

Generating the fermion mass hierarchy at the TeV scale

Claudio Andrea Manzari

Institute for Advanced Study

Based on [hep-ph:2602.17754](#)

N. Arkani-Hamed, C. Figueiredo, L. Hall, C.A. Manzari

PERIODIC TABLE

Observed order in chemical elements led to discovery of atomic model and quantum mechanics.

PubChem

| Atomic Number | | 17 | | 35.45 | | Atomic Mass, u | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------------------|-----------|----|-----------|----------|--------------|----------------|--------------|----|-----------|-----------|----------------------|----|-----------|----|-----------|----|-----------|----|-------------|----|-------------|-----|-----------|-----|-------------|-----|-----------|-----|------------|-----|-----------|-----|-----------|----|--------|
| Name | | Cl | | Chlorine | | Halogen | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Symbol | | Cl | | Chlorine | | Halogen | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Chemical Group Block | | Cl | | Chlorine | | Halogen | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1.0080 | 1 | H | Hydrogen | Nonmetal | 18 | 4.00260 | 2 | He | Helium | Noble Gas | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 7.0 | 4 | 9.012183 | 2 | Li | Lithium | Alkali Metal | 4 | Be | Beryllium | Alkaline Earth Me... | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 22.989... | 12 | 24.305 | 3 | Na | Sodium | Alkali Metal | 12 | Mg | Magnesium | Alkaline Earth Me... | | | | | | | | | | | | | | | | | | | | | | | | |
| 19 | 39.0983 | 20 | 40.08 | 21 | 44.95591 | 22 | 47.867 | 23 | 50.9415 | 24 | 51.996 | 25 | 54.93804 | 26 | 55.84 | 27 | 58.93319 | 28 | 58.693 | 29 | 63.55 | 30 | 65.4 | 31 | 69.723 | 32 | 72.63 | 33 | 74.92159 | 34 | 78.97 | 35 | 79.90 | 36 | 83.80 |
| 37 | 85.468 | 38 | 87.62 | 39 | 88.90584 | 40 | 91.22 | 41 | 92.90637 | 42 | 95.95 | 43 | 96.90636 | 44 | 101.1 | 45 | 102.9055 | 46 | 106.42 | 47 | 107.868 | 48 | 112.41 | 49 | 114.818 | 50 | 118.71 | 51 | 121.760 | 52 | 127.6 | 53 | 126.9045 | 54 | 131.29 |
| 55 | 132.90... | 56 | 137.33 | 57 | 178.49 | 58 | 180.9479 | 59 | 183.84 | 60 | 186.207 | 61 | 190.2 | 62 | 192.22 | 63 | 195.08 | 64 | 196.96... | 65 | 200.59 | 66 | 204.383 | 67 | 207 | 68 | 208.98... | 69 | 208.98... | 70 | 209.98... | 71 | 212.01... | | |
| 87 | 223.01... | 88 | 226.02... | 89 | 227.02... | 90 | 232.038 | 91 | 231.03... | 92 | 238.0289 | 93 | 237.04... | 94 | 244.06... | 95 | 243.06... | 96 | 247.07... | 97 | 247.07... | 98 | 251.07... | 99 | 252.0830 | 100 | 257.0... | 101 | 258.0... | 102 | 259.1... | 103 | 266.1... | | |
| 57 | 138.9055 | 58 | 140.116 | 59 | 140.90... | 60 | 144.24 | 61 | 144.91... | 62 | 150.4 | 63 | 151.964 | 64 | 157.2 | 65 | 158.92... | 66 | 162.500 | 67 | 164.93... | 68 | 167.26 | 69 | 168.93... | 70 | 173.05 | 71 | 174.9668 | | | | | | |
| 89 | Actinium | 90 | Thorium | 91 | Protactinium | 92 | Uranium | 93 | Neptunium | 94 | Plutonium | 95 | Americium | 96 | Curium | 97 | Berkelium | 98 | Californium | 99 | Einsteinium | 100 | Fermium | 101 | Mendelevium | 102 | Nobelium | 103 | Lawrencium | | | | | | |

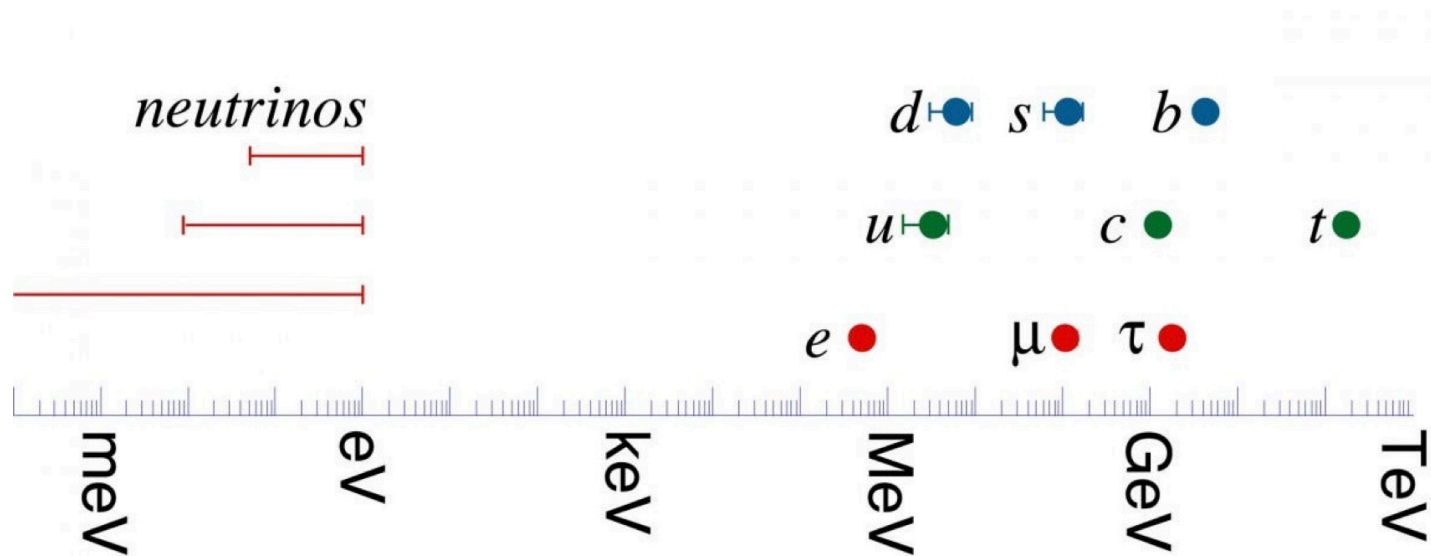
PERIODIC TABLE OF FERMIONS

Similarly the observed properties of fermions could encode information about deeper organization principle

