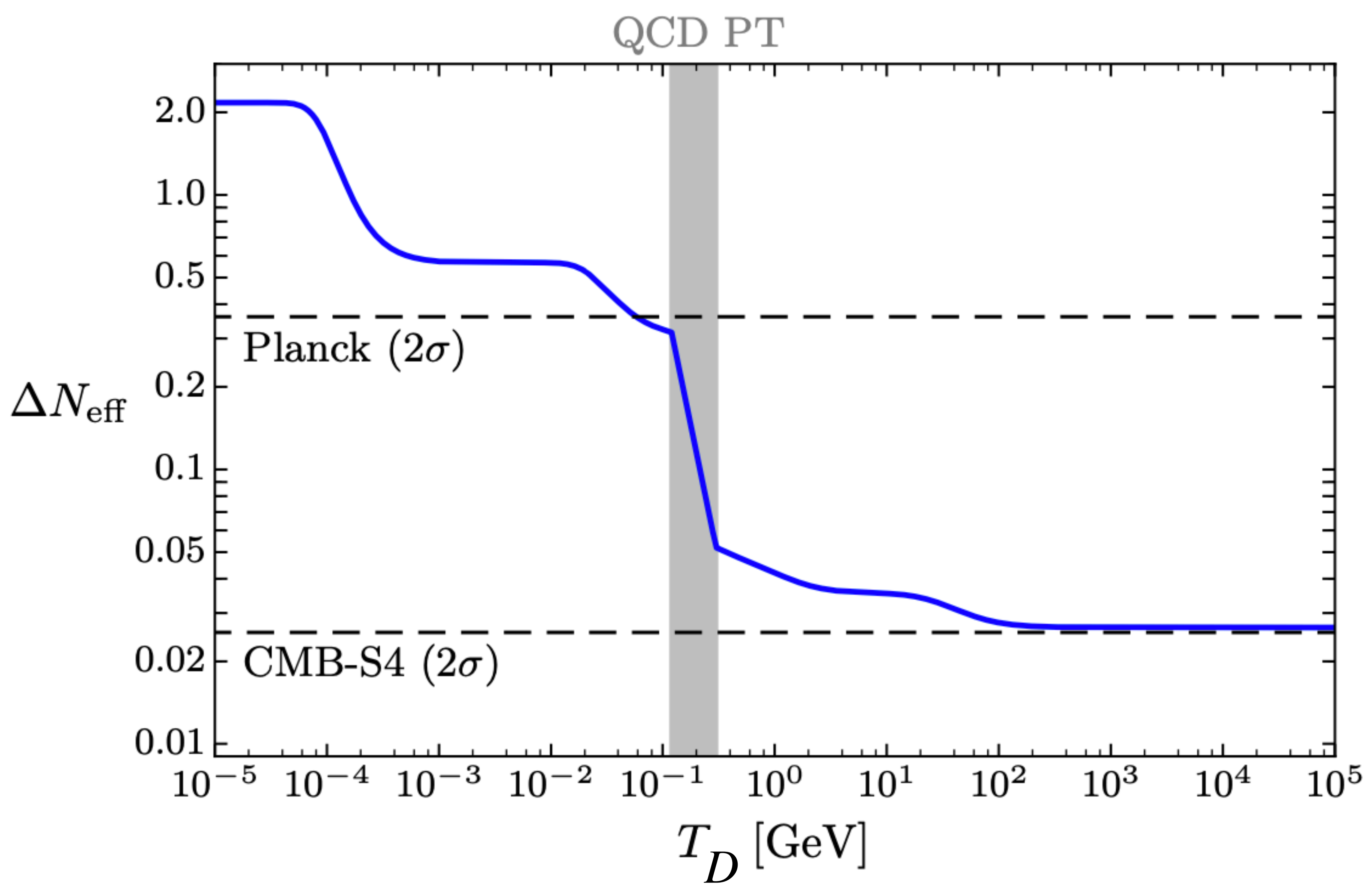
Baumann et al. 1604.08614

Axion hot dark matter bound, reliably

Luca Di Luzio,[1,*] Guido Martinelli,[2,†] and Gioacchino Piazza[3,‡]

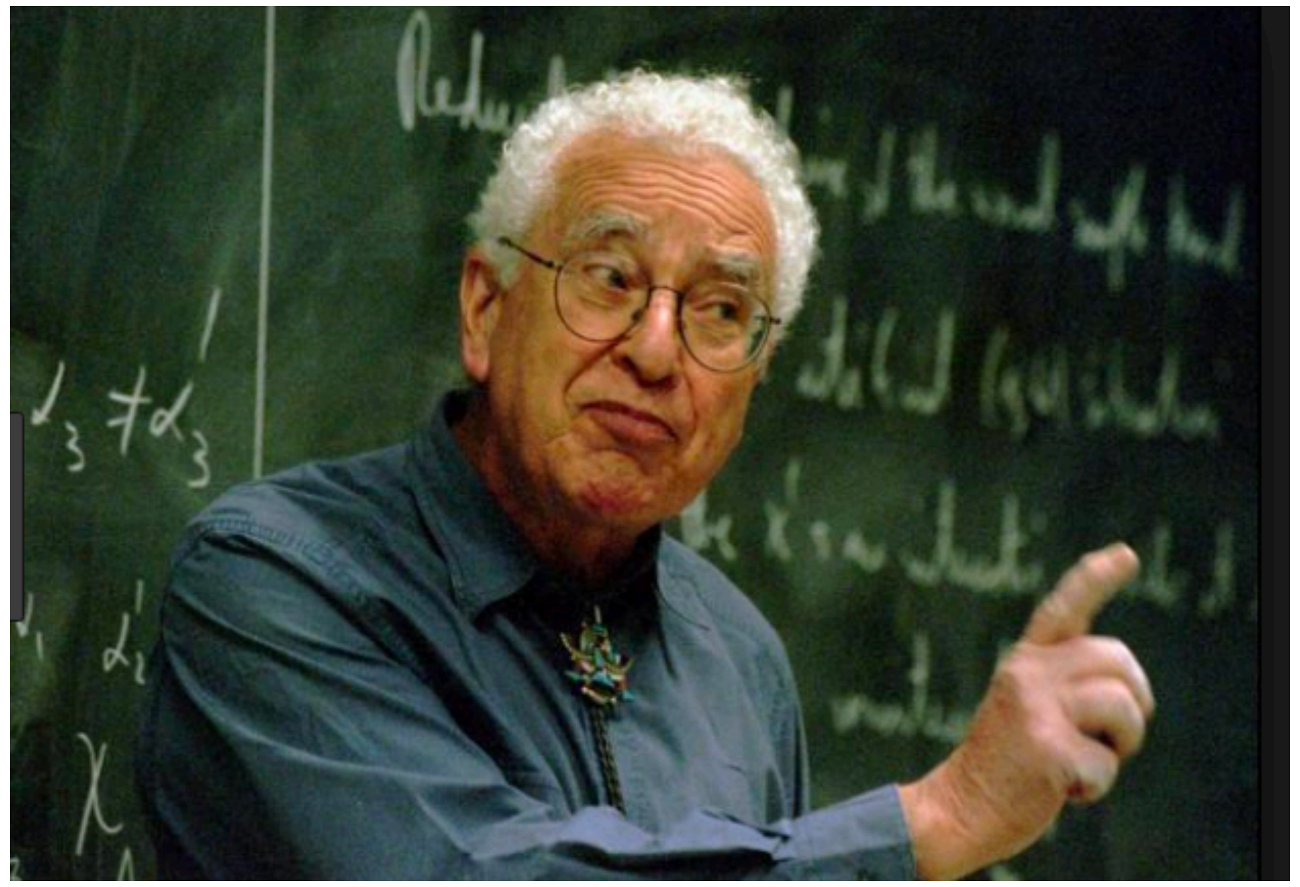
[1] *DESY, Notkestraße 85, D-22607 Hamburg, Germany*
[2] *Physics Department and INFN Sezione di Roma La Sapienza, Piazzale Aldo Moro 5, 00185 Roma, Italy*
[3] *IJCLab, Pôle Théorie (Bât. 210), CNRS/IN2P3 et Université Paris-Saclay, 91405 Orsay, France*

Outline



1. **Strong CP problem & The Axion**
2. **Axion-Pion Effective Lagrangian: Leading Order**
3. **Axion Hot Dark Matter bound, LO**
4. **Why HDM bound is not reliable:**
    1. **Full axion-pion Lagrangian at NLO**
    2. $a\pi \leftrightarrow \pi\pi$ **thermalization rate at NLO**
5. **The need for a reliable bound**

Strong CP problem



"Any process which is not forbidden by a conservation law actually does take place with appreciable probability"

[M. Gell-Mann, 1956]

Lorentz & Gauge inv. + QCD vacuum structure = $S \to S_{\text{eff}} = S + i\theta \frac{g^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a$

CP-V: Expectation $\bar{\theta} \sim \mathcal{O}(1)$, but $|\bar{\theta}| \lesssim 10^{-10}$ from **nEDM**

[C. Abel et al. 2001.11966]

WHY?

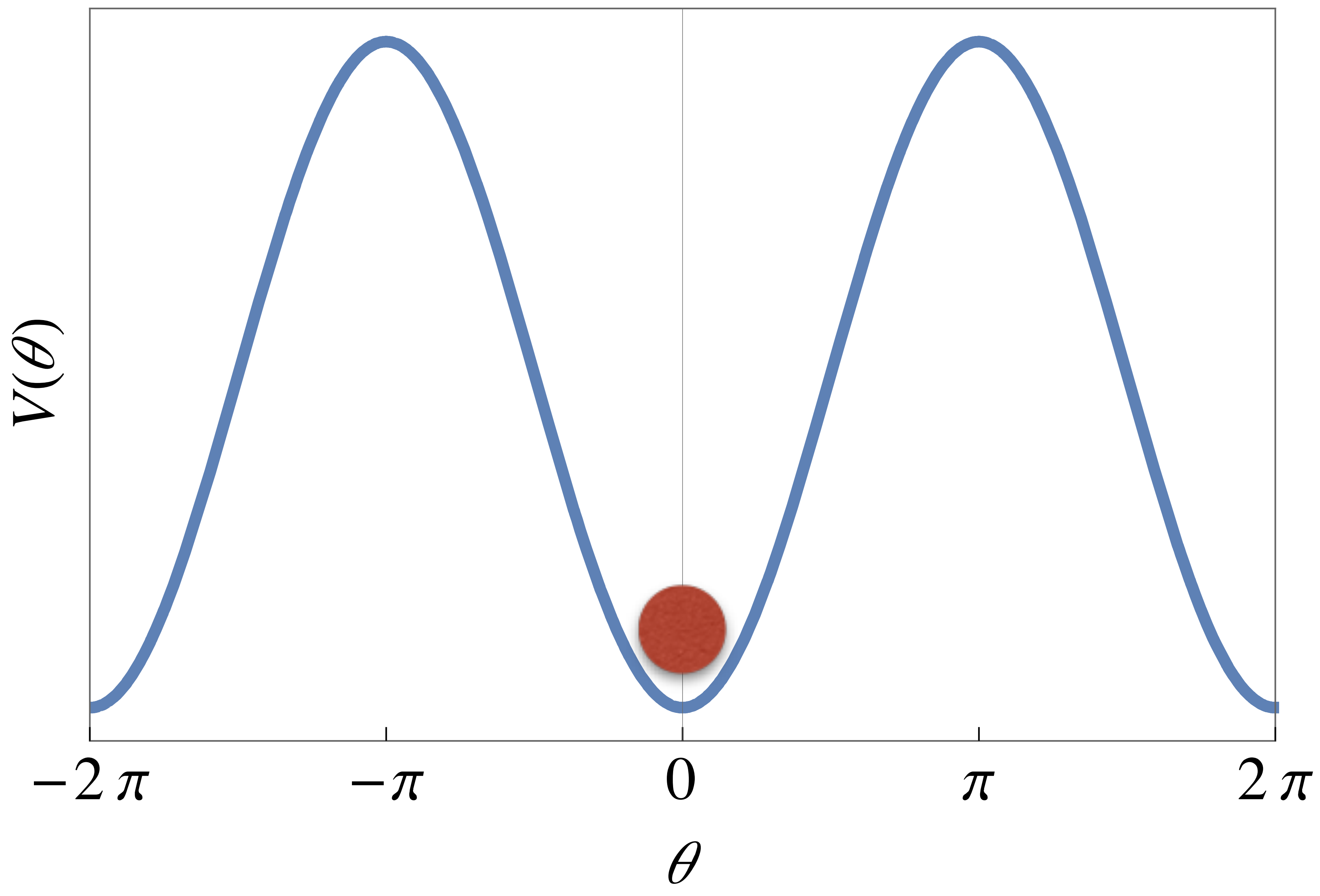
The Axion

- Pseudo-Goldstone Boson of a spontaneously broken global symmetry, with QCD anomaly.
- Solves the Strong CP problem: θ-term dynamically set to zero

$$\delta\mathcal{L}_{\rm QCD} = \theta\,\frac{\alpha_s}{8\pi} G\tilde{G}\ , \ |\theta| \lesssim 10^{-10}$$

$$\theta \to \frac{a}{f_a} \quad \text{with} \quad \langle a \rangle = 0$$

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\,\frac{m_\pi^2 f_\pi^2}{f_a^2}$$



