Energy-Dependent Neutrino Mixing Parameters at Oscillation Experiments

based on 2108.11961, in collaboration with
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Vedran Brdar

Renormalization and β Function

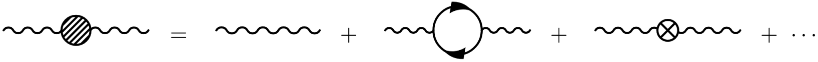
- QED:

$$\mathcal{L}_{QED} = \bar{\Psi}_0(i\partial\!\!\!/ - m_0)\Psi_0 - \frac{1}{4}F_{0,\mu\nu}F^{0,\mu\nu} - e_0\bar{\Psi}_0\gamma^\mu\Psi_0 A_{0,\mu}$$

$$\mathcal{L}_{QED} = \overbrace{(1+\delta_\Psi)}^{Z_\Psi}\bar{\Psi}_r i\partial\!\!\!/\Psi_r - (m_r+\delta_m)\bar{\Psi}_r\Psi_r - \frac{1}{4}\overbrace{(1+\delta_A)}^{Z_A}F_{r,\mu\nu}F^{r,\mu\nu} - \underbrace{(1+\delta_e)}_{Z_e}e_r\bar{\Psi}_r\gamma^\mu\Psi_r A_{r,\mu}$$

$$e_r = Z_e^{-1}Z_\Psi Z_A^{1/2}e_0$$

$$\beta = \mu\frac{\partial e_r}{\partial\mu} = \frac{\partial}{\partial\log\mu}\sqrt{Z_A}e_0 = \frac{e_0}{2\sqrt{Z_A}}\frac{\partial Z_A}{\partial log_\mu} \simeq e_r\frac{\partial\delta_A}{\partial log_\mu^2}$$

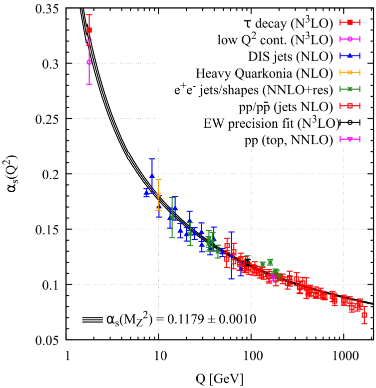


$$\delta_A = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\left(\frac{2}{\epsilon} - \gamma + \log 4\pi - log\,\mu^2\right)$$

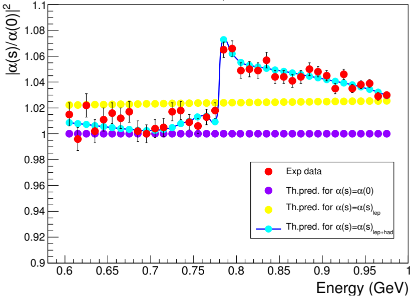
Running Couplings

$$\beta_{QED} = \mu \frac{\partial e_r(\mu)}{\partial \mu} = \frac{e_r(\mu)^3}{12\pi^2}$$

PDG



KLOE-2 collaboration, 1609.06631

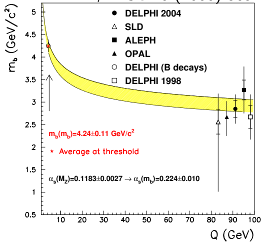


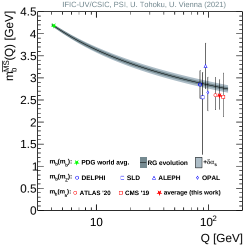
$$\beta_{QCD} = \frac{g_s^3}{16\pi^2}\left(-\frac{11}{3}N_c + \frac{2}{3}N_f\right)$$

Further Examples for Running in SM

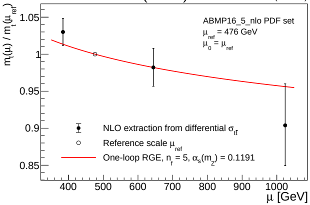
- parameters in the Standard Model and Beyond are energy dependent

