Electromagnetism and Gravity with Continuous Spin

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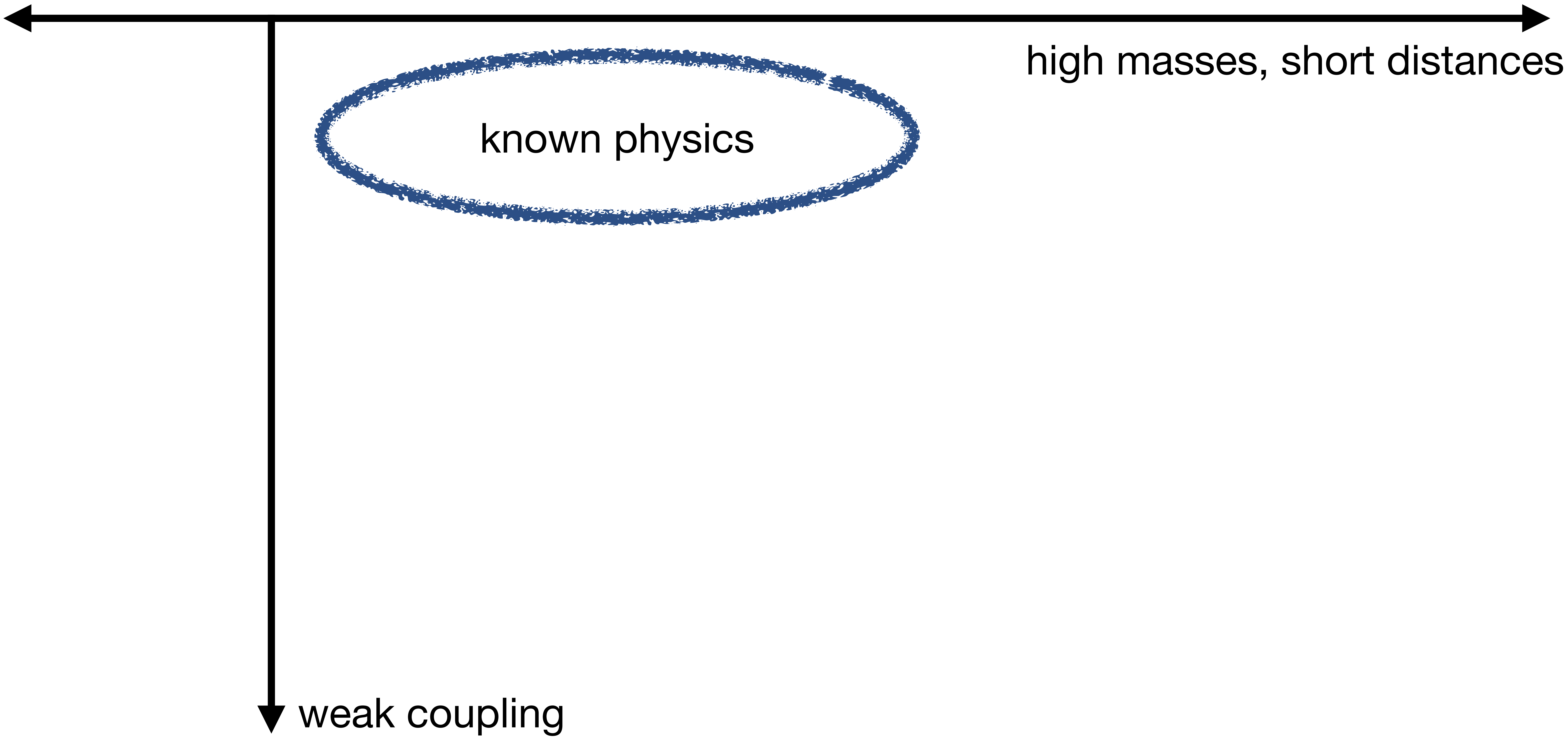
SLAC

Hunting Invisibles Physics Seminar — November 7, 2023

arXiv:2303.04816 (JHEP), with Philip Schuster and Natalia Toro

My focus is finding new experimental methods to probe beyond the Standard Model.



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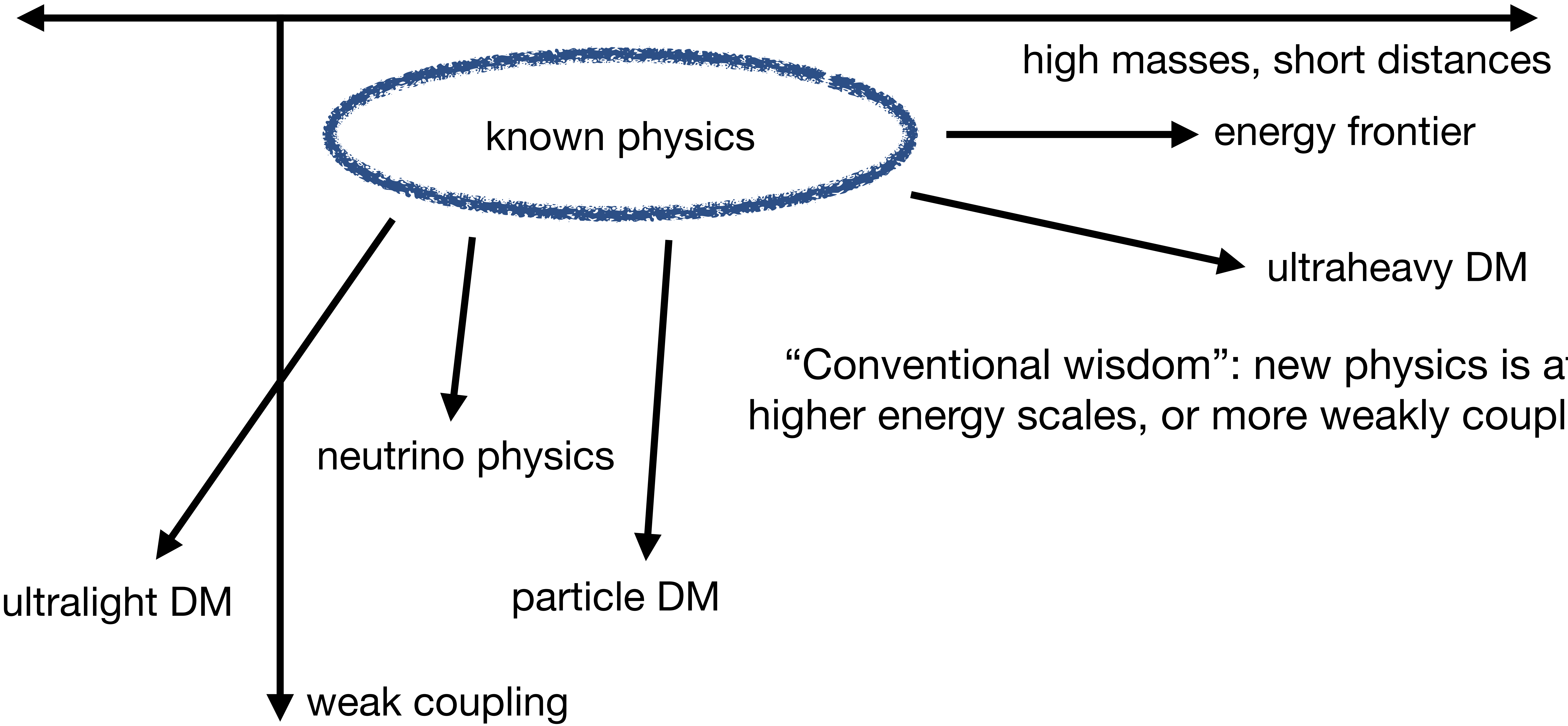
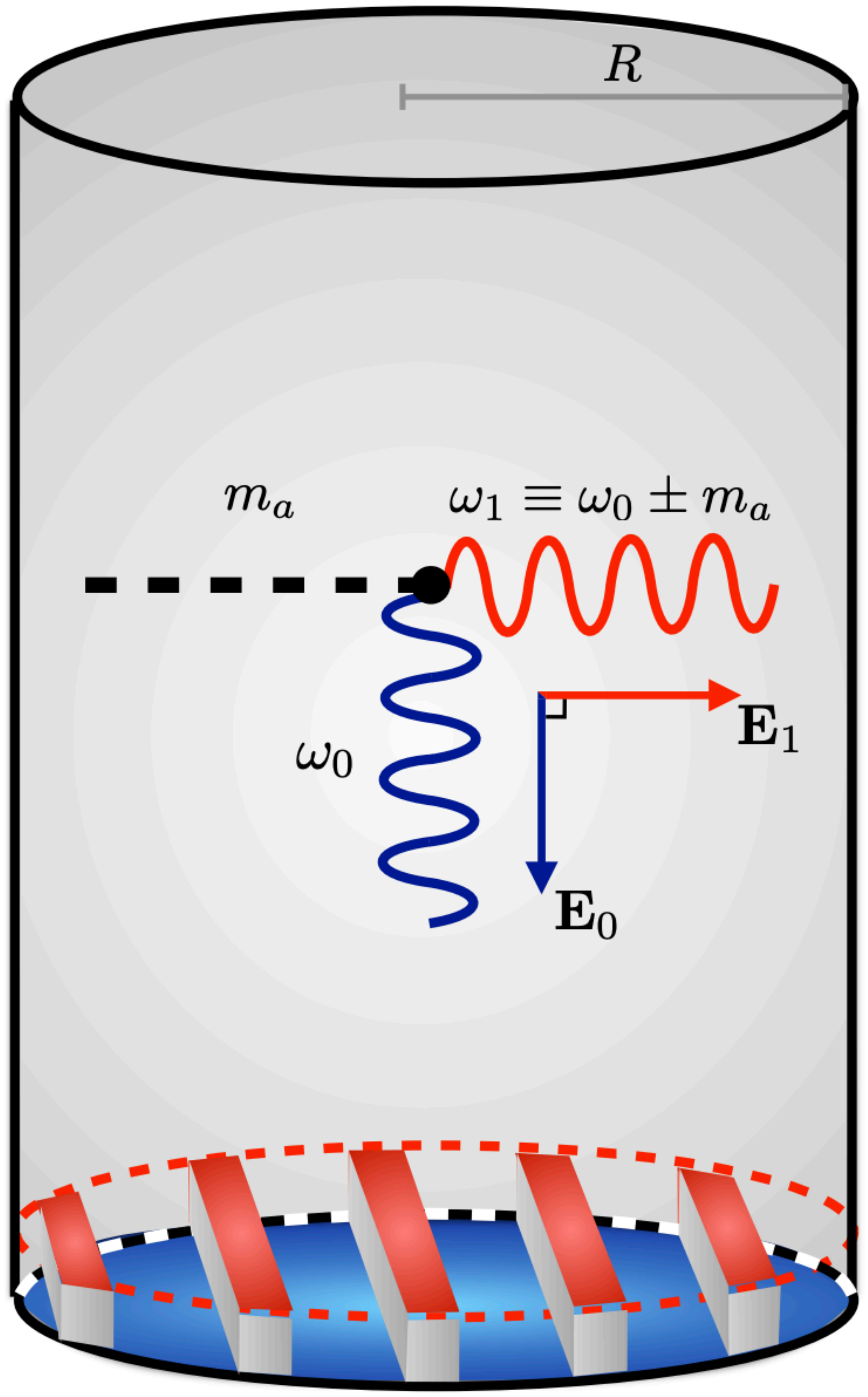
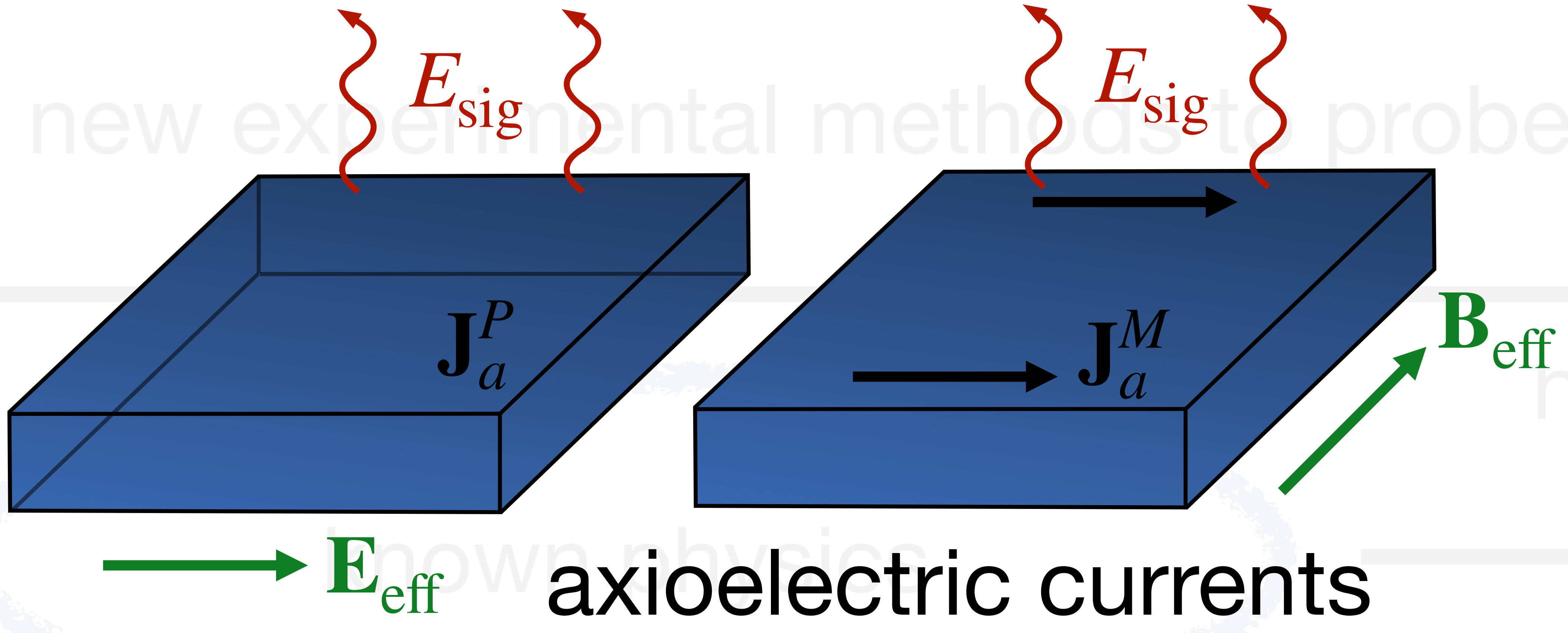