- Rich phenomenology (Dark Matter, Astrophysics, Cosmology)

Axion-Pion Effective Lagrangian: Leading Order

$E < \Lambda_{QCD}$

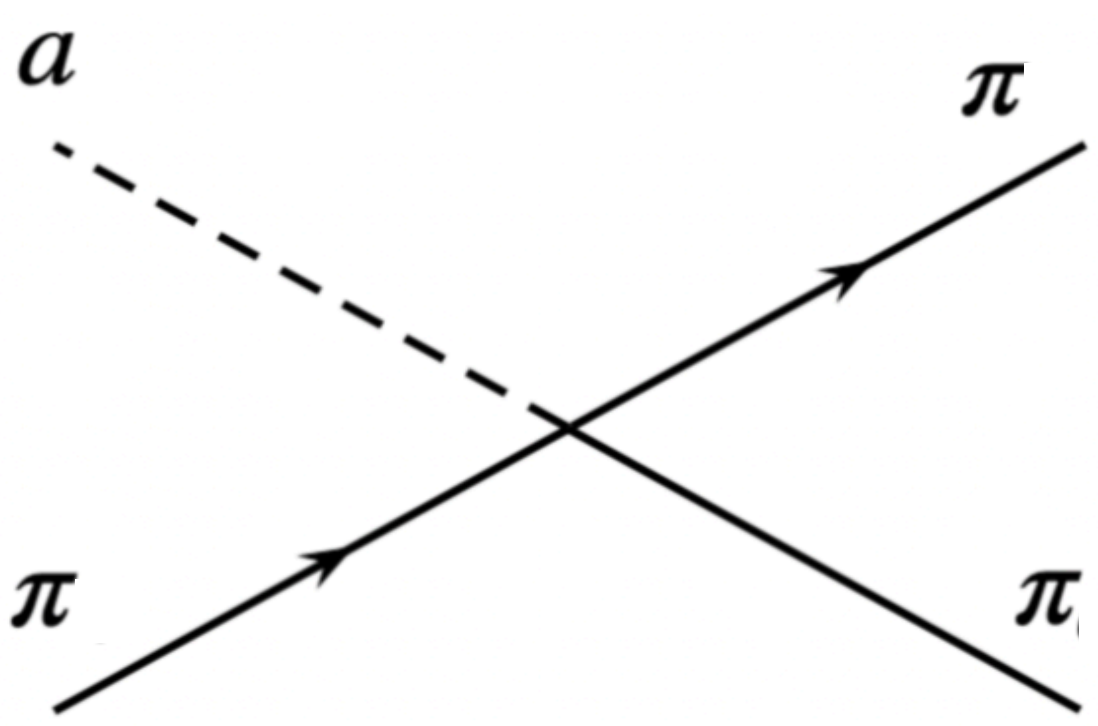
$$\mathcal{L}_a^{\chi} = \frac{f_\pi^2}{4} Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2} Tr\left[c_q \sigma^a\right] J_\mu^a$$

[H. Georgi, D. B. Kaplan, L. Randall, Phys. Lett. B **169** (1986)]

$$\begin{cases} U = e^{i\pi^a\sigma^a/f_\pi} \\ \chi = 2B_0\, e^{i\frac{a}{2f_a}Q_a} M_q e^{i\frac{a}{2f_a}Q_a} \end{cases} \qquad J_\mu^a = \frac{i}{4} f_\pi^2 Tr\left[\sigma^a\{U, (D^\mu U)^\dagger\}\right]$$

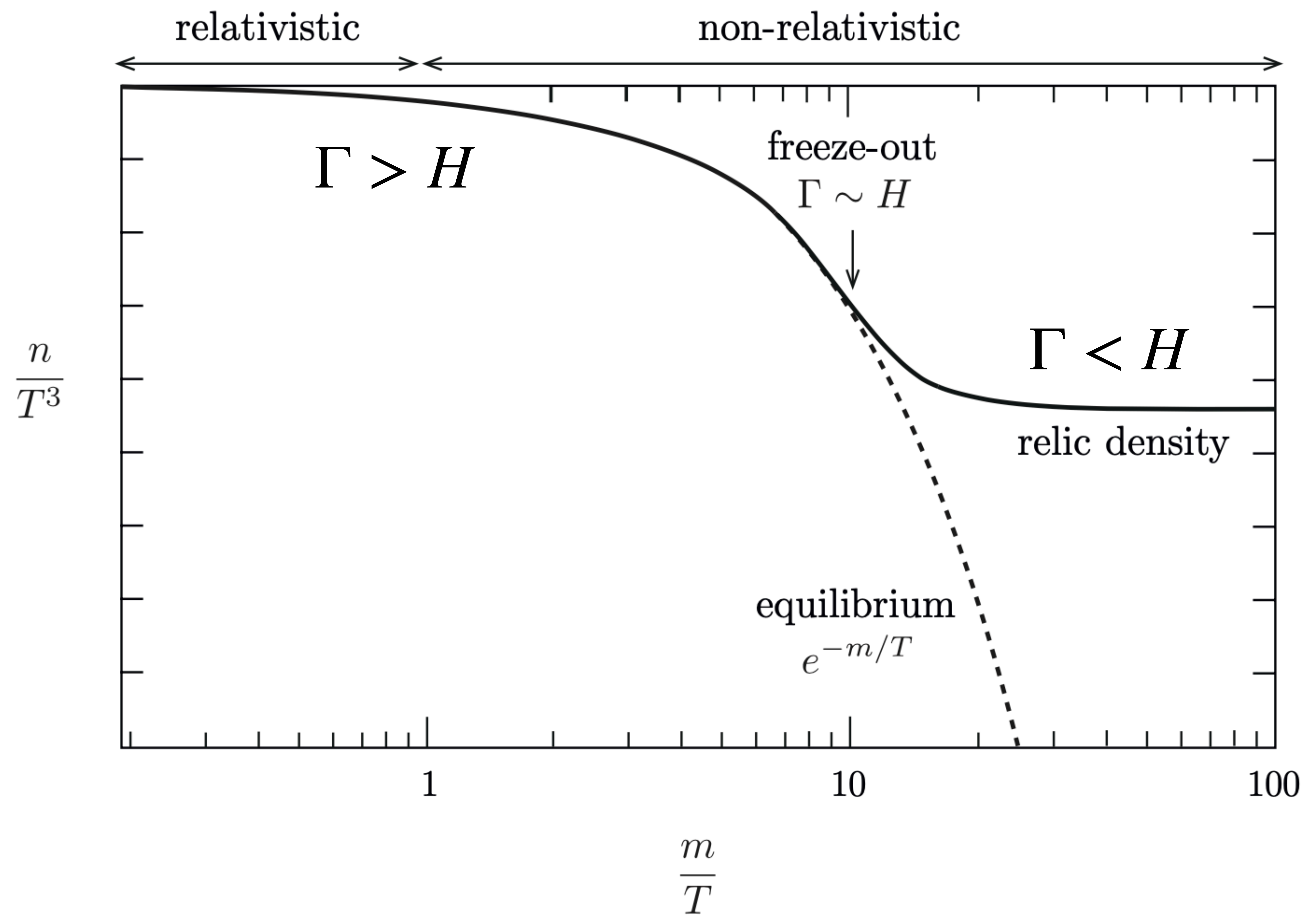
$$\mathcal{L}_{a\pi}^{(\mathrm{LO})} = \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a\left(2\partial_\mu\pi_0\pi_+\pi_- - \pi_0\partial_\mu\pi_+\pi_- - \pi_0\pi_+\partial_\mu\pi_-\right)$$

$$C_{a\pi} = \frac{1}{3}\left(\frac{m_d - m_u}{m_u + m_d} + c_d^0 - c_u^0\right)$$



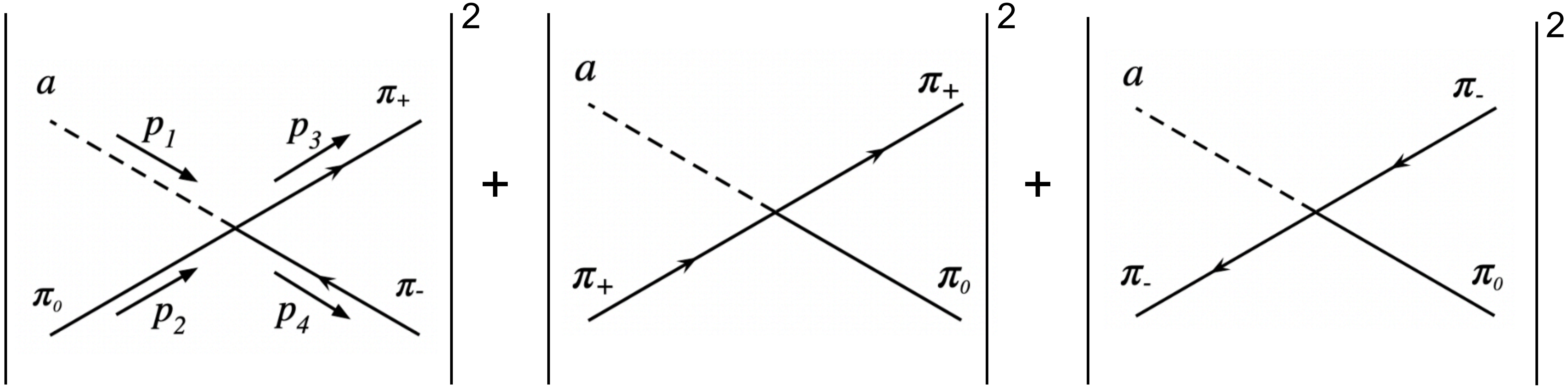
Axion thermal production in the Early Universe

- The evolution of the axions number density is controlled by the ratio Γ/H



Leading order scattering amplitude

$$\mathcal{L}_{a\pi}^{(\mathrm{LO})} = \frac{C_{a\pi}}{f_a f_\pi} \partial_\mu a \Big(2\partial_\mu \pi_0 \pi_+ \pi_- - \pi_0 \partial_\mu \pi_+ \pi_- - \pi_0 \pi_+ \partial_\mu \pi_- \Big)$$



$$\sum |\mathcal{M}|^2_{\mathrm{LO}} = \left(\frac{C_{a\pi}}{f_a f_\pi} \right)^2 \frac{9}{4} \left[s^2 + t^2 + u^2 - 3m_\pi^4 \right]$$

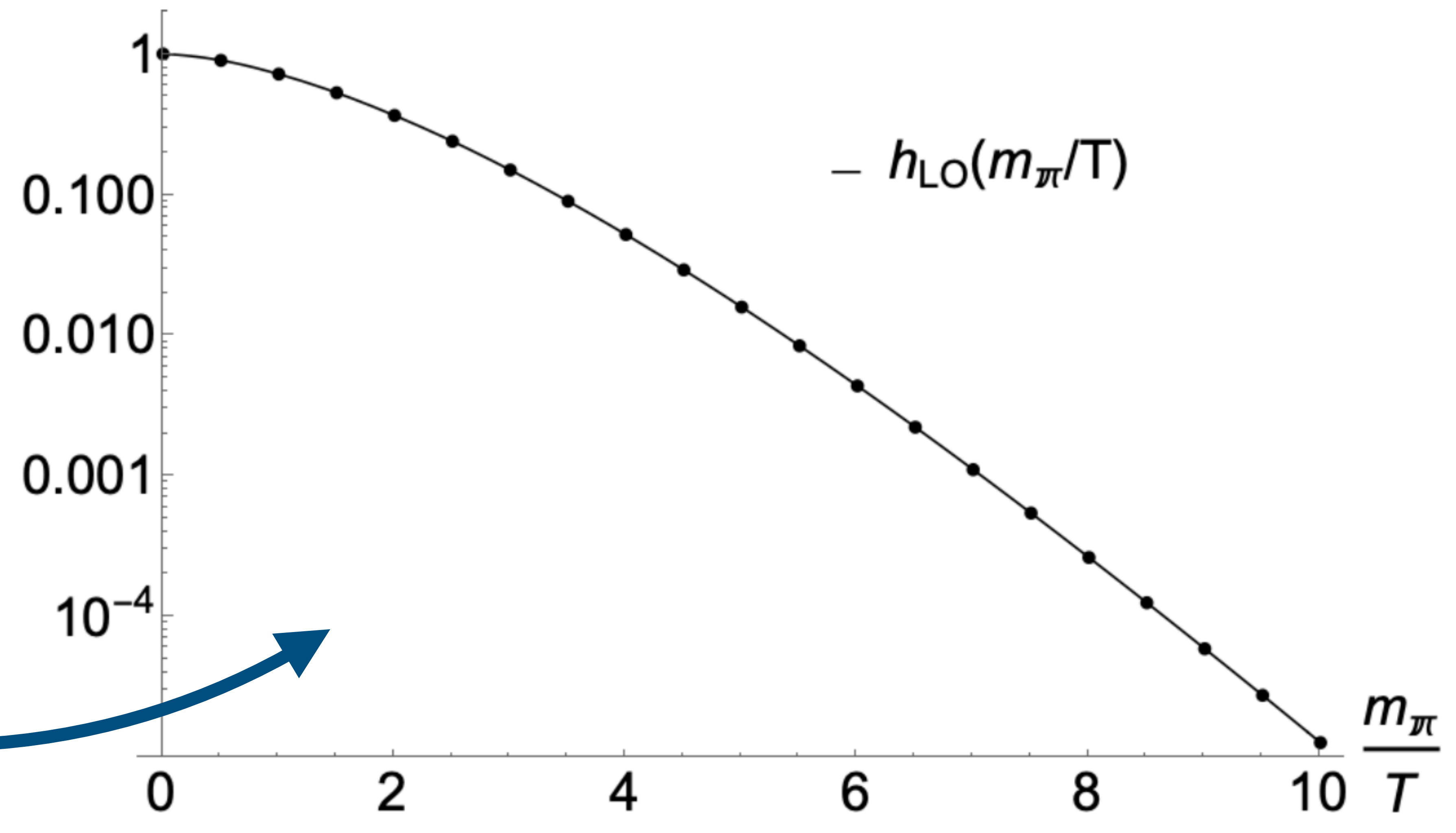
Thermal scattering rate

$$\Gamma = \frac{1}{n_a^{\rm eq}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \sum |\mathcal{M}|^2$$
$$(2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p_4) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4} \left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

