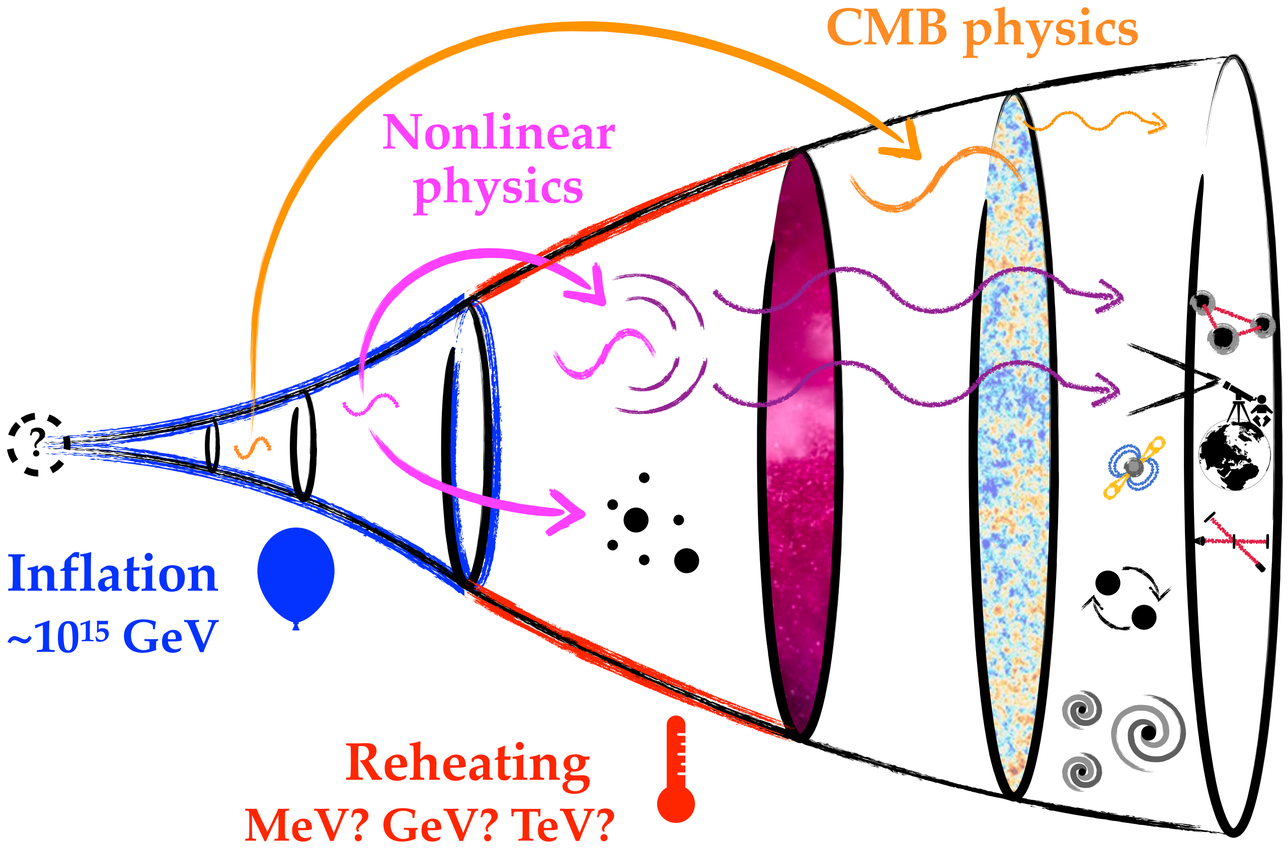
Gravitational waves from primordial fluctuations



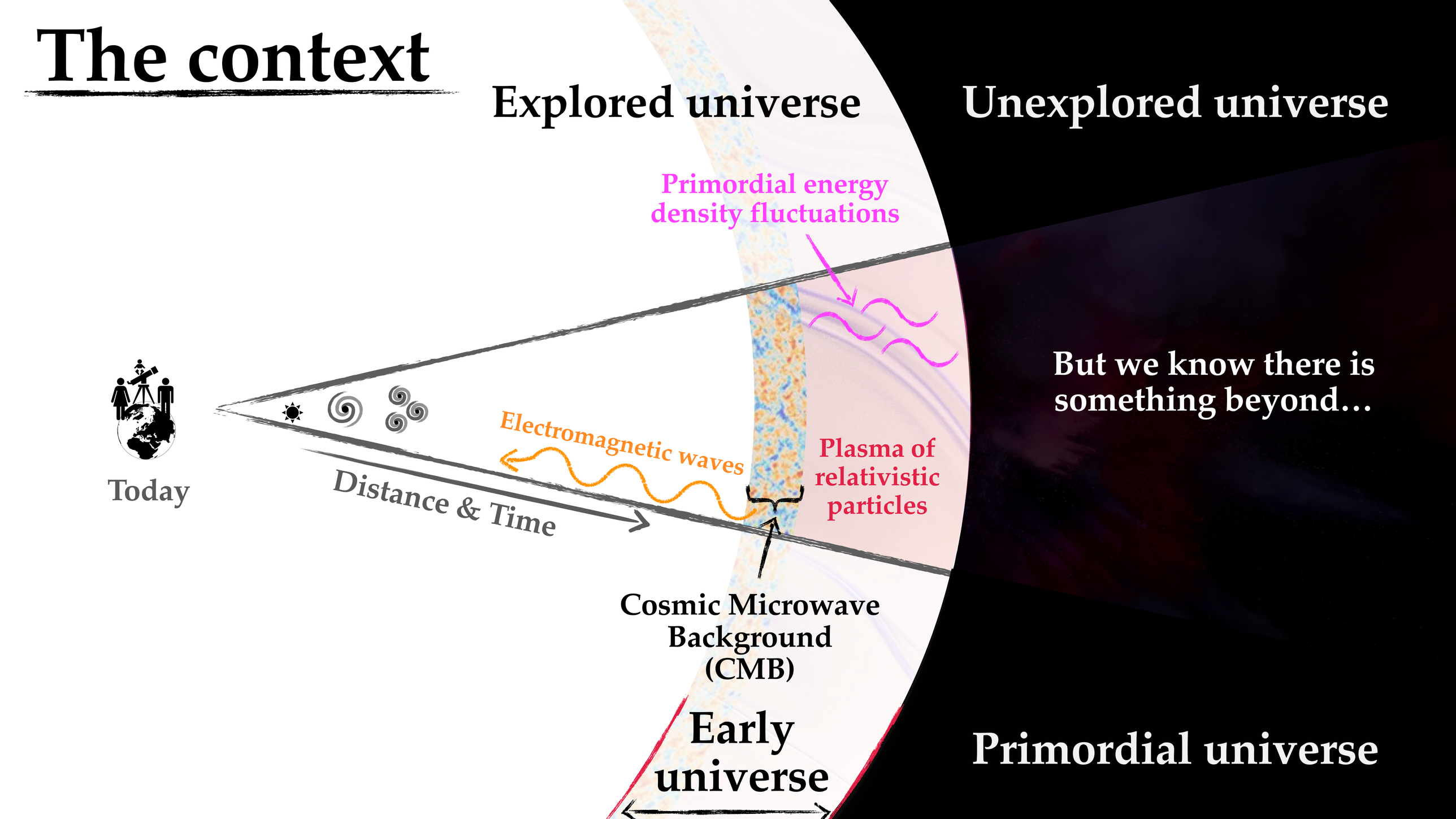
by **Guillem Domènech**
(ITP Hannover)

