Generating the fermion mass hierarchy at the TeV scale

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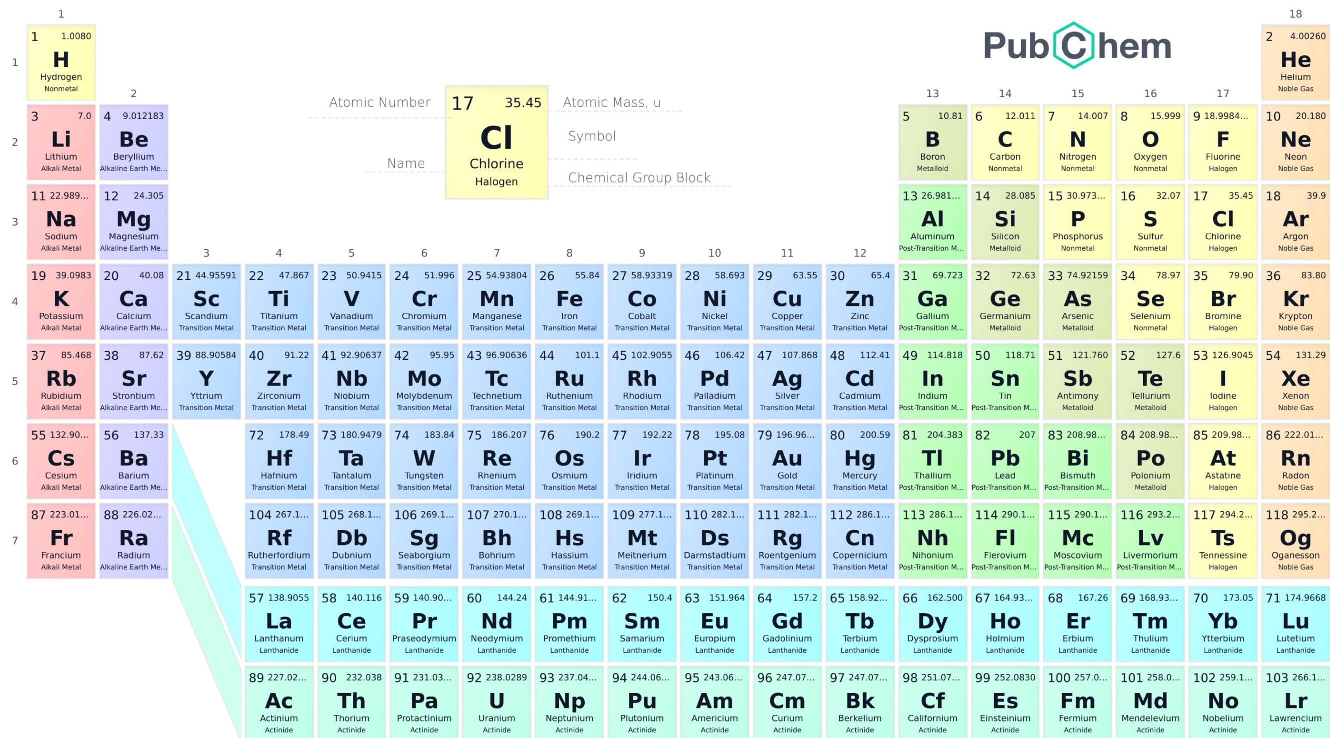
Asymmetry
Essential Asymmetries of Nature

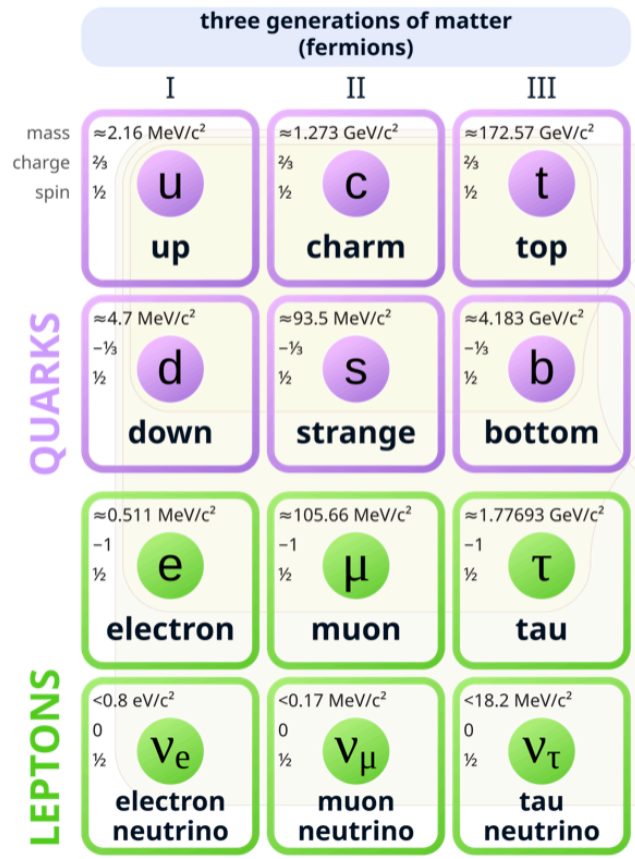
PERIODIC TABLE

Observed order in chemical elements led to discovery of atomic model and quantum mechanics.