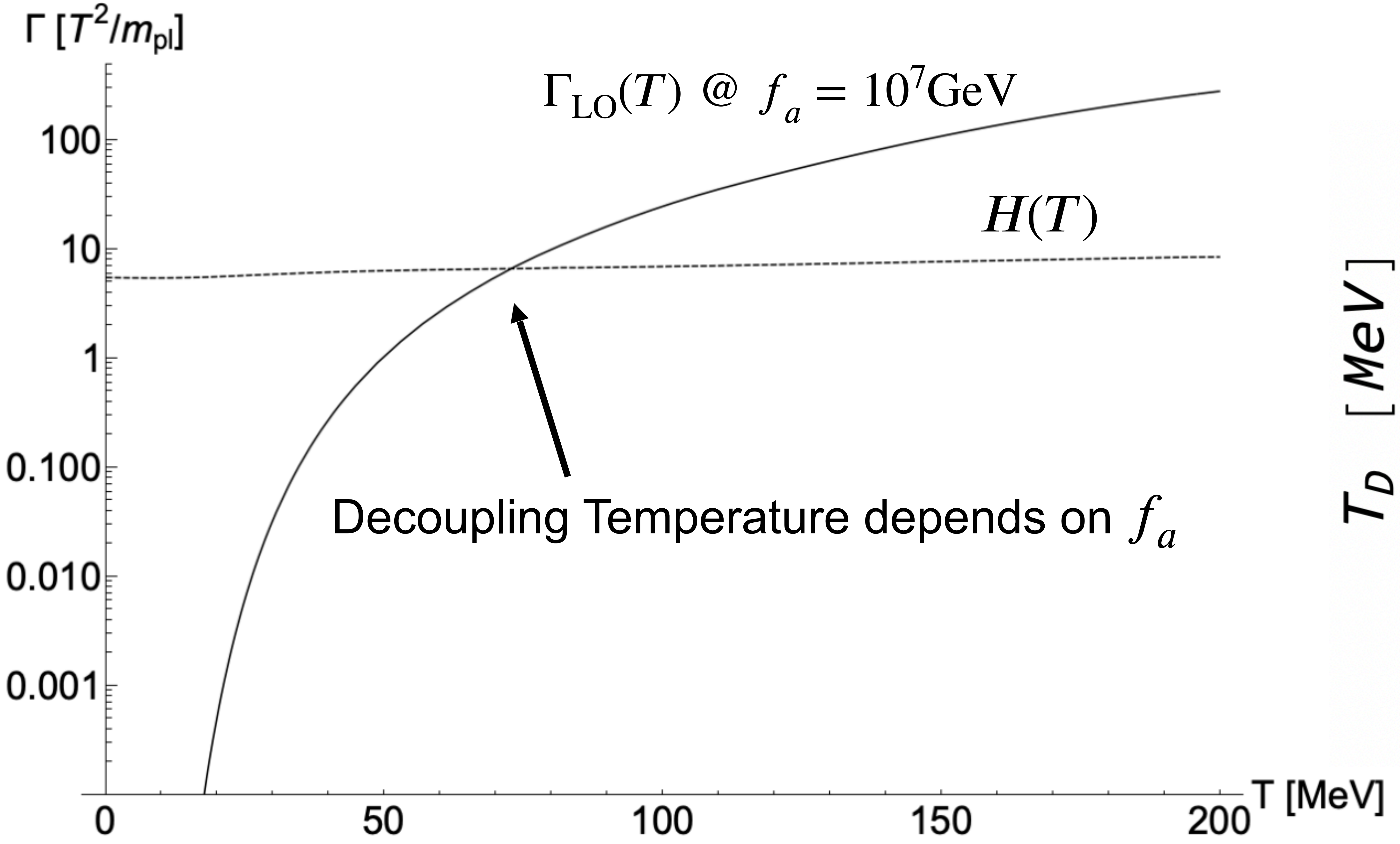
Numerically integrating:

$$\Gamma(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\rm LO}(m_\pi/T)$$

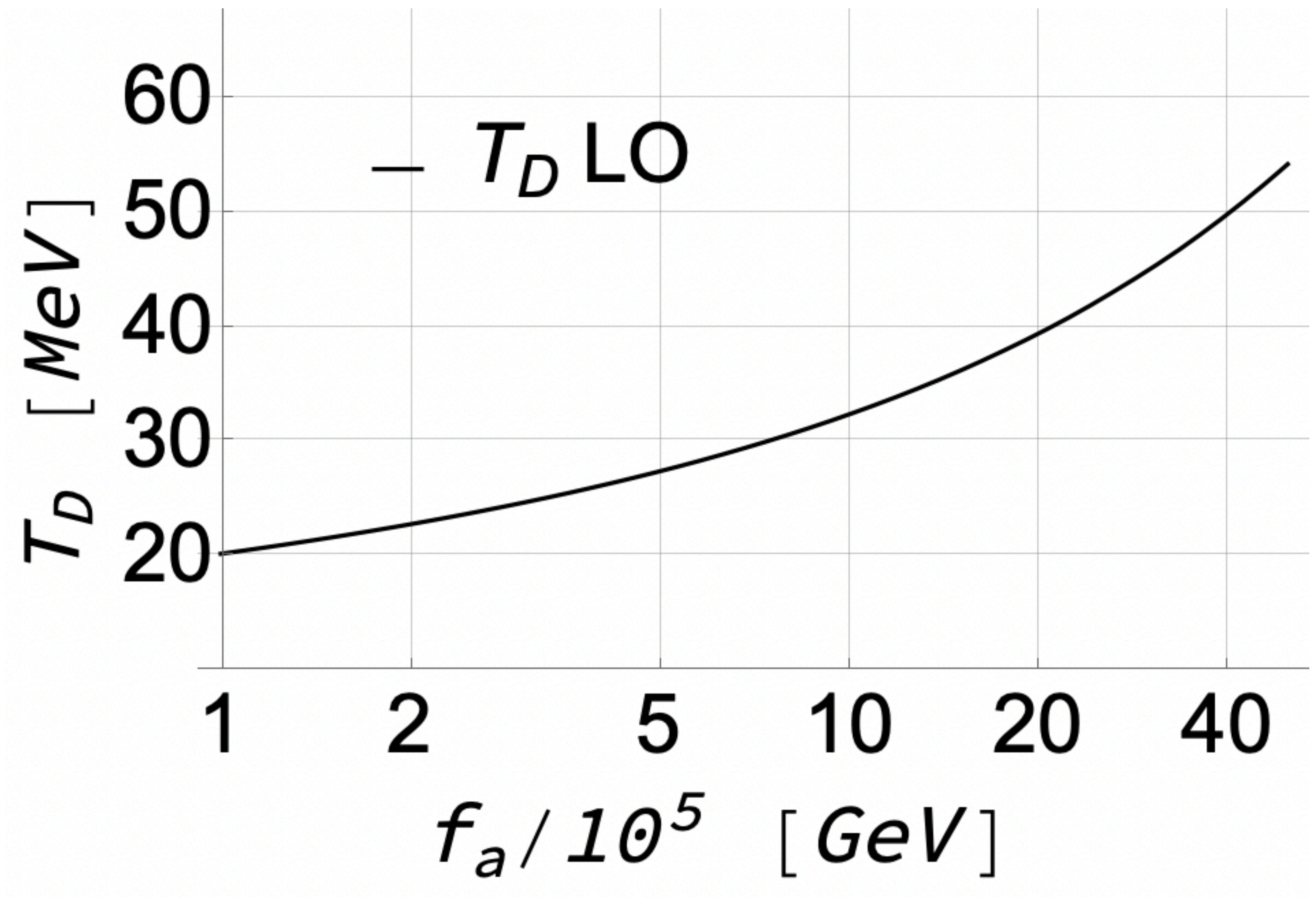


see also:
[Chang, Choi, hep-ph/9306216]
[Hannestad, Mirizzi, Raffelt, hep-ph/0504059]

Γ vs H



$\Gamma\ [T^2/m_{pl}]$
100
10
1
0.100
0.010
0.001
0
50
100
150
200
T [MeV]
$\Gamma_{LO}(T)$ @ $f_a = 10^7$GeV
$H(T)$
Decoupling Temperature depends on f_a

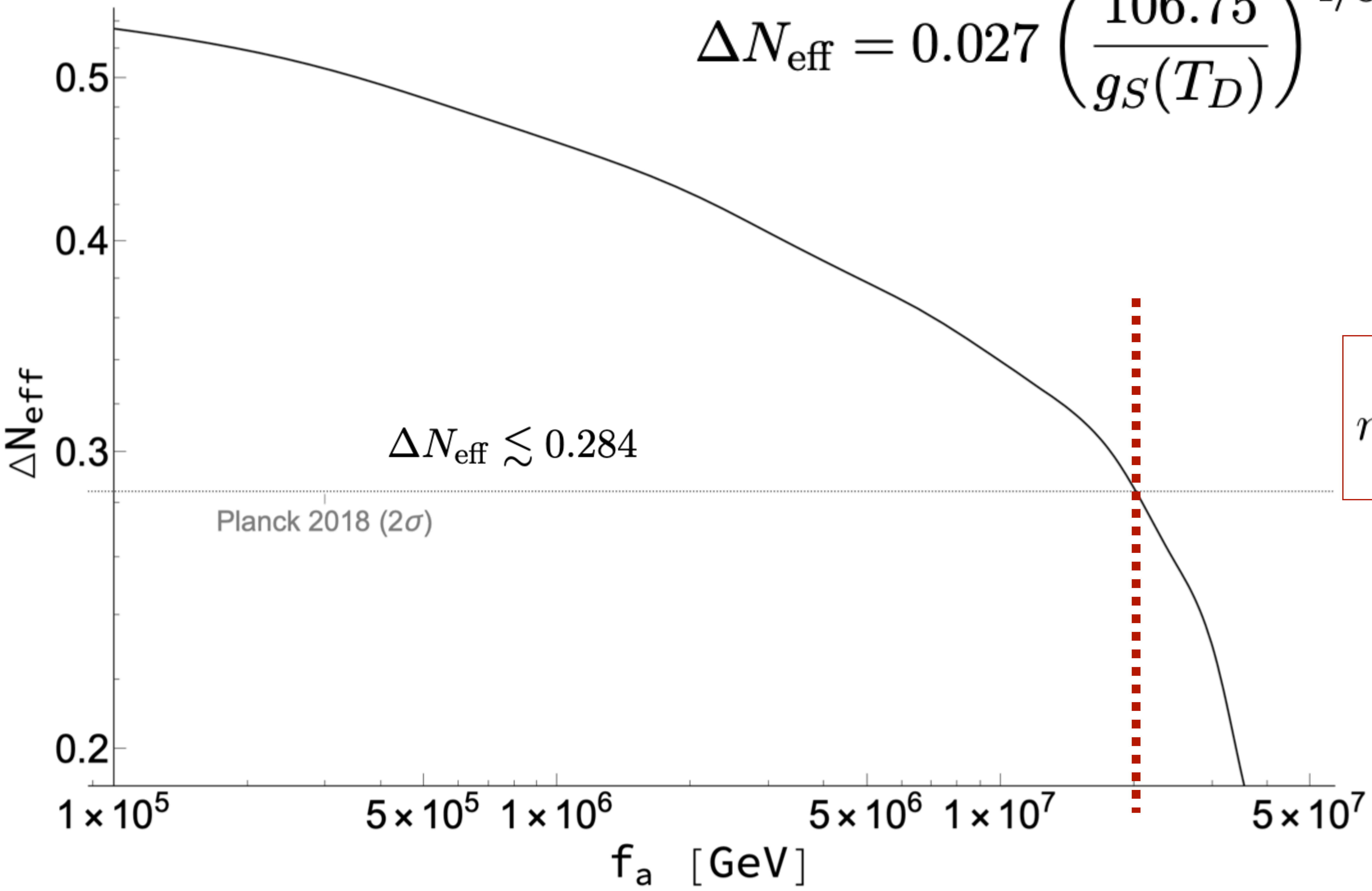


T_D LO
T_D [MeV]
60
50
40
30
20
1
2
5
10
20
40
$f_a / 10^5$ [GeV]

Hot Dark Matter bound on Axion mass, LO

Axion contribution to the Number of relativistic species

$$\Delta N_{\rm eff} = 0.027 \left(\frac{106.75}{g_S(T_D)}\right)^{4/3}$$



$$f_a \gtrsim 2.03 \times 10^7 \text{ GeV}$$

$$m_a \simeq 5.7 \left(\frac{10^{12}}{f_a\ [\text{GeV}]}\right) \mu\text{eV} \lesssim 0.28 \text{ eV}$$

Naïve way to set the bound.