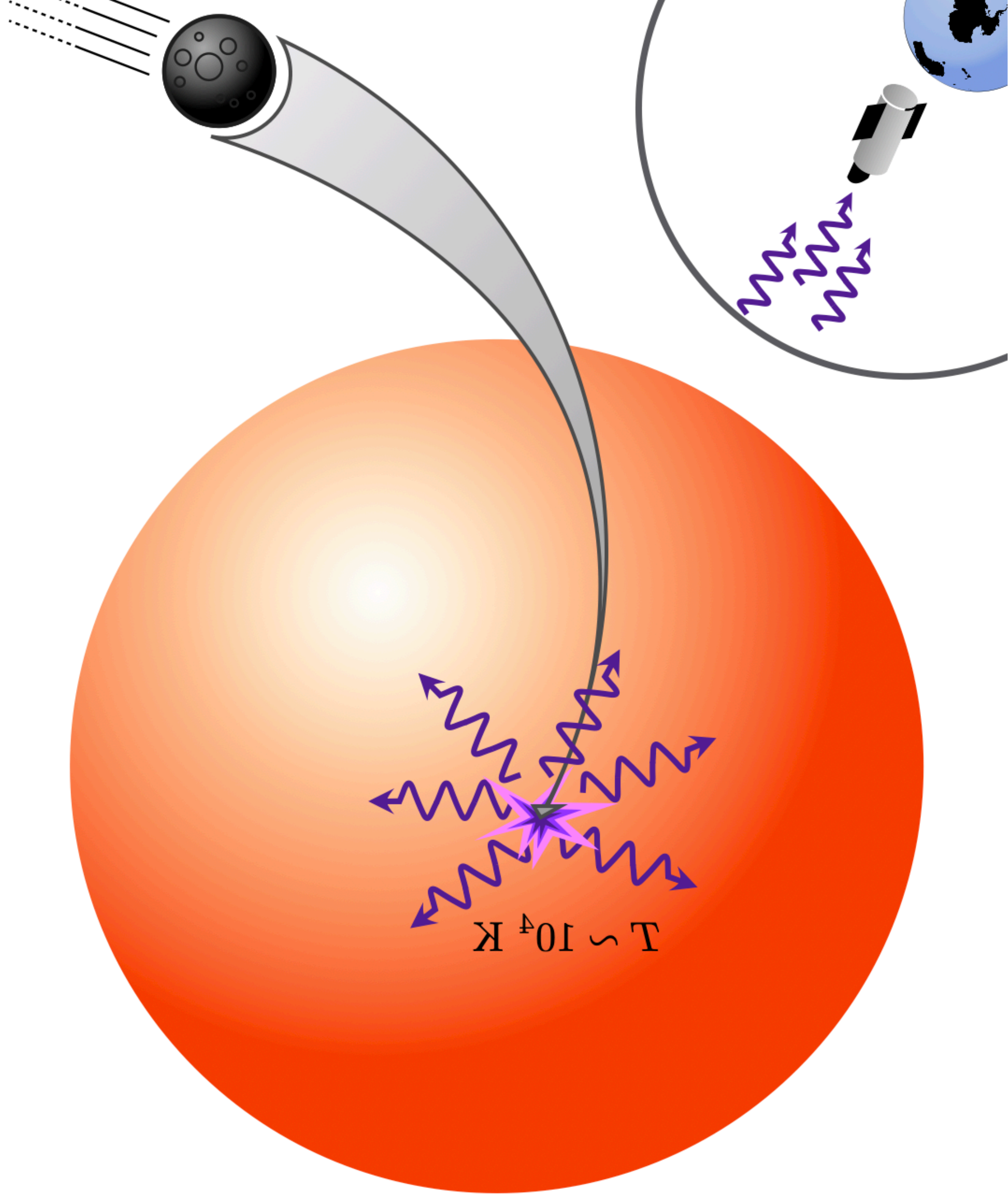
“Conventional wisdom”: new physics is at higher energy scales, or more weakly coupled



axion upconversion

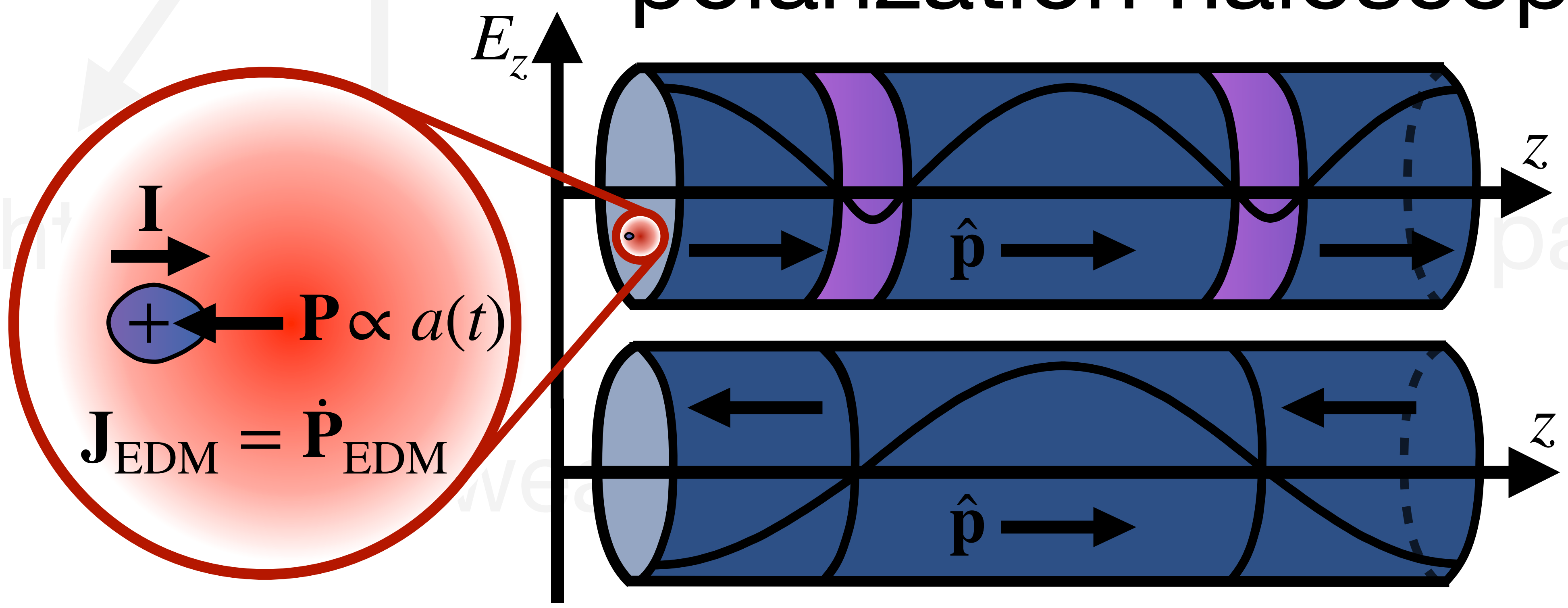


axioelectric currents

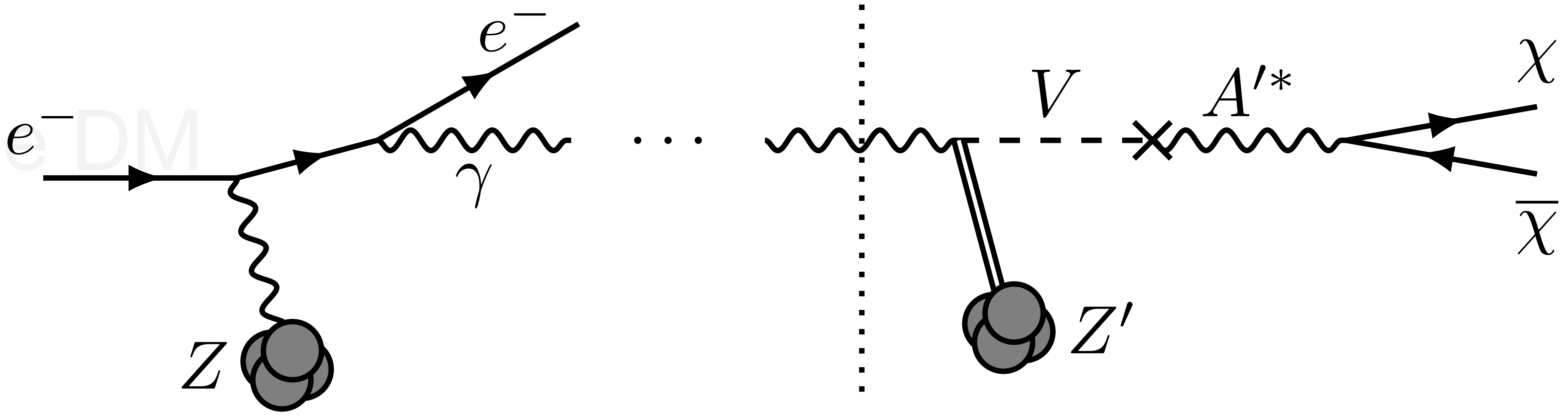


stellar shock transients

(accordingly, much of my work has been proposing new ways to search for dark matter, at a variety of mass scales)



polarization haloscopes



invisible meson decays

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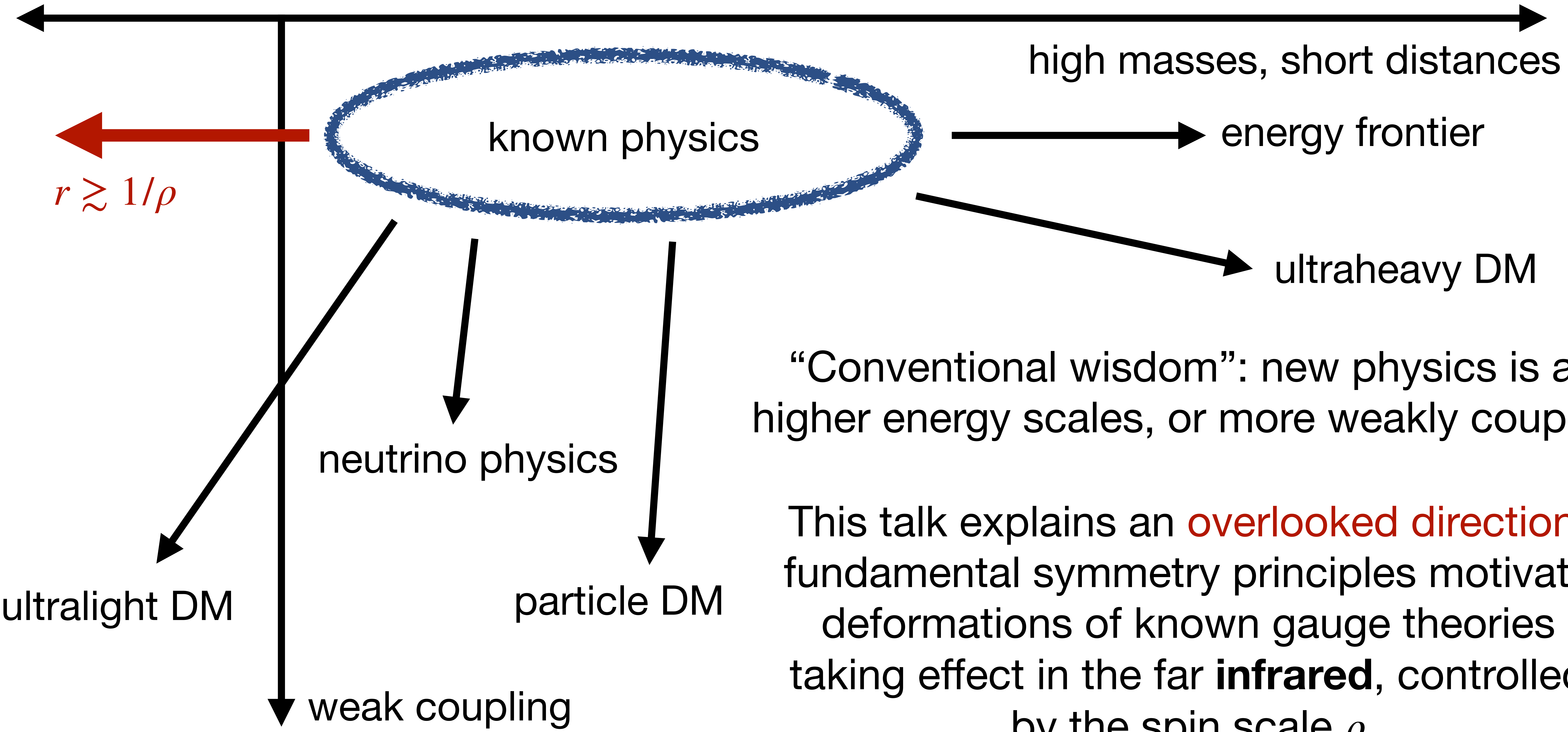

"Conventional wisdom": new physics is at higher energy scales, or more weakly coupled

This talk explains an overlooked direction: fundamental symmetry principles motivate deformations of known gauge theories taking effect in the far **infrared**, controlled by the spin scale ρ

Classifying Particles by Mass and Spin Scale

States transform under translations P^μ and rotations/boosts $J^{\mu\nu}$

Particle states with definite momentum obey $P^\mu \;\; k, \sigma\rangle = k^\mu \;\; k, \sigma\rangle$

Little group transformations $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J_{\nu\rho} k_\sigma$ affect only internal state σ

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Different types of particles classified by $P^2 = m^2$ and $W^2 = -\rho^2$

What is the physical meaning of the spin scale ρ?