DELPHI, EPJC 46 (2006) 569

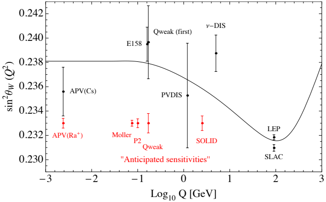




CMS PLB 803(2020)135263 35.9 fb^{-1} (13 TeV)



Davoudiasl et al., arXiv:1507.00352

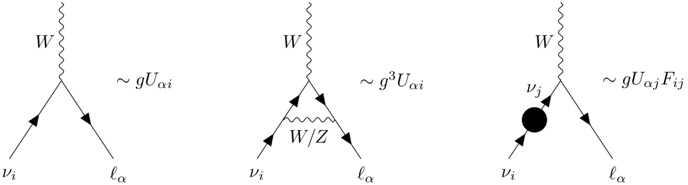


What about neutrino sector?

Energy Dependence of the PMNS Matrix

$$\nu_\alpha(Q^2) = U_{\alpha i}(Q^2)\nu_i$$

$\alpha = e, \mu, \tau$
$i, j = 1, 2, 3$

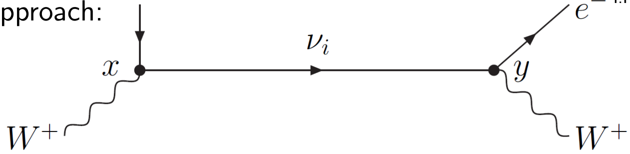


- when higher-order quantum effects are included, do $U_{\alpha i}$ matrix elements change relative to one another?
- higher-order electroweak corrections lead to very minor effects but in neutrino mass models $U_{\alpha i}$ can change in a flavor-dependent way
- this was already extensively studied for many models with heavy BSM degrees of freedom, see *e.g.* Antusch et al. (JHEP 03 (2005) 024) Casas et al. (NPB 573 (2000)) Goswami et al. (PRD 80 (2009)) Balaji et al. (PLB 481 (2000))
- we are however interested here in relatively light new physics where masses of new particles are comparable or lighter with respect to the neutrino energies at various experiments

Connection to Neutrino Experiments

M. Beuthe, arXiv:hep-ph/0109119
I.P. Volobuev, arXiv:1703.08070

QFT approach:



- amplitude: $\sum_i U^*_{\alpha i}\, e^{-i\frac{m_i^2 - p^2}{2|\vec{p}|}L}\, U_{\beta i} \to P_{\alpha\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\, e^{-i\frac{m_j^2 - m_k^2}{2|\vec{p}|}L}$

PRODUCTION: contribution to the amplitude should be Lorentz invariant; in the rest frame of decaying pion $E = m_\pi \to U_{\alpha i} = U_{\alpha i}(Q_p^2 = m_\pi^2)$

DETECTION: $U_{\beta i}(Q_d^2)$ where Q_d^2 has no dependence on m_π^2

PROPAGATION: neutrino is on shell $(Q^2 = p_\nu^2 = m_\nu^2 \approx 0)$
$\Longrightarrow m_i$ in formula is the mass at $\sqrt{Q^2} = m_i$

Neutrino Oscillations in Vacuum

2 flavors:

$$U(Q^2) = \begin{pmatrix} \cos\theta(Q^2) & \sin\theta(Q^2) \\ -\sin\theta(Q^2) & \cos\theta(Q^2) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\tilde{\beta}(Q^2)} \end{pmatrix}$$

$\theta(Q_p^2) \equiv \theta_p, \quad \theta(Q_d^2) \equiv \theta_d, \quad \text{and} \quad \tilde{\beta}(Q_d^2) - \tilde{\beta}(Q_p^2) \equiv \beta$

Grossman, PLB 359 (1995)

$$P_{\mu e} = \sin^2(\theta_p - \theta_d) + \sin 2\theta_p \sin 2\theta_d \sin^2\left(\frac{\Delta m^2 L}{4E} + \frac{\beta}{2}\right)$$