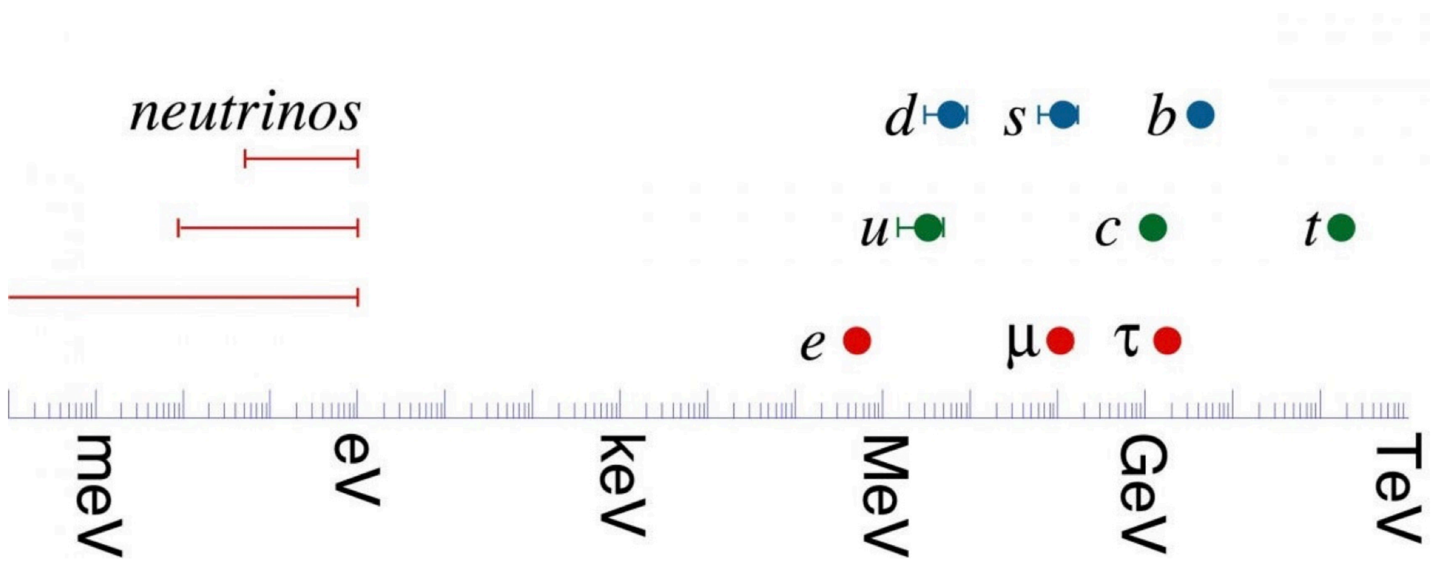
PERIODIC TABLE OF FERMIONS

Similarly the observed properties of fermions could encode information about deeper organization principle



OPEN QUESTION: The Flavor Puzzle

- Why do we observe such a bizarre pattern of fermion masses and mixing angles?



- Why do fermions come in 3 generations?
- What is the origin of neutrimo masses?
- What is the origin of CP violation?
- ...

Anthropic Principle

- Masses of light fermions important for nuclear physics?

- May play a role, but unlikely to solve the problem entirely

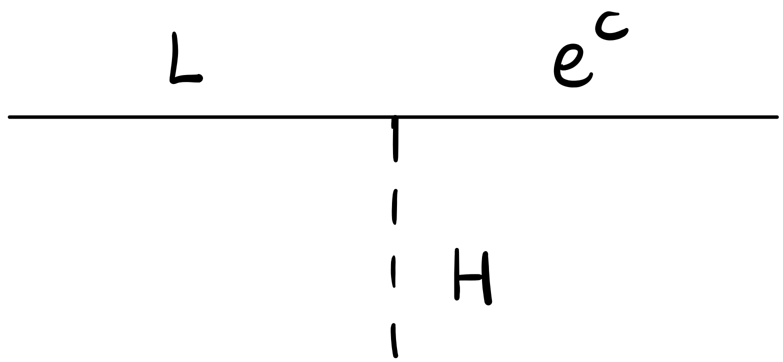
Connection with other open questions

- EW hierarchy problem
- Matter-Antimatter asymmetry
- Strong CP problem

Flavor in the SM

Fermion masses arise through interaction with the Higgs boson

$$L = (\nu\,,\,e)$$
$$H = (H^+, H^0)$$



$$\mathcal{L} \supset y_e L e^c H^\dagger \xrightarrow[\langle H\rangle = (0,v)]{} y_e v \; e\, e^c$$

$y_e v = m_e$

- Large Number of Parameters in the SM.

QUARK SECTOR

$$\begin{array}{cc} & \begin{pmatrix} u^c & c^c & t^c \end{pmatrix} \\ \begin{pmatrix} u \\ c \\ t \end{pmatrix} & \underbrace{\begin{pmatrix} & & \\ & 3\times 3 & \\ & & \end{pmatrix}}_{M_u} \end{array} \qquad \begin{array}{cc} & \begin{pmatrix} d^c & s^c & b^c \end{pmatrix} \\ \begin{pmatrix} d \\ s \\ b \end{pmatrix} & \underbrace{\begin{pmatrix} & & \\ & 3\times 3 & \\ & & \end{pmatrix}}_{M_d} \end{array}$$

9 complex parameters

9 complex parameters

We can diagonalize the mass matrices

$$M_U = U_u M_u^{diag} U_{u^c} \; ; \qquad M_D = U_d M_d^{diag} U_{d^c} \; ;$$

$$\mathcal{L} \supset d_i d_i^c \frac{y_i}{\sqrt{2}} (v+h) + \bar{u}_i \bar{\sigma}^\mu d_j W_\mu \left[\frac{g_2}{\sqrt{2}} \left(V_{CKM}\right)_{ij} \right]$$

$$V_{CKM} = U_u^\dagger U_d \quad \text{complex } 3\times 3 \text{ matrix}$$

OBSERVABLES

6 masses

3 mixing angles, 1 phase

TOWARD A THEORY OF FLAVOR

① Has to be predictive: parameters < observables;

② Has to explain the observed hierarchies;

How light can the NP scale be?

Are there mechanisms testable at colliders?

- Generally theories of flavor have to leave at high scales.

FCNC : $\Lambda \gtrsim 10^6$ GeV

Ex. : $\Delta F = 2$
$(K - \bar{K})$

$$H_{eff} = \sum_{i=1}^{5} \frac{C_i}{\Lambda^2} Q_i + \sum_{i=1}^{3} \frac{\tilde{C}_i}{\Lambda^2} \tilde{Q}_i$$

$$Q_1 = (s^\dagger_\alpha \sigma_\mu d_\alpha)(s^\dagger_\beta \sigma^\mu d_\alpha)$$

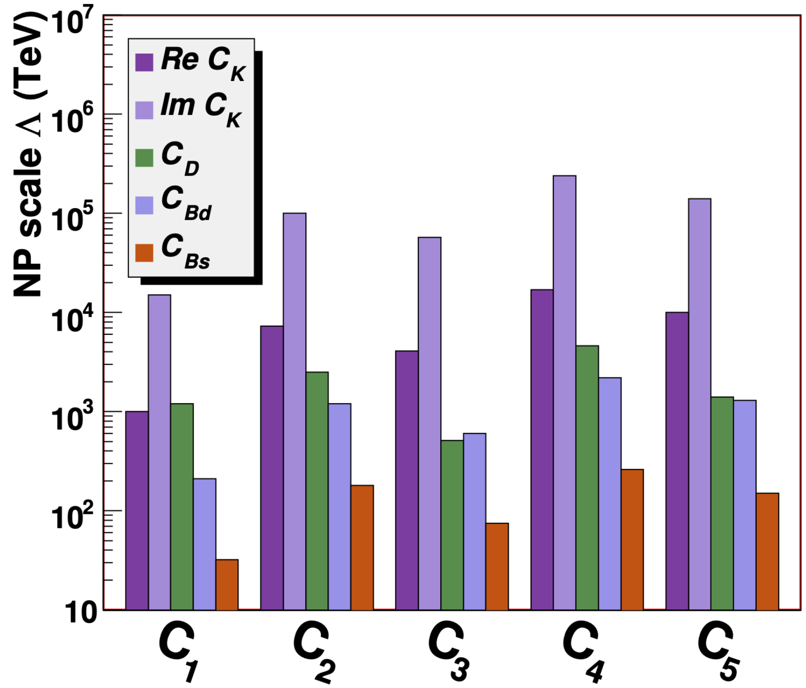
$$Q_2 = (s^c_\alpha \, d_\alpha)(s^c_\beta \, d_\beta)$$