| three generations of matter (fermions) | | | |
|--|--|--|--|
| | I | II | III |
| mass | $\approx 2.16 \text{ MeV}/c^2$ | $\approx 1.273 \text{ GeV}/c^2$ | $\approx 172.57 \text{ GeV}/c^2$ |
| charge | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |
| spin | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| QUARKS | u up | c charm | t top |
| | d down | s strange | b bottom |
| LEPTONS | $\approx 0.511 \text{ MeV}/c^2$ | $\approx 105.66 \text{ MeV}/c^2$ | $\approx 1.77693 \text{ GeV}/c^2$ |
| | -1 | -1 | -1 |
| | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| | e electron | μ muon | τ tau |
| | $< 0.8 \text{ eV}/c^2$ | $< 0.17 \text{ MeV}/c^2$ | $< 18.2 \text{ MeV}/c^2$ |
| | 0 | 0 | 0 |
| | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| | ν_e electron neutrino | ν_μ muon neutrino | ν_τ tau neutrino |

OPEN QUESTION: The Flavor Puzzle

- Why do we observe such a bizarre pattern of fermion masses and mixing angles?



- Why do fermions come in 3 generations?
- What is the origin of neutrino masses?
- What is the origin of CP violation?
- . . .

Anthropic Principle

- Masses of light fermions important for nuclear physics?
- May play a role, but unlikely to solve the problem entirely

Connection with other open questions

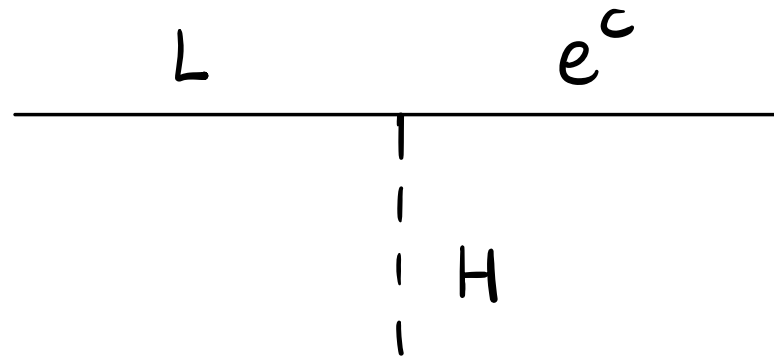
- EW hierarchy problem
- Matter-Antimatter asymmetry
- Strong CP problem

Flavor in the SM

Fermion masses arise through interaction with the Higgs boson

$$L = (\nu, e)$$

$$H = (H^+, H^0)$$



$$L \supset y_e L e^c H^+ \xrightarrow{\langle H \rangle = (0, v)} y_e v e e^c$$

$y_e v = m_e$

- Large Number of Parameters in the SM.

QUARK SECTOR

$$\begin{matrix} & (u^c & c^c & t^c) \\ \begin{pmatrix} u \\ c \\ t \end{pmatrix} & \begin{pmatrix} & & \\ & 3 \times 3 & \\ & & \end{pmatrix} \\ & \underbrace{\hspace{10em}} \\ & M_u \end{matrix}$$

9 complex
parameters

$$\begin{matrix} & (d^c & s^c & b^c) \\ \begin{pmatrix} d \\ s \\ b \end{pmatrix} & \begin{pmatrix} & & \\ & 3 \times 3 & \\ & & \end{pmatrix} \\ & \underbrace{\hspace{10em}} \\ & M_d \end{matrix}$$

9 complex
parameters

We can diagonalize the mass matrices

$$M_u = U_u M_u^{\text{diag}} U_u^c ; \quad M_D = U_d M_d^{\text{diag}} U_d^c ;$$

$$\mathcal{L} \supset d_i d_i^c \frac{y_i}{\sqrt{2}} (v+h) + \bar{u}_i \bar{\sigma}^\mu d_j W_\mu \left[\frac{g_2}{\sqrt{2}} (V_{CKM})_{ij} \right]$$

$$V_{CKM} = U_u^\dagger U_d \quad \text{complex} \\ 3 \times 3 \text{ matrix}$$

OBSERVABLES

6 masses

3 mixing angles, 1 phase

TOWARD A THEORY OF FLAVOR

- ① Has to be predictive: parameters < observables;
- ② Has to explain the observed hierarchies;

How light can the NP scale be?

Are these mechanisms testable at colliders?

- Generally theories of flavor have to leave at high scales.

$$\text{FCNC: } \Lambda \gtrsim 10^6 \text{ GeV}$$

$$\text{Ex.: } \Delta F=2 \\ (k-\bar{k})$$

$$H_{\text{eff}} = \sum_{i=1}^5 \frac{C_i}{\Lambda^2} Q_i + \sum_{i=1}^3 \frac{\tilde{C}_i}{\Lambda^2} \tilde{Q}_i$$

$$Q_1 = (s_\alpha^\dagger \sigma_\mu d_\alpha)(s_\beta^\dagger \sigma^\mu d_\beta)$$