HiDDeN ITN Webinar
Seminar Feb 13th, 2024

Leibniz Universität Hannover

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

The context

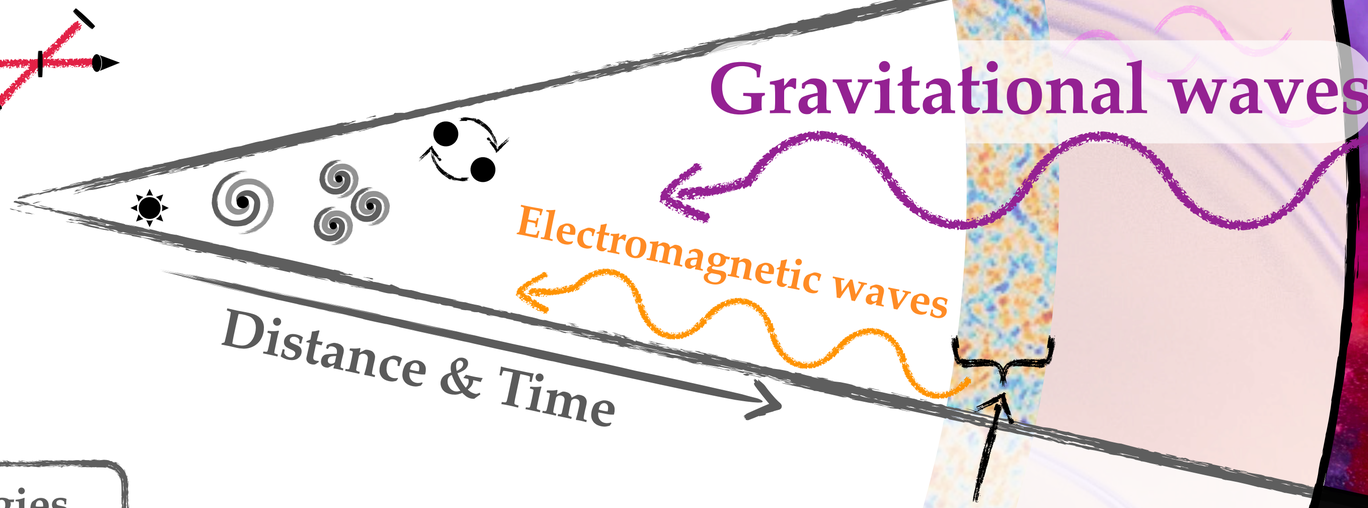

Explored universe
Unexplored universe
Primordial energy density fluctuations
But we know there is something beyond…
Today
Electromagnetic waves
Distance & Time
Plasma of relativistic particles
Cosmic Microwave Background (CMB)
Early universe
Primordial universe