$$Q_3 = (s^c_\alpha \, d_\beta)(s^c_\beta \, d_\alpha)$$

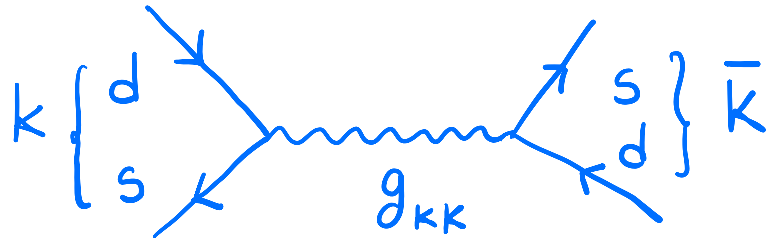
$$Q_4 = (s^c_\alpha \, d_\alpha)(s^\dagger_\beta \, d^{c\dagger}_\beta)$$

$$Q_5 = (s^c_\alpha \, d_\beta)(s^\dagger_\beta \, d^{c\dagger}_\alpha)$$

$$\tilde{Q} \overset{c}{\longleftrightarrow} Q$$

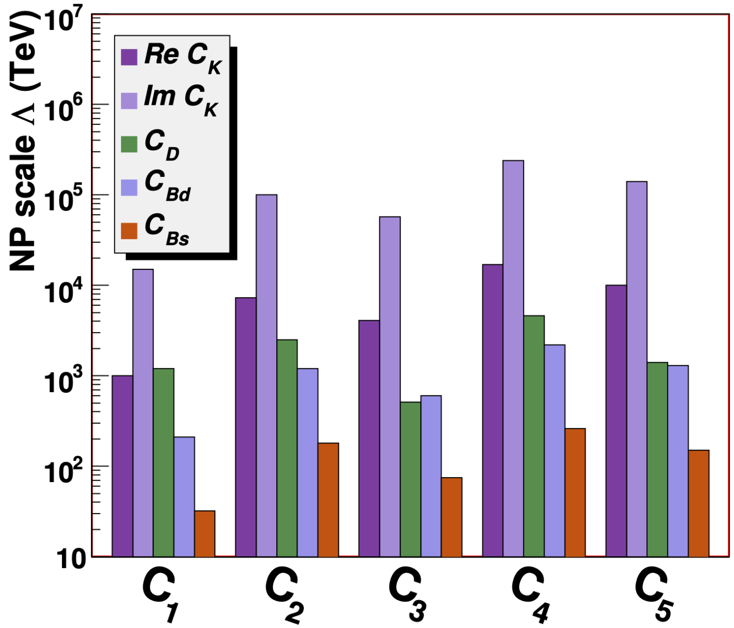


Ex: 5D Warped RS Models



$$C_4 \sim \frac{1}{M_{g_{KK}}^2} \, g_3^{*2} \, \frac{m_d m_s}{v^2} \in \mathbb{C}$$

$$\Rightarrow \; M_{g_{KK}} \gtrsim 20 \text{ TeV}$$



- It would be great to have theories of Flavor where the NP scale has to be close to the TeV scale!

LESS AMBITIOUS BUT A GOOD STARTING POINT

- Are there simple theory of Flavor where NP can be close to the TeV scale?

GOALS:

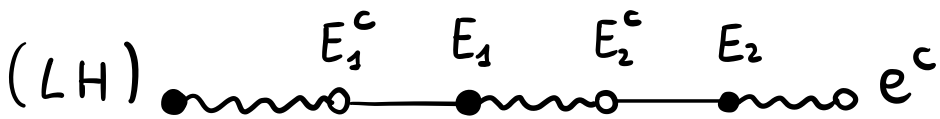
- Generate hierarchies without "small" parameters;
- Flavor constraints on NP at or below TeV scale. NP possible to detect at current or near-future colliders.

Main Idea

- Theories where the "light" NP are VL fermions with the same charges of SM field.
- Flavor hierarchies are generated through chains of VL fermions

Example

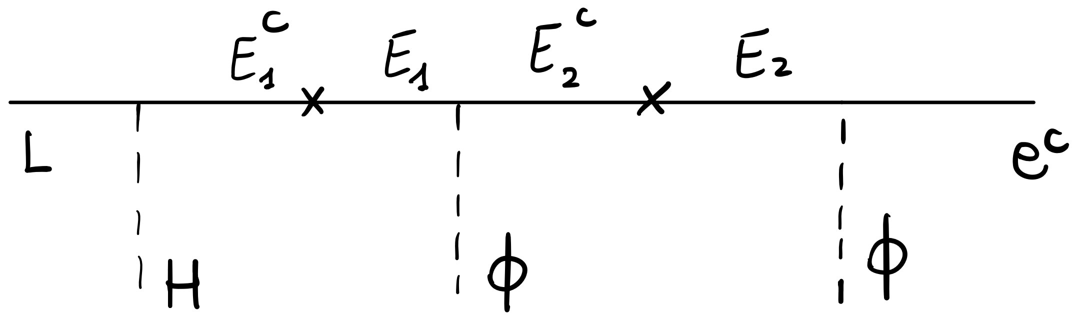
$$\mathcal{L} \supset M E_i^c E_i + \varepsilon M E_i^c E_j + \varepsilon M e^c E + \lambda L E^c H^\dagger ; \qquad \varepsilon = 0.1$$



$$\approx \frac{\varepsilon^3 \cancel{M^5}}{\cancel{M^5}} L e^c H$$

$\lambda = \varepsilon$ for simplicity. NOT NEEDED

SIMILAR TO FROGGATT-NIELSEN



$$\mathcal{L} \supset H \underbrace{\left(\frac{\phi}{M}\right)}_{\varepsilon}^{n} L\, e^c$$

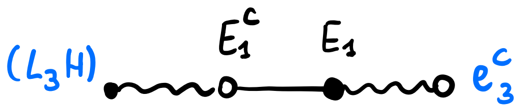
- Here the lightest NP are scalars (Laurer, Nir, Seiberg ~ TeV scale with $\varepsilon \sim 10^{-3}$)
- We won't see the flavor mechanism until we see the heavy fermions