See:
[A. Melchiorri, O. Mena, and A. Slosar, arXiv:0705.2695]

But… is ChPT valid?

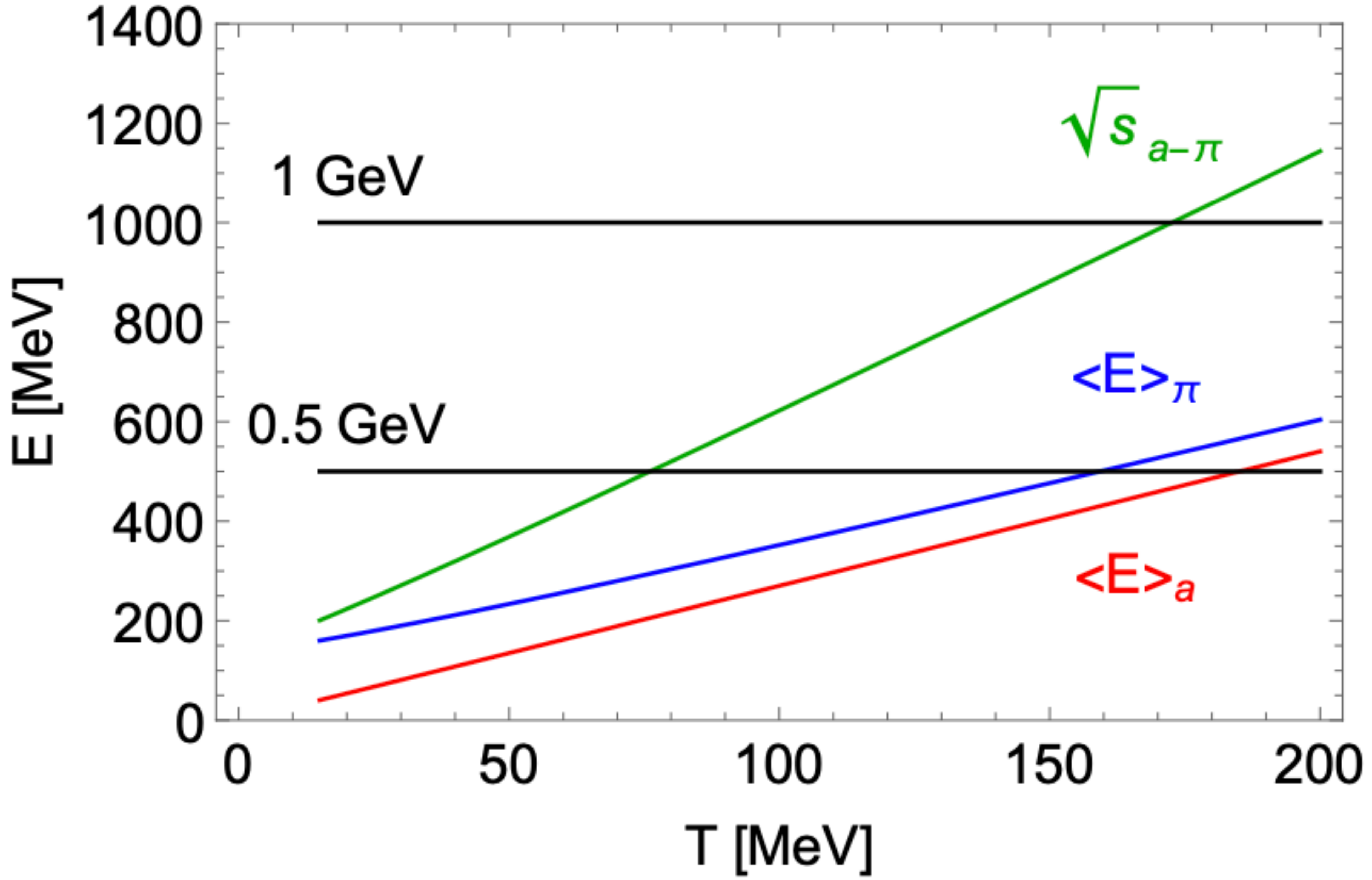
The mean energy of π, a at $T \simeq 100$ MeV is $\langle E \rangle \equiv \rho/n \simeq 350$ MeV, 270 MeV

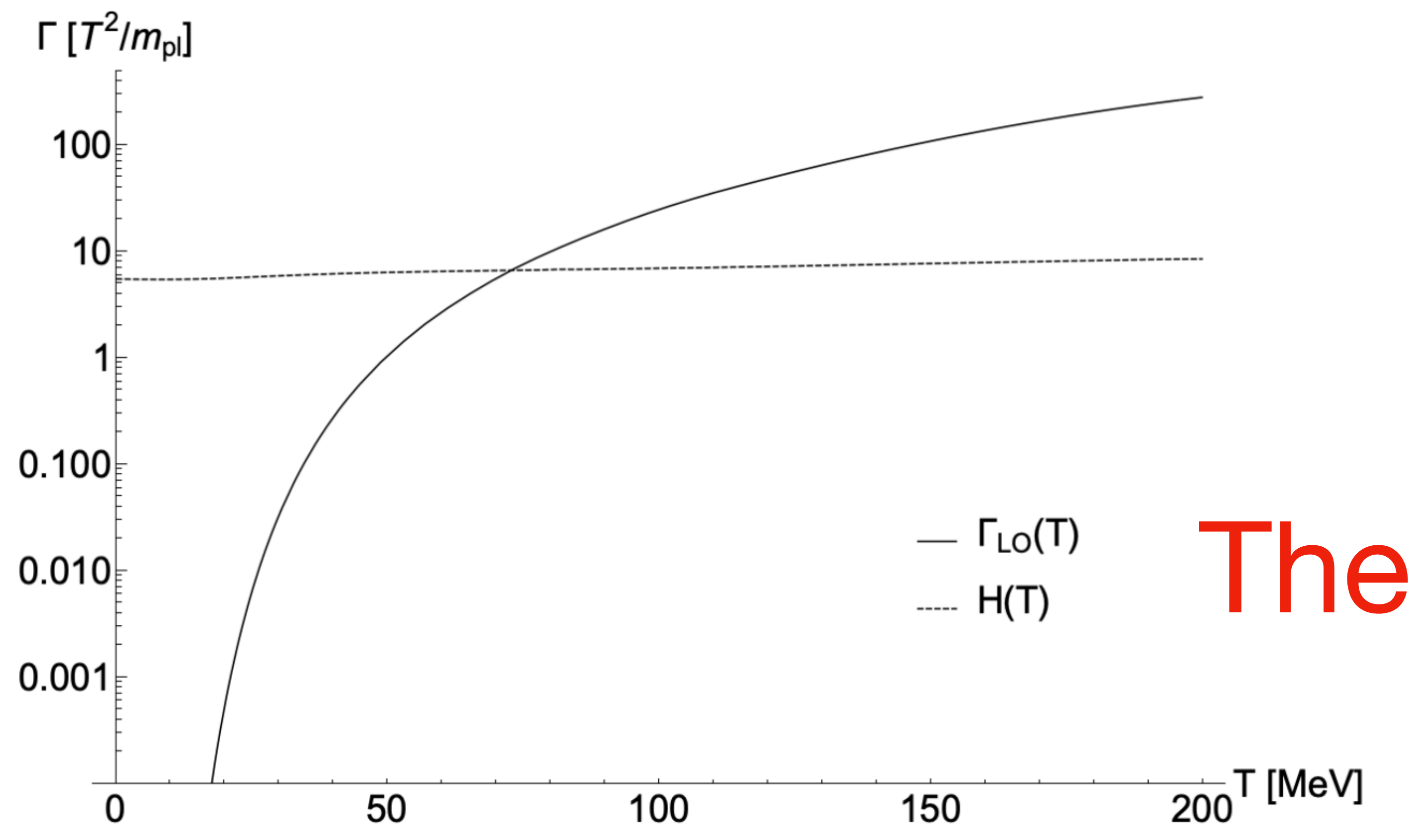
BUT

2-flavour EFT is valid for $E \lesssim 460$ MeV

[Donoghue et al., PhysRevD.86.014025]

$$\langle E \rangle(T) \sim \frac{\rho(T)}{n(T)}$$





The bound extracted from ChPT is not reliable

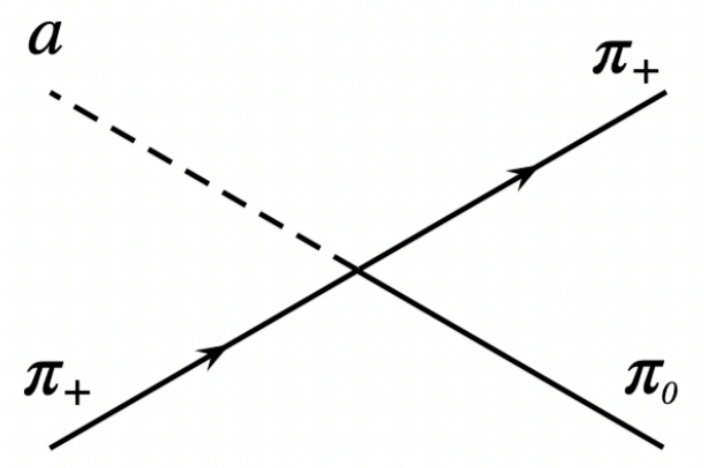
Computation of the NLO rate

Axion-Pion scattering: Next-to-Leading Order

Ingredients

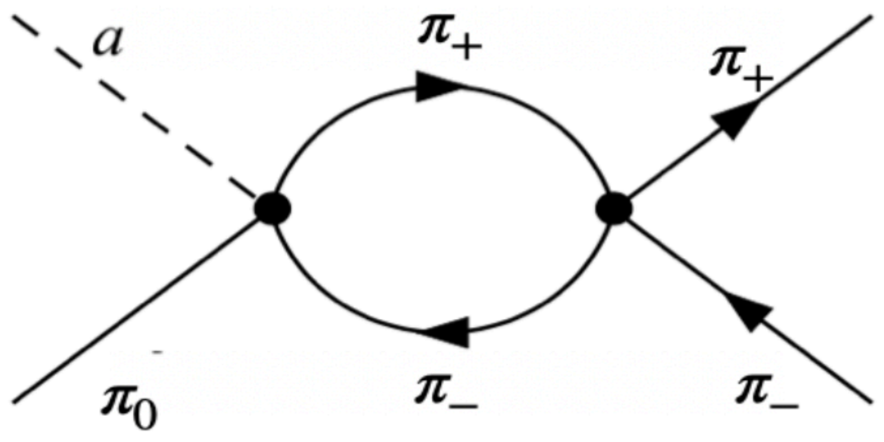
$a\pi \to \pi\pi$ Axion-pion scattering at NLO

LO scattering amplitude



Tree-level graph from NLO Lagrangian and loop amplitudes from LO Lagrangian

Tree-level amplitudes from NLO $\mathcal{L}_{\rm NLO}$



1-loop amplitudes from LO

$O\left(p^4\right)$ Operators

$$
\begin{aligned}
\mathcal{L}_{\rm NLO} = &\frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\
&+ \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right] \\
&+ l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu} U f_L^{\mu\nu} U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)\right] \\
&+ i\frac{l_6}{2}\mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\
&- \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1 + h_3}{4}\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1 - h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\
&\left. + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)
\end{aligned}
$$

$$\mathcal{L}_a^\chi \supset \frac{\partial^\mu a}{f_a}\mathrm{Tr}\frac{1}{2}\left[c_q \sigma^a\right] J^a_\mu$$