Classifying Particles by Mass and Spin Scale

For $m^2 > 0$, representations are spin S massive particles

States are $|k, h\rangle$ for helicity $h = -S, \ldots, S$, which is not Lorentz invariant

Boosts mix helicities by amount determined by $\rho = m\sqrt{S(S+1)}$

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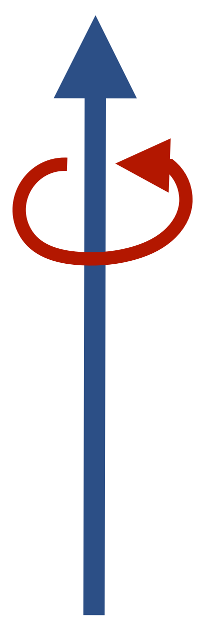
For $m^2 = 0$, states are still indexed by helicity $|k, h\rangle$

Spin scale again determines how helicity varies under boosts

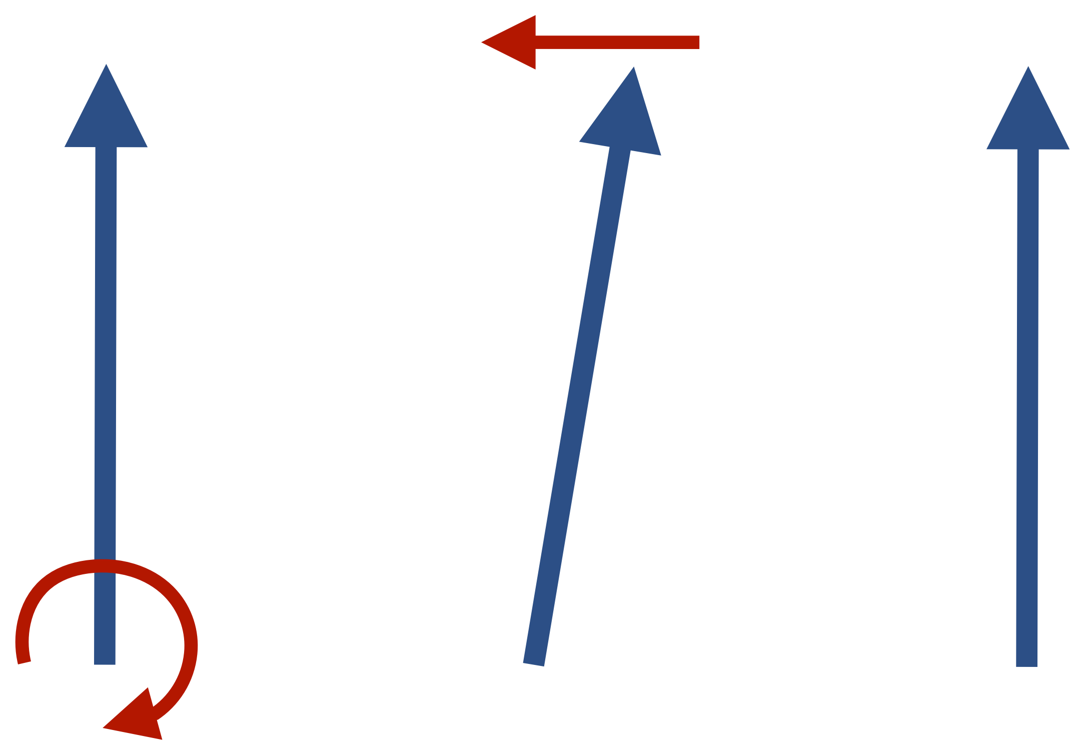
The Massless Little Group

For a massless particle, $k^\mu = (\omega, 0, 0, \omega)$, little group generators are



$$R = J_z$$

$$R \;\; k, h\rangle = h \;\; k, h\rangle$$



$$T_1 \propto J_x + K_y$$

$$T_2 \propto J_y - K_x$$

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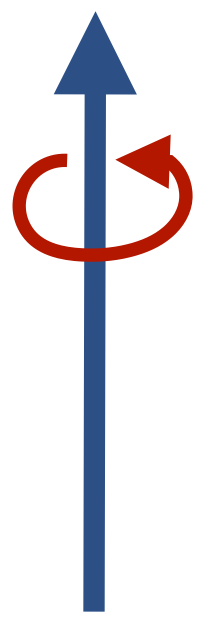

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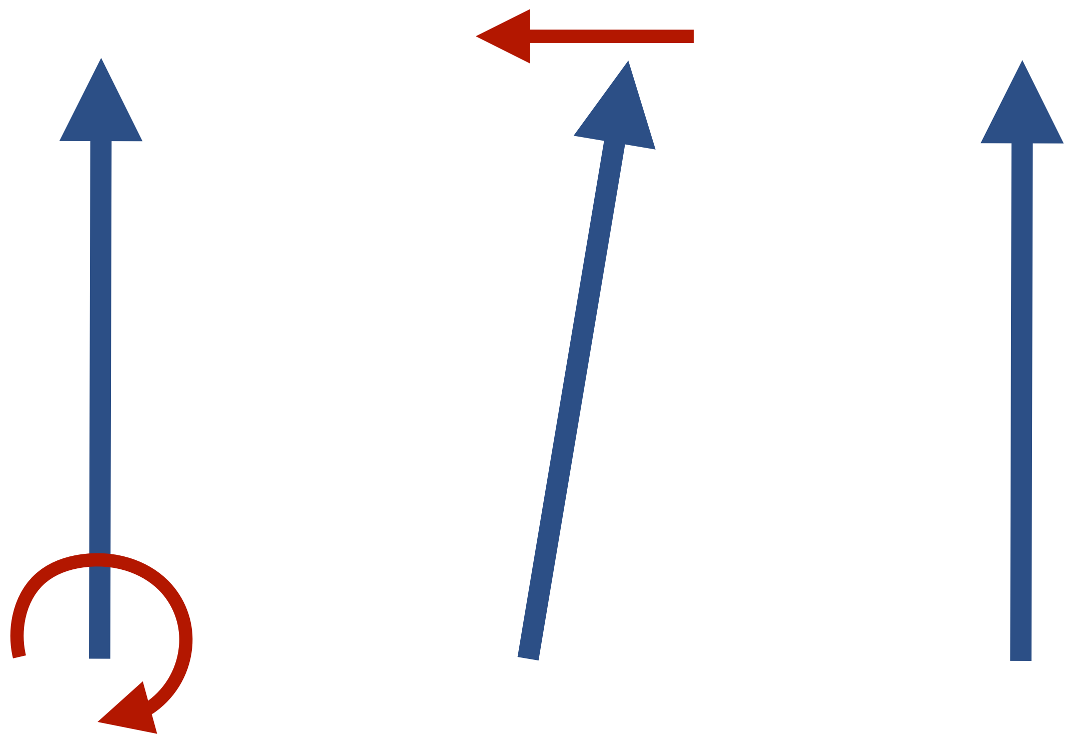

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Defining $T_\pm = T_1 \pm iT_2$, commutation relations imply

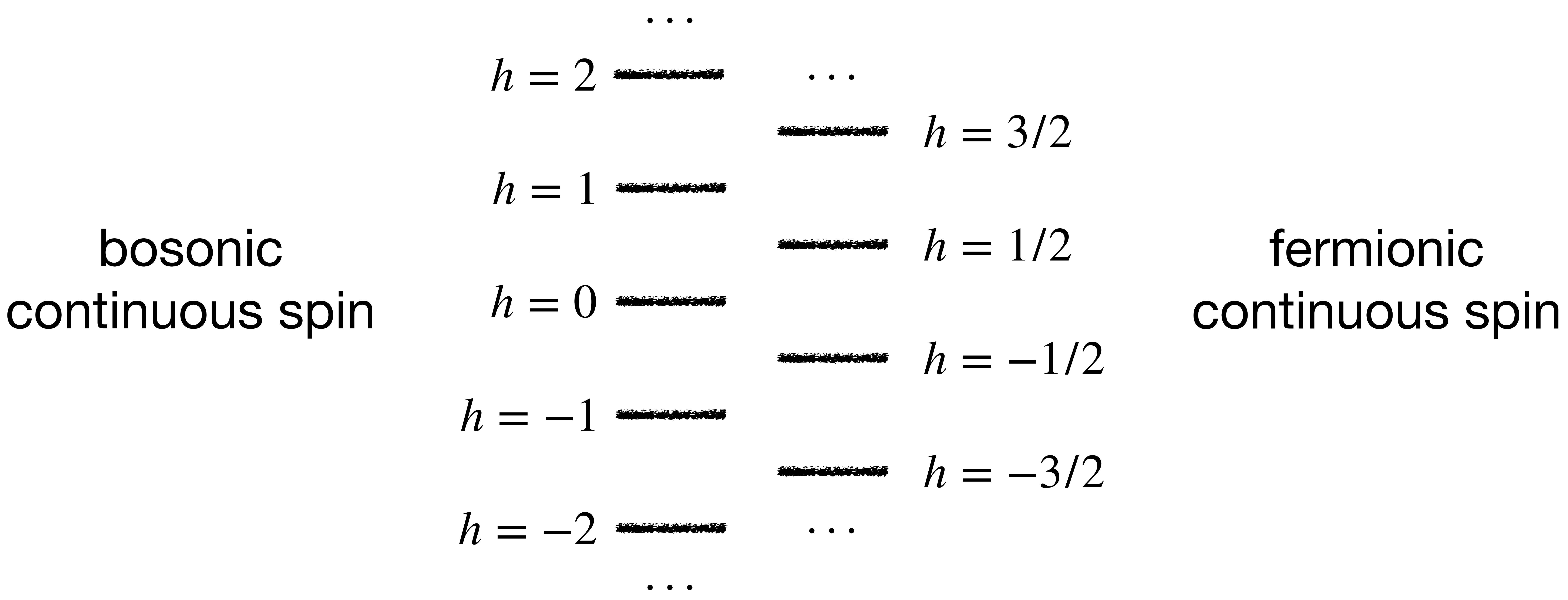
$$T_\pm \;\; k, h\rangle = \rho \;\; k, h \pm 1\rangle$$

Generic result is an **infinite** ladder of integer-spaced helicities!

Allowed Helicities for Massless Particles

Generic massless particle representation has continuous-valued spin scale ρ

Since h is always integer or half-integer, gives two options, known since 1930s:



(plus supersymmetric, (A)dS, higher/lower dimension variants)

Allowed Helicities for Massless Particles

If we set $\rho = 0$, recover a single helicity h (related to $-h$ by CPT symmetry)

Focus on bosonic case, which can mediate long-range $1/r^2$ forces

$h = 0$

$h = 1$

$h = 2$

$h = 3$

$\ldots$

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Focus on bosonic case, which can mediate long-range $1/r^2$ forces

- $h = 0$ massless scalar (requires fine-tuning)
- $h = 1$ photon (minimal coupling to conserved charge)
- $h = 2$ graviton (minimal coupling to stress-energy)
- $h = 3$ higher spin (no minimal couplings allowed)
- $\dots$

Role of each h in nature well-understood from general arguments from 1960s

Why Not Consider Continuous Spin?

Ruled out by Weinberg soft theorems?

Theorems rely on Lorentz invariant h

Generalize to good soft factors for $\rho \neq 0$

Schuster and Toro, JHEP (2013) 104/105

Incompatible with field theory?

Simple free gauge theory found

Schuster and Toro, PRD (2015)

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Infinite set of h's leads to infinities in scattering/cosmology/astrophysics/…?

All but one h decouples in the $\rho \to 0$ limit!

$h = 0$ scalar-like (recovers Yukawa theory)

$h = 1$ vector-like (recovers electromagnetism)

$h = 2$ tensor-like (recovers linearized gravity)

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Theory predicts ρ-dependent deviations from electromagnetism and general relativity

The value of ρ is unknown, and only experiment can determine it

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Force in a radiation background: $$\frac{\mathbf{F}}{q} = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \left(\frac{\rho v_\perp}{2\omega} \right)^2 \left(\mathbf{E}_\perp + \frac{\mathbf{E}}{2} \right) + \ldots$$

Significant correction when particles travel distance $\gtrsim 1/\rho$, total result always well-behaved

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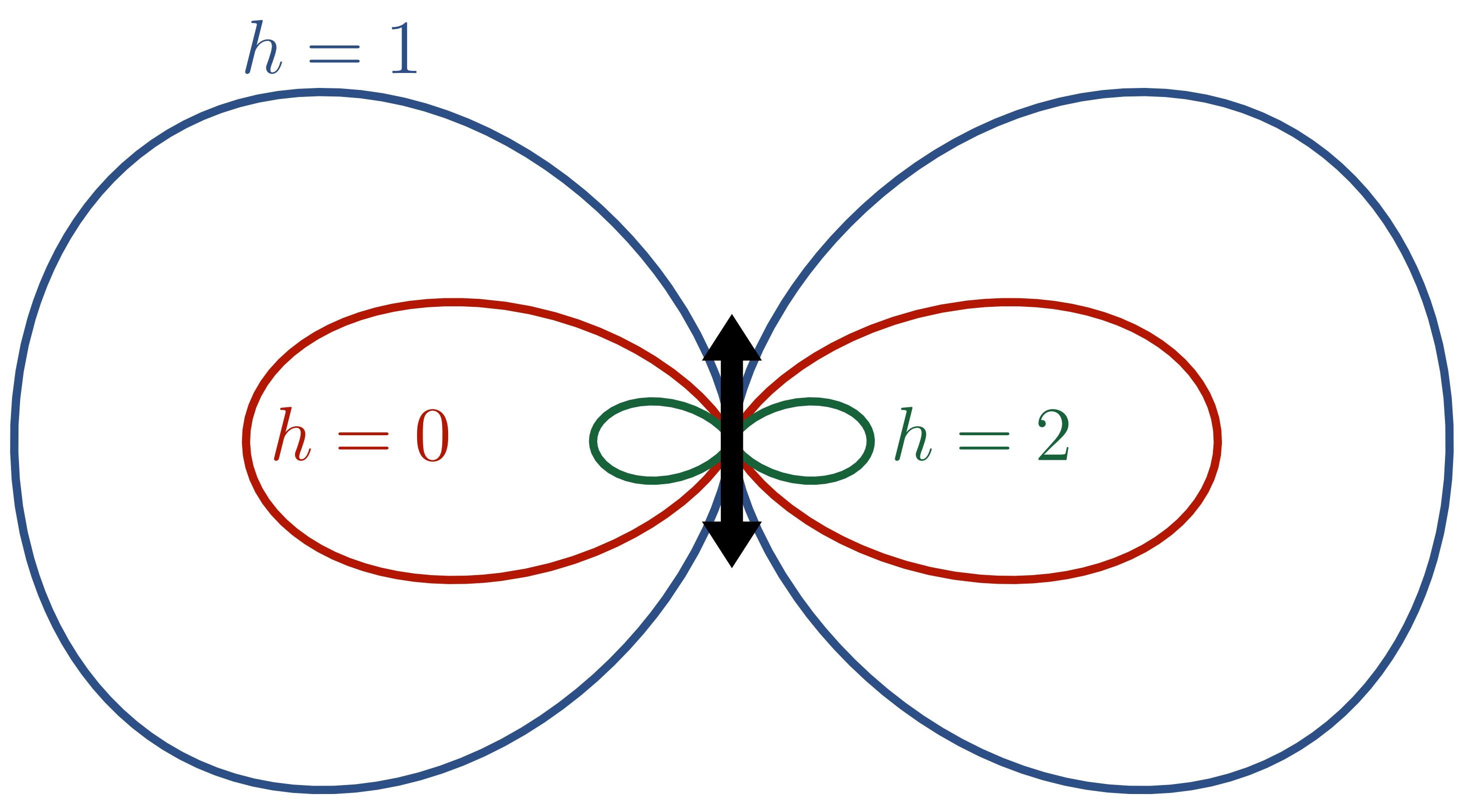
Radiation from an oscillating particle:



$$P = P_{\text{Larmor}} \times \begin{cases} (\rho\ell)^2/40 + \ldots & h = 0 \\ 1 - 3\,(\rho\ell)^2/20 + \ldots & h = \pm 1 \\ (\rho\ell)^2/80 + \ldots & h = \pm 2 \end{cases}$$