- β appears due to the CP-violating couplings in the new physics sector and it appears also in Dirac neutrino models
- β "shifts" the oscillation phase: $\Delta m^2 L/2E \to \Delta m^2 L/2E + \beta$

For $\epsilon_\theta = \theta_d - \theta_p \ll 1$ and $\beta = \epsilon_\beta + \mathcal{O}(\epsilon_\beta^2) \ll 1$:

$$P_{\mu e} = \epsilon_\theta^2 + \mathcal{O}(\epsilon_\theta^4) + \left[\sin^2 2\theta_d - \sin 4\theta_d \epsilon_\theta + \mathcal{O}(\epsilon_\theta^2)\right]\left[\sin^2\left(\frac{\Delta m^2 L}{4E}\right) + \frac{\epsilon_\beta}{2}\sin\left(\frac{\Delta m^2 L}{2E}\right) + \mathcal{O}(\epsilon_\beta^2)\right]$$

- In the zero-baseline limit, $L \to 0$, new-physics appears at $\mathcal{O}(\epsilon_\theta^2, \epsilon_\beta^2)$
- for a finite baseline, $\mathcal{O}(\epsilon_\theta, \epsilon_\beta)$

Neutrino Oscillations in Vacuum

3 flavors:

- CP-odd phases β, α, $\delta(Q_p^2)$, $\delta(Q_d^2)$
- $P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}}$ gives simple and useful results

$$\epsilon_{ij} \equiv \theta_{ij}(Q_d^2) - \theta_{ij}(Q_p^2),\ \epsilon_\delta = \delta(Q_d^2) - \delta(Q_p^2),\ \epsilon_\alpha = \alpha,\ \epsilon_\beta = \beta \qquad \Delta_{ij} \equiv \Delta m_{ij}^2 L/2E$$

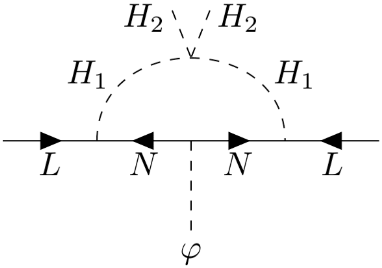
$$P_{\mu e} - P_{\bar{\mu}\bar{e}} \simeq -8\, J\, \Delta_{21} \sin^2\left(\frac{\Delta_{31}}{2}\right)\left[1 + \left(2\frac{\epsilon_{12}}{\sin 2\theta_{12}} + \epsilon_\alpha \frac{c_\delta}{s_\delta}\right)\frac{\cot(\Delta_{31}/2)}{\Delta_{21}}\right]$$

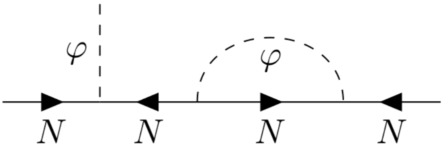
- in the $\delta \to 0$ limit, RG induced CP violation is present

$$P_{ee} - P_{\bar{e}\bar{e}} \simeq (\epsilon_\beta - \epsilon_\delta)\sin^2 2\theta_{13}\sin\Delta_{31} - \epsilon_\alpha(s_{12}^2 \sin^2 2\theta_{13}\sin\Delta_{31} - \sin^2 2\theta_{12}\sin\Delta_{21})$$

- apparent but not actual CPT viaolation $P(\nu_\alpha(Q_p^2) \to \nu_\alpha(Q_d^2)) = P(\bar{\nu}_\alpha(Q_d^2) \to \bar{\nu}_\alpha(Q_p^2))$, is satisfied
- for T2K and NOvA matter effects are included in our analysis

The Model





Scotogenic-like realization

$$-\mathcal{L}_\nu^{(1)} = \overline{L} Y_\nu \tilde{H}_1 N_R + \varphi \overline{N_R^c} Y_N N_R + \text{h.c.}$$