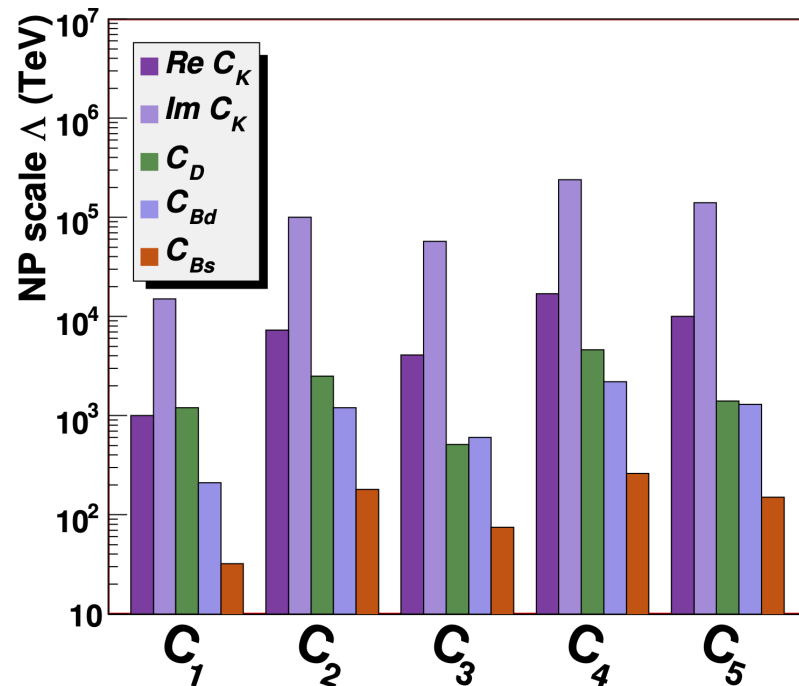
$$Q_2 = (s_\alpha^c d_\alpha)(s_\beta^c d_\beta)$$

$$Q_3 = (s_\alpha^c d_\beta)(s_\beta^c d_\alpha)$$

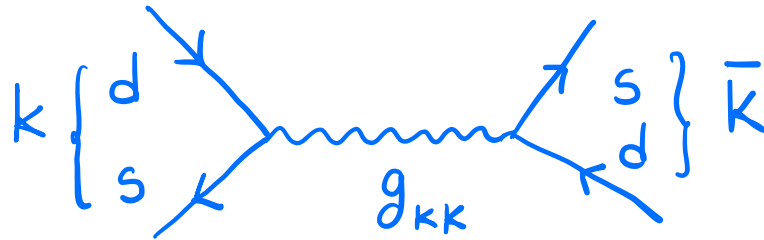
$$Q_4 = (s_\alpha^c d_\alpha)(s_\beta^\dagger d_\beta^{ct})$$

$$Q_5 = (s_\alpha^c d_\beta)(s_\beta^\dagger d_\alpha^{ct})$$

$$\tilde{Q} \xleftrightarrow{c} Q$$

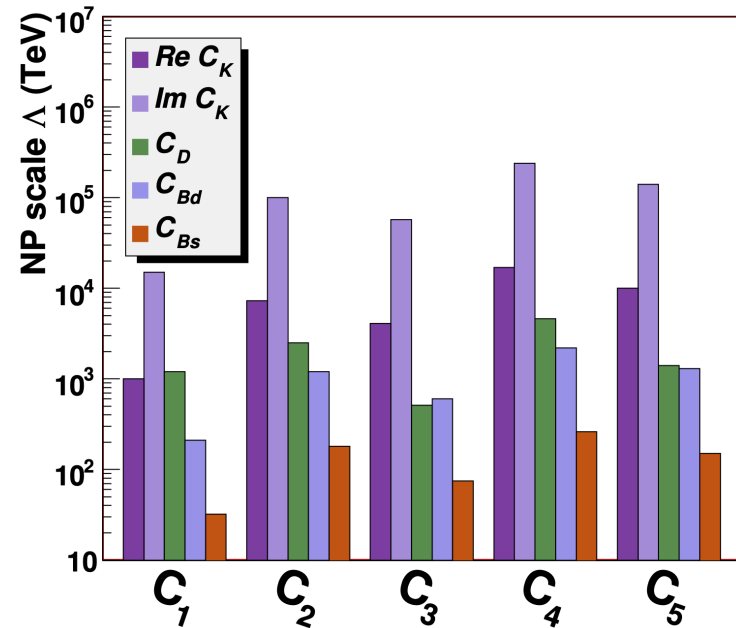


Ex: 5D Warped RS Models



$$C_4 \sim \frac{1}{M_{g_{KK}}^2} g_3^{*2} \frac{m_d m_s}{\sqrt{2}} \in \mathbb{C}$$

$$\Rightarrow M_{g_{KK}} \gtrsim 20 \text{ TeV}$$



- It would be great to have theories of Flavor where the NP scale has to be close to the TeV scale!

LESS AMBITIOUS BUT A GOOD STARTING POINT

- Are there simple theory of Flavor where NP can be close to the TeV scale?

GOALS:

- Generate hierarchies without "small" parameters;
- Flavor constraints on NP at or below TeV scale.
NP possible to detect at current or near-future colliders.

MAIN IDEA

- Theories where the "light" NP are VL fermions with the same charges of SM field.
- Flavor hierarchies are generated through chains of VL fermions

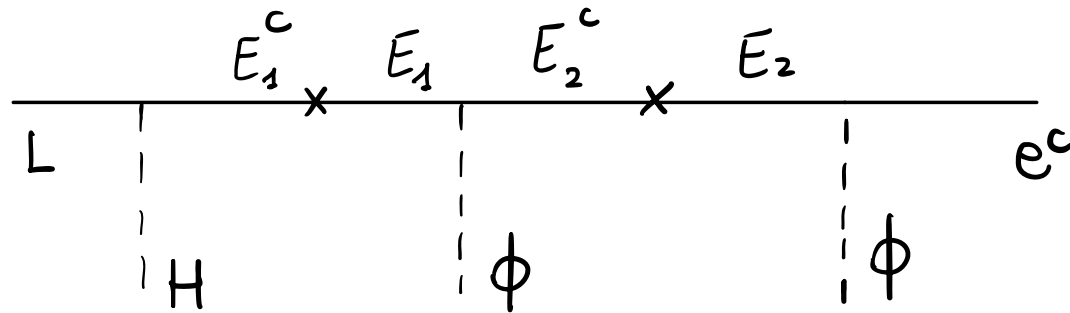
EXAMPLE

$$\mathcal{L} \supset M E_i^c E_i + \epsilon M E_i^c E_j + \epsilon M e^c E + \lambda L E^c H^\dagger; \quad \epsilon = 0.1$$

$$(LH) \bullet \overset{E_1^c}{\text{---}} \bullet \overset{E_1}{\text{---}} \bullet \overset{E_2^c}{\text{---}} \bullet \overset{E_2}{\text{---}} \bullet e^c \approx \frac{\epsilon^3 M^5}{M^5} L e^c H$$