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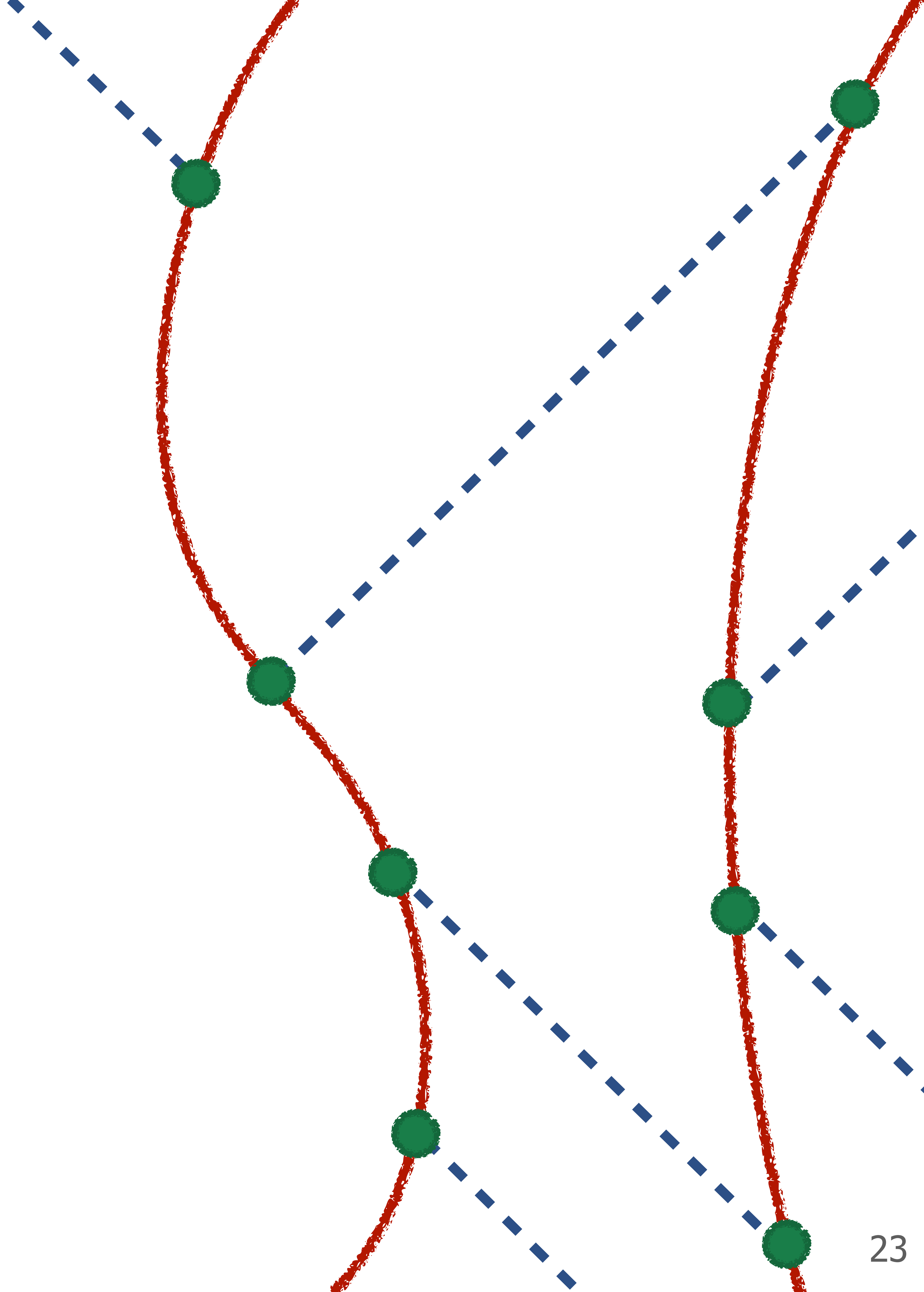
For the model builder: “because it’s novel”

A new infrared deformation of gauge theories, which may shed light on long-distance physics (dark matter, cosmic acceleration)

A new type of spacetime symmetry based on a bosonic superspace, possibly relevant for tuning problems (hierarchy, cosmological constant)

Outline

- **Free continuous spin fields**
- Coupling to matter particles
- Physics with continuous spin



Free Fields for Massless Particles

Tricky even for $\rho = 0$, by mismatch of field and particle degrees of freedom

scalar $h = 0$	scalar field ϕ, no extra components
photon $h = \pm 1$	vector field A_μ, $4 - 2 = 2$ extra components
	must use action with gauge symmetry $\delta A_\mu = \partial_\mu \alpha$

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	must use action with gauge symmetry $\delta A_\mu = \partial_\mu \alpha$
graviton $h = \pm 2$	sym. tensor field $h_{\mu\nu}$, $10 - 2 = 8$ extra components
	must use action with gauge symmetry $\delta h_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}$
higher spin $h > 2$	sym. tensor field $\phi_{\mu_1 \ldots \mu_h}$, many extra components

Given complexity of higher h, constructing a continuous spin field seems intractable!

Introducing Vector Superspace

A field in "vector superspace" (x^μ, η^μ) has tensor components of all ranks

$$\Psi(\eta, x) = \phi(x) + \sqrt{2}\,\eta^\mu A_\mu(x) + (2\eta^\mu\eta^\nu - g^{\mu\nu}(\eta^2 + 1))\,h_{\mu\nu}(x) + \dots.$$

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flat metric

just suggestive notation: these components not necessarily related to electromagnetic potential or metric perturbation

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Simple expression has free Lagrangian for each tensor field simultaneously!

$$\mathscr{L}[\Psi] = \frac{1}{2} \int_\eta \delta'(\eta^2 + 1)(\partial_x \Psi)^2 + \frac{1}{2}\, \delta(\eta^2 + 1)(\Delta\Psi)^2 \qquad \Delta = \partial_x \cdot \partial_\eta$$

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Integration measure normalized by $\displaystyle\int_\eta \delta(\eta^2+1) \equiv 1$

Symmetry and basic integration properties fix all other integrals, e.g. $\displaystyle\int_\eta \delta'(\eta^2+1) = 1, \int_\eta \delta'(\eta^2+1)\,\eta^\mu\eta^\nu = -\frac{1}{2}g^{\mu\nu}, \dots$

Recovering Familiar Actions

$$\mathscr{L}[\phi] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\,\delta(\eta^2+1)(\partial_x \cdot \partial_\eta \Psi)^2 \bigg|_{\Psi=\phi}$$

Recovering Familiar Actions

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More generally, we recover the linearized Einstein-Hilbert action, and higher-rank Fronsdal actions, with no mixing

Recovering Familiar Dynamics

One equation of motion for all helicities:

$$\delta'(\eta^2 + 1)\, \partial_x^2\, \Psi - \frac{1}{2}\, \Delta\, (\delta(\eta^2 + 1)\, \Delta\Psi) = 0 \qquad \begin{cases} \partial^2 \phi = 0 \\ \partial^2 A^\mu - \partial^\mu(\partial \cdot A) = 0 \\ \cdots \end{cases}$$

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$$\begin{cases} \partial^2\phi = 0 \\ \partial^2 A^\mu - \partial^\mu(\partial\cdot A) = 0 \\ \cdots \end{cases}$$

One gauge transformation for all helicities:

$$\delta\Psi = (\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)\Delta)\,\alpha(\eta,x)$$

$$\begin{cases} \delta A^\mu = \partial^\mu\alpha \\ \delta h^{\mu\nu} = \partial^{(\mu}\alpha^{\nu)} \\ \cdots \end{cases}$$

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One mode expansion for all helicities:

$$\Psi_{k,h} = e^{-ik\cdot x}\,(\eta\cdot\epsilon_\pm)^{\,h}$$

$$\begin{cases} \phi_k = e^{-ik\cdot x} \\ A_k^\mu = e^{-ik\cdot x}\epsilon_\pm^\mu \\ \cdots \end{cases}$$

Turning on the Spin Scale

All previous results can be generalized to arbitrary ρ by taking $\Delta = \partial_x \cdot \partial_\eta + \rho$

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All previous results can be generalized to arbitrary ρ by taking $\Delta = \partial_x \cdot \partial_\eta + \rho$

Still get one mode of each helicity, but now the action, equation of motion, gauge transformations, and plane waves all mix tensor ranks, e.g.

$$\mathscr{L} \supset \rho \left(\frac{1}{\sqrt{2}} \phi \, \partial_\mu A^\mu + \ldots \right) + \rho^2 \left(-\frac{1}{4} \phi \, h^\mu_\mu + \ldots \right)$$

$$\Psi_{k,h} = e^{-ik\cdot x} \, e^{-i\rho\eta\cdot q} \, (\eta \cdot \epsilon_\pm)^{\,h} \qquad\qquad q \cdot k = 1$$

Because of the infinite tower of mixing terms, tensor expansion is complicated and physically opaque, while vector superspace description remains simple

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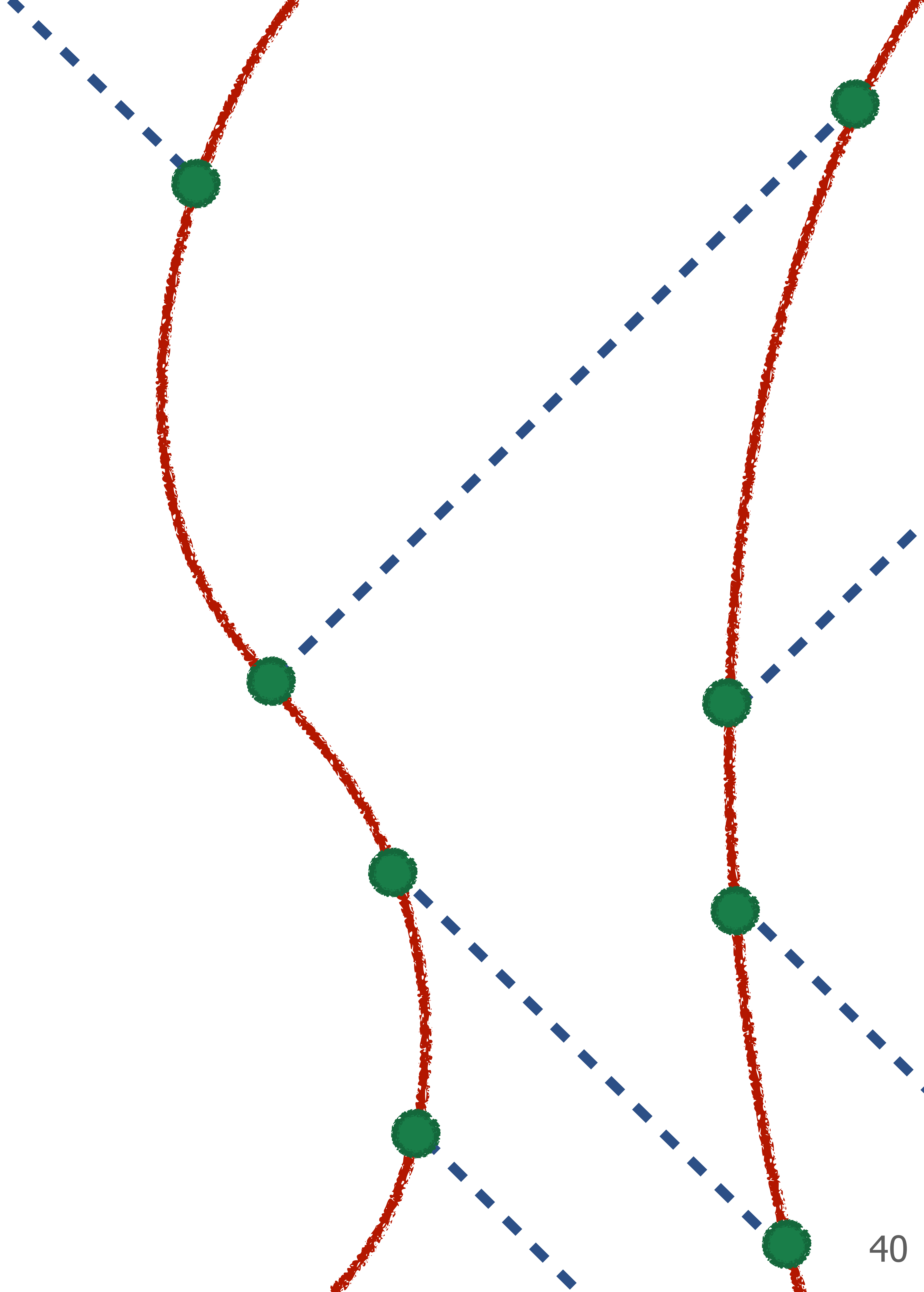

Coupling Currents to Fields

Couple the continuous spin field to a current by

$$\mathscr{L}_{\text{int}} = \int_\eta \delta'(\eta^2 + 1)\, J(\eta, x)\, \Psi(\eta, x)$$

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$$\mathscr{L}_{\text{int}} = \int_\eta \delta'(\eta^2 + 1)\, J(\eta, x)\, \Psi(\eta, x) = \phi J - A_\mu J^\mu + h_{\mu\nu} J^{\mu\nu} + \dots$$

Recover familiar results by tensor decomposition

$$J(\eta, x) = J(x) - \sqrt{2}\, \eta^\mu J_\mu(x) + (2\eta^\mu \eta^\nu + g^{\mu\nu})\, J_{\mu\nu}(x) + \dots$$

Preserving continuous spin gauge symmetry implies relations between $J, J_\mu, J_{\mu\nu}, \dots$

Tensors are **not** conserved! Instead, nonzero divergence related to other tensors

Digression: Connecting to the Hierarchy Problem

A simplified framing: minimally coupled scalars are not naturally light

Minimally coupled massless scalar receives large mass corrections $\delta m^2 \sim \Lambda_{\mathrm{UV}}^2$

Goldstone bosons like axions have mass protected by shift symmetry — but requires derivative couplings

Continuous spin fields achieve both at once: a minimal coupling $\mathscr{L} \supset \phi J$, and a mass protected by gauge symmetry!

Protecting Scalar Masses: Bottom Up

To see this, truncate the tensor expansion at order ρ assuming ϕ dominates

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\rho}{\sqrt{2}} (\phi\, \partial_\mu A^\mu) + \ldots$$

Gauge symmetry is $\delta A_\mu = \partial_\mu \epsilon / \sqrt{2}$, $\delta\phi = \rho\epsilon$, forbidding scalar mass term!

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But we can simultaneously have a minimal coupling for the scalar

$$\mathscr{L} \supset \phi J - A_\mu J^\mu + \ldots$$

which preserves gauge symmetry at order ρ provided $\partial_\mu J^\mu = -\rho J / \sqrt{2}$

Continuous spin theory is the extension of this idea to all orders in ρ!
A leading conserved current comes with infinite tower of nonconserved currents

Protecting Scalar Masses: Top Down

A deeper perspective: the full action, with coupling to $J(\eta, x)$, is invariant under the bosonic superspace translation $\delta x^\mu = \omega^{\mu\nu}\eta_\nu$

Corresponds to tensorial conserved charge $i\eta^{[\mu}\partial_x^{\nu]}$ which mixes modes separated by **integer** helicity — a new exception to Coleman-Mandula

Transfers the protection of massless vectors, tensors, etc. to massless scalars

Of course, much more work needed to see if something like this can protect the mass of the Higgs at the quantum level

Currents From Matter Particles

In familiar theories, the current from a matter particle is local to its worldline $z^\mu(\tau)$

For spinless particles, the minimal couplings are:

$$\left.\begin{aligned} J(x) &= g \int d\tau\, \delta^4(x - z(\tau)) \\ J^\mu(x) &= e \int d\tau\, \delta^4(x - z(\tau))\, \frac{dz^\mu}{d\tau} \\ T^{\mu\nu}(x) &= m \int d\tau\, \delta^4(x - z(\tau))\, \frac{dz^\mu}{d\tau}\frac{dz^\nu}{d\tau} \end{aligned}\right\} \text{ should be } \rho \to 0 \text{ limit of } \left\{\begin{aligned} &\text{scalar-like current} \\ &\text{vector-like current} \\ &\text{tensor-like current} \end{aligned}\right.$$

Must be incorporated into full current $J(\eta, x)$ satisfying $\delta(\eta^2 + 1)\,(\partial_x \cdot \partial_\eta + \rho)\, J(\eta, x) = 0$

Locality and Causality

In Fourier space, current and gauge invariance condition are

$$J(\eta, k) = \int d\tau\, e^{ik\cdot z(\tau)} f(\dot{z}, k, \eta) \qquad\qquad (-ik\cdot\partial_\eta + \rho) f \approx 0$$

Our currents are generically **not** localized to the worldline!