KEY POINT: SOFT MASS TERMS

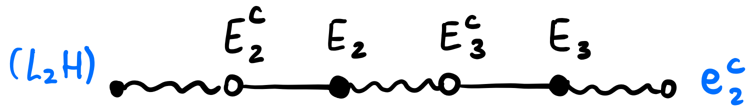
LEPTON SECTOR (charged leptons, diagonal mass matrix)

$$\mathcal{L} \supset M E_i^c E_i + \varepsilon M E_i^c E_j + \varepsilon M e^c E + \lambda L E^c H^\dagger ; \qquad \varepsilon = 0.1$$

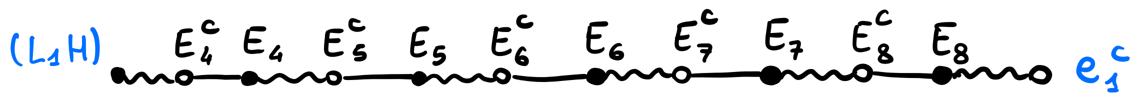
$y_\tau \sim \varepsilon^2$



$y_\mu \sim \varepsilon^3$



$y_e \sim \varepsilon^6$



$$\Longrightarrow \quad y_e = \begin{pmatrix} \varepsilon^6 & & \\ & \varepsilon^3 & \\ & & \varepsilon^2 \end{pmatrix}$$

$$
\begin{pmatrix} e_1 & e_2 & e_3 & E_1 \ldots E_n \end{pmatrix}
\left(\begin{array}{c:c}
O_{3\times 3} & \lambda_{3\times n} H \\
\hdashline
(\varepsilon M)_{n\times 3} & \begin{matrix} M & & \varepsilon M \\ & M & \\ & & M \\ \varepsilon M & & M \end{matrix}
\end{array}\right)
\begin{pmatrix} e^c_1 \\ e^c_2 \\ e^c_3 \\ E^c_1 \\ \vdots \\ E^c_n \end{pmatrix}
$$

We can rotate $(e^c, E^c) \to (\tilde{e}^c, \tilde{E}^c)$ to diagonalize M

$$
\mathcal{L} \supset M \tilde{E}^c_i E_i + \tilde{\lambda} L \tilde{E}^c H^\dagger + y L \tilde{e}^c H^\dagger
$$

$$
y \sim \begin{pmatrix} \varepsilon^6 & & \\ & \varepsilon^3 & \\ & & \varepsilon^2 \end{pmatrix}
$$

- <u>In the SM this is not possible!</u>

Allow to have only $O(1)$ or $O(\varepsilon)$ entries

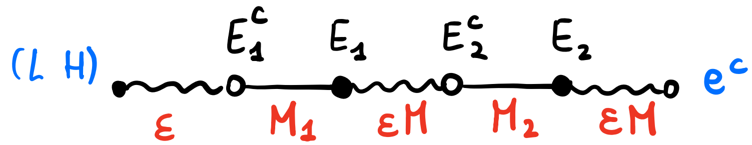
$$Y = \begin{pmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon \\ \varepsilon & \varepsilon & 1 \end{pmatrix} \qquad \mathrm{Det}(Y) = \underbrace{y_1 \cdot y_2 \cdot y_3}_{\text{eigenvalues}} \gtrsim \varepsilon^3$$

But we know

$$\mathrm{Det}(Y_u) \sim 10^{-7} \sim \varepsilon^7$$
$$\mathrm{Det}(Y_d) \sim 10^{-10} \sim \varepsilon^{10}$$
$$\mathrm{Det}(Y_e) \sim 10^{-11} \sim \varepsilon^{11}$$

If E can be at the TeV scale:

$$\tilde{\lambda}_i L \tilde{E}_i^c H$$



$$\tilde{\lambda}_1 \sim \varepsilon \qquad \tilde{\lambda}_2 \sim \frac{\varepsilon^2 M^2}{M_2^2 - M_1^2}$$

$$E \longrightarrow e\, H$$

$$\Gamma_i \sim \left(\frac{\tilde{\lambda}_i}{0.2}\right)^2 \left(\frac{M}{650\ \mathrm{GeV}}\right) \mathrm{GeV}$$

(a) NO DEGENERACY

$$\frac{\Gamma(E_1 \to e\ H)}{\Gamma(E_2 \to e\ H)} \sim \frac{\varepsilon}{\varepsilon^2} \sim \frac{1}{\varepsilon}$$

States further down the chain may be difficult to probe

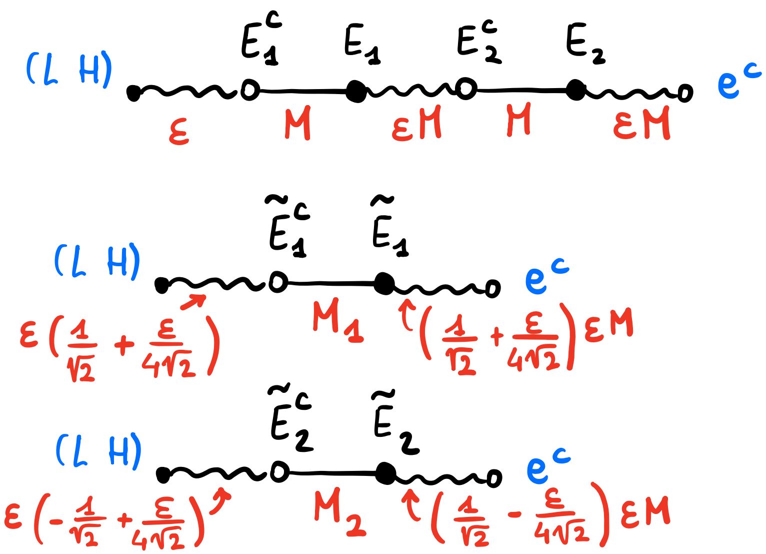
$$c\tau \sim 10^{2p-15}\,\mathrm{cm} \left(\frac{0.1}{\varepsilon}\right)^{2p} \cdot \left(\frac{\mathrm{TeV}}{M}\right)$$