	$U(1)_L$	$\mathbb{Z}_2$
L	0	+
H_2	0	+
H_1	+1	−
N_R	+1	−
ϕ	−2	+

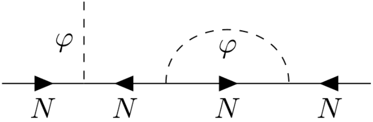
for $M_N^i = Y_N^i v_\varphi/\sqrt{2} \ll M_{H,A}$, $\quad M_\nu \simeq \frac{v_\varphi}{16\sqrt{2}\pi^2} Y_\nu Y_N Y_\nu^T \ln \frac{M_H^2}{M_A^2}$

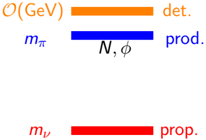
The Model

$$H = \sum_i \frac{m_i^2}{2E} |\nu_i\rangle\langle\nu_i| + \sqrt{2} G_F N_e |\nu_e(Q^2 = 0)\rangle\langle\nu_e(Q^2 = 0)|$$

- at Q_p^2 scale mixing parameters are sampled using NuFIT values
- $Y_N \sim \mathcal{O}(1)$
- Y_ν is obtained using Casas-Ibarra parametrization

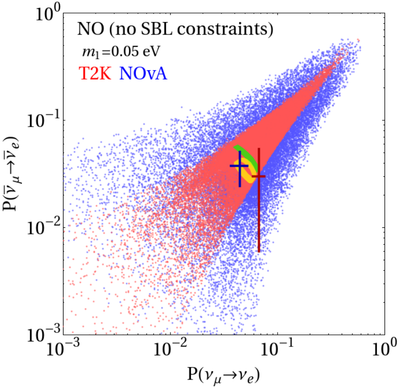
$$16\pi^2 \beta(Y_N) \equiv 16\pi^2 \frac{dY_N}{d \ln |Q|} = 4 Y_N \left[Y_N^2 + \frac{1}{2} \mathrm{Tr}(Y_N^2) \right]$$

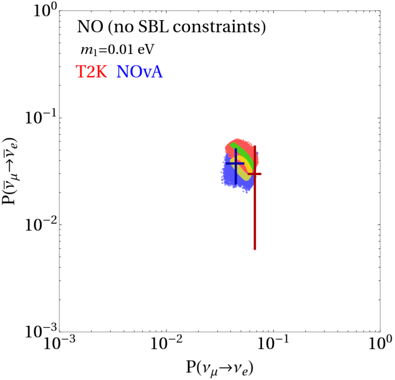




- at Q_d^2 scale, Y_N and hence $M_\nu(Q_d^2)$ is found
- diagonalize M_ν to get PMNS matrix at higher scale

RGE Effect





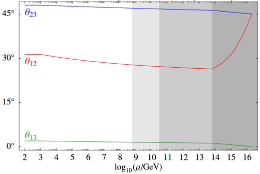
Ohlsson, Zhou, Nature Commun. 5 (2014) 5153

$$8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\theta_{12} \approx -C_\kappa y_\tau^2 \frac{m_1^2}{\Delta m_{21}^2} s_{23}^2 \sin 2\theta_{12}$$

$$8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\theta_{13} \approx -C_\kappa y_\tau^2 \frac{m_1^2}{\Delta m_{31}^2} c_{23}^2 \sin 2\theta_{13}$$

$$8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\theta_{23} \approx -C_\kappa y_\tau^2 \frac{m_1^2}{\Delta m_{31}^2} \sin 2\theta_{23}$$

Antusch et al. (JHEP 03 (2005) 024)