Locality and Causality

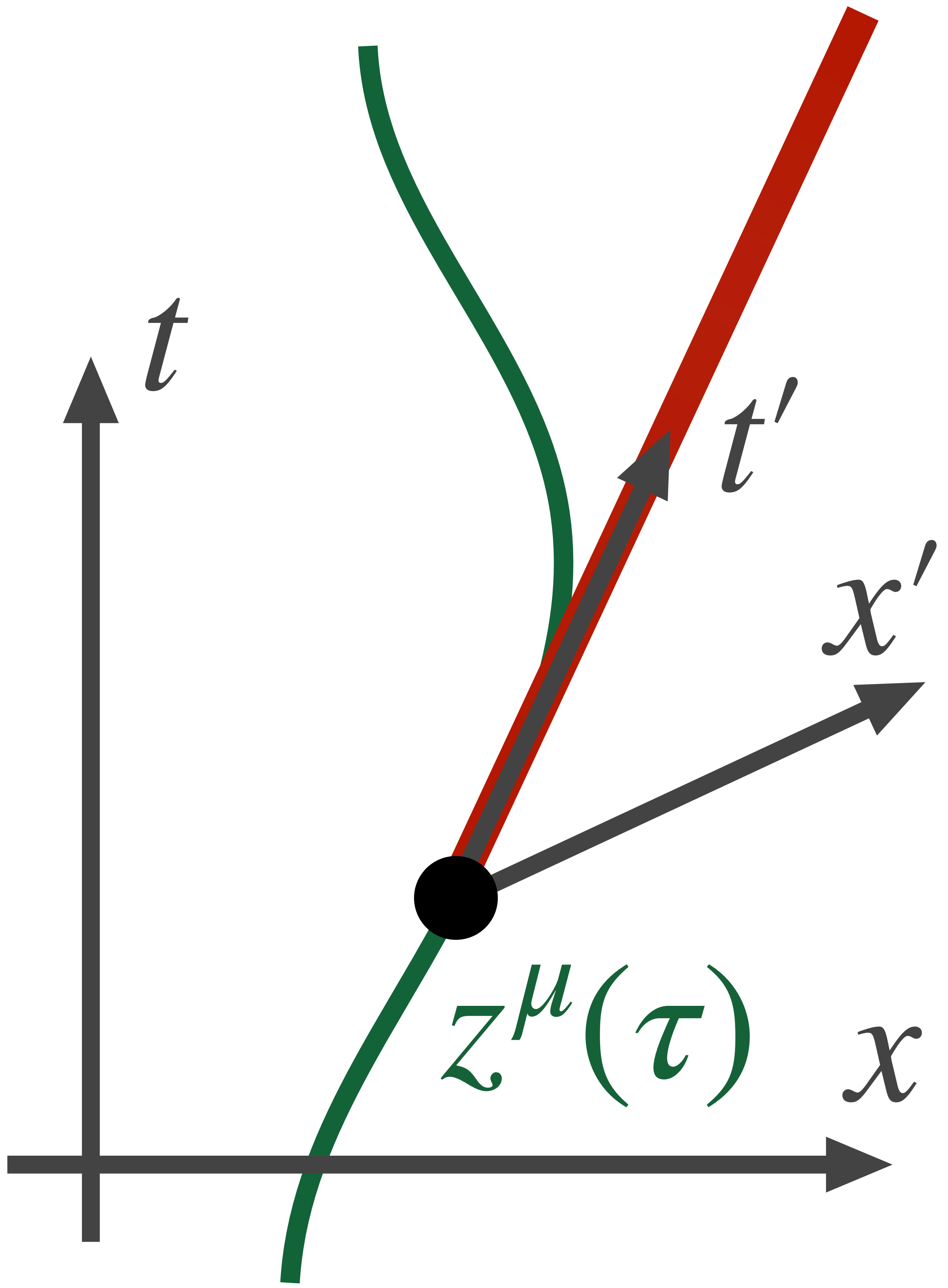
In Fourier space, current and gauge invariance condition are

$$J(\eta, k) = \int d\tau \, e^{ik\cdot z(\tau)} f(\dot{z}, k, \eta) \qquad (-ik\cdot\partial_\eta + \rho) f \approx 0$$

Our currents are generically **not** localized to the worldline!

For example, $f = e^{-i\rho\eta\cdot\dot{z}/k\cdot\dot{z}}$ produces a scalar-like current with "wake" confined in future (or past) light cone

This can yield causal particle dynamics; could emerge from integrating out fields in a manifestly local description



A Universality Result

Key technical result: all currents can be decomposed as

$$f = e^{-i\rho\eta\cdot\dot{z}/k\cdot\dot{z}}\,\hat{g}(k\cdot\dot{z}) + \mathscr{D}X$$

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First term contains the key physics of nonzero spin scale ρ

Many physical observables are **universal**: determined by only ρ and $\hat{g}$, where

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ e\,k\cdot\dot{z} & \text{vector-like current} \\ m\,(k\cdot\dot{z})^2 + \dots & \text{tensor-like current} \end{cases}$$

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"Shape" terms are proportional to the equation of motion operator

Like familiar contact terms, these couplings can be completely removed by field redefinition

All valid currents found in earlier works were pure shape terms, with $\hat{g} = 0$

Extracting the Physics

From the action $S[\Psi, z_i^\mu(\tau)]$ we can compute any desired classical observable:

"Integrate out" Ψ
(plug solution into action)

↓

Matter interaction potential

$$V(\mathbf{r}_i, \mathbf{v}_i, \ldots)$$

Depends on "shape" terms

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Matter interaction potential

$V(\mathbf{r}_i, \mathbf{v}_i, \ldots)$

Depends on "shape" terms

Find z_i equation of motion
with given Ψ

↓

Matter forces in background

$\mathbf{F}[\Psi]$

Universal

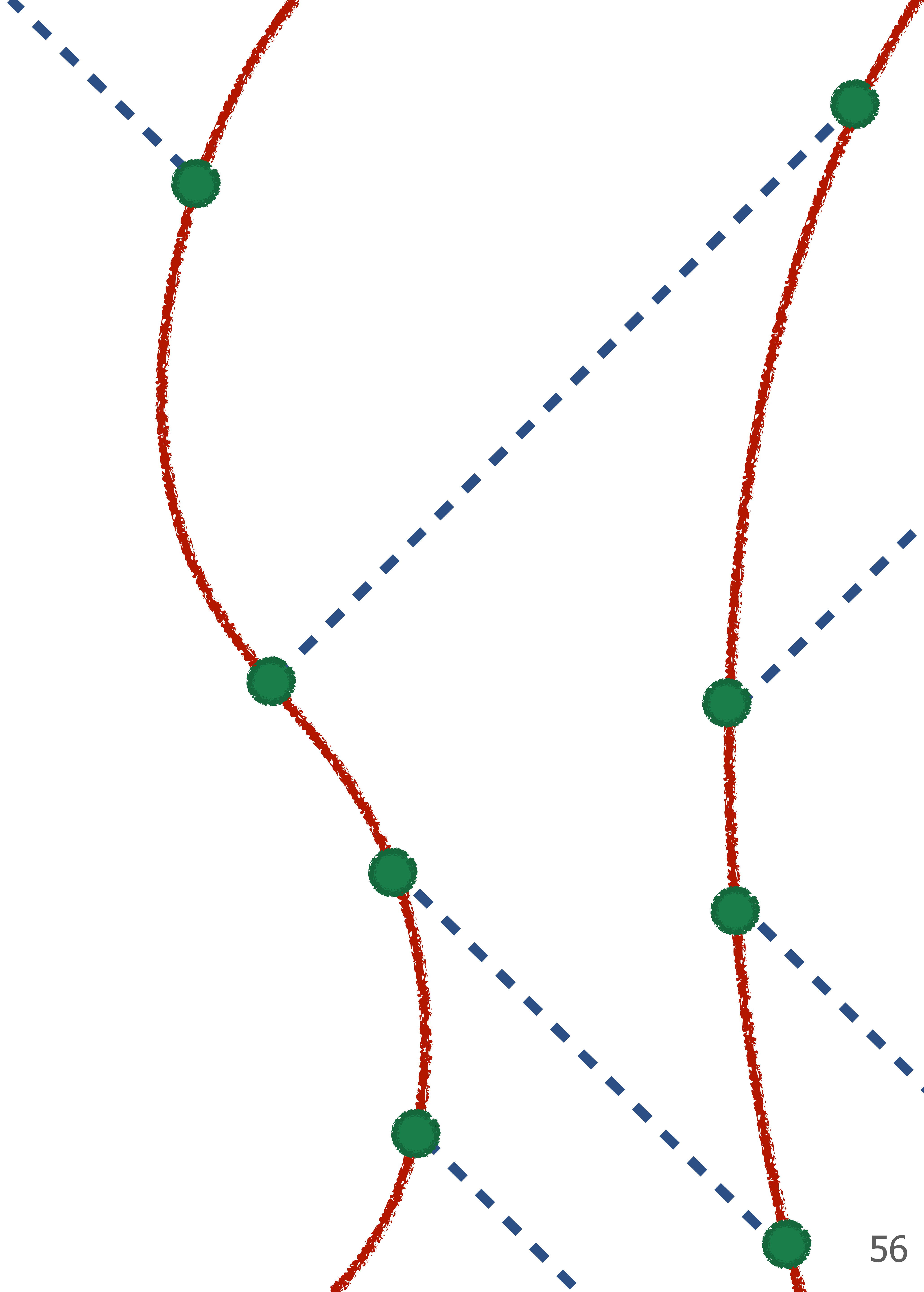
Extracting the Physics

From the action $S[\Psi, z_i^\mu(\tau)]$ we can compute any desired classical observable:

“Integrate out” Ψ (plug solution into action)	Find z_i equation of motion with given Ψ	Solve Ψ equation of motion with given trajectories
↓	↓	↓
Matter interaction potential $V(\mathbf{r}_i, \mathbf{v}_i, \ldots)$	Matter forces in background $\mathbf{F}[\Psi]$	Radiation emission $dP_h/d\Omega$
Depends on “shape” terms	**Universal**	**Universal**

Outline

- Free continuous spin fields
- Coupling to matter particles
- **Physics with continuous spin**



Forces in Background Fields

Force on particle with vector-like current in background of frequency ω, helicity h:

$$\frac{\mathbf{F}_{h=0}}{q} = \frac{\rho}{\omega} \frac{\dot{\phi}\, \mathbf{v}_\perp}{2} + \ldots$$

$$\frac{\mathbf{F}_{h=\pm 1}}{q} = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \left(\frac{\rho}{\omega}\right)^2 \left(\frac{\mathbf{v}_\perp(\mathbf{v}_\perp \cdot \mathbf{E})}{4} + \frac{v_\perp^2\, \mathbf{E}}{8}\right) + \ldots$$

$$\frac{\mathbf{F}_{h=\pm 2}}{q} = \frac{\rho}{\omega} \frac{\dot{h}_+ (v_x \hat{\mathbf{x}} - v_y \hat{\mathbf{y}})}{4} + \ldots$$

Corrections controlled by $\rho v/\omega$, and as $\rho \to 0$ other helicities decouple

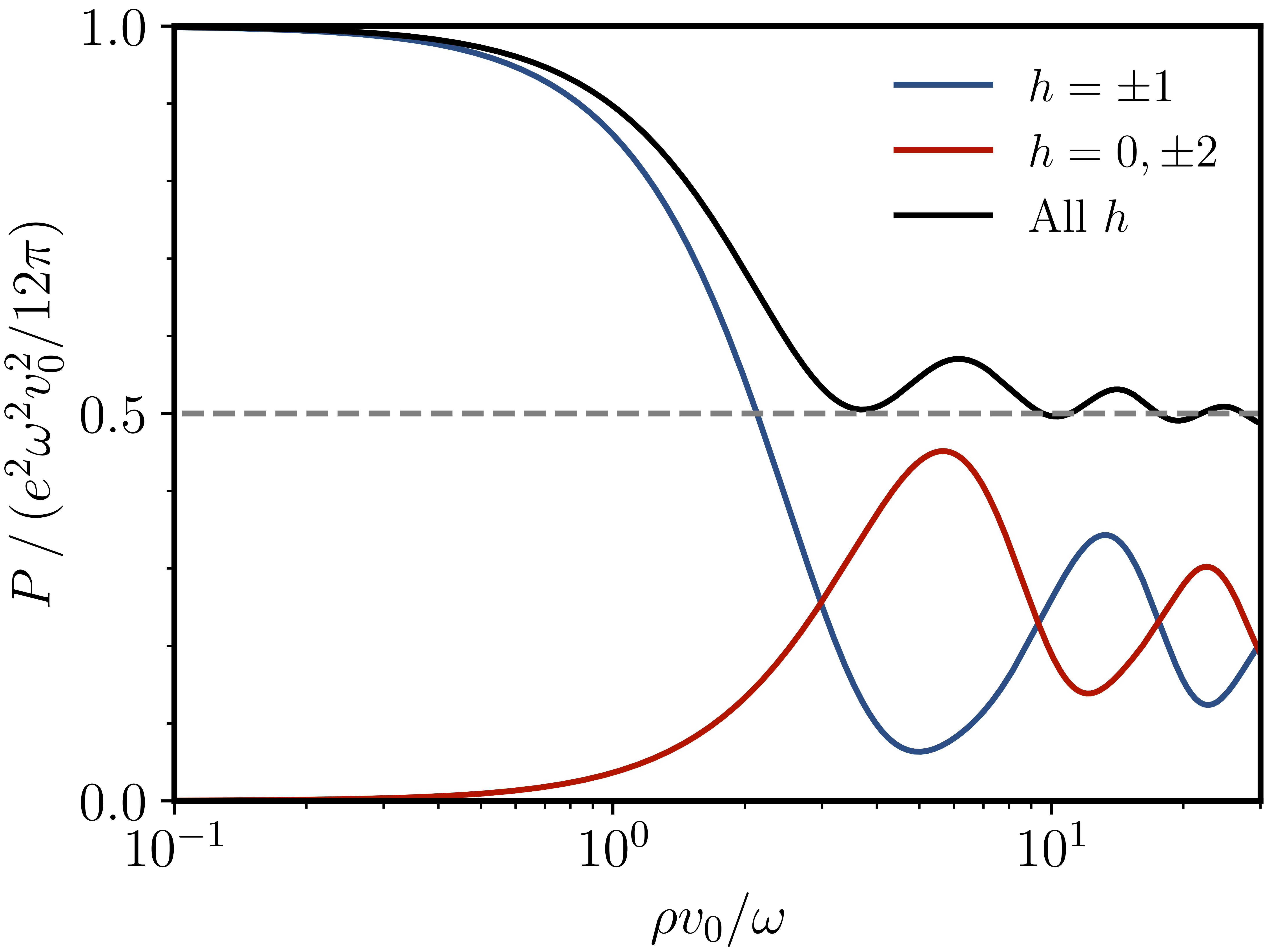
Full expressions have Bessel functions, convergent at large arguments

Radiation From Oscillating Particle

Consider motion with amplitude $\ell = v_0/\omega$, and vector-like current

Radiation emitted in all helicities, and at large $\rho\ell$ many helicities contribute, but total power radiated is finite!