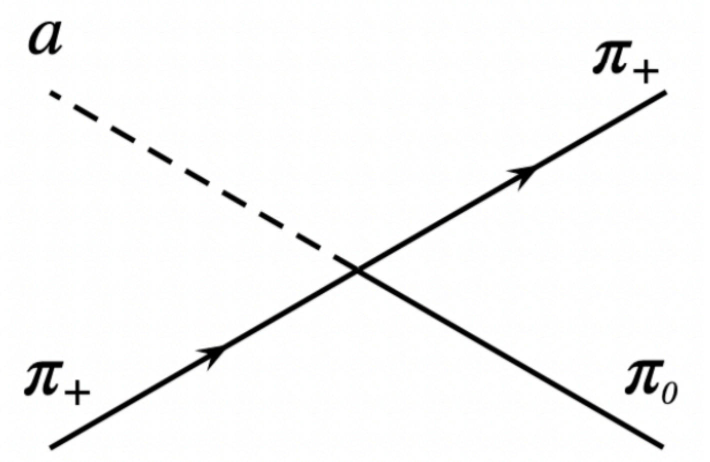
NLO chiral axial current J^a_μ

Axion-Pion scattering: Next-to-Leading Order

Ingredients

$a\pi \to \pi\pi$ Axion-pion scattering at NLO

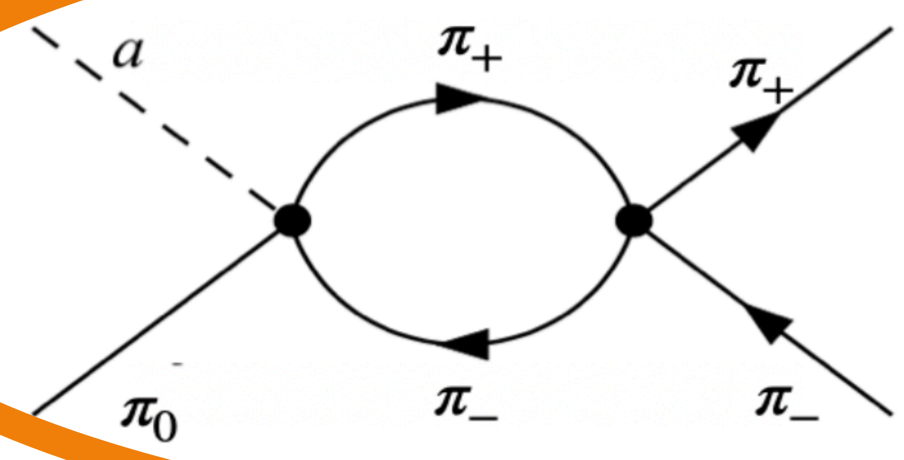
LO scattering amplitude



Tree-level graph from NLO Lagrangian and loop

NEW!

Tree-level amplitudes from NLO $\mathcal{L}_{\text{NLO}}$



1-loop amplitudes from LO

$O\left(p^4\right)$ Operators

$$\begin{aligned}\mathcal{L}_{\text{NLO}} = & \frac{l_1}{4}\left\{\text{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\text{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\text{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\ & + \frac{l_3}{16}\left[\text{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\text{Tr}\left[D_\mu U\left(D^\mu\chi\right)^\dagger + D_\mu\chi\left(D^\mu U\right)^\dagger\right] \\ & + l_5\left[\text{Tr}\left(f^R_{\mu\nu}U f_L^{\mu\nu}U^\dagger\right) - \frac{1}{2}\text{Tr}\left(f^L_{\mu\nu}f_L^{\mu\nu} + f^R_{\mu\nu}f_R^{\mu\nu}\right)\right] \\ & + i\frac{l_6}{2}\text{Tr}\left[f^R_{\mu\nu}D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\ & - \frac{l_7}{16}\left[\text{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{4}\text{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1-h_3}{16}\left\{\left[\text{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\ & \left. + \left[\text{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\text{Tr}\left(\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\text{Tr}\left(f^L_{\mu\nu}f_L^{\mu\nu} + f^R_{\mu\nu}f_R^{\mu\nu}\right)\end{aligned}$$

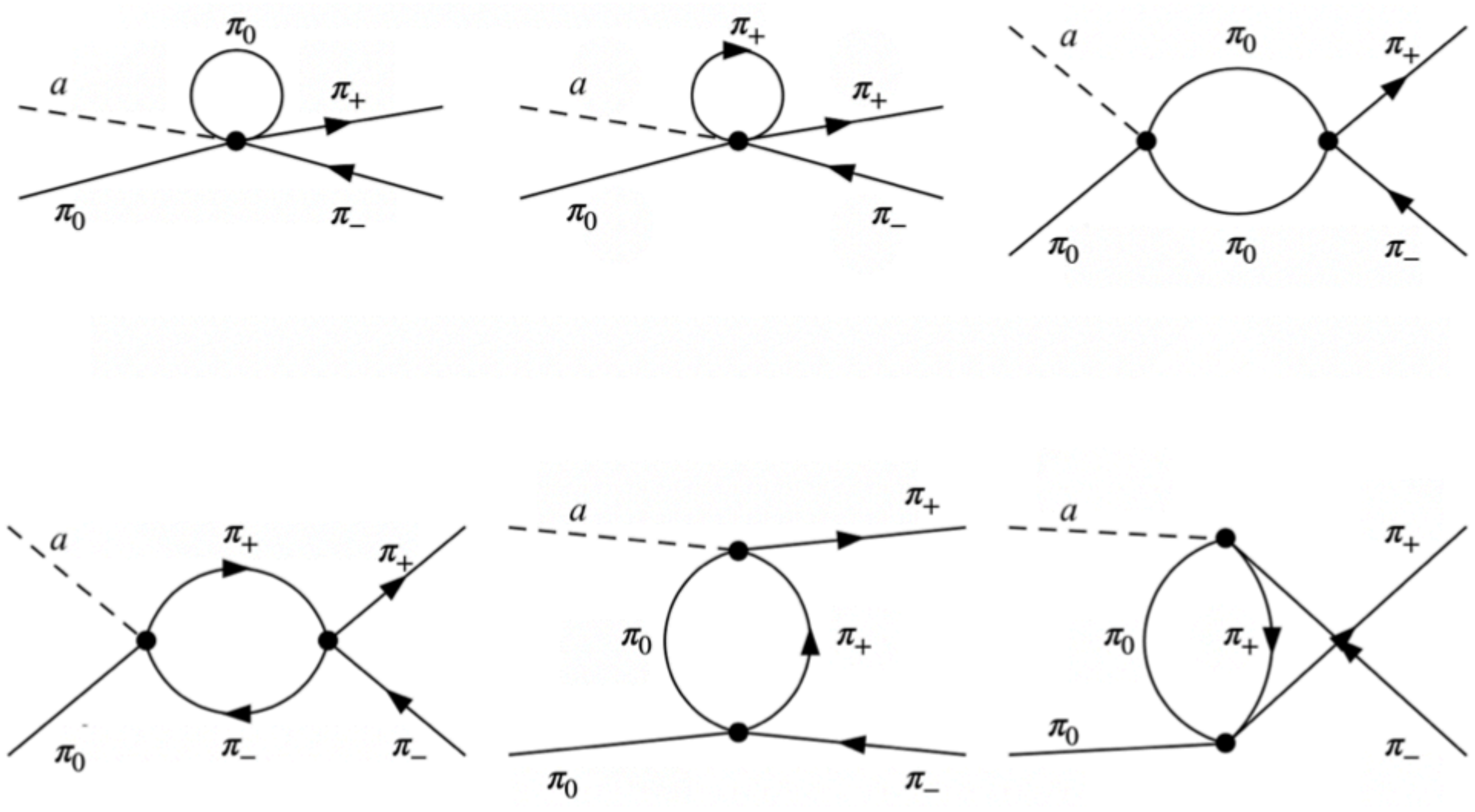
$$\mathcal{L}_a^\chi \supset \frac{\partial^\mu a}{f_a}\text{Tr}\frac{1}{2}\left[c_q\sigma^a\right]J^a_\mu$$

NLO chiral axial current J^a_μ

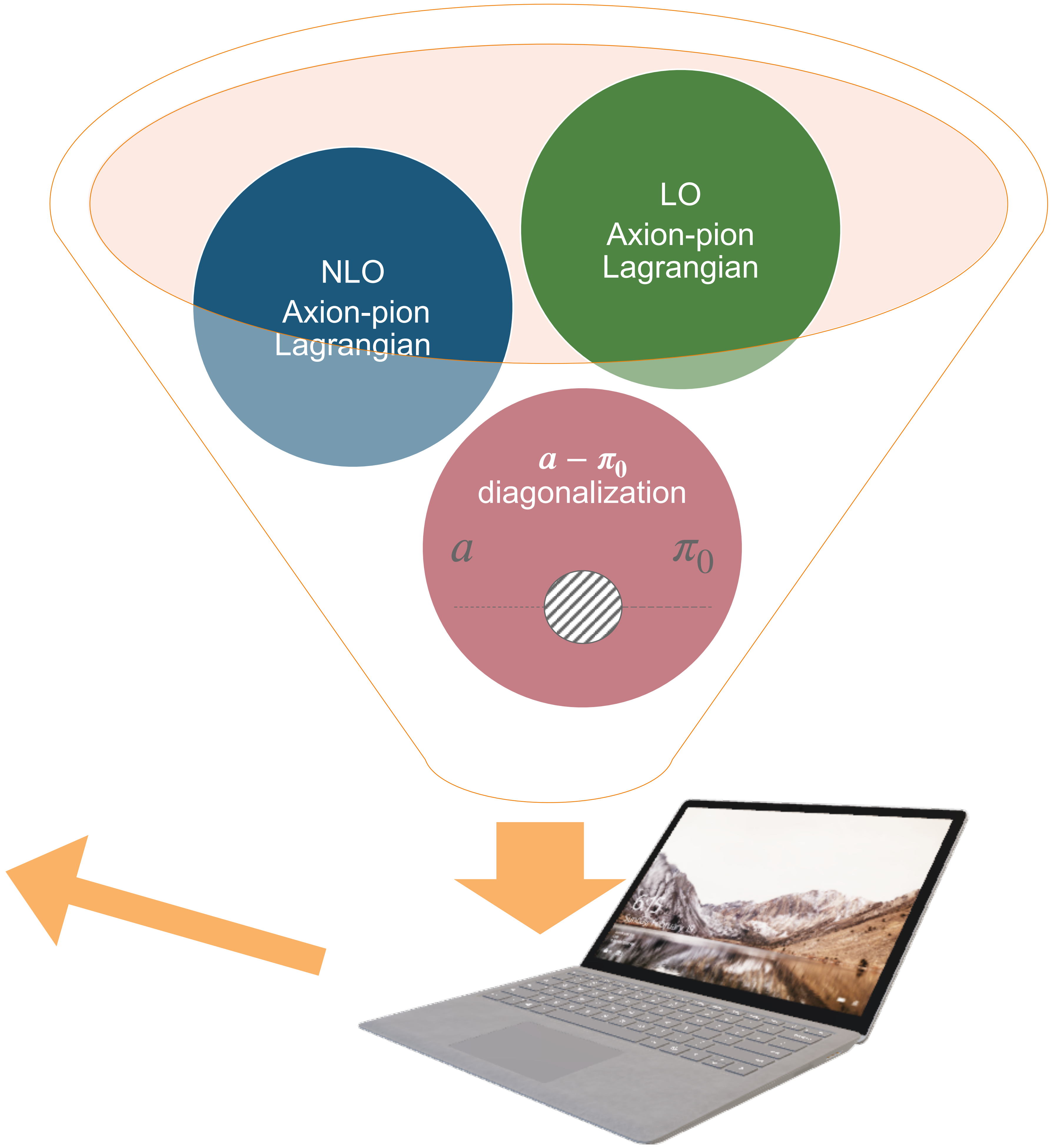
Computing axion-pion scattering at NLO

Computation of the full $a\pi \to \pi\pi$ amplitude (FeynRules, FeynArts, FeynCalc)