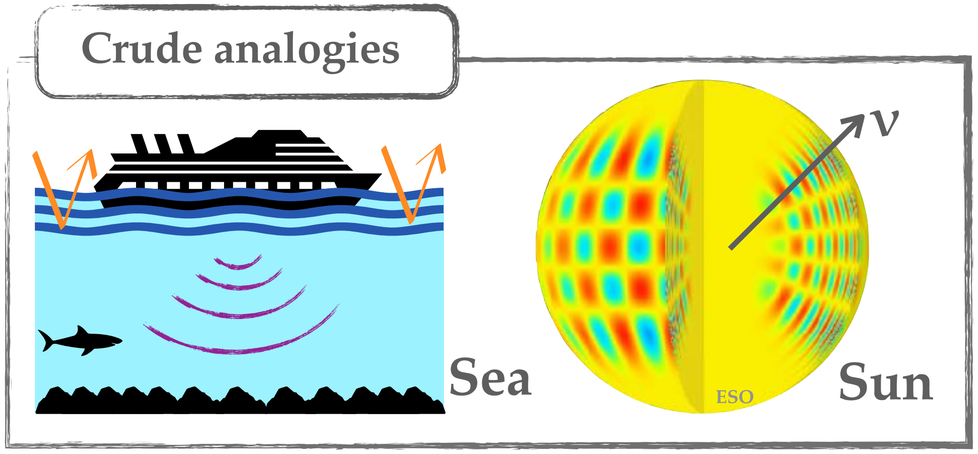
The context

Explored universe

Unexplored universe

Gravitational waves can probe the **unexplored** universe





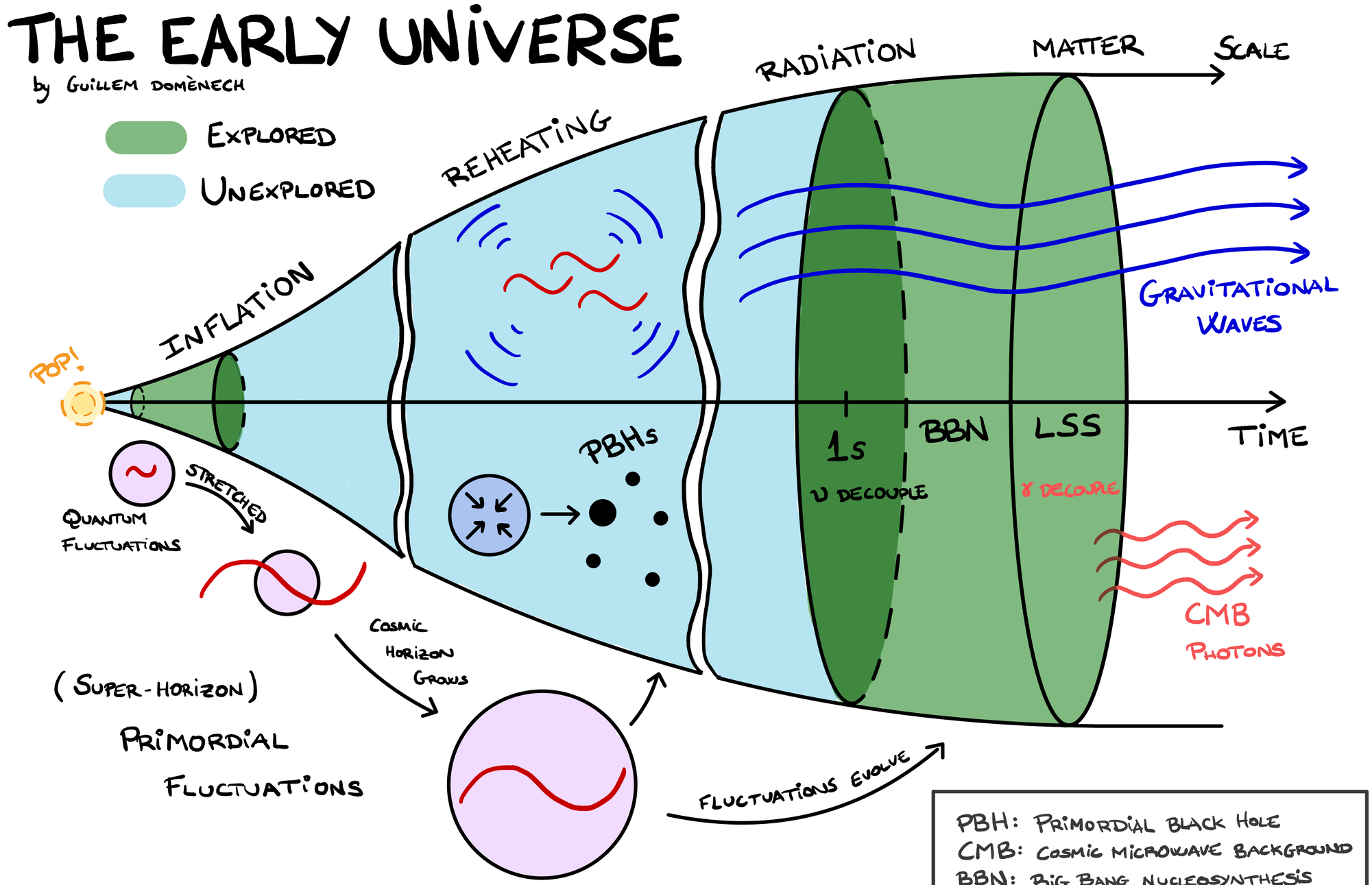


Image from: https://domenechcosmo.netlify.app/post/induced-gws/

General (ambitious) questions

- What is **dark matter**? Particles or (Primordial) **Black Holes**?
- How can we further test **cosmic inflation** and **gravity**?
- (New) **gravitational waves probes**: all BHs astrophysical? **PTAs**?
- How can we test beyond SM physics in the very early universe?

  For **T>4 MeV** we are not so sure.

Why care?

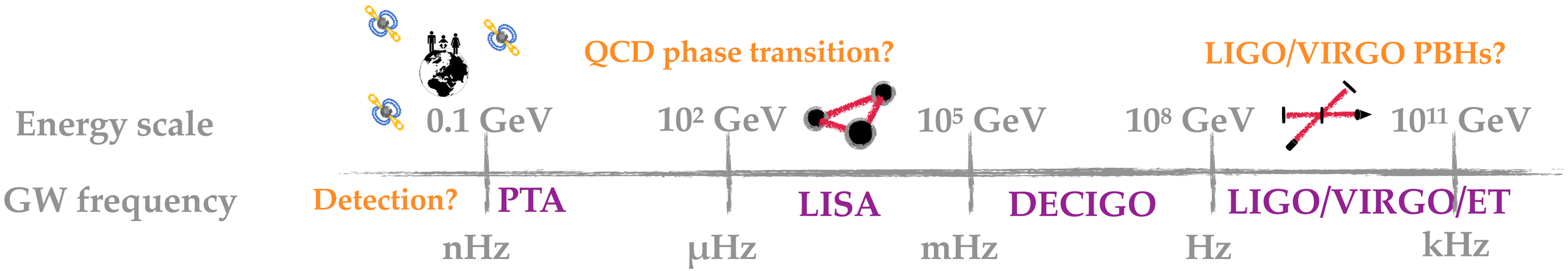
We will be probing **unexplored regimes**:

Inflation, reheating, …

Expected **astrophysical** signals but maybe **cosmological** too!

New tests of **gravity** and **particle physics** at extremely high energies

Phase transitions, cosmic strings, axions, …



Take note!

Good for astrophysics too!

Exhausting all possible cosmic GW spectra
=
Only a "clean" astrophysical signal

Primordial black holes are nice!

[Review: Sasaki+1801.05235]

Candidates for:
dark matter, some of LIGO BH mergers, seeds of SMBH, OGLE planet mass BH, etc.

Most common mechanism: **collapse of large primordial fluctuations**

[Carr & Hawking 1974]