(b) DEGENERATE FERMIONS

$\tilde{\lambda}_i \sim \varepsilon \quad \forall\, i$

- All fermions maximally coupled
- Mass splitting $O(\varepsilon M)$

IMPORTANT: SM Yukawa couplings are still the same



$$\begin{pmatrix} e \\ E_1 \\ E_2 \end{pmatrix}^T \begin{pmatrix} 0 & \varepsilon H & 0 \\ 0 & M & \varepsilon M \\ \varepsilon M & 0 & M \end{pmatrix} \begin{pmatrix} e^c \\ E_1^c \\ E_2^c \end{pmatrix}$$

$$M_1 \sim M - \frac{\varepsilon M}{2}$$

$$M_2 \sim M + \frac{\varepsilon M}{2}$$

$$y \sim \varepsilon^3 \,!$$

QUARK SECTOR

- Quark masses (6)
- CKM angles (3) There are a large number of theories.
- CKM phase (1)

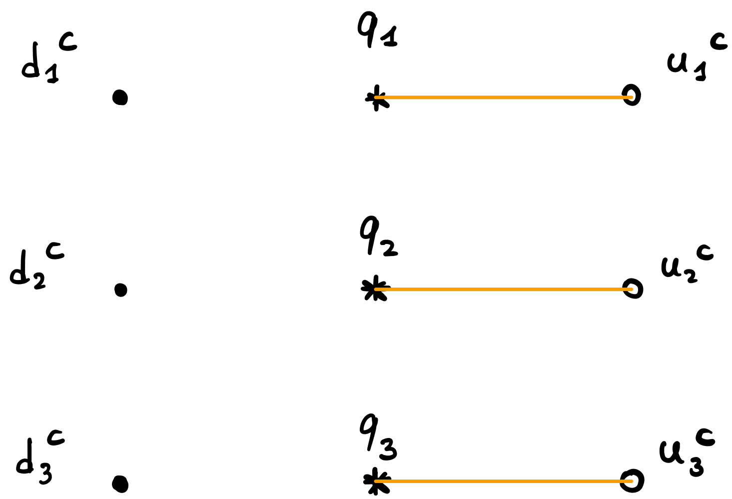
Let require 9 real parameters and 1 phase:

- 6 + 3 (only 3 patterns)
- 4+5 (≤ 20 patterns)

$\boxed{6+3}$

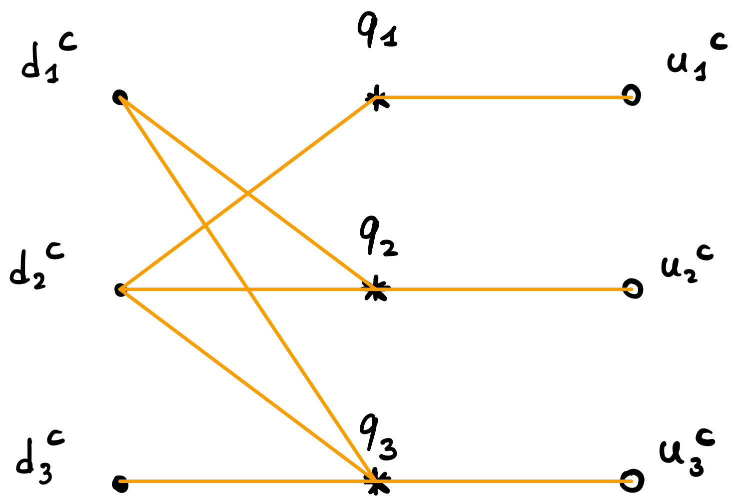
d_1^c •	q_1 $*$	∘ u_1^c
d_2^c •	q_2 $*$	∘ u_2^c
d_3^c •	q_3 $*$	∘ u_3^c

$\boxed{6 + 3}$



λ_u has 3 parameters

$\boxed{6+3}$

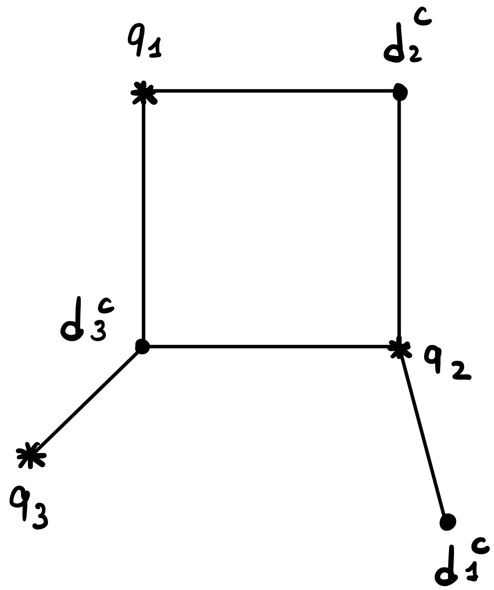


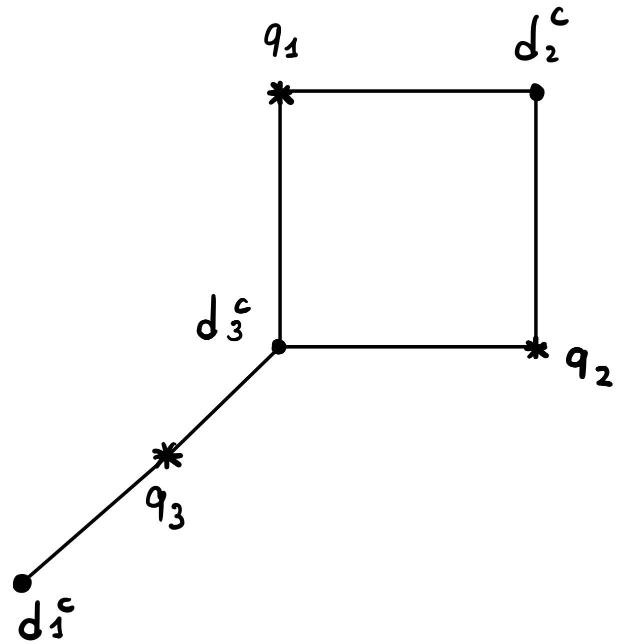
λ_u has 3 parameters

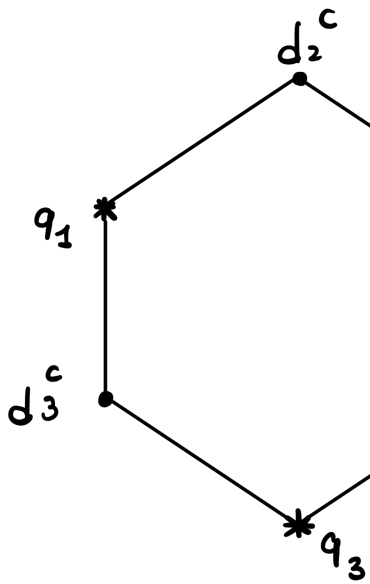
- each vertex touched once to have rank-3
- loop to have a phase