$\lambda = \epsilon$ for simplicity. NOT NEEDED

SIMILAR TO FROGGATT-NIELSEN



$$L \supset H \underbrace{\left(\frac{\phi}{M} \right)^n}_{\varepsilon} L e^c$$

- Here the lightest NP are scalars (Laurer, Nir, Seiberg)
 $\sim \text{TeV}$ scale
 with $\varepsilon \sim 10^{-3}$
- We won't see the flavor mechanism until we see the heavy fermions

KEY POINT: SOFT MASS TERMS

LEPTON SECTOR (charged leptons, diagonal mass matrix)

$$\mathcal{L} \supset M E_i^c E_i + \varepsilon M E_i^c E_j + \varepsilon M e^c E + \lambda L E^c H^\dagger; \quad \varepsilon = 0.1$$

$$y_\tau \sim \varepsilon^2 \quad (L_3 H) \text{---} E_1^c \text{---} E_1 \text{---} e_3^c$$

$$y_\mu \sim \varepsilon^3 \quad (L_2 H) \text{---} E_2^c \text{---} E_2 \text{---} E_3^c \text{---} E_3 \text{---} e_2^c$$

$$y_e \sim \varepsilon^6 \quad (L_1 H) \text{---} E_4^c \text{---} E_4 \text{---} E_5^c \text{---} E_5 \text{---} E_6^c \text{---} E_6 \text{---} E_7^c \text{---} E_7 \text{---} E_8^c \text{---} E_8 \text{---} e_1^c$$

$$\Rightarrow y_e = \begin{pmatrix} \varepsilon^6 & & \\ & \varepsilon^3 & \\ & & \varepsilon^2 \end{pmatrix}$$

$$(e_1 \ e_2 \ e_3 \ E_1 \dots E_n) \left(\begin{array}{c|ccc} O_{3 \times 3} & & & \lambda_{3 \times n} H \\ \hline & M & & \epsilon M \\ & & M & \\ & \epsilon M & & M \end{array} \right) \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \\ E_1^c \\ \vdots \\ E_n^c \end{pmatrix}$$

We can rotate $(e^c, E^c) \rightarrow (\tilde{e}^c, \tilde{E}^c)$ to diagonalize M

$$\mathcal{L} \supset M \tilde{E}_i^c E_i + \tilde{\lambda} L \tilde{E}^c H^\dagger + y L \tilde{e}^c H^\dagger$$

$$y \sim \begin{pmatrix} \epsilon^6 & & \\ & \epsilon^3 & \\ & & \epsilon^2 \end{pmatrix}$$

- In the SM this is not possible!

Allow to have only $O(1)$ or $O(\epsilon)$ entries

$$Y = \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}$$

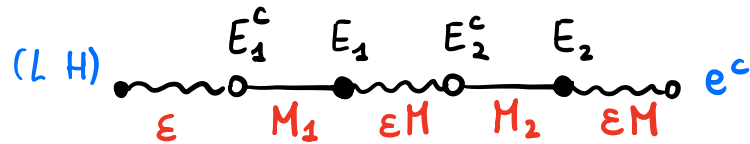
$$\text{Det}(Y) = \underbrace{y_1 \cdot y_2 \cdot y_3}_{\text{eigenvalues}} \geq \epsilon^3$$

But we know

$$\begin{aligned} \text{Det}(Y_u) &\sim 10^{-7} \sim \epsilon^7 \\ \text{Det}(Y_d) &\sim 10^{-10} \sim \epsilon^{10} \\ \text{Det}(Y_e) &\sim 10^{-11} \sim \epsilon^{11} \end{aligned}$$

If E can be at the TeV scale:

$$\tilde{\lambda}_i L \tilde{E}_i^c H$$



$$\tilde{\lambda}_1 \sim \epsilon$$

$$\tilde{\lambda}_2 \sim \frac{\epsilon^2 M^2}{M_2^2 - M_1^2}$$

$$E \rightarrow e H$$

$$\Gamma_i \sim \left(\frac{\tilde{\lambda}_i}{0.2} \right)^2 \left(\frac{M}{650 \text{ GeV}} \right) \text{ GeV}$$

(a) NO DEGENERACY

$$\frac{\Gamma(E_1 \rightarrow e H)}{\Gamma(E_2 \rightarrow e H)} \sim \frac{\epsilon}{\epsilon^2} \sim \frac{1}{\epsilon}$$

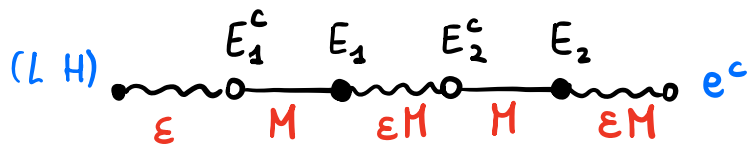
States further down the chain may be difficult to probe

$$c\tau \sim 10^{2p-15} \text{ cm} \left(\frac{0.1}{\epsilon} \right)^{2p} \cdot \left(\frac{\text{TeV}}{M} \right)$$

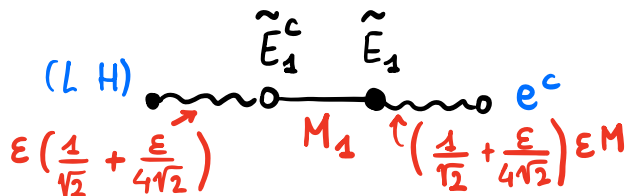
(b) DEGENERATE FERMIONS

- $\tilde{\lambda}_i \sim \epsilon \quad \forall i$ - All fermions maximally coupled
 - Mass splitting $O(\epsilon M)$