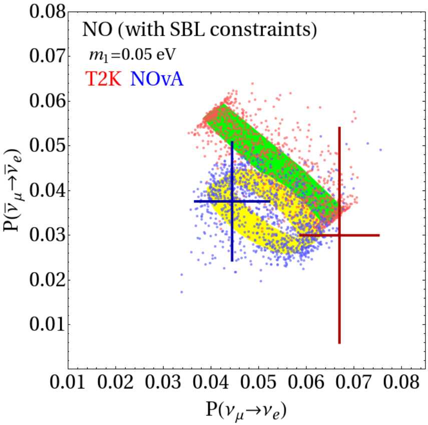


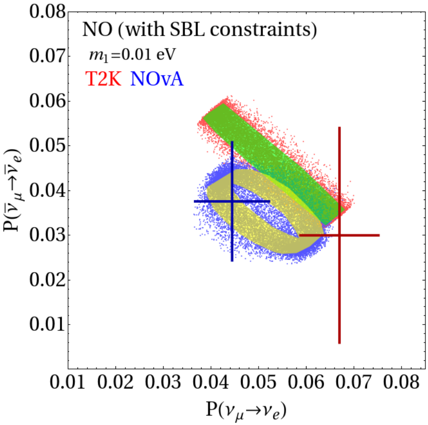
Constraints from Short Baseline Experiments

- RG running leads to zero baseline effects since $U(Q_p^2)U^\dagger(Q_d^2) \neq \mathbb{1}$
- experiments with high average neutrino energy are especially sensitive due to the larger difference between $Q_p^2 = m_\pi^2$ and Q_d^2
- while we found successful explanations or LSND and MiniBooNE, constraints from short baseline experiments rule out such possibilities

Experiment	E (GeV)	$\sqrt{Q_d^2}$ (GeV)	channel	constraint
ICARUS	17	3.94	$\nu_\mu \to \nu_e$	3.4×10^{-3}
CHARM-II	24	4.70	$\nu_\mu \to \nu_e$	2.8×10^{-3}
NOMAD	47.5	6.64	$\nu_\mu \to \nu_e$	7.4×10^{-3}
			$\nu_\mu \to \nu_\tau$	1.63×10^{-4}
NuTeV	250	15.30	$\nu_\mu \to \nu_e$	5.5×10^{-4}
			$\nu_e \to \nu_\tau$	0.1
			$\nu_\mu \to \nu_\tau$	9×10^{-3}

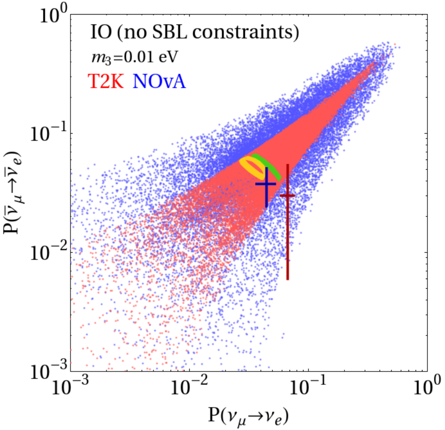
Constraints from Short Baseline Experiments

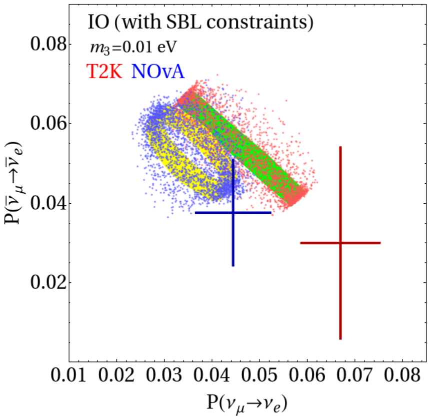




- short baseline constraints remove parameter points with strongest running

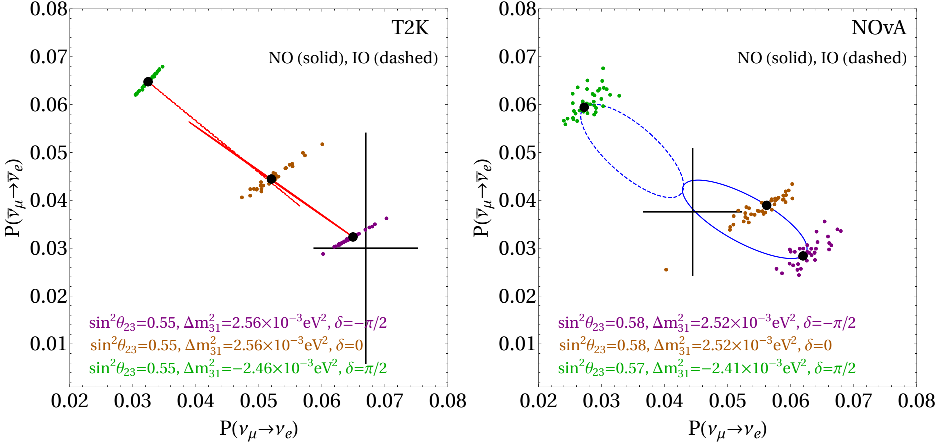
Inverted Ordering





- for fixed lightest neutrino mass, RG running in the inverted ordering is stronger than in the normal one