Connection to (or test of) cosmic inflation

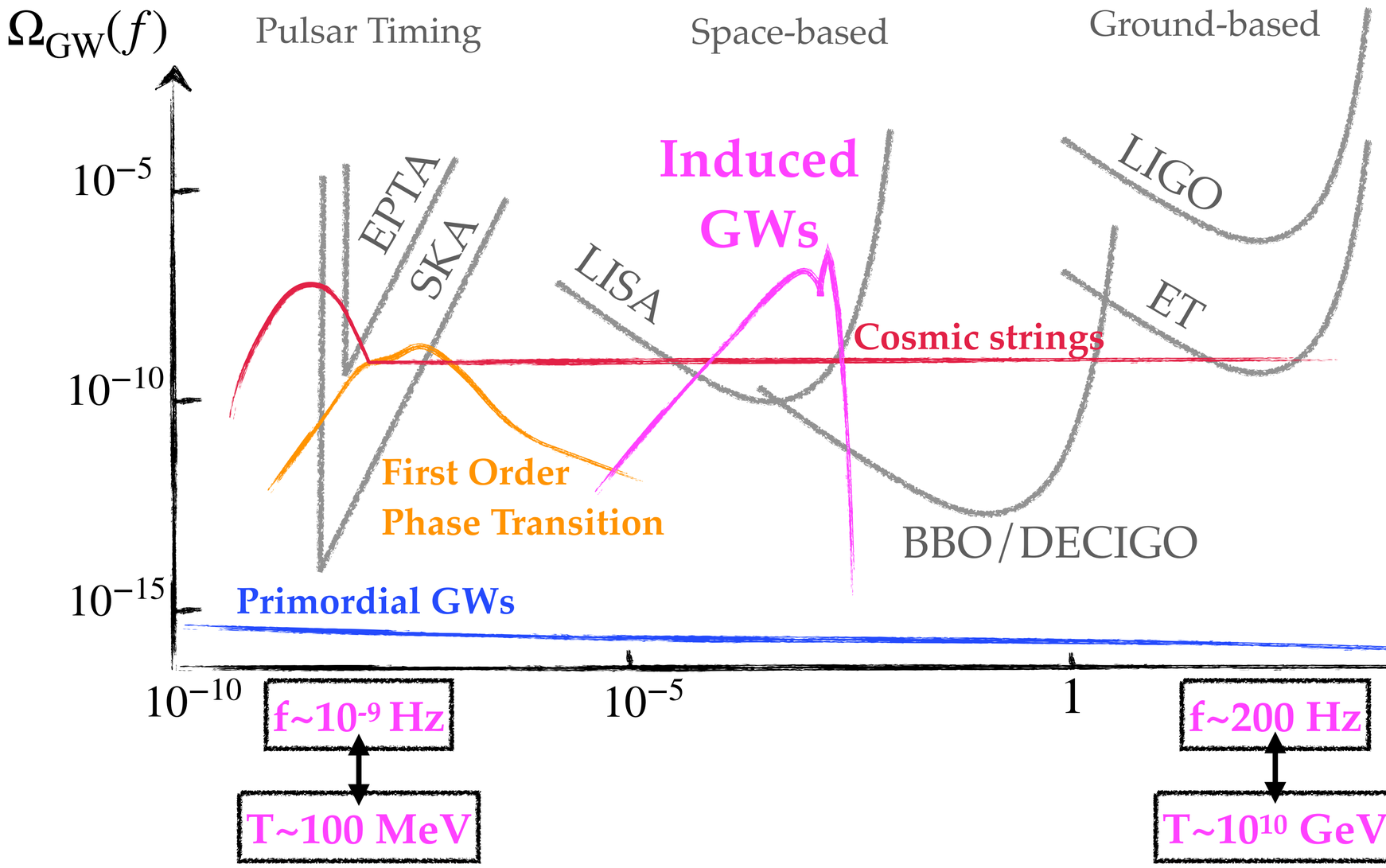
Predicts (nice) observable GW signal (induced GWs)

[Review: Domènech 2109.01398]

PBH may also form by long-range Yukawa interactions in the early universe… [Domènech+2304.13053]

Gravitational wave (primordial) cosmology

For more see e.g.
Guzzetti+1605.01615
Caprini+1801.04268
Kuroyanagi+1807.00786



Related to detector noise

$$\rho_{GW} \sim \langle \dot{h}_{ij}\dot{h}^{ij} \rangle$$

$$\Omega_{\rm GW}(f) = \frac{1}{3H_0^2 M_{\rm pl}^2}\frac{d\rho_{GW}}{d\ln f}$$

Disclaimer: all these spectra are sensitive to the expansion history of the early universe

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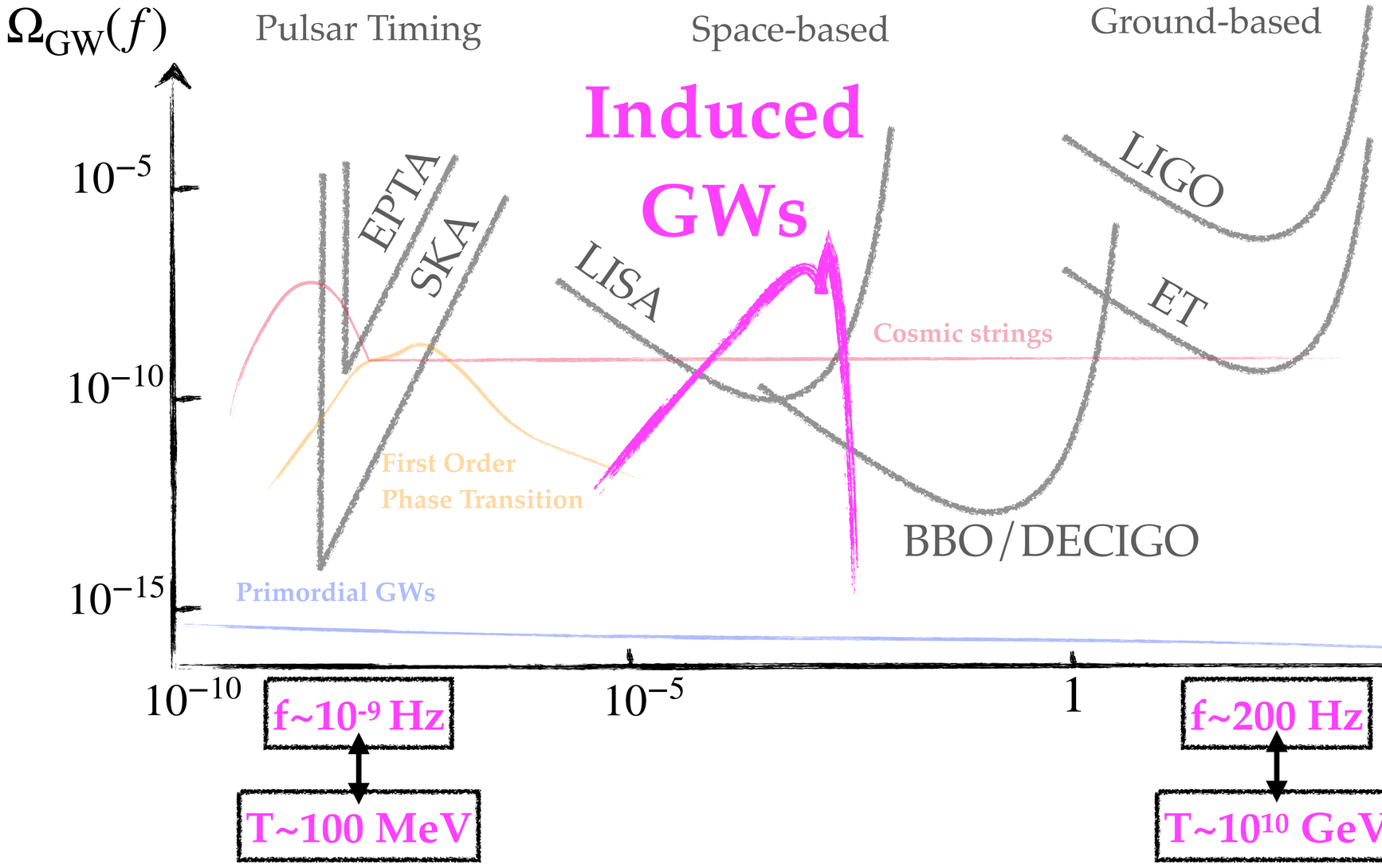