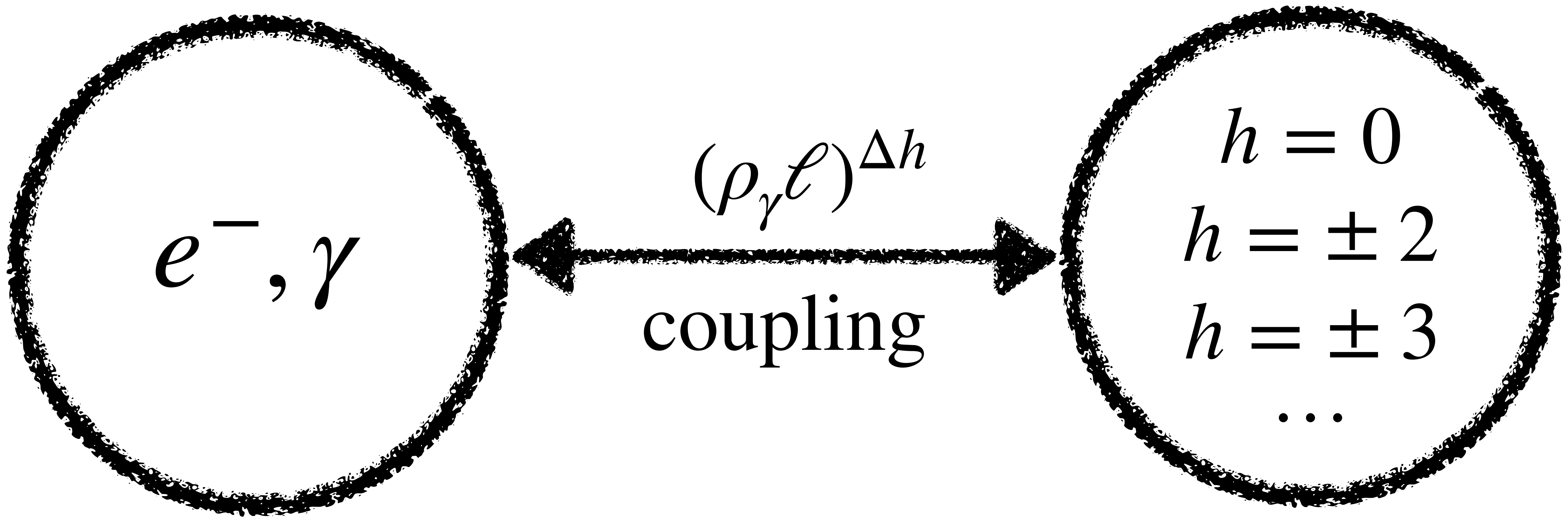
Variant of this calculation recovers previously known soft emission amplitudes



Probing The Spin Scale of the Photon

For vector-like currents, $h = \pm 1$ could be observed photon

Other helicities are weakly coupled "dark radiation"

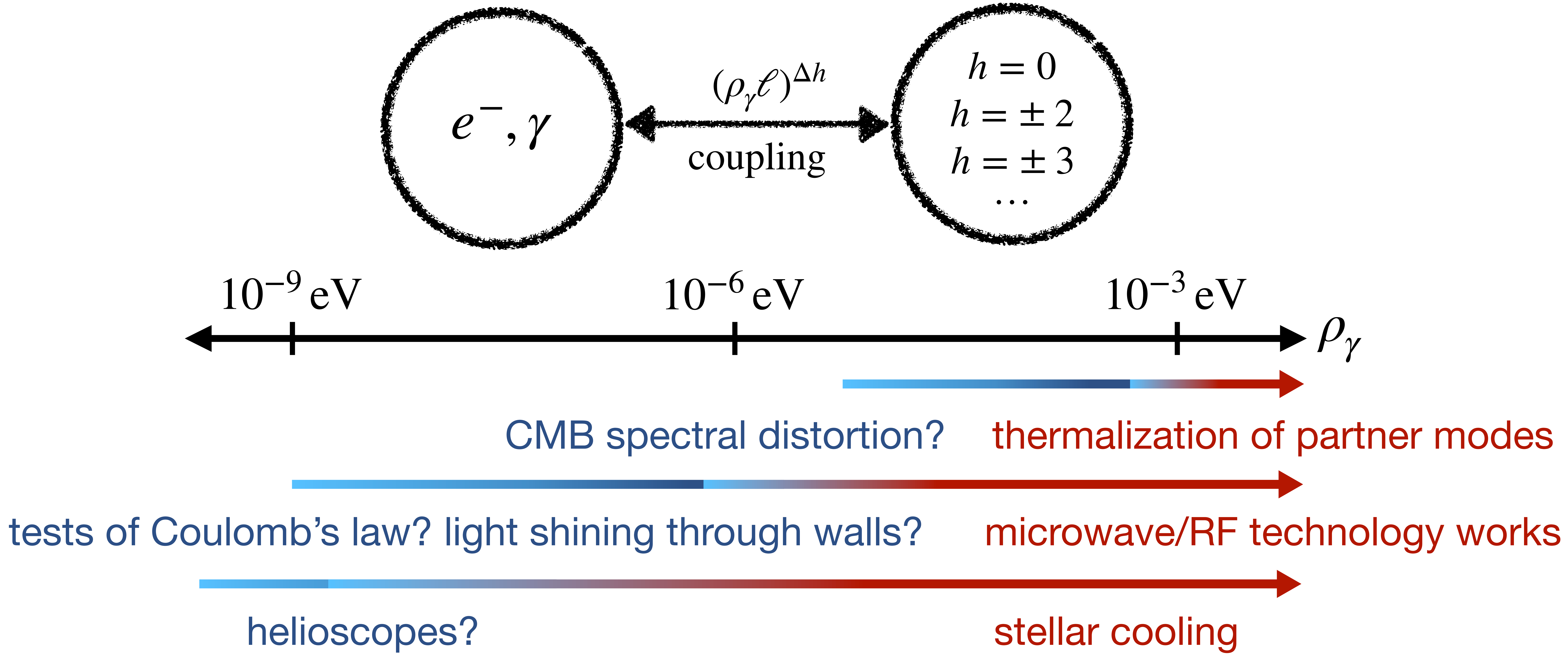


Familiar electron and photon

Photon partner polarizations

Sensitivity of various probes can be readily calculated

Probing The Spin Scale of the Photon



Very rough, preliminary estimates!

Continuous Spin and Gravity

We need to distinguish two distinct possibilities

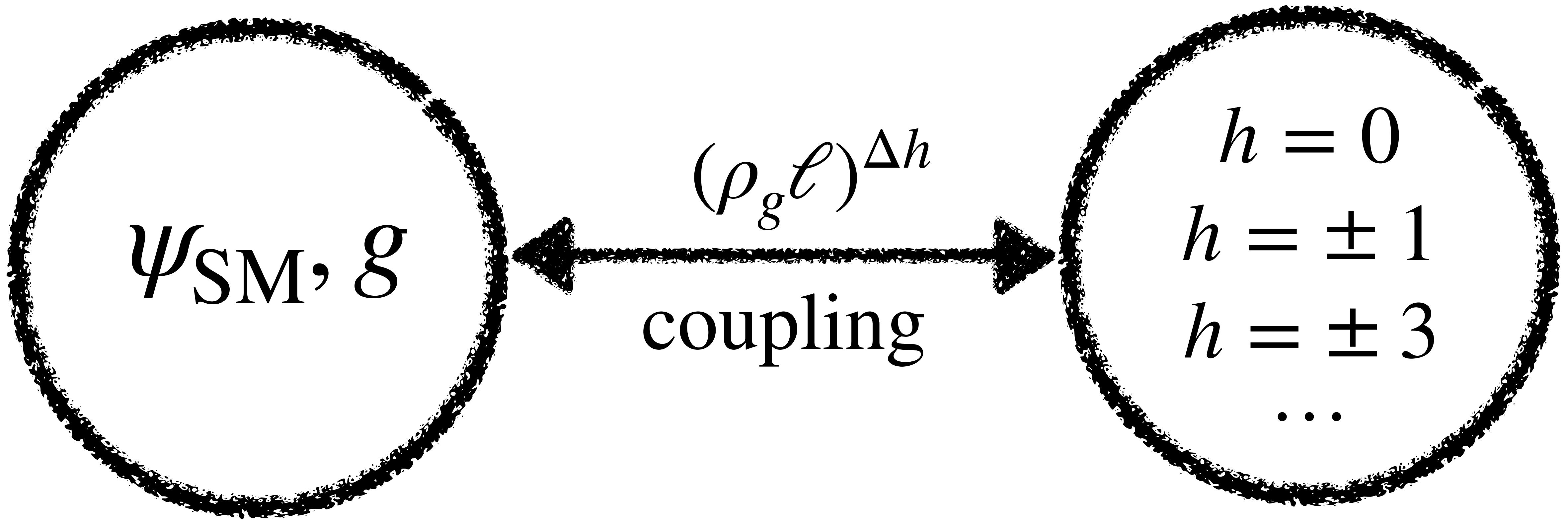
The graviton is the $h = 2$ part of a continuous spin particle (which implies infrared modifications of gravity)

The photon is the $h = 1$ part of a continuous spin particle (which then needs to be coupled to ordinary gravity)

(technically, both could be true at once, though this seems unlikely)

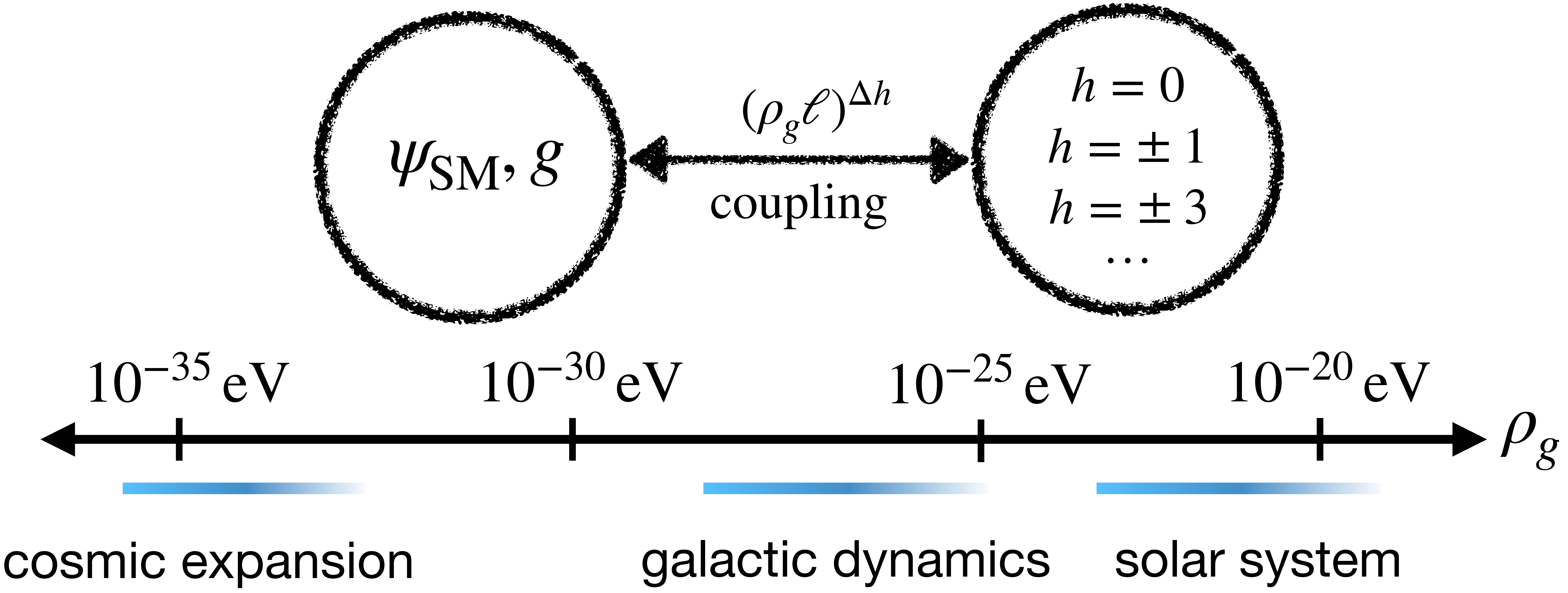
Probing The Spin Scale of the Graviton

A tensor-like current yields linearized gravity with ρ-dependent corrections



Linearized theory enough for many observables, though strong field effects require understanding self-interactions of continuous spin fields, and nonlinear generalization of their gauge symmetry

Probing The Spin Scale of the Graviton



Certain scales motivated by potential deviations from inverse square force

Can also probe universal deviations from gravitational radiation physics

Continuous Spin Fields in Curved Spacetime

We focused on couplings to matter fields in flat spacetime,
but continuous spin particles known to exist in dS and AdS

To couple Ψ to gravity, start with its conserved stress-energy tensor:

$$T^{\mu\nu} = -\, g^{\mu\nu}\mathscr{L} + \int_\eta \delta'(\eta^2+1)\,\partial^\mu\Psi\,\partial^\nu\Psi - \frac{1}{2}\,\partial_\eta^\mu\,(\delta(\eta^2+1)\Delta\Psi)\partial^\nu\Psi$$

By itself, this breaks Ψ's gauge symmetry — need to find consistent completion

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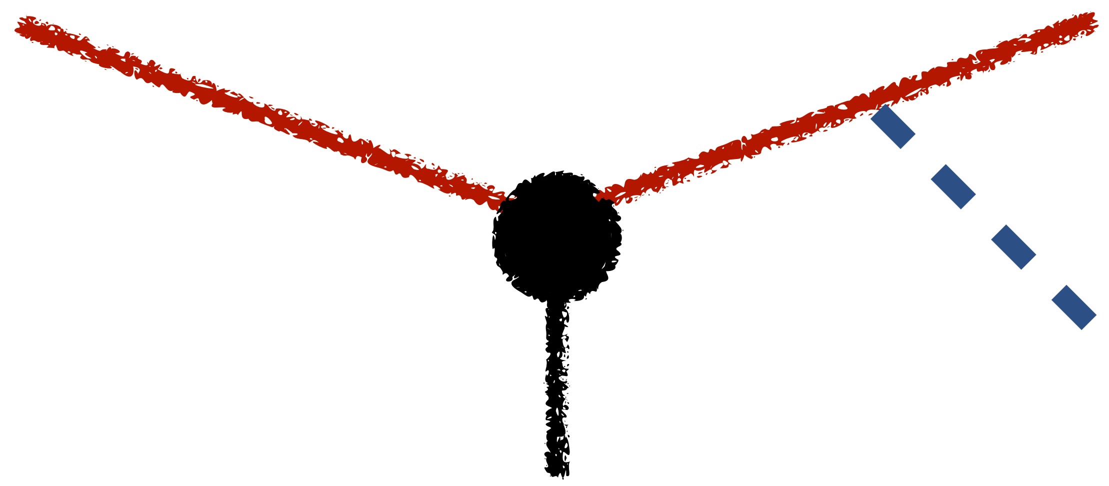
What about Hawking radiation of CSPs?