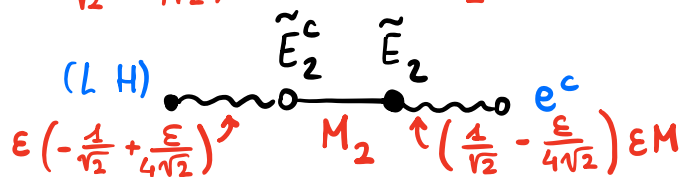
IMPORTANT: SM Yukawa couplings are still the same



$$\begin{pmatrix} e \\ E_1 \\ E_2 \end{pmatrix}^T \begin{pmatrix} 0 & \epsilon H & 0 \\ 0 & M & \epsilon M \\ \epsilon M & 0 & M \end{pmatrix} \begin{pmatrix} e^c \\ E_1^c \\ E_2^c \end{pmatrix}$$



$$M_1 \sim M - \frac{\epsilon M}{2}$$



$$M_2 \sim M + \frac{\epsilon M}{2}$$

$$y \sim \epsilon^3!$$

QUARK SECTOR

- Quark masses (6)
- CKM angles (3) There are a large number of theories.
- CKM phase (1)

Let require 9 real parameters and 1 phase:

- 6 + 3 (only 3 patterns)
- 4 + 5 (≤ 20 patterns)

$$6 + 3$$

$$d_1^c \bullet$$

$$q_1 *$$

$$o u_1^c$$

$$d_2^c \bullet$$

$$q_2 *$$

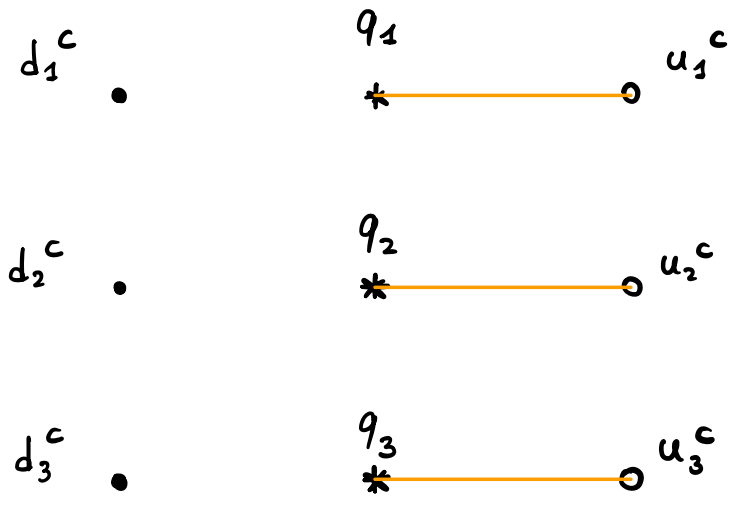
$$o u_2^c$$

$$d_3^c \bullet$$

$$q_3 *$$

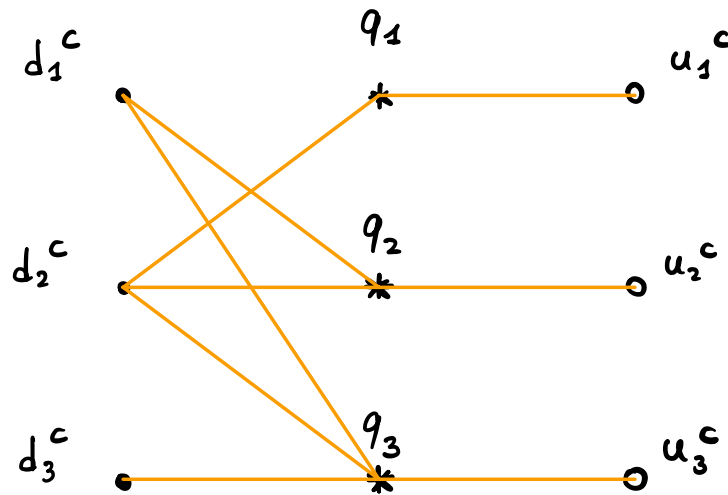
$$o u_3^c$$

$$6 + 3$$



λ_u has 3 parameters

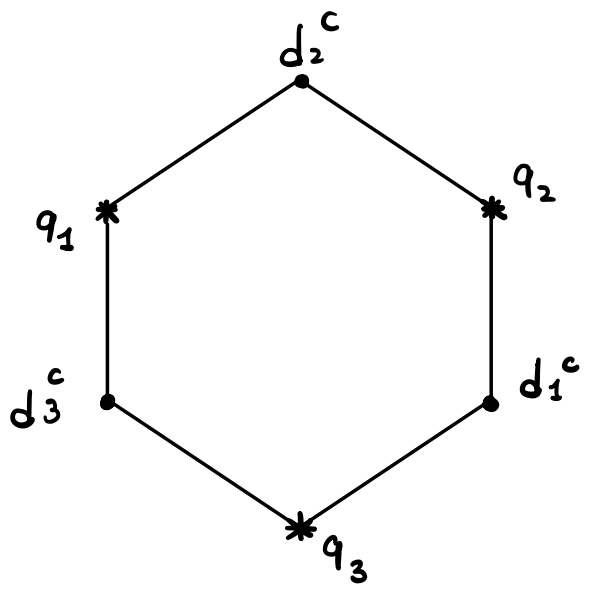
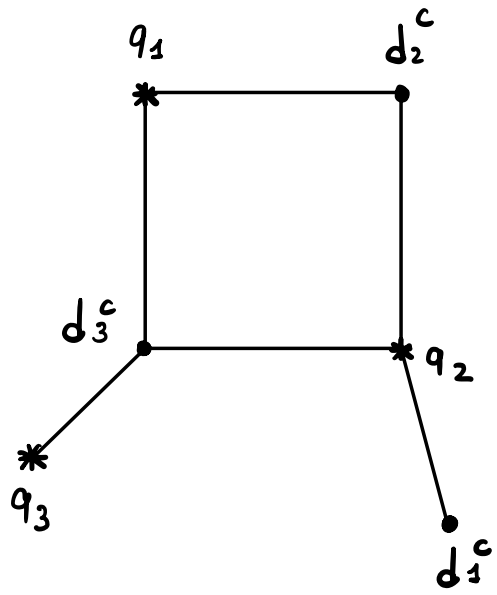
6 + 3