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Overview

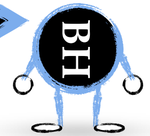
1. Bird's eye view on Induced GWs

2. General results on induced GWs

3. GWs from the PBH dominated universe

Overview

1. Bird's eye view on Induced GWs



2. General results on induced GWs

3. GWs from the PBH dominated universe

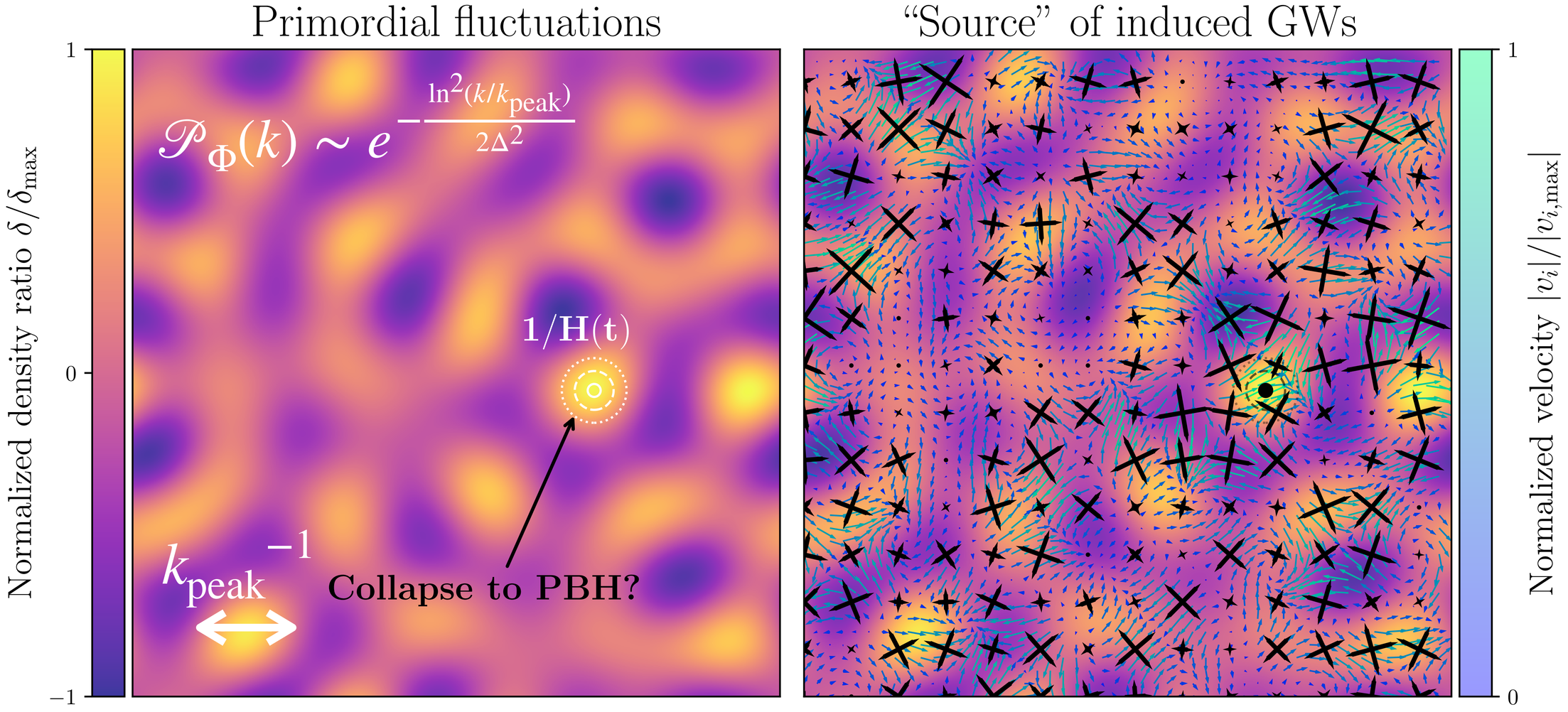
Induced GWs history

- First pointed out by K. Tomita in 1971 [*Prog. Theor. Phys.* 45, 1747 (1971)]
- Followed by Matarrese, Pantano, Saez in 1993 [*Phys.Rev.Lett.* 72 (1994) 320-323]
- Also Matarrese, Mollerach, Bruni in 1997 [*Phys.Rev.D* 58 (1998) 043504]
- Then Ananda, Clarkson and Wands in 2006 [gr-qc/0612013]
- And Baumann, Ichiki, Steinhardt and Takahashi in 2007 [hep-th/0703290]
- Saito and Yokoyama in 2008: **induced GWs <=> PBHs!** [0812.4339]
- …After the first LIGO detection the publication number keeps growing!

[Sorry for missing all the other works… they don't fit here, not even mines! More in my review paper: 2109.01398]