$\boxed{6+3}$

Only 3 possibilities
+ permutations of rows & columns

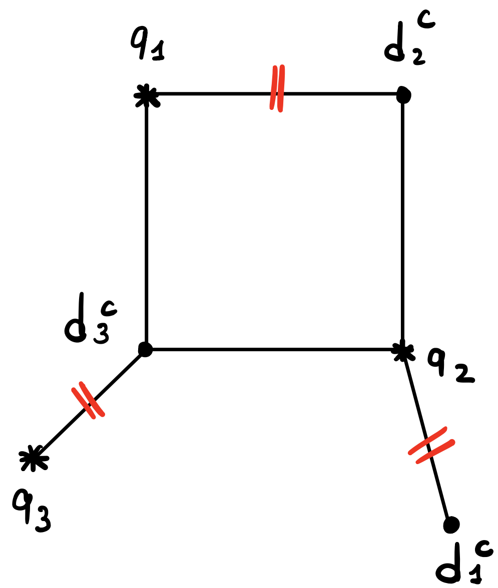






$\boxed{6+3}$

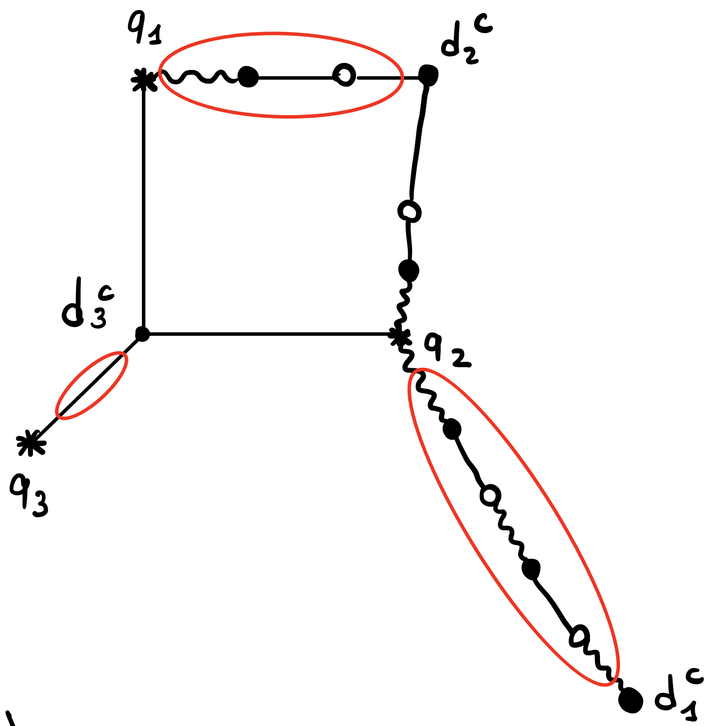
Only 3 possibilities
+ permutations of rows & columns



Determinant: combinations of segments touching each vertex once and only once

$$\det(\lambda_d) = \lambda_d^{12}\,\lambda_d^{21}\,\lambda_d^{33}$$

Opening the chain



$$y_d = \begin{pmatrix} 0 & \# & \# \\ \# & \# & \# \\ 0 & 0 & \# \end{pmatrix}$$

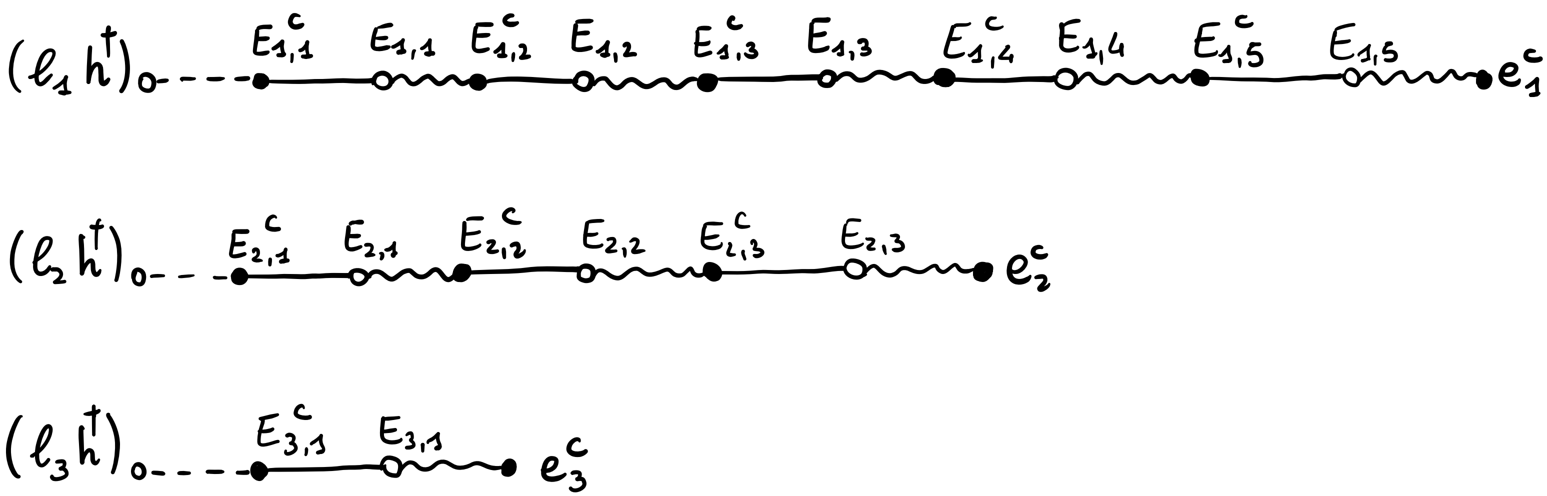
The determinant is read out in the same way.

EXAMPLE

$\lambda \sim \varepsilon \sim 0.1$

Lepton sector

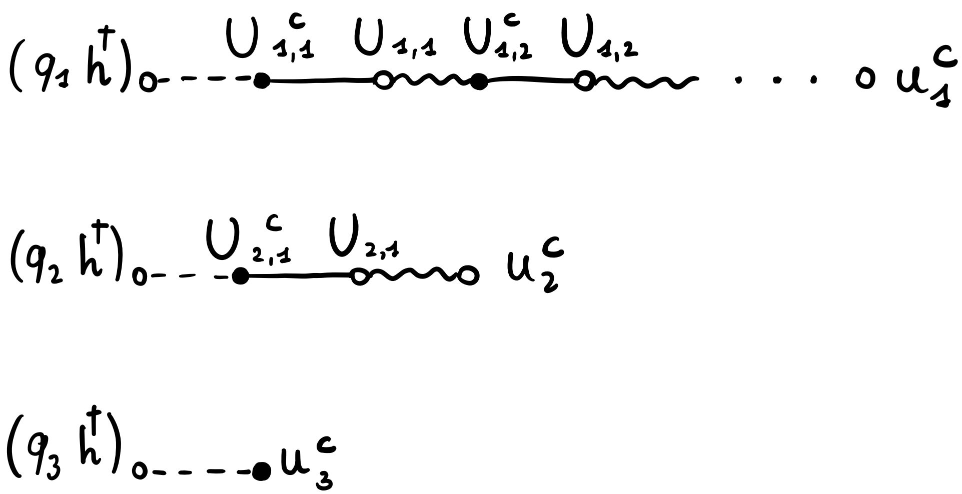
$$\lambda_\ell \sim \begin{bmatrix} a_1 \varepsilon^6 & & \\ & a_2 \varepsilon^4 & \\ & & a_3 \varepsilon^2 \end{bmatrix}$$