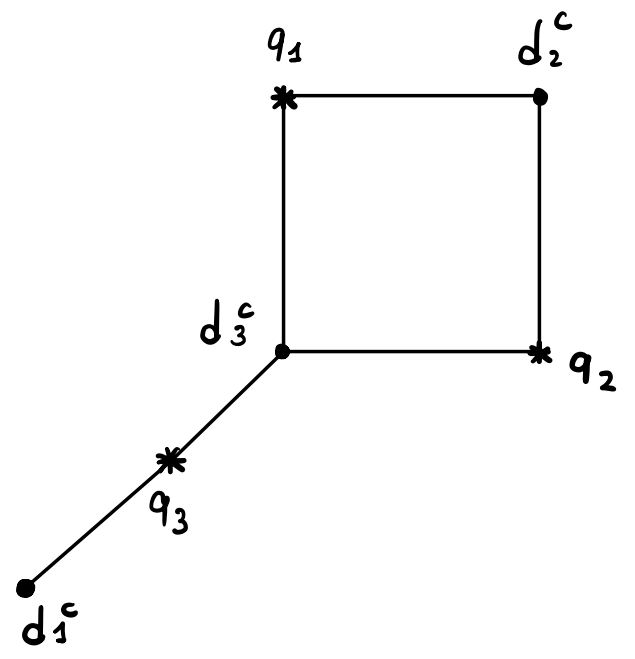
λ_u has 3 parameters

- each vertex touched once to have rank-3
- loop to have a phase

6 + 3

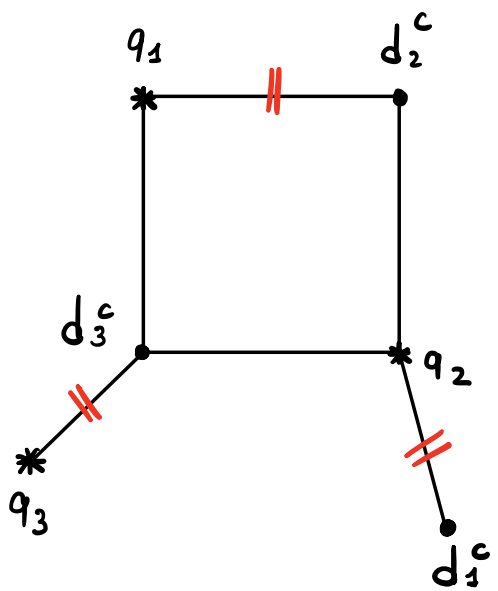


Only 3 possibilities
+ permutations of rows & columns



$$6 + 3$$

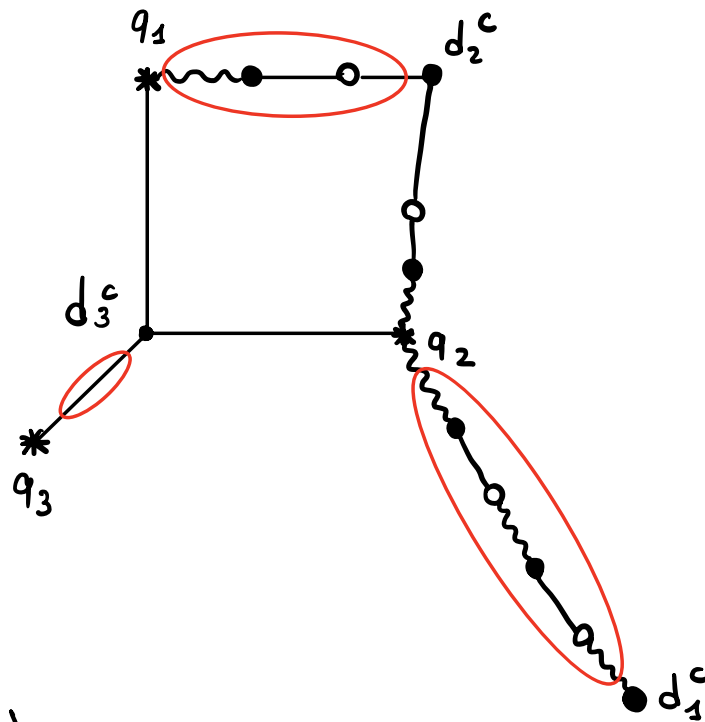
Only 3 possibilities
+ permutations of rows & columns



Determinant: combinations of segments touching each vertex once and only once

$$\det(\lambda_d) = \lambda_d^{12} \lambda_d^{21} \lambda_d^{33}$$

Opening the chain



$$y_d = \begin{pmatrix} 0 & \# & \# \\ \# & \# & \# \\ 0 & 0 & \# \end{pmatrix}$$

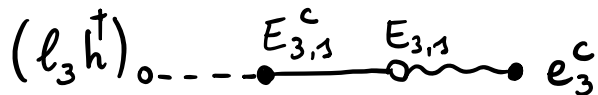
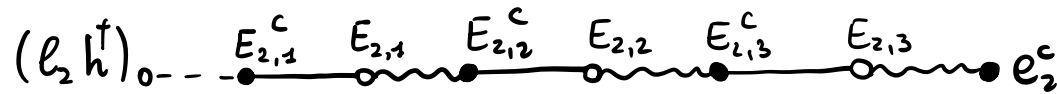
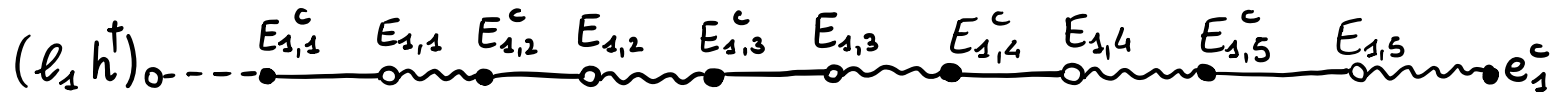
The determinant is read out in the same way.