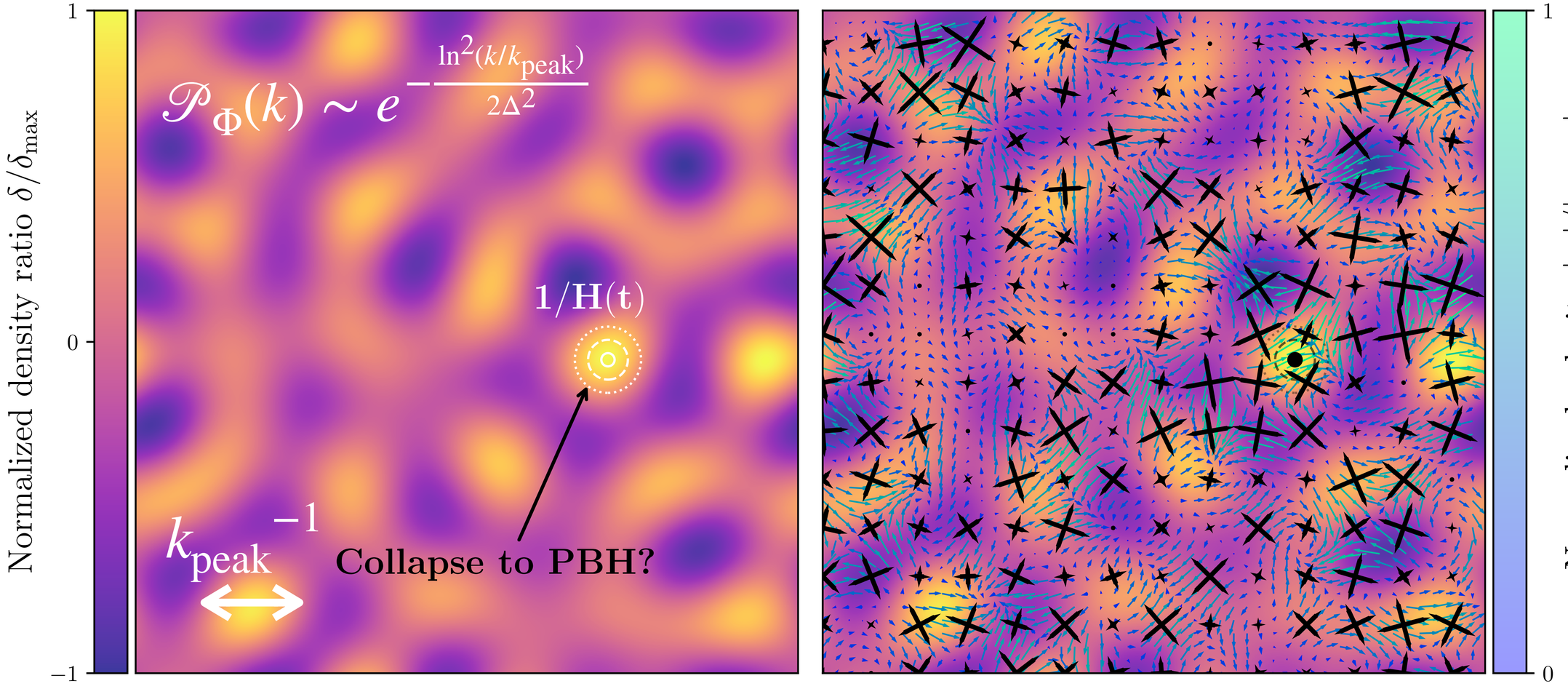
Secondary GWs

$$h_{ij}'' + 2\mathscr{H} h_{ij}' - \Delta h_{ij} = \left[4\partial_i \Phi \partial_j \Phi + 2a^2(\rho + P) v_i v_j \right]^{\mathrm{TT}}$$



3 main messages

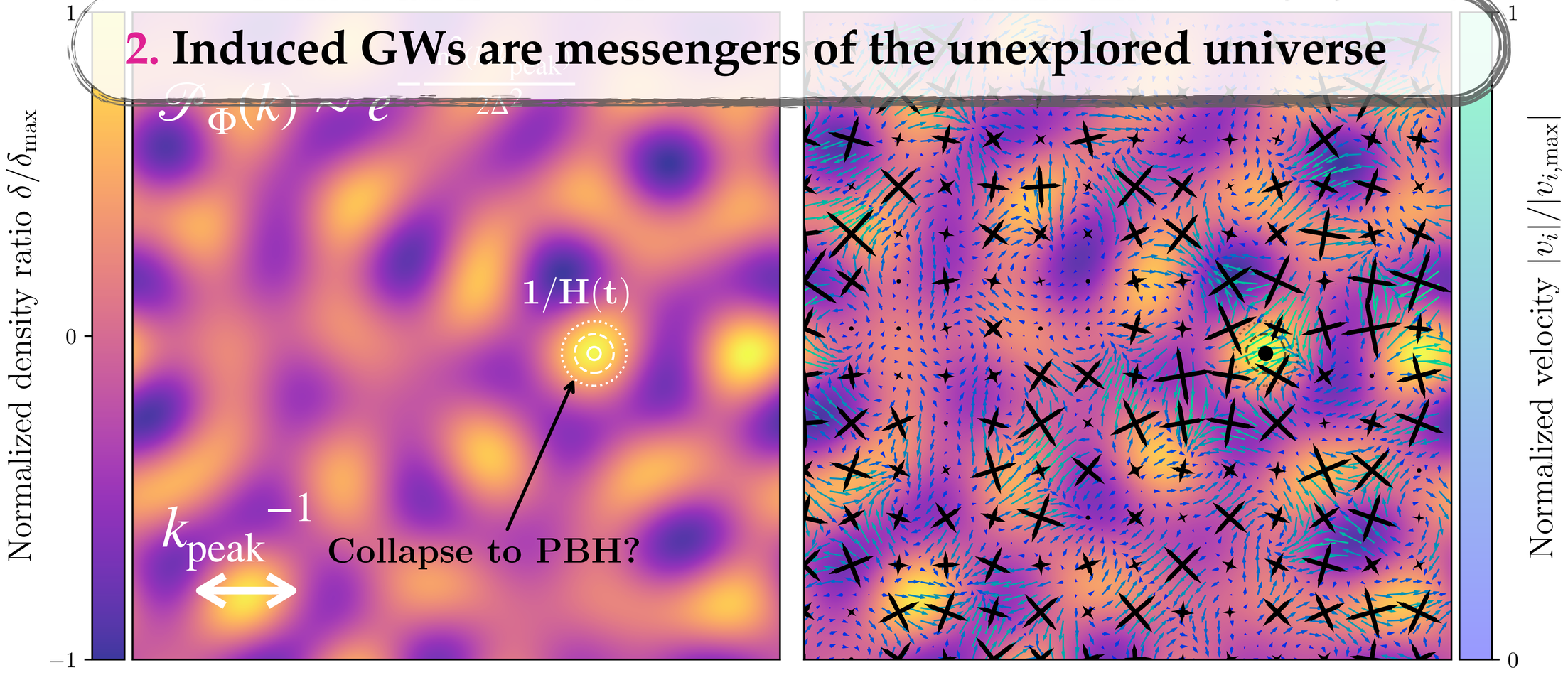
1. Any spacetime fluctuation induces Gravitational Waves