Up-quark sector

$\lambda \sim \varepsilon \sim 0.1$

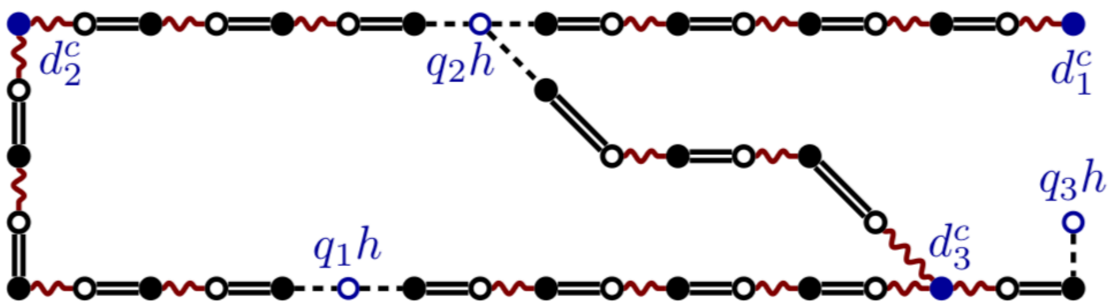
$$y_u \sim \begin{bmatrix} a_1 \varepsilon^5 & & \\ & a_2 \varepsilon^2 & \\ & & a_3 \end{bmatrix}$$



Down-quark sector

$\lambda \sim \varepsilon \sim 0.1$

$$y_d = \begin{bmatrix} 0 & \varepsilon^5 & \varepsilon^5 \\ \varepsilon^5 & \varepsilon^4 e^{i\theta} & \varepsilon^4 \\ 0 & 0 & \varepsilon^2 \end{bmatrix}$$



New Physics Scale

How does the theory look like in the mass basis?

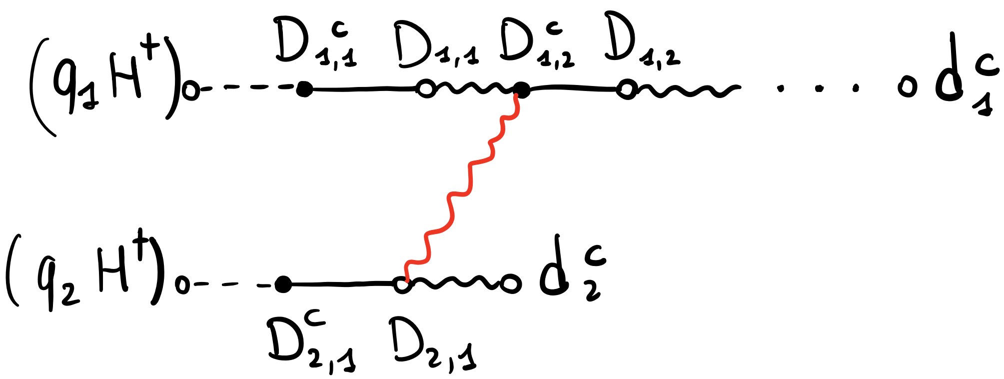
SM + Heavy fermions coupled to h, Z, W, A

How light can this NP be ?

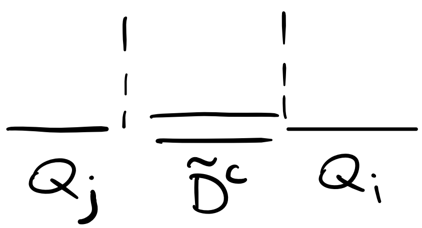
TeV scale !

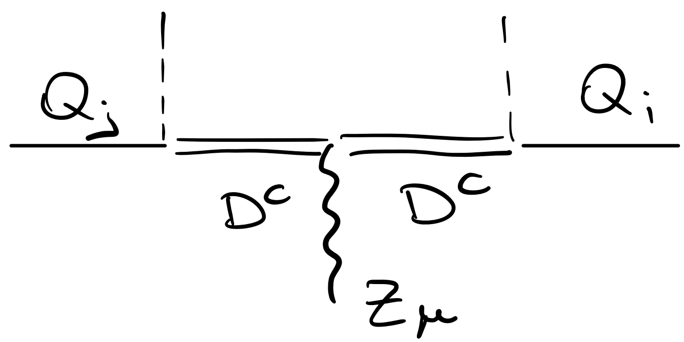
- Direct Searches
- Indirect Constraints (deviation from SM)

New Physics Scale



After rotating (d^c, D^c) $\mathcal{L} \supset M \tilde{D}^c_i D_i + \tilde{\lambda} Q \tilde{D}^c H^\dagger + y Q \tilde{d}_c H^\dagger$





New Physics Scale

At the EW scale:

$$\mathcal{L} \supset d_i d^c_j h \left[\frac{y_{ij}}{\sqrt{2}} - \frac{3}{4} v^2 \left(U_d \frac{\tilde{\lambda}^\dagger \tilde{\lambda}}{M^2} U_{d^c} \right)_{ij} \right]$$

$$+ \bar{d}_i \bar{\sigma}^\mu d_j Z_\mu \left[- \frac{g_2}{c_W} \left(T_3 - Q s_W^2 \right) \delta_{ij} + \frac{g_2}{c_W} T_3 v^2 \left(U_d^\dagger \frac{\tilde{\lambda}^\dagger \tilde{\lambda}}{M^2} U_d \right)_{ij} \right]$$

$$+ \bar{u}_i \bar{\sigma}^\mu d_j W_\mu \left[\frac{g_2}{\sqrt{2}} \left(V_{CKM} \right)_{ij} - \frac{g_2}{2\sqrt{2}} v^2 \left(U_u^\dagger \frac{\tilde{\lambda}^\dagger \tilde{\lambda}}{M^2} U_d \right)_{ij} \right]$$