When $\rho \to 0$, recover infinite number of conventional degrees of freedom,
but radiation rate **not** infinite because of falling greybody factors for higher h modes!

Warm up calculation: lab frame response of Unruh-de Witt detector

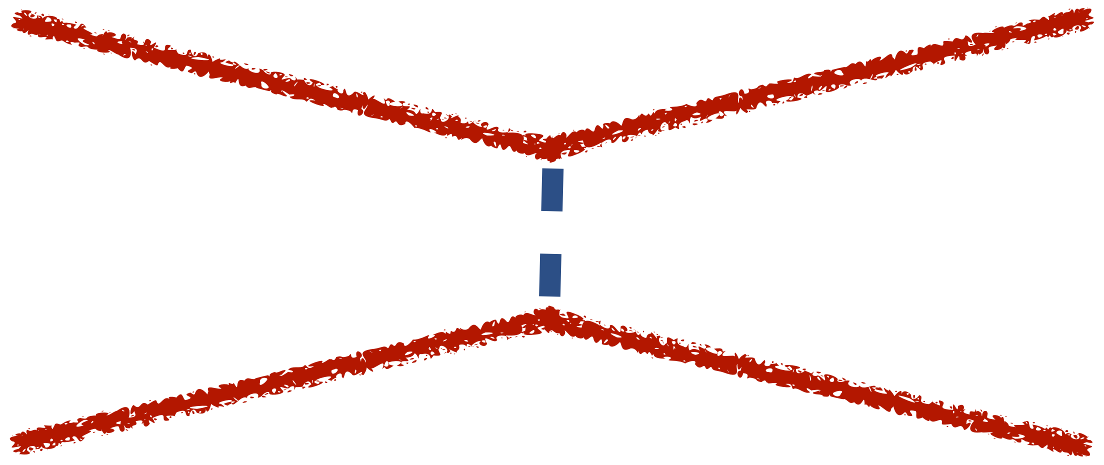
Scattering Amplitudes

Scattering amplitudes computable with path integral (worldline formalism)



CSP emission straightforward; recovers soft factors

CSP-matter scattering only involves external CSPs,
so is universal (recently computed)

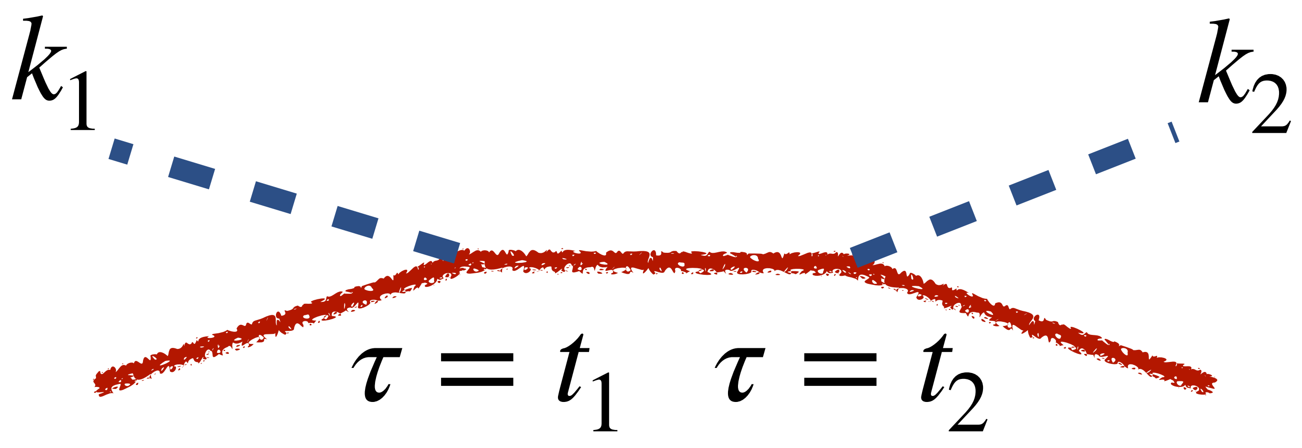


CSP exchange not universal,
but obeys tree-level unitarity

Unitarity at loop level unknown, may place
constraints on current (key next step)

Setting Up the Path Integral

Use the "string inspired" first quantized worldline formalism:



$$\mathscr{M} \sim \int \mathscr{D}[z(\tau)] \left(\prod_i \int dt_i \, V(k_i, t_i) \right) \exp \left(i \int d\tau \, \dot{z}^2/4 \right)$$

for a scalar massless matter particle, where the V's are vertex operators

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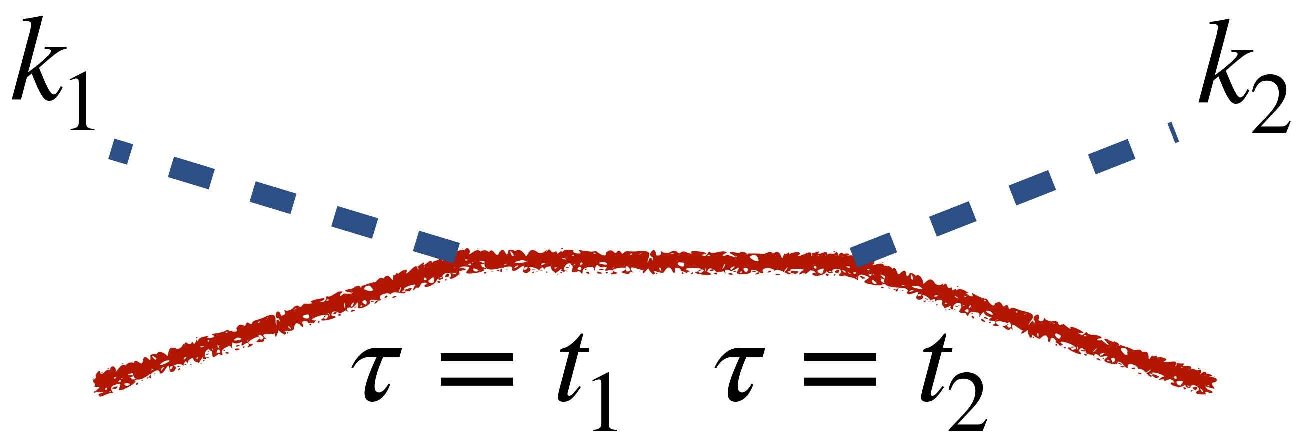

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for a scalar massless matter particle, where the V’s are vertex operators

Vertex operators can be read off from the action:

$$V(k,t) \sim g \, e^{ik\cdot z(t)} \qquad V_\mu(k,t) \sim q \, e^{ik\cdot z(t)} \frac{dz_\mu}{dt} \qquad V(\eta, k, t) \sim q \, e^{ik\cdot z(t)} \frac{e^{-i\rho\eta\cdot\dot{z}/k\cdot\dot{z}}}{-i\rho/k\cdot\dot{z}}$$

scalar — vector — vector-like CSP

(shape terms drop out for on-shell CSPs)

CSP-Matter Compton Scattering

Define useful "intermediate" momenta:

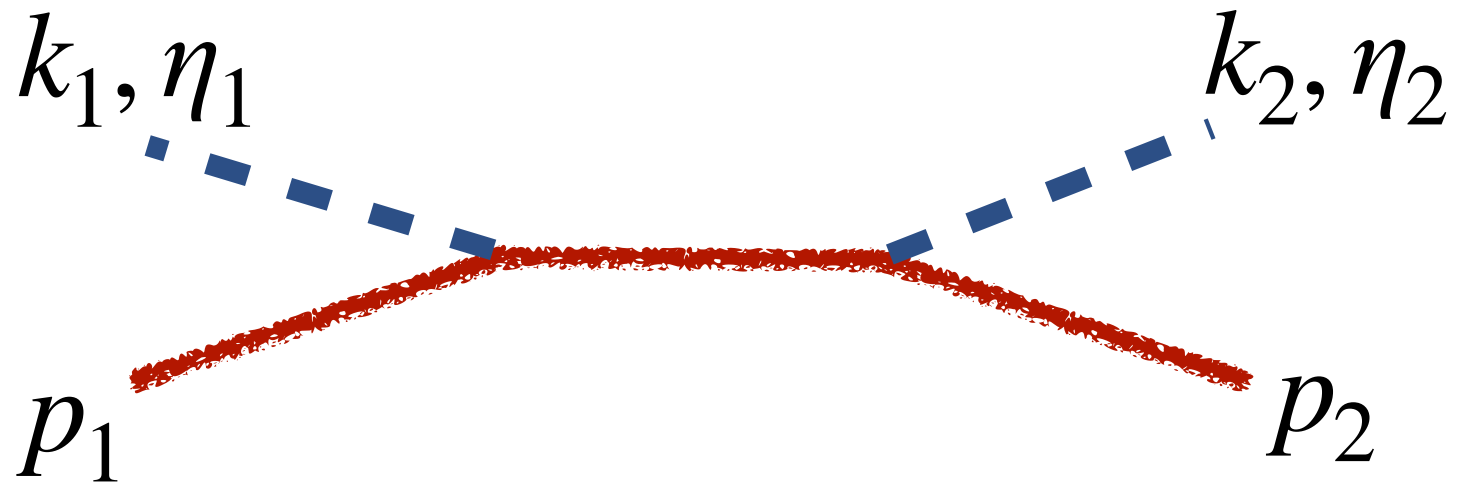
$$P_1 = p_1 + p_2 + xk_2 \qquad P_2 = p_1 + p_2 + xk_1$$

Result of the path integral is

(2308.16218)

$$\mathscr{M} \sim \int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1 \cdot P_1}{k_1 \cdot P_1} k_1\right) \cdot \left(\eta_2 - \frac{\eta_2 \cdot P_2}{k_2 \cdot P_2} k_2\right) \exp\left(i\rho\left(\frac{\eta_1 \cdot P_1}{k_1 \cdot P_1} - \frac{\eta_2 \cdot P_2}{k_2 \cdot P_2}\right)\right)$$

CSP-Matter Compton Scattering



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Reduces to usual scalar QED result for $\rho = 0$ and external $h = 1$ states

In general, ρ must be in numerator, so momenta must be in denominator

But apparently pathological higher order poles in series expansion resum into a benign essential singularity — amplitude bounded even in deep infrared limit!

CSP Loop Corrections

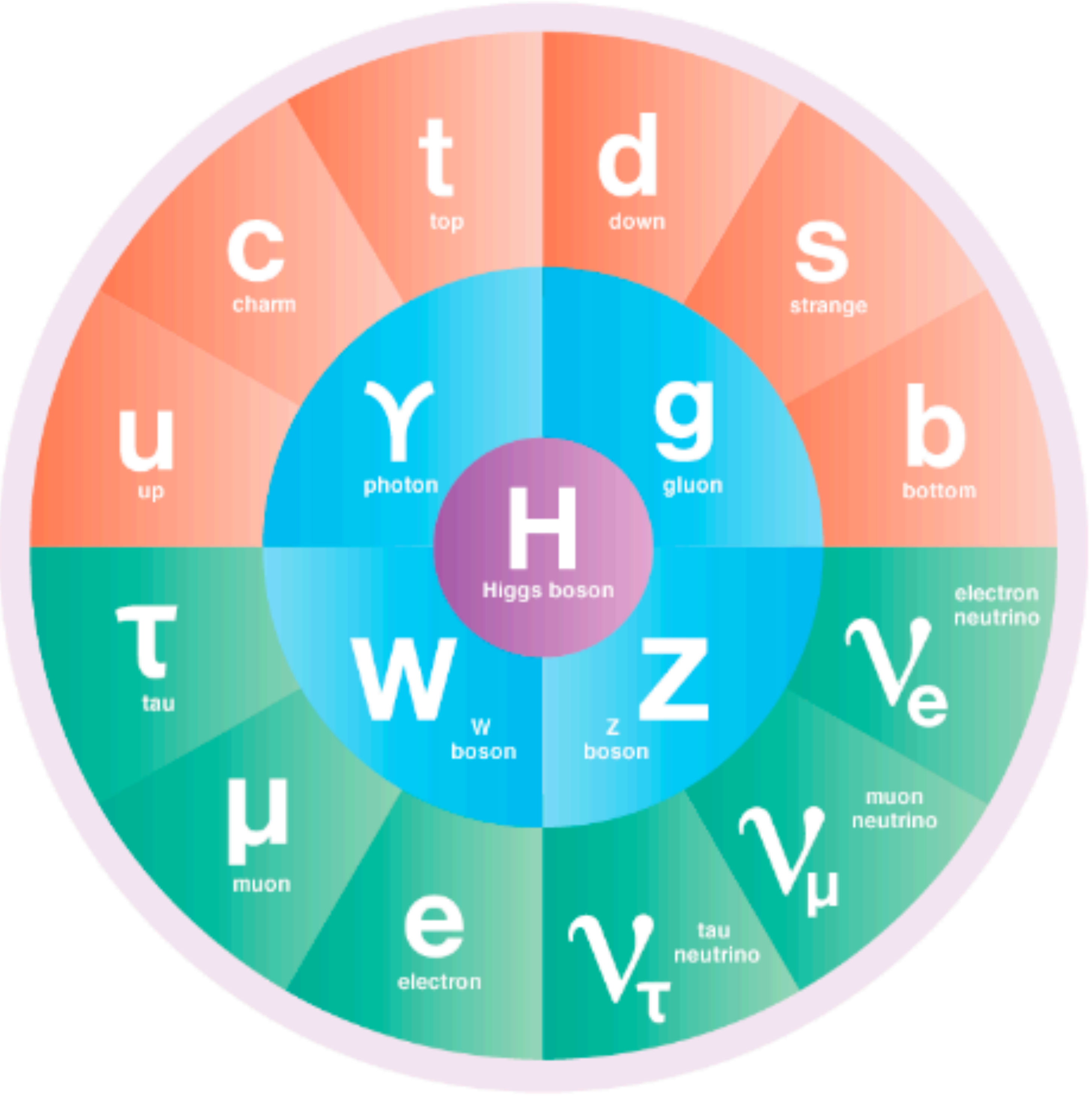
Easiest next step is to extend to a closed matter loop: check unitarity and renormalization of CSP mass

Technical issue: singular contractions of vertex operators are ill-defined distributions, depend on regularization prescription

Conceptual puzzle: when $\rho = 0$, a scalar's mass is renormalized, but when $\rho \neq 0$ a scalar-like CSP's mass should be protected by gauge symmetry!

At the amplitude level, the order of limits $\rho \to 0$ and $k^2 \to 0$ matters!

A Continuous Spin Standard Model?