3 main messages

1. Any spacetime fluctuation induces Gravitational Waves

2. Induced GWs are messengers of the unexplored universe

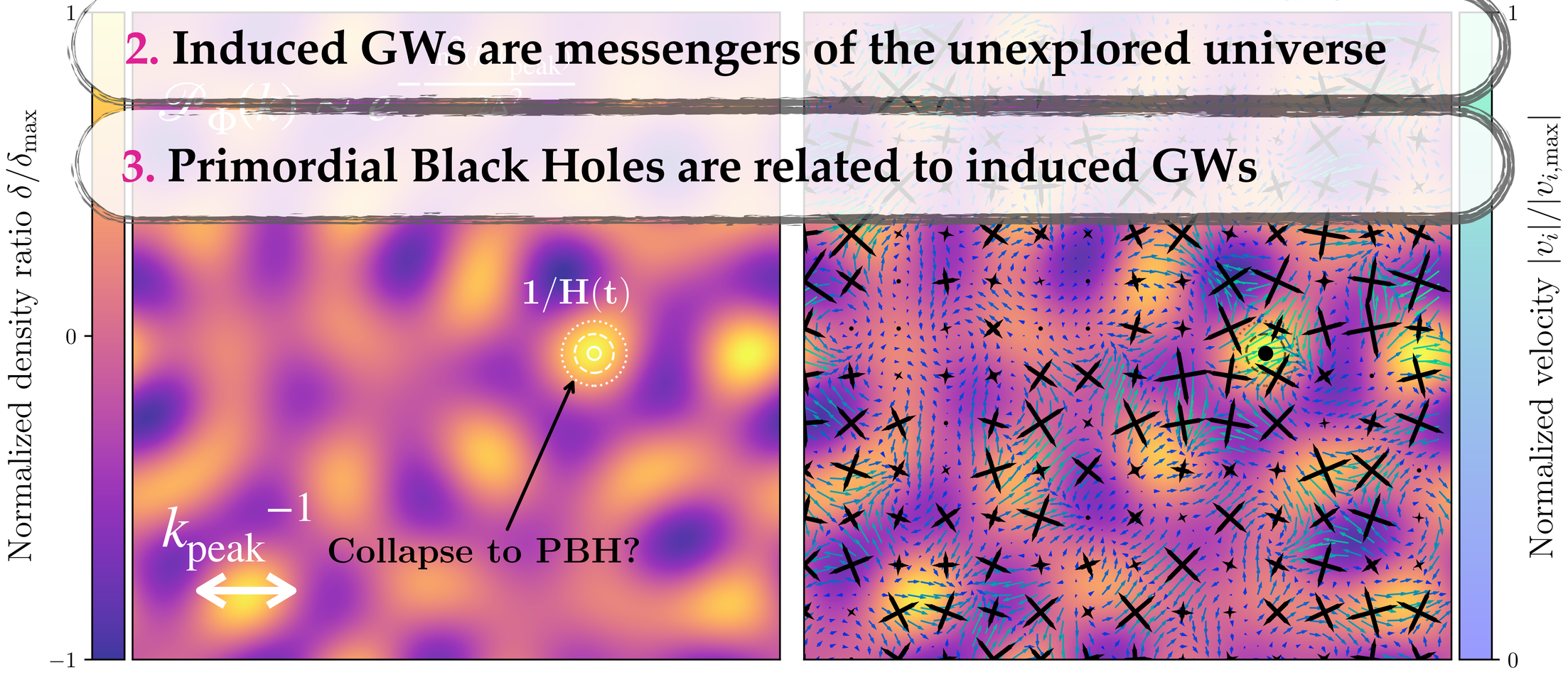


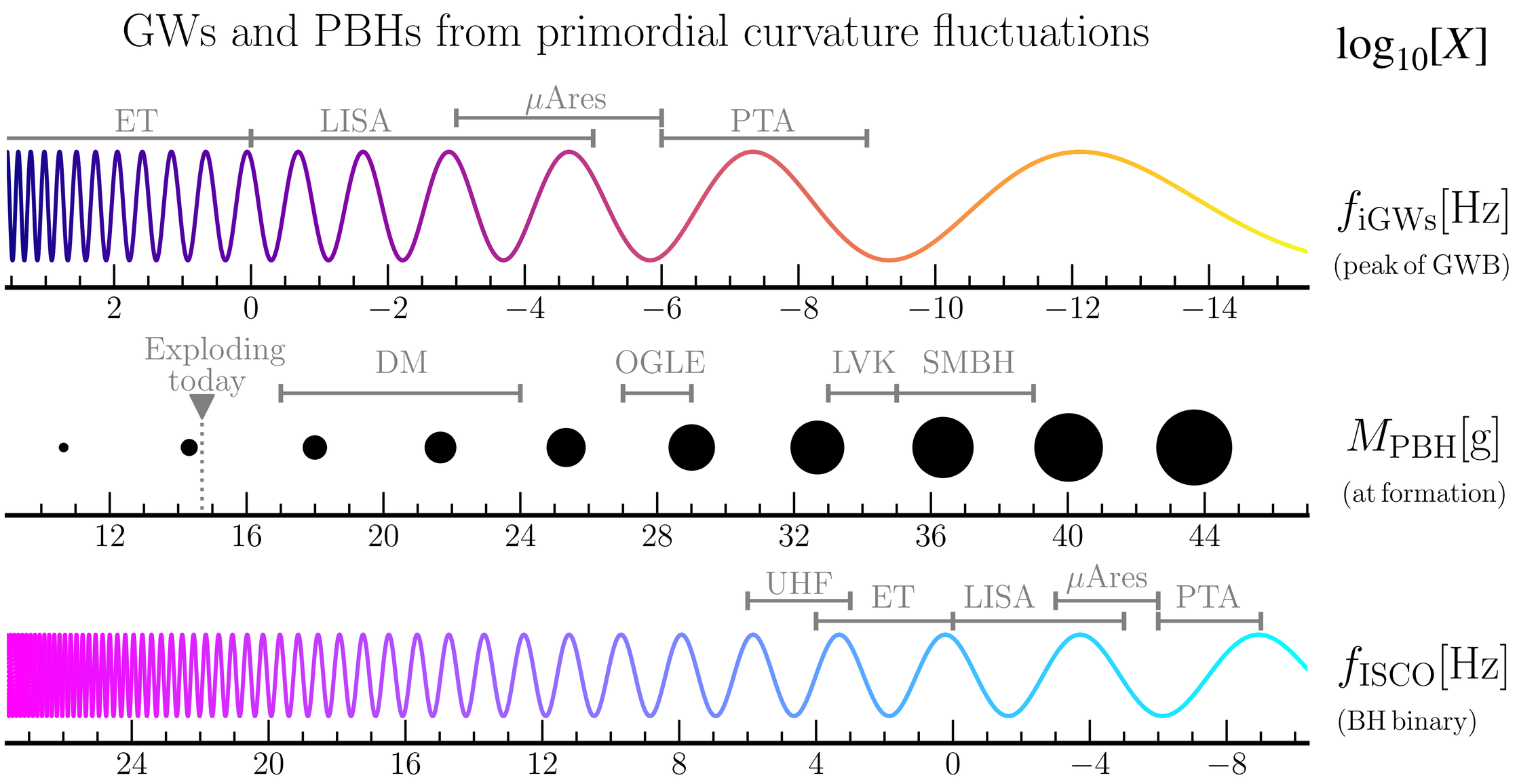
3 main messages

1. Any spacetime fluctuation induces Gravitational Waves

2. Induced GWs are messengers of the unexplored universe

3. Primordial Black Holes are related to induced GWs




GWs and PBHs from primordial curvature fluctuations
$\log_{10}[X]$
ET
LISA
μAres
PTA
$f_{\rm iGWs}$[Hz]
(peak of GWB)
2
0
−2
−4
−6
−8
−10
−12
−14
Exploding today
DM
OGLE
LVK
SMBH
$M_{\rm PBH}$[g]
(at formation)
12
16
20
24
28
32
36
40
44
UHF
ET
LISA
μAres
PTA
$f_{\rm ISCO}$[Hz]
(BH binary)
24
20
16
12
8
4
0
−4
−8

Order of magnitude estimate

After inflation: **1st order:** **Free wave propagating** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} = 0$