- 18 loop diagrams for the 3 channels



…



Amplitudes

- After renormalization & including NLO corrections to m_π, f_π

$$\mathcal{M}^{\rm NLO}_{a\pi_0\to\pi_+\pi_-} = \frac{C_{a\pi}}{192\pi^2 f_\pi^3 f_a}\Bigg\{15m_\pi^2(u+t) - 11u^2 - 8ut - 11t^2 - 6\overline{\ell_1}\left(m_\pi^2 - s\right)\left(2m_\pi^2 - s\right)$$
$$- 6\overline{\ell_2}\left(-3m_\pi^2(u+t) + 4m_\pi^4 + u^2 + t^2\right) + 9m_\pi^4\overline{\ell_3} + 18\overline{\ell_4}m_\pi^2(m_\pi^2 - s) + 576\pi^2\ell_7 m_\pi^4\left(\frac{m_d - m_u}{m_d + m_u}\right)^2$$
$$+ 3\left[3\sqrt{1 - \frac{4m_\pi^2}{s}}\, s\left(m_\pi^2 - s\right)\ln\left(\frac{\sqrt{s\left(s - 4m_\pi^2\right)} + 2m_\pi^2 - s}{2m_\pi^2}\right)\right.$$
$$+ \sqrt{1 - \frac{4m_\pi^2}{t}}\left(m_\pi^2(t - 4u) + 3m_\pi^4 + t(u - t)\right)\ln\left(\frac{\sqrt{t\left(t - 4m_\pi^2\right)} + 2m_\pi^2 - t}{2m_\pi^2}\right)$$
$$\left.+\sqrt{1 - \frac{4m_\pi^2}{u}}\left(m_\pi^2(u - 4t) + 3m_\pi^4 + u(t - u)\right)\ln\left(\frac{\sqrt{u\left(u - 4m_\pi^2\right)} + 2m_\pi^2 - u}{2m_\pi^2}\right)\right]\Bigg\}$$
$$+ \frac{4\ell_7 m_\pi^2 m_d\left(s - 2m_\pi^2\right) m_u\left(m_d - m_u\right)}{f_\pi^3 f_a\left(m_d + m_u\right)^3},$$

- Other pionic channels obtained via $s \to t, u$

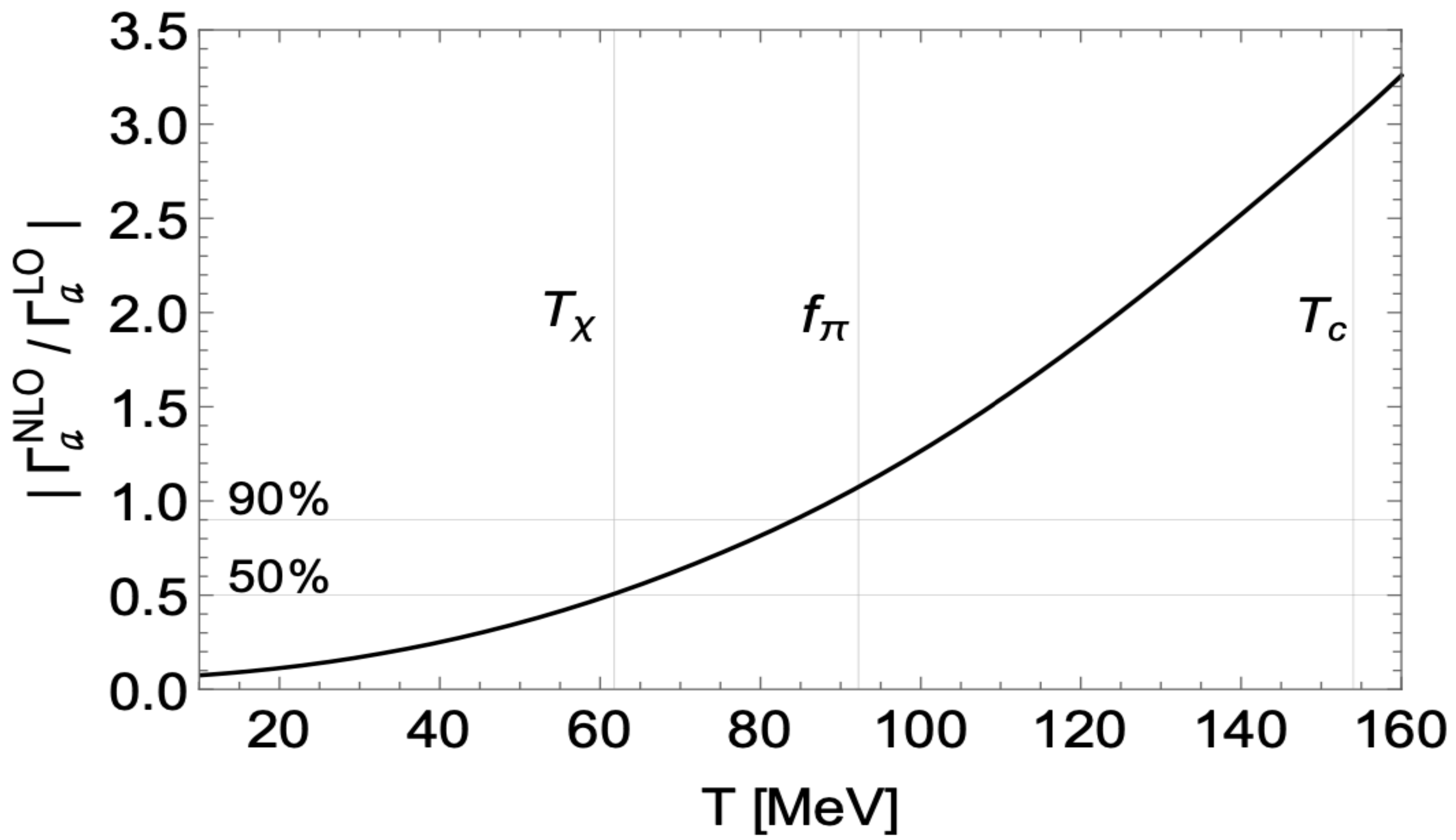
NLO Thermalization rate

$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO}]$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.212\, T^5 \left[h_{\rm LO}(m_\pi/T) - 2.80\frac{T^2}{f_\pi^2}\, h_{\rm NLO}(m_\pi/T)\right]$$

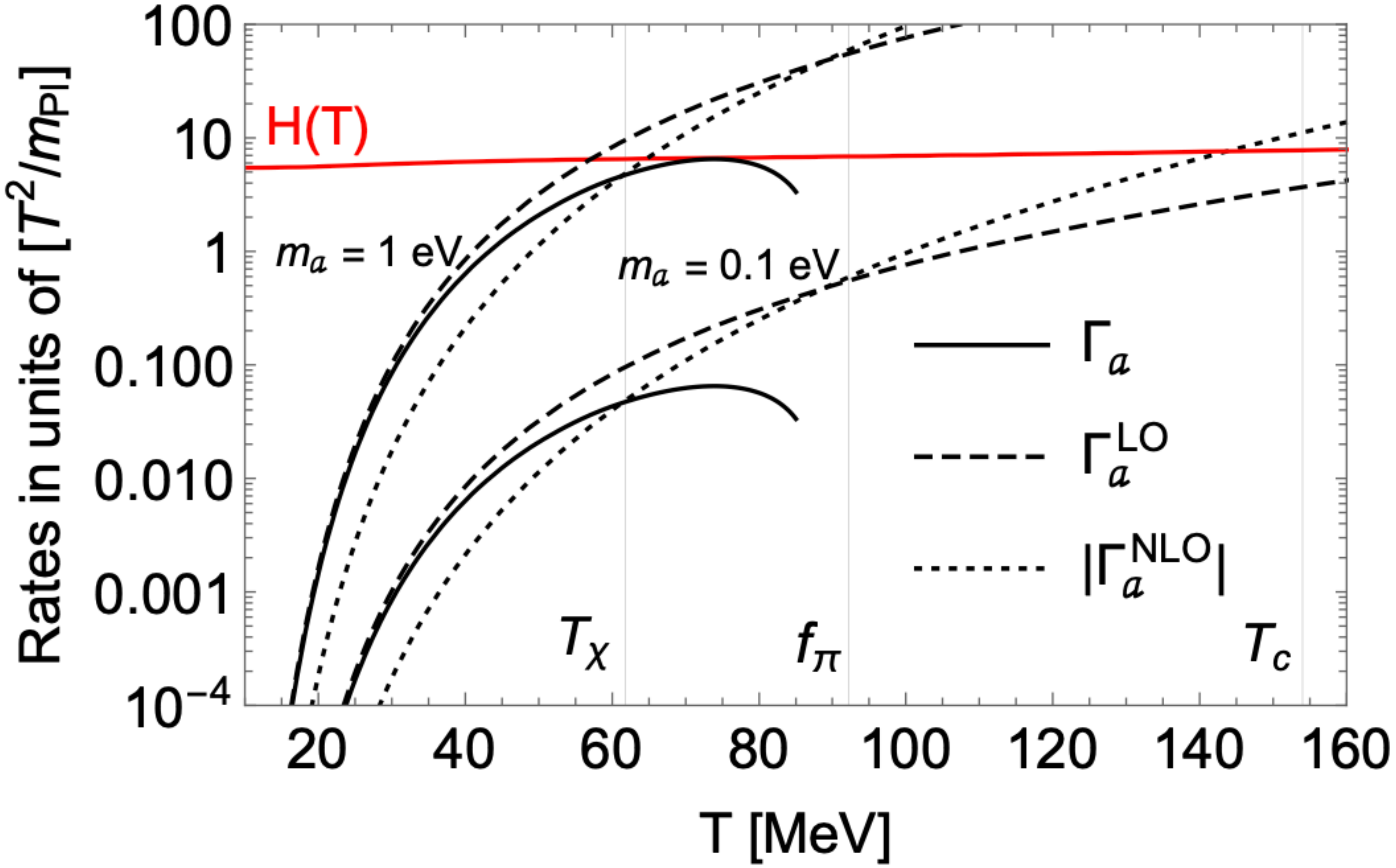
Different T-behavior between LO and NLO contributions

Correction of $\sim 50\,\%$ already at $T \simeq 60$ MeV
Convergence problem!



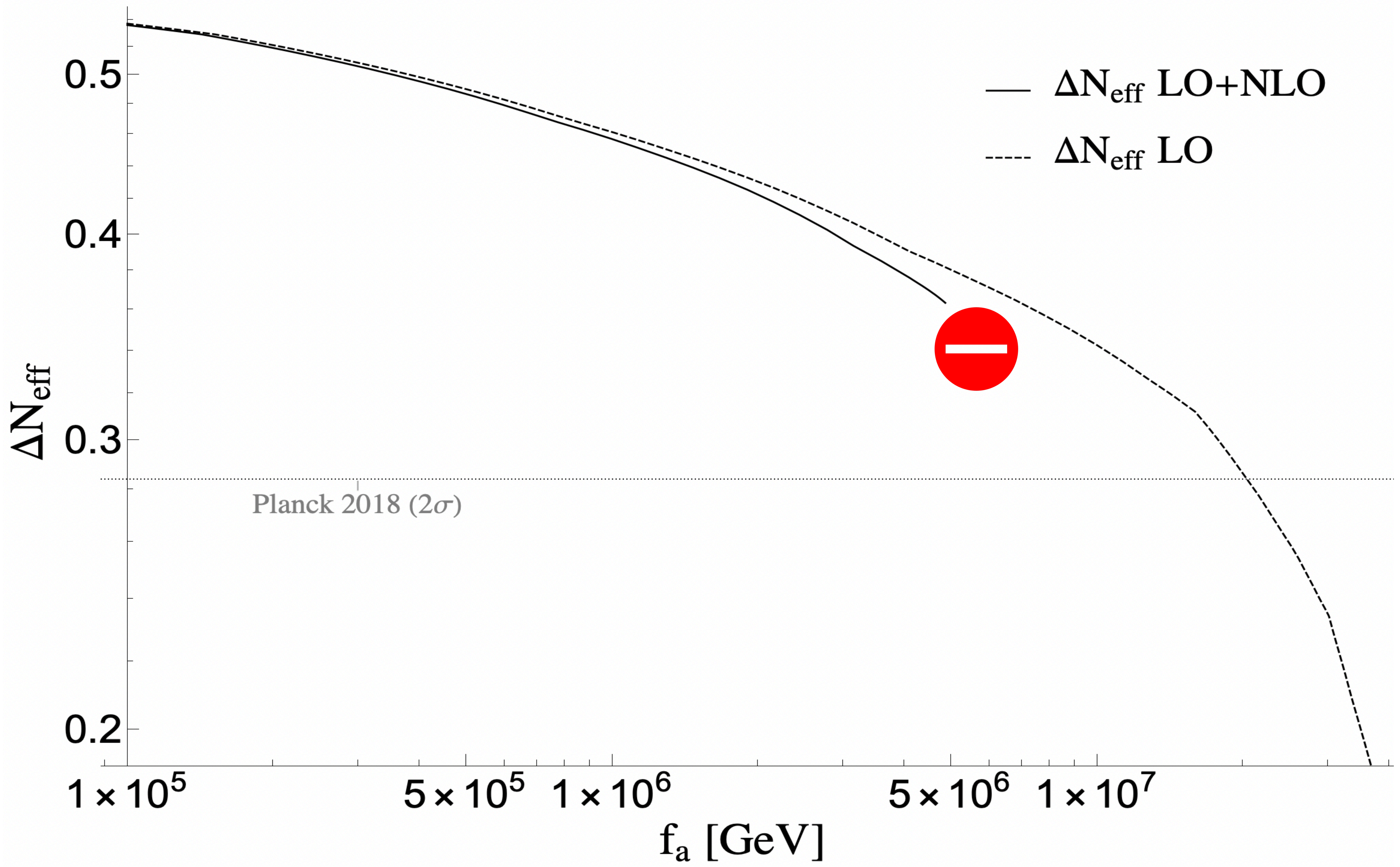
Γ vs H , **NLO**

- $m_a = 1$ eV: the most conservative HDM bound
- $m_a = 0.1$ eV: typical reach of future CMB-S4 experiments
- $T_\chi \sim 62$ MeV: boundary of validity of the chiral expansion



$\Delta N_{\rm eff}$ including NLO correction

For $f_a \gtrsim 4.9 \times 10^6$ GeV the decoupling temperature cannot be extracted in χPT. The chiral expansion of the *axion-pion theory does not converge* in the region of temperatures considered.