Bi-probabilities



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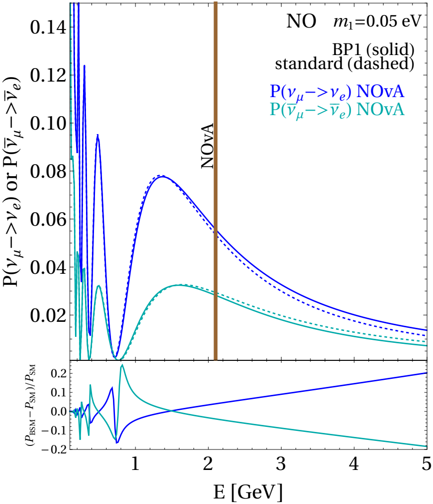
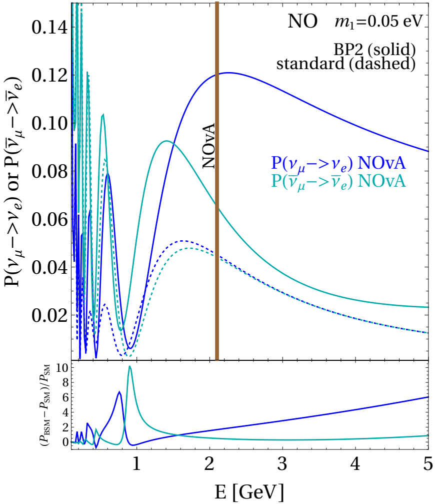
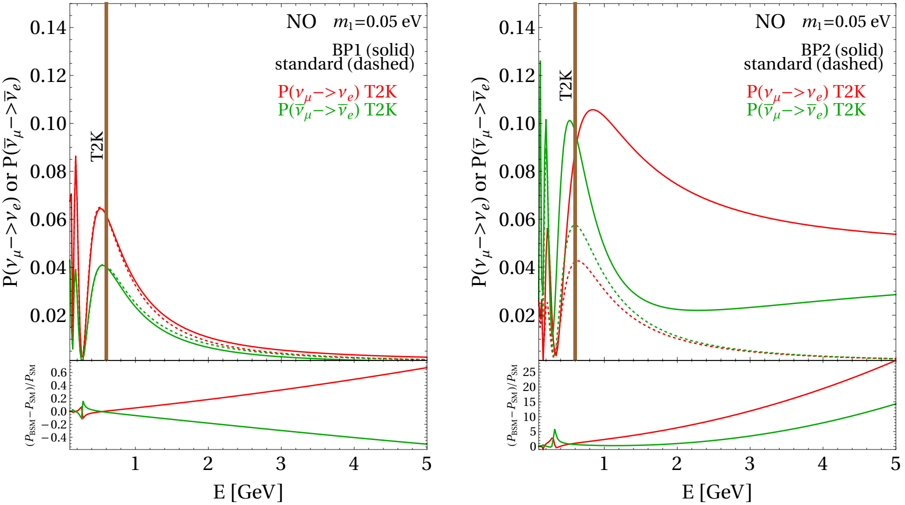
Oscillation Probabilities – NOvA