EXAMPLE

$$\lambda \sim \epsilon \sim 0.1$$

Lepton sector

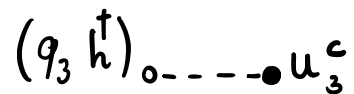
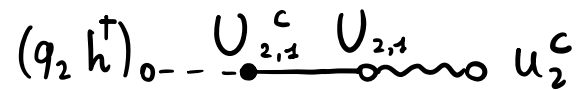
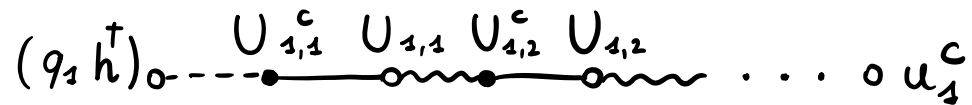
$$\lambda_\ell \sim \begin{bmatrix} a_1 \epsilon^6 & & \\ & a_2 \epsilon^4 & \\ & & a_3 \epsilon^2 \end{bmatrix}$$



Up - quark sector

$$\lambda \sim \epsilon \sim 0.1$$

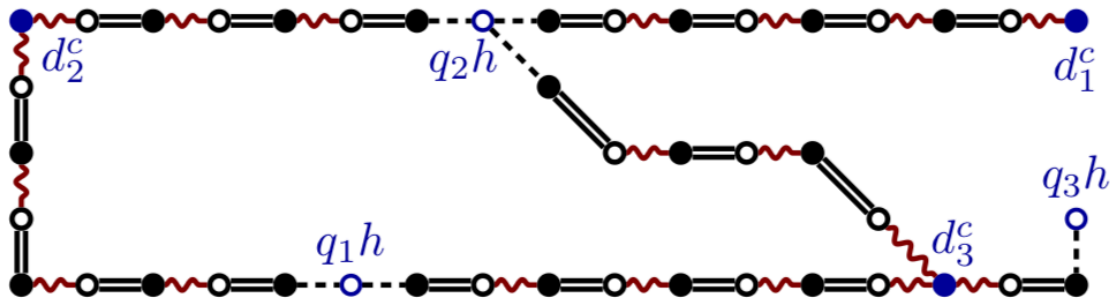
$$y_u \sim \begin{bmatrix} a_1 \epsilon^5 & & & \\ & a_2 \epsilon^2 & & \\ & & & \\ & & & a_3 \end{bmatrix}$$



Down-quark sector

$$\lambda \sim \epsilon \sim 0.1$$

$$y_d = \begin{bmatrix} 0 & \epsilon^5 & \epsilon^5 \\ \epsilon^5 & \epsilon^4 e^{i\theta} & \epsilon^4 \\ 0 & 0 & \epsilon^2 \end{bmatrix}$$



New Physics Scale

How does the theory look like in the mass basis?

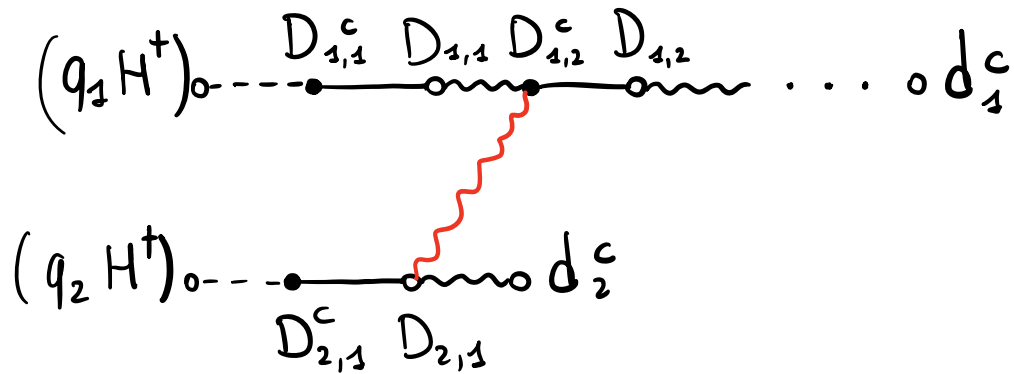
SM + Heavy fermions coupled to h, Z, W, A

How light can this NP be?

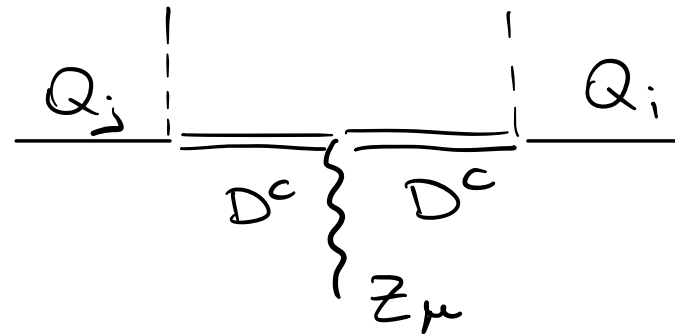
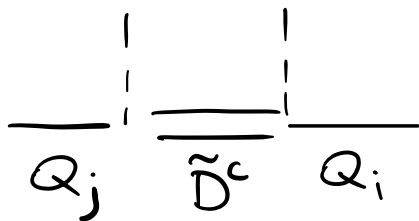
TeV scale!

- Direct Searches
- Indirect Constraints (deviation from SM)

New Physics Scale



After rotating (d^c, D^c) $\mathcal{L} = M \tilde{D}_i^c D_i + \tilde{\lambda} Q \tilde{D}^c H^\dagger + y Q \tilde{d}_c H^\dagger$



New Physics Scale

At the EW scale:

$$\begin{aligned} \mathcal{L} \supset & \bar{d}_i d_j^c h \left[\frac{y_{ij}}{\sqrt{2}} - \frac{3}{4} v^2 \left(U_d \frac{\tilde{\lambda}^+ \tilde{\lambda}}{M^2} U_d^c \right)_{ij} \right] \\ & + \bar{d}_i \bar{\sigma}^\mu d_j Z_\mu \left[-\frac{g_2}{c_w} (T_3 - Q s_w^2) \delta_{ij} + \frac{g_2}{c_w} T_3 v^2 \left(U_d^\dagger \frac{\tilde{\lambda}^+ \tilde{\lambda}}{M^2} U_d \right)_{ij} \right] \\ & + \bar{u}_i \bar{\sigma}^\mu d_j W_\mu \left[\frac{g_2}{\sqrt{2}} (V_{CKM})_{ij} - \frac{g_2}{2\sqrt{2}} v^2 \left(U_u^\dagger \frac{\tilde{\lambda}^+ \tilde{\lambda}}{M^2} U_d \right)_{ij} \right] \end{aligned}$$