Conclusions

- The present HDM bound $m_a \lesssim 0.2\ \mathrm{eV}$ is not reliable;
- In the mass range of interest, $m_a \in [0.1,1]\ \mathrm{eV}$, the decoupling temperature and consequently the axion HDM bound cannot be extracted within the chiral Lagrangian, since thermal effects push energies above EFT scale ;
- A reliable computation is needed to set targets for future CMB surveys (inclusion of ρ meson, lattice QCD, …)

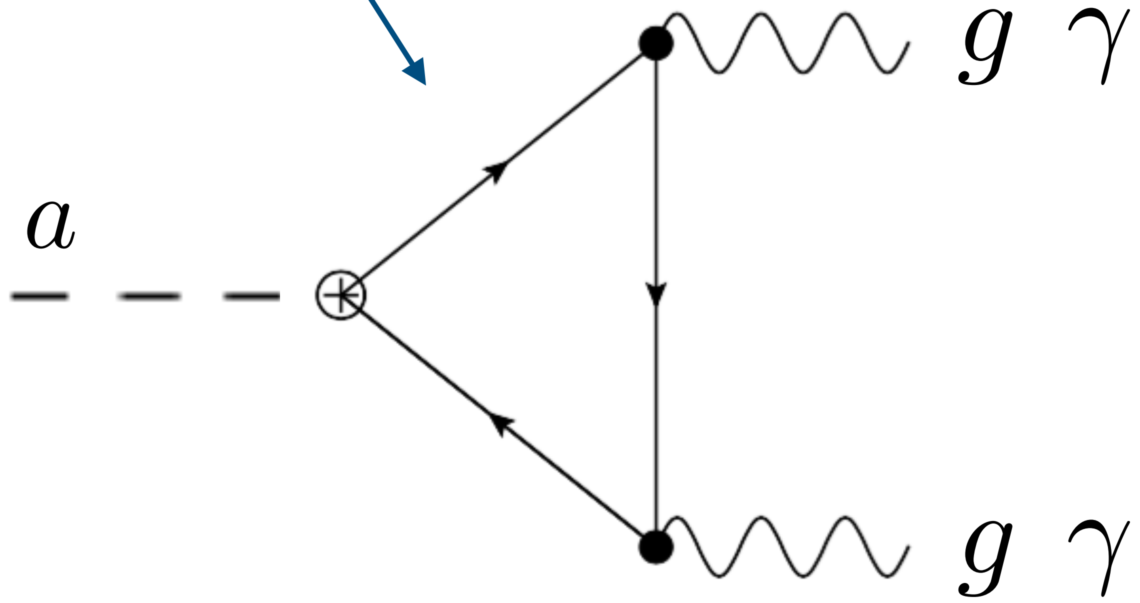
Thanks for the attention!

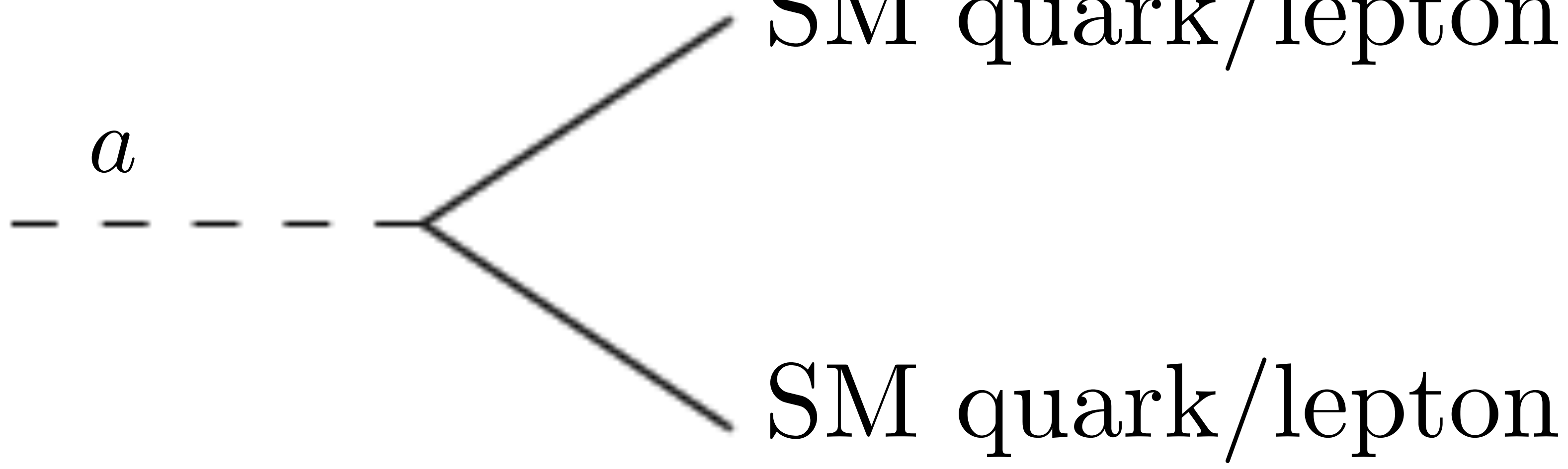
Backup

Axion EFT

$E > \Lambda_{QCD}$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\widetilde{G} + \frac{1}{4}g^0_{a\gamma}aF\widetilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c^0_q\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + h.c.$$





Axion Chiral Lagrangian, LO

$E > \Lambda_{QCD}$

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial_\mu a) + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\widetilde{G} + \frac{1}{4}g^0_{a\gamma}aF\widetilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c^0_q\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + h.c.$$

1. Rotate away $aG\widetilde{G}$ through $q \to e^{i\gamma_5 \frac{a}{2f_a}Q_a}q$,
2. Write an EFT with Pions and Axions as GB

$E < \Lambda_{QCD}$

$$\mathcal{L}_a^\chi = \frac{f_\pi^2}{4}Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2}Tr\left[c_q\sigma^a\right]J^a_\mu$$

With $U = e^{i\pi^a\sigma^a/f_\pi}$ and $\chi = 2B_0\, e^{i\frac{a}{2f_a}Q_a}M_q e^{i\frac{a}{2f_a}Q_a}$

Axion Chiral Lagrangian, NLO

$$\mathcal{L}_{\text{NLO}} = \frac{l_1}{4}\left\{\text{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\text{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\text{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right]$$
$$+\frac{l_3}{16}\left[\text{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\text{Tr}\left[D_\mu U\left(D^\mu\chi\right)^\dagger + D_\mu\chi\left(D^\mu U\right)^\dagger\right]$$
$$+ l_5\left[\text{Tr}\left(f^R_{\mu\nu}U f^{\mu\nu}_L U^\dagger\right) - \frac{1}{2}\text{Tr}\left(f^L_{\mu\nu}f^{\mu\nu}_L + f^R_{\mu\nu}f^{\mu\nu}_R\right)\right]$$
$$+ i\frac{l_6}{2}\text{Tr}\left[f^R_{\mu\nu}D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right]$$
$$-\frac{l_7}{16}\left[\text{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{4}\text{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1-h_3}{16}\left\{\left[\text{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right.$$
$$\left.+\left[\text{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\text{Tr}\left(\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\text{Tr}\left(f^L_{\mu\nu}f^{\mu\nu}_L + f^R_{\mu\nu}f^{\mu\nu}_R\right)$$

[J. Gasser and H. Leutwyler, Annals Phys. **158** (1984)]
[S. Scherer, arXiv:hep-ph/0210398]

And, differentiating the Lagrangian with respect to the external fields…

Axial current to NLO

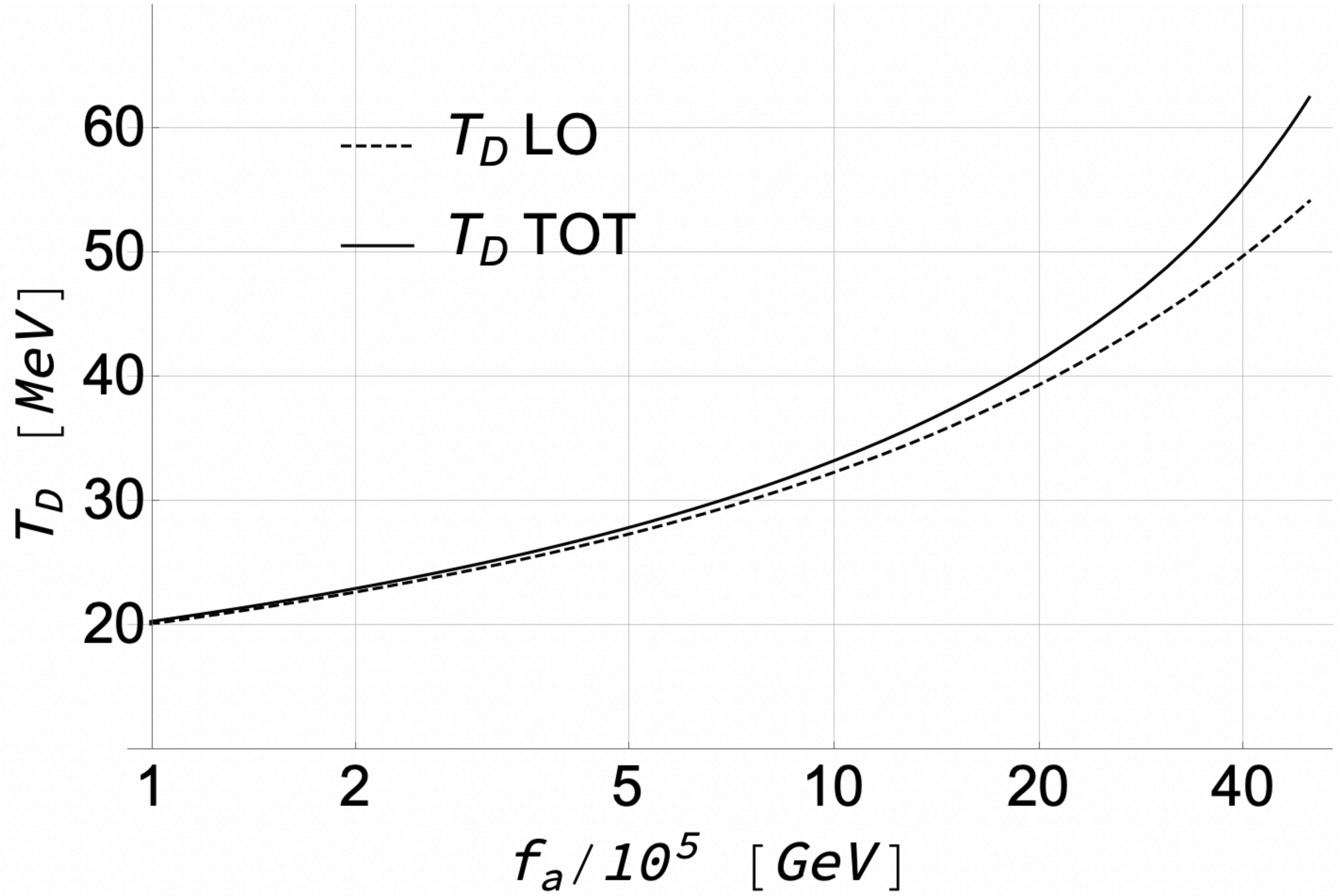
$$\frac{\partial^\mu a}{4f_a}\mathrm{Tr}\left[c_q\sigma^a\right]\left(J_\mu^{a\ (\mathrm{LO})} + J_\mu^{a\ (\mathrm{NLO})}\right)$$