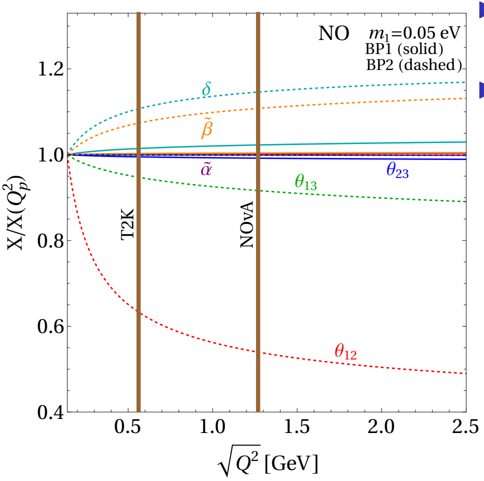


- BP1 best fits T2K and NOvA data and BP2 is strongly disfavoured by both short and long baseline experiments

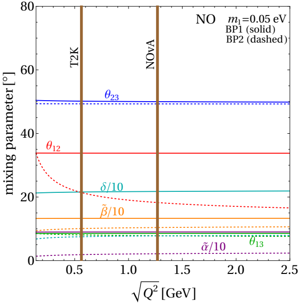
Oscillation Probabilities – T2K



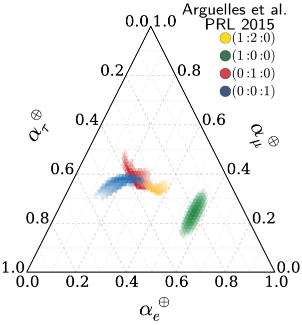
RG Evolution of the Mixing Parameters

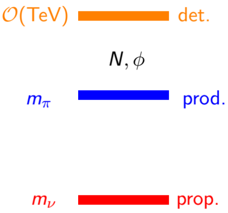


- the strongest effects are in running of θ_{12}
- variation of θ_{12} relative to the other mixing angles θ_{13} and θ_{23} is enhanced by $|\Delta\theta_{12}/\Delta\theta_{13}|$, $|\Delta\theta_{12}/\Delta\theta_{23}| \propto |\Delta m^2_{31}/\Delta m^2_{21}|$



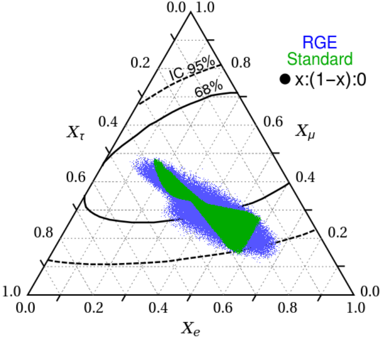
Ultra-High Energy Neutrinos - Flavor Ratios





- detected neutrinos are incoherent superposition of mass eigenstates

$$P_{\alpha\beta} = \sum_{j=1}^{3} \left|U_{\alpha j}(Q_p^2)\right|^2 \left|U_{\beta j}(Q_d^2)\right|^2$$



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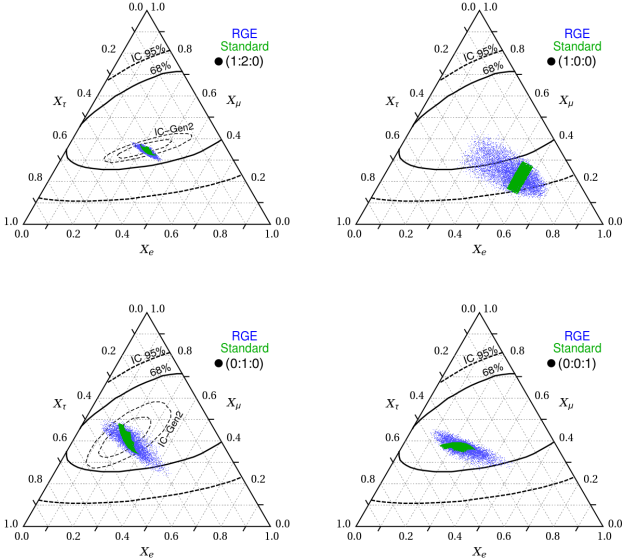

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- all of this can be induced by light new physics sector that does not need to be produced at experiments but only impacts through quantum corrections that induce non-trivial energy depedence of $U(Q^2)$
- renormalization group evolution of the mixing parameters can induce observable effects at T2K and NOvA and in the flavor composition of ultra-high energy neutrinos