$Z_\mu \bar{d}^c \bar{\sigma}^\mu d^c$ and $A_\mu J^\mu$ as in the SM

EW Precision

- Flavor diagonal shift $\propto \tilde{\lambda}^2 = \varepsilon^2$

$$\delta g_Z \sim \frac{\varepsilon^2}{M^2} v^2 \quad \Rightarrow \quad M \gtrsim \left(\frac{\varepsilon}{0.2}\right) \text{TeV}$$

LHC

One order of magnitude better at FCC

- Flavor off-diagonal shift further suppressed

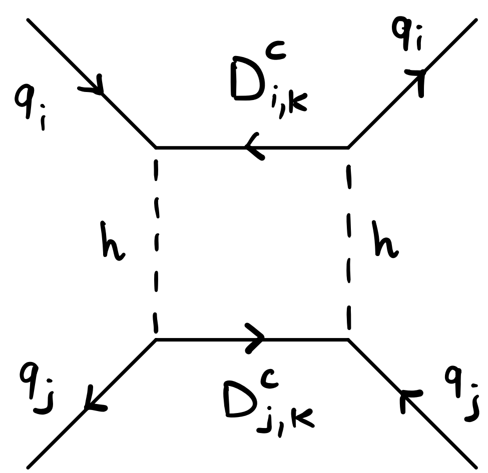
Similar situation in $\mu \to e\gamma$, $\mu \to e$, $\ell \to 3\ell'$, which impose

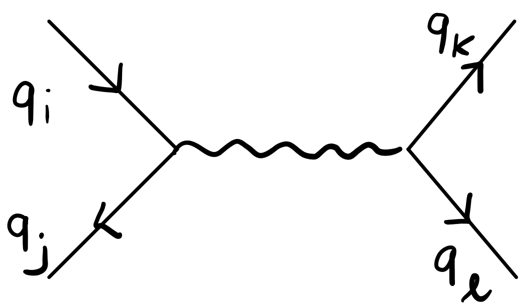
$$\hat{\lambda}_i \hat{\lambda}_j \lesssim 10^{-6} \quad \text{for} \quad M \sim \text{TeV}$$

Therefore we need links between chains for from L, Q sites.

FCNC

Two leading effects





Integrating out U^c, D^c, H we get $C_{ij}\,(q_i^\dagger \sigma^\mu q_i)(q_j^\dagger \sigma_\mu q_j)$

$$C_{ij} \propto |\lambda_i|^2 \times |\lambda_j|^2 \times \left[\frac{2}{128\pi^2 M^2} + \frac{g_2^2 v^4}{8 c_w^2 m_Z^2 M^4} \right]$$

After CKM rotation (assuming $U_d \sim V_{CKM}$)

$$C_1 \propto \left(\sum_{i=1}^{3} |\lambda_i|^2 V_{i1} V_{i2}^* \right)^2 \left[\frac{2}{128\pi^2 M^2} + \frac{g_2^2 v^4}{8 c_w^2 m_Z^2 M^4} \right]$$

$$\mathrm{Re}[C_1] \lesssim (10^4\,\mathrm{TeV})^{-2} \Rightarrow M \gtrsim \left[\underbrace{\frac{\lambda}{0.2}}_{Z} , \underbrace{\left(\frac{\lambda}{0.2}\right)^2}_{\text{box}} \right] \mathrm{TeV}$$

Note CP violation from CKM.

But imagine $(\bar{s}\,\sigma^\mu d)(\bar{s}\,\sigma_\mu d)$ is generated directly through couplings with D^c

$$C \sim \frac{\lambda^4}{128\pi^2 M^2} \lesssim (10^4\ \mathrm{TeV})^{-2}$$

$$\Rightarrow M \gtrsim 10 \left(\frac{\lambda}{0.2}\right)^2 \mathrm{TeV}$$

Still close to TeV

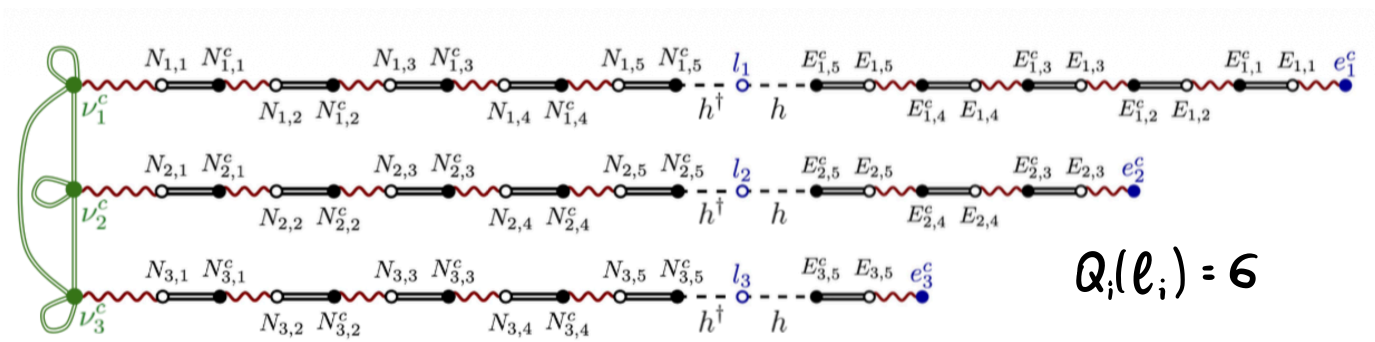
NEUTRINO MASSES

Flavor symmetry $G_F = U(1)^3$

Majorana mass term

$$\mathcal{L}_{G_F} = \lambda \ell E^c H^\dagger + M E E^c + \lambda \ell N^c H + M N N^c + \tilde{m} \nu^c \nu^c$$

$$\mathcal{L}_{G_F\,\mathrm{viol.}} = \mu E_i E_j^c + \mu E e^c + \mu N_i N_j^c$$



$Q_i(\ell_i) = 6$

$$m_\nu \simeq y_e^2 \frac{v^2}{M} \approx 0.1\ \mathrm{eV}\left(\frac{\mathrm{TeV}}{M}\right) \qquad \theta_{ij} \approx 1$$

Conclusion

- Flavor hierarchies through fermion chains
- Simple theories where flavor mechanism appears at TeV scale
- We may see a bunch of VLF coupled $O(\varepsilon)$ to $Hq/H\ell$
- I didn't have time to mention: $U(2)$, $U(1)^{15}$, unification, $\bar{\theta}$ 1-loop, etc.

Work in progress:

- Studying the space of theories;
- Studying the structure of $\arg\det(M)$; [Nelson-Barr + improvement on Flavor]
- Studying detection prospects.