Would ultimately want to embed a continuous spin photon within the electroweak sector

Need to give continuous spin fields mass

Need nonabelian continuous spin gauge symmetry

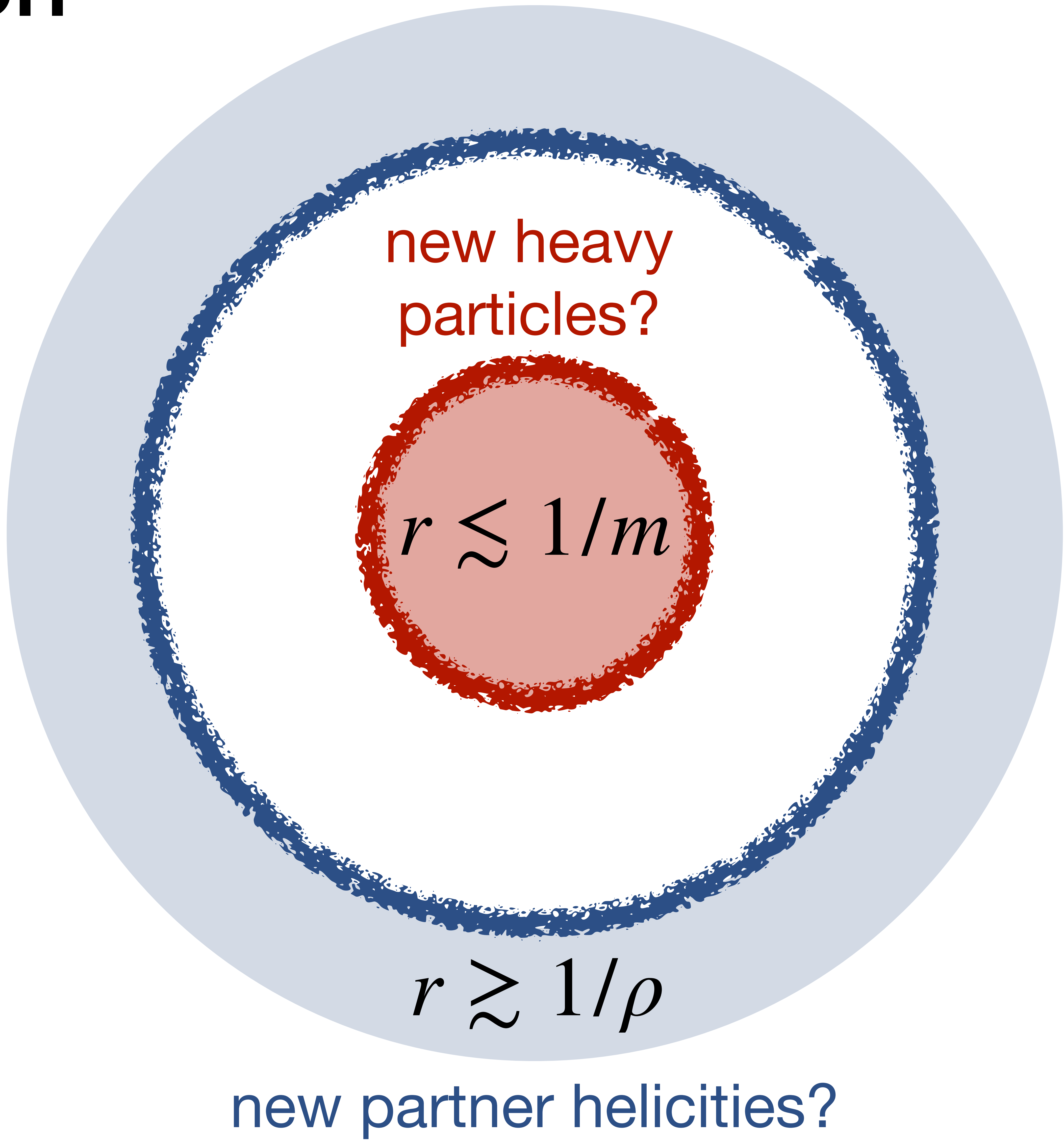
As a first step, consider a Stuckelberg mass term $m^2\Psi^2/2$

Yields massive weakly coupled partner polarizations; natural dark matter candidate?

Conclusion

Lorentz symmetry implies massless particles have a spin scale ρ — **is it zero or not**?

For nonzero ρ, there are calculable, universal predictions which can be immediately tested!



Conclusion

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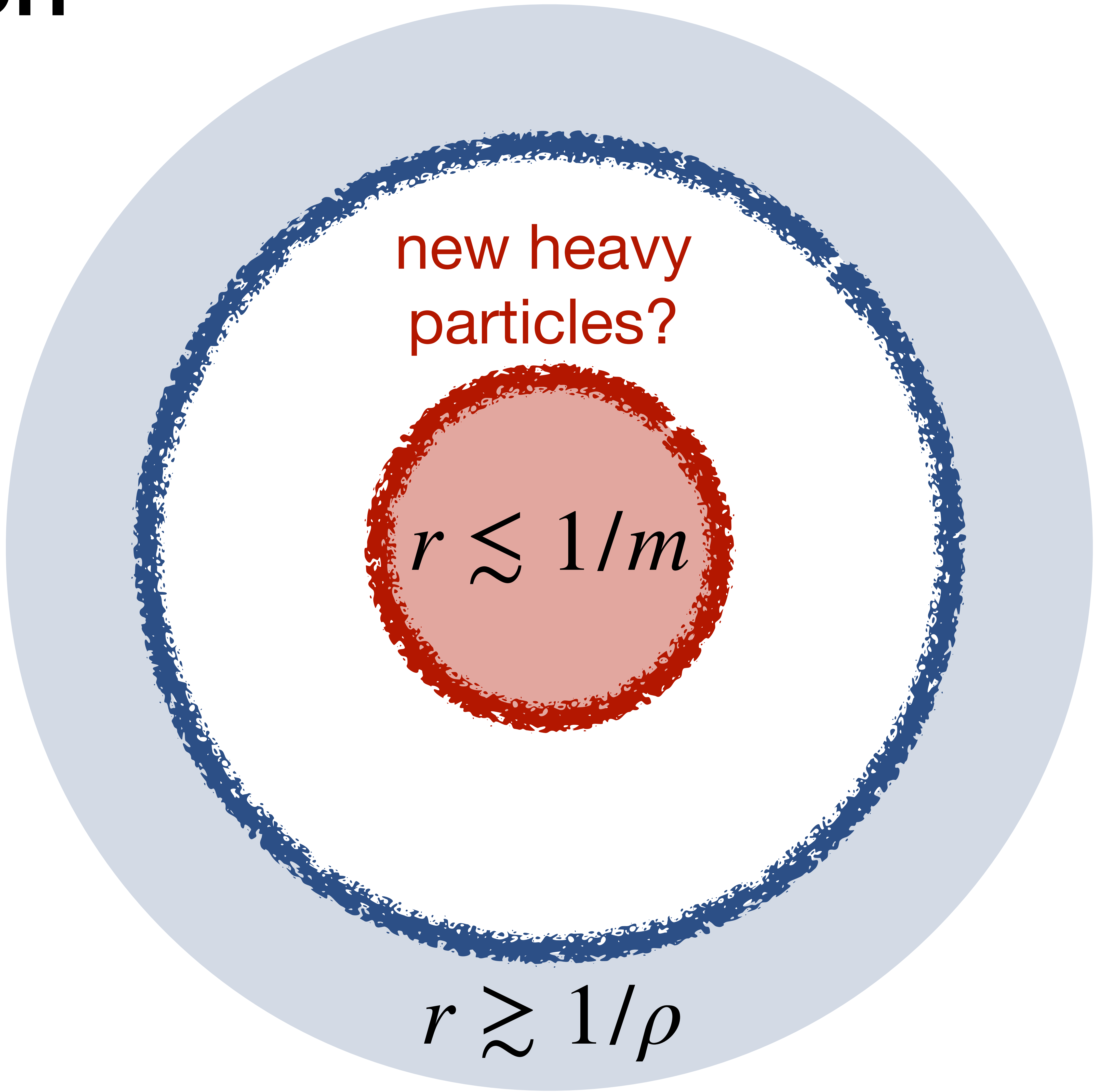
For nonzero ρ, there are calculable, universal predictions which can be immediately tested!

Grand theory question: are there fully consistent analogues of the full Standard Model and/or general relativity at nonzero ρ?

If so, they are effective theories at both long and short distances — which can shed light on a variety of fundamental problems



new partner helicities?

Extra Slides

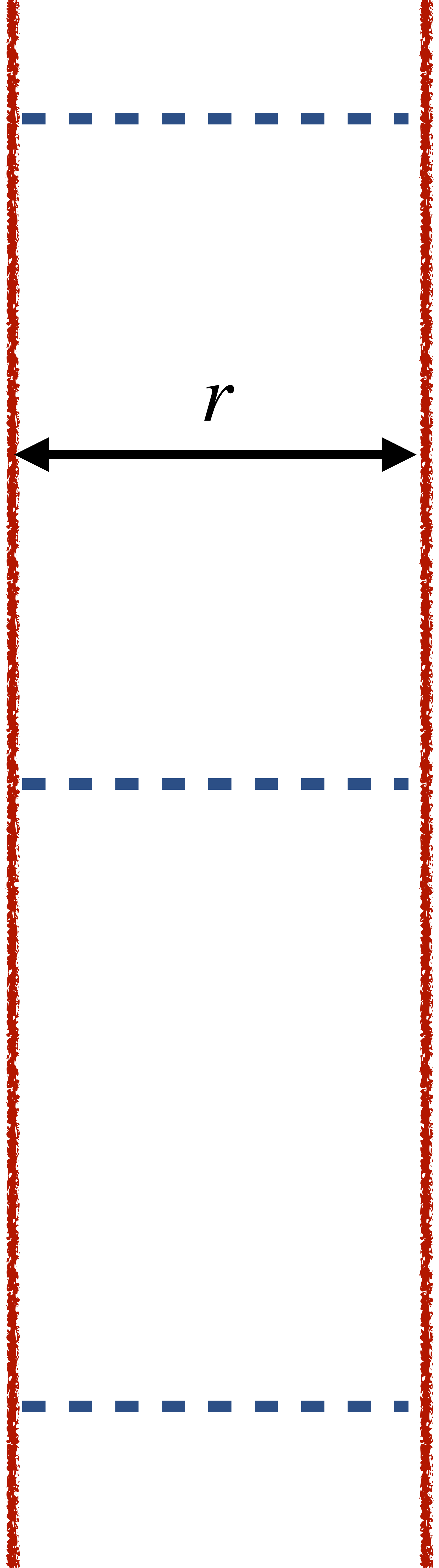
Static Potentials

Static potentials can exhibit deviations at long distances:

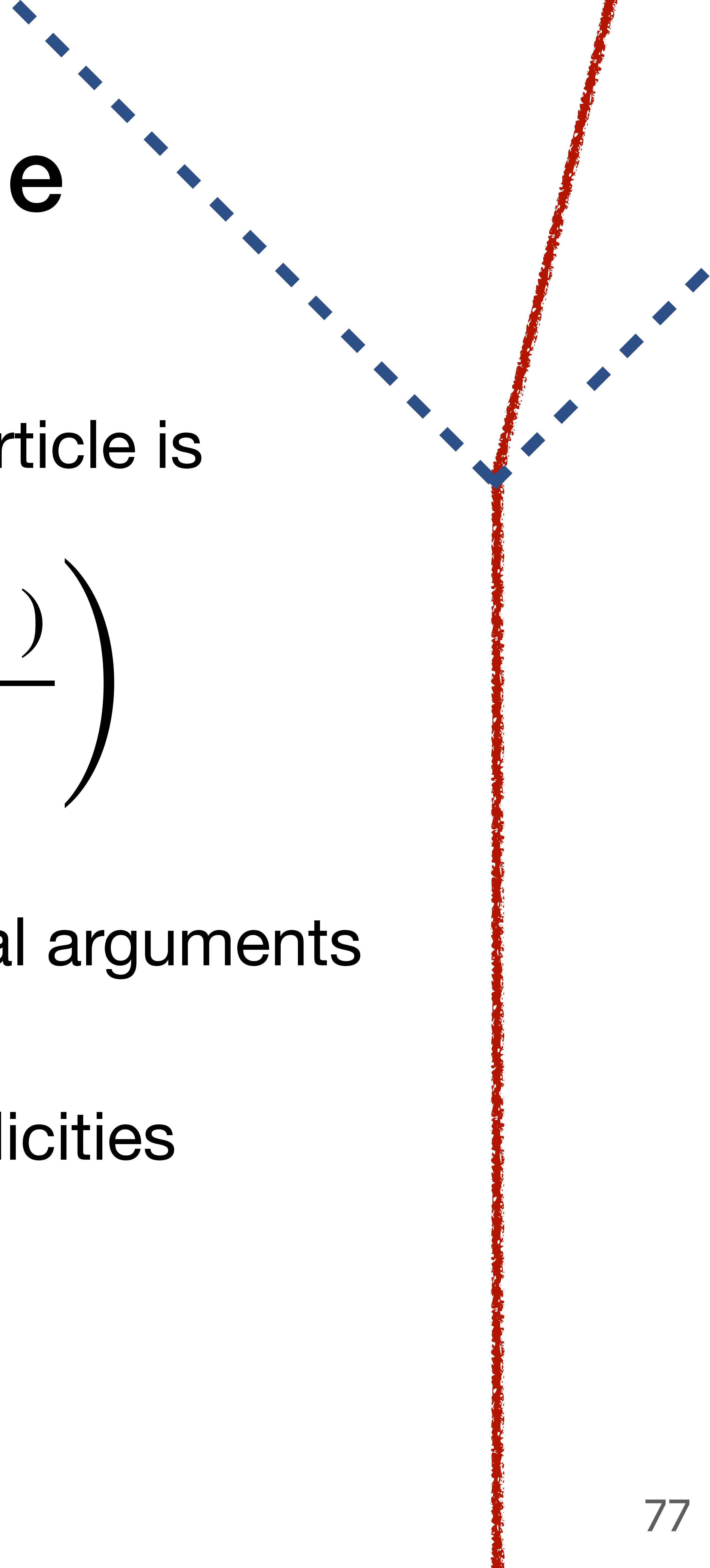
$$V(r) = \frac{g^2}{4\pi r}\left(1 - c_1 \rho r + c_2 (\rho r)^2 + \ldots\right)$$

Coefficients depend on current: vanish for simplest currents, but for general currents can cause force to flip sign at large distances

Similar results for vector-like currents; can also find velocity-dependent potentials (e.g. corrections to magnetic interaction)



Radiation From Kicked Particle



For any scalar-like current, radiation amplitude from a kicked particle is

$$a_{h,k} \propto g \left(\frac{\tilde{J}_h(\rho \quad \epsilon_- \cdot p / k \cdot p \quad)}{k \cdot p} - \frac{\tilde{J}_h(\rho \quad \epsilon_- \cdot p' / k \cdot p' \quad)}{k \cdot p'} \right)$$

which exactly matches soft emission amplitudes fixed by general arguments

Same agreement for vector-like currents; in both cases other helicities decouple as $\rho \to 0$