$Z_\mu \bar{d}^c \bar{\sigma}^\mu d^c$ and $A_\mu J^\mu$ as in the SM

EW Precision

- Flavor diagonal shift $\propto \tilde{\lambda}^2 = \epsilon^2$

$$\delta g_z \sim \frac{\epsilon^2}{M^2} v^2 \Rightarrow M \gtrsim \left(\frac{\epsilon}{0.2} \right) \text{TeV}$$

LHC
One order of magnitude better at FCC

- Flavor off-diagonal shift further suppressed

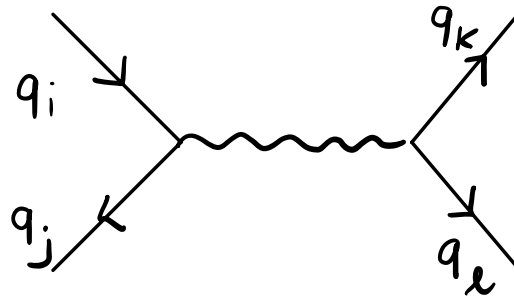
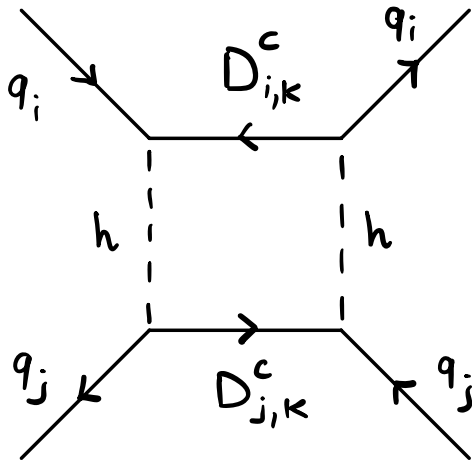
Similar situation in $\mu \rightarrow e\gamma$, $\mu \rightarrow e$, $e \rightarrow 3e'$, which impose

$$\hat{\lambda}_i \hat{\lambda}_j \lesssim 10^{-6} \text{ for } M \sim \text{TeV}$$

Therefore we need links between chains for from L, Q sites.

FCNC

Two leading effects



Integrating out U^c, D^c, H we get $C_{ij} (q_i^\dagger \sigma^\mu q_i) (q_j^\dagger \sigma_\mu q_j)$

$$C_{ij} \propto |\lambda_i|^2 \times |\lambda_j|^2 \times \left[\frac{2}{128 \pi^2 M^2} + \frac{g_2^2 V^4}{8 c_w^2 m_{\frac{Z}{2}}^2 M^4} \right]$$

After CKM rotation (assuming $U_d \sim V_{CKM}$)

$$C_1 \propto \left(\sum_{i=1}^3 |\lambda_i|^2 V_{i1} V_{i2}^* \right)^2 \left[\frac{2}{128 \pi^2 M^2} + \frac{g_2^2 V^4}{8 c_w^2 m_Z^2 M^4} \right]$$

$$\text{Re}[C_1] \leq (10^4 \text{ TeV})^{-2} \Rightarrow M \gtrsim \left[\frac{\lambda}{0.2}, \left(\frac{\lambda}{0.2} \right)^2 \right] \text{ TeV}$$

↑ ↑
Z box

Note CP violation from CKM.

But imagine $(\bar{s} \sigma^\mu d)(\bar{s} \sigma_\mu d)$ is generated directly through couplings with D^c

$$C \sim \frac{\lambda^4}{128 \pi^2 M^2} \lesssim (10^4 \text{ TeV})^2$$

$$\Rightarrow M \gtrsim 10 \left(\frac{\lambda}{0.2} \right)^2 \text{ TeV}$$

Still close to TeV

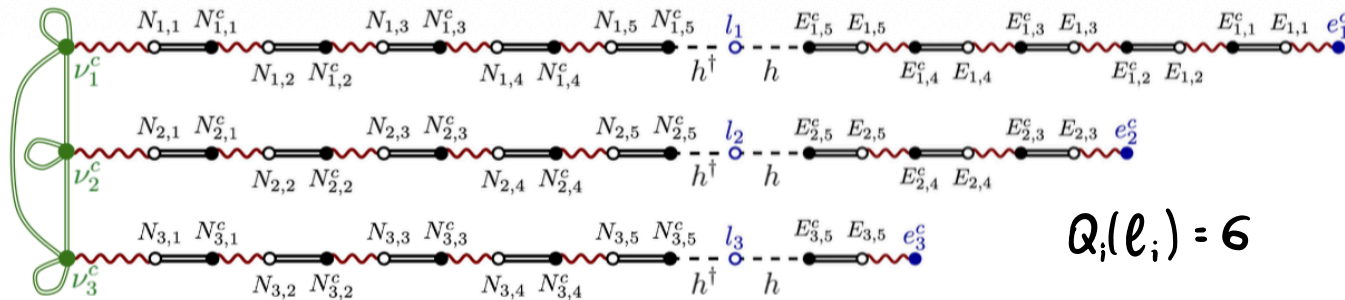
NEUTRINO MASSES

Flavor symmetry $G_F = U(1)^3$

Majorana
 \uparrow mass term

$$\mathcal{L}_{GF} = \lambda \ell E^c H^\dagger + M E E^c + \lambda \ell N^c H + M N N^c + \tilde{m} \nu^c \nu^c$$

$$\mathcal{L}_{GF \text{ viol.}} = \mu E_i E_j^c + \mu E e^c + \mu N_i N_j^c$$



$$m_\nu \approx y_e^2 \frac{v^2}{M} \approx 0.1 \text{ eV} \left(\frac{\text{TeV}}{M} \right)$$

$$\theta_{ij} \approx 1$$

Conclusion

- Flavor hierarchies through fermion chains
- Simple theories where flavor mechanism appears at TeV scale
- We may see a bunch of VLF coupled $O(\epsilon)$ to $Hq/H\ell$
- I didn't have time to mention: $U(2), U(1)^{15}$, unification, $\bar{\Theta}$ 1-loop, etc.

Work in progress:

- Studying the space of theories;
- Studying the structure of $\arg\det(\mathcal{M})$; [Nelson-Barr + improvement on Flavor]
- Studying detection prospects.