↓

$$J^{\mu,a}_{A\ (\mathrm{NLO})} = +i\frac{l_1}{2}\left\langle\sigma^a\left\{D^\mu U^\dagger, U\right\}\right\rangle\left\langle D_\nu U D^\nu U^\dagger\right\rangle$$
$$+i\frac{l_2}{4}\left\langle\sigma^a\left\{D^\nu U^\dagger, U\right\}\right\rangle\left\langle D^\mu U D_\nu U^\dagger + D_\nu U D^\mu U^\dagger\right\rangle$$
$$-i\frac{l_4}{8}\left\langle\sigma^a\left\{D^\mu U, \chi^\dagger\right\} - \sigma^a\left\{U, D^\mu\chi^\dagger\right\} + \sigma^a\left\{D^\mu\chi, U^\dagger\right\} - \sigma^a\left\{\chi, D^\mu U^\dagger\right\}\right\rangle$$
$$+\frac{l_6}{4}\left\langle f^R_{\mu\nu}\left[\sigma^a, D^\nu U\right]U^\dagger + f^R_{\mu\nu}U\left[D^\nu U^\dagger, \sigma^a\right] + f^L_{\mu\nu}U^\dagger\left[\sigma^a, D^\nu U\right] + f^L_{\mu\nu}\left[D^\nu U^\dagger, \sigma^a\right]U\right\rangle$$

T_d vs $\mathrm{f_a}$

Decoupling temperature for the LO and LO+NLO case, as a function of f_a

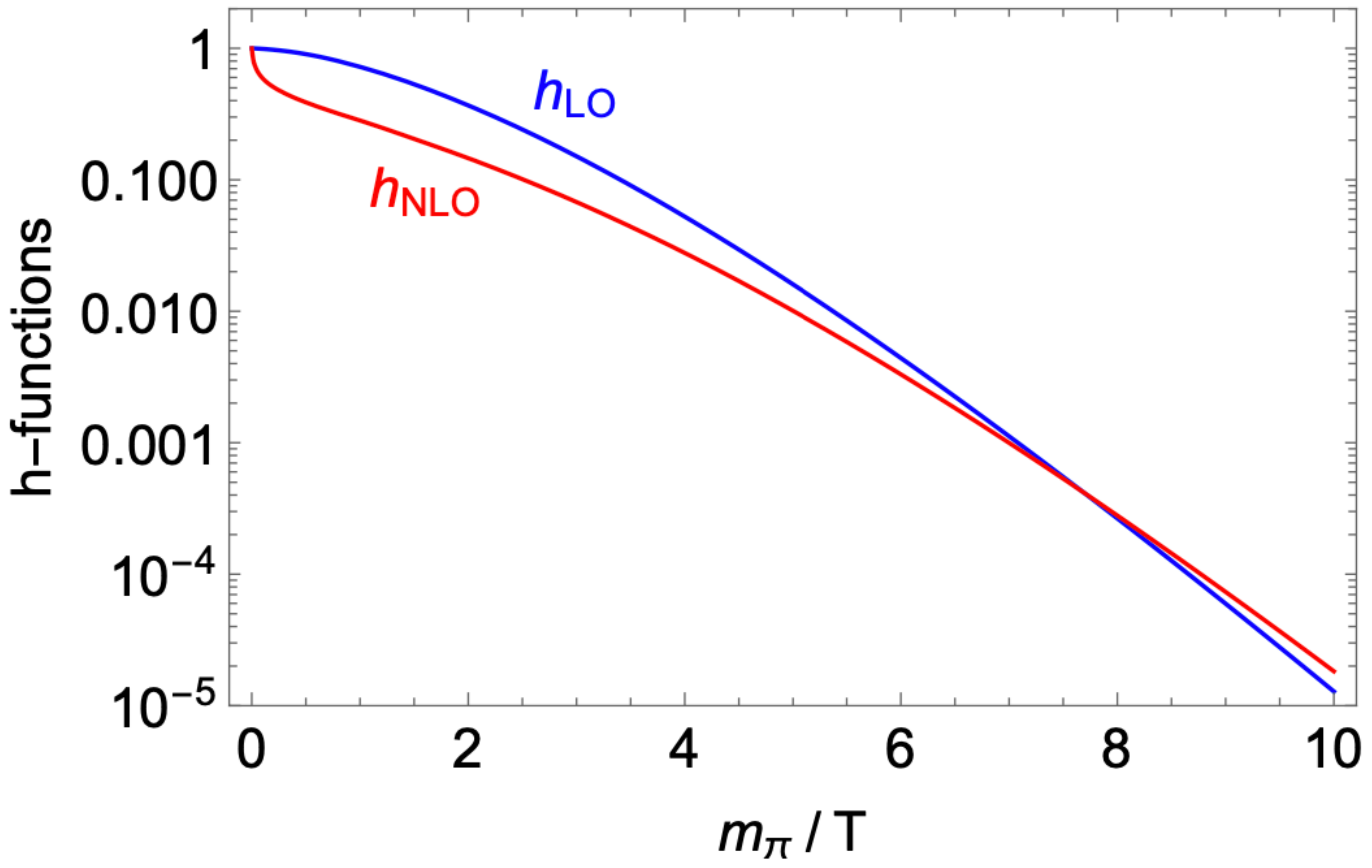


$$T_D < T_\chi$$

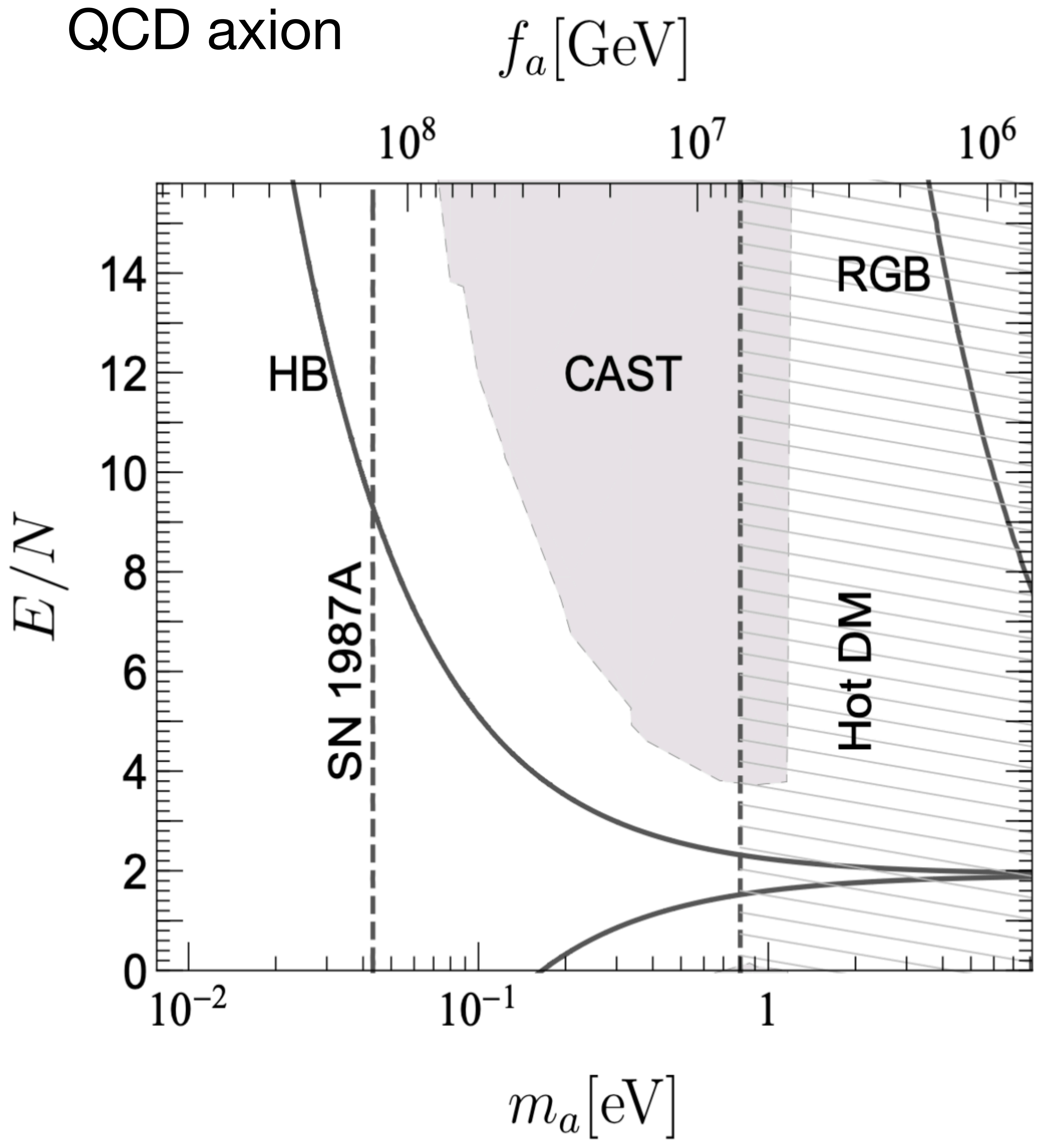
if

$$f_a \lesssim 4.9 \times 10^6 \text{ GeV}$$
$$m_a \gtrsim 1.16 \text{ eV}$$

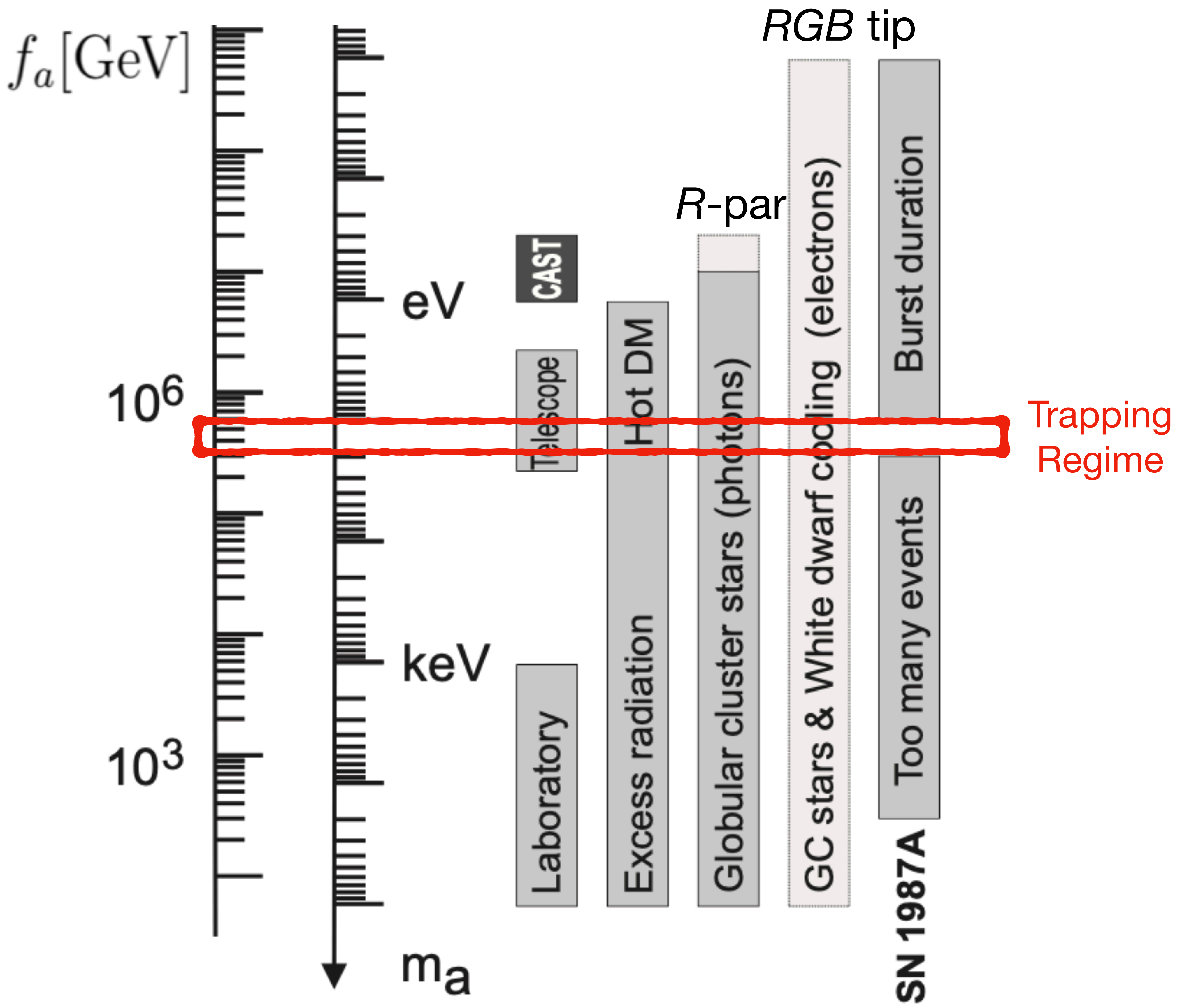
h functions



ASTRO Bounds



[Di Luzio et al., Phys. Rept. **870** (2020)]



- $g^0_{ae} = 0$ in KSVZ models
- SN bound not solid from astrophysics [1907.05020]
- $g_{a\gamma}$ can be accidentally suppressed [1610.07593]