Curvature perturbation

2nd order: **Massless field with source** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{\ ab}(\partial_a\Phi\partial_b\Phi)$

Energy density of GWs $\rho_{GW} \sim \langle \dot{h}_{ij}\dot{h}^{ij} \rangle$

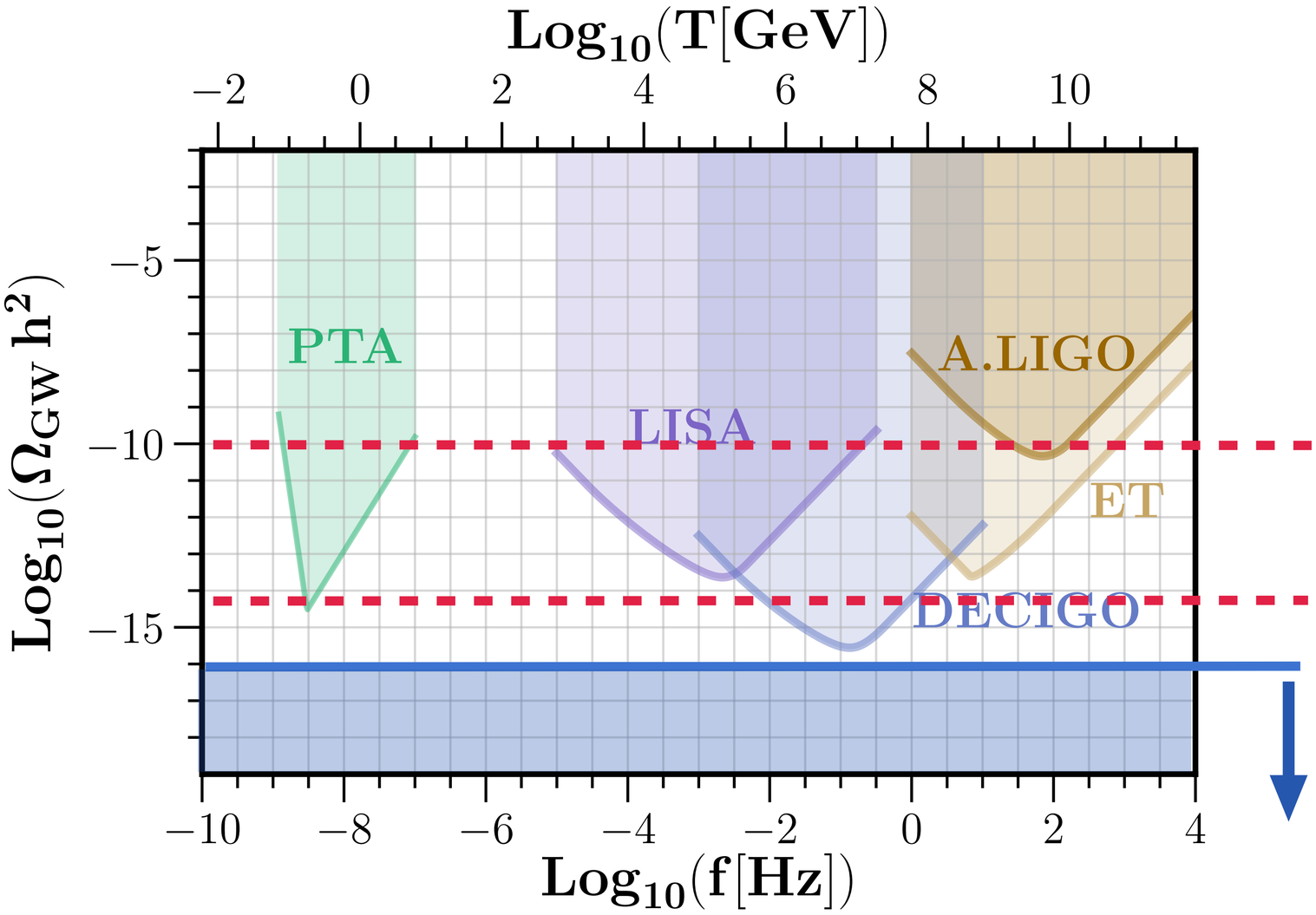
$$\Omega_{\rm GW}(k) = \frac{1}{3H^2M_{\rm pl}^2}\frac{d\rho_{GW}}{d\ln k} = \frac{k^2}{12\mathscr{H}^2}\mathscr{P}_h(k,\tau)$$

$$\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2(k \gg k_{\rm CMB})$$

Density ratio of radiation today $\Omega_{r,0}$~4x10^{-5}

Good estimate for GW spectrum's peak amplitude in radiation domination

Order of magnitude estimate



$\Omega_{\mathrm{GW}}^{\mathrm{induced}} \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2(k \gg k_{\mathrm{CMB}})$

$\mathscr{P}_{\mathscr{R}} > 10^{-2}$ **+ Detectable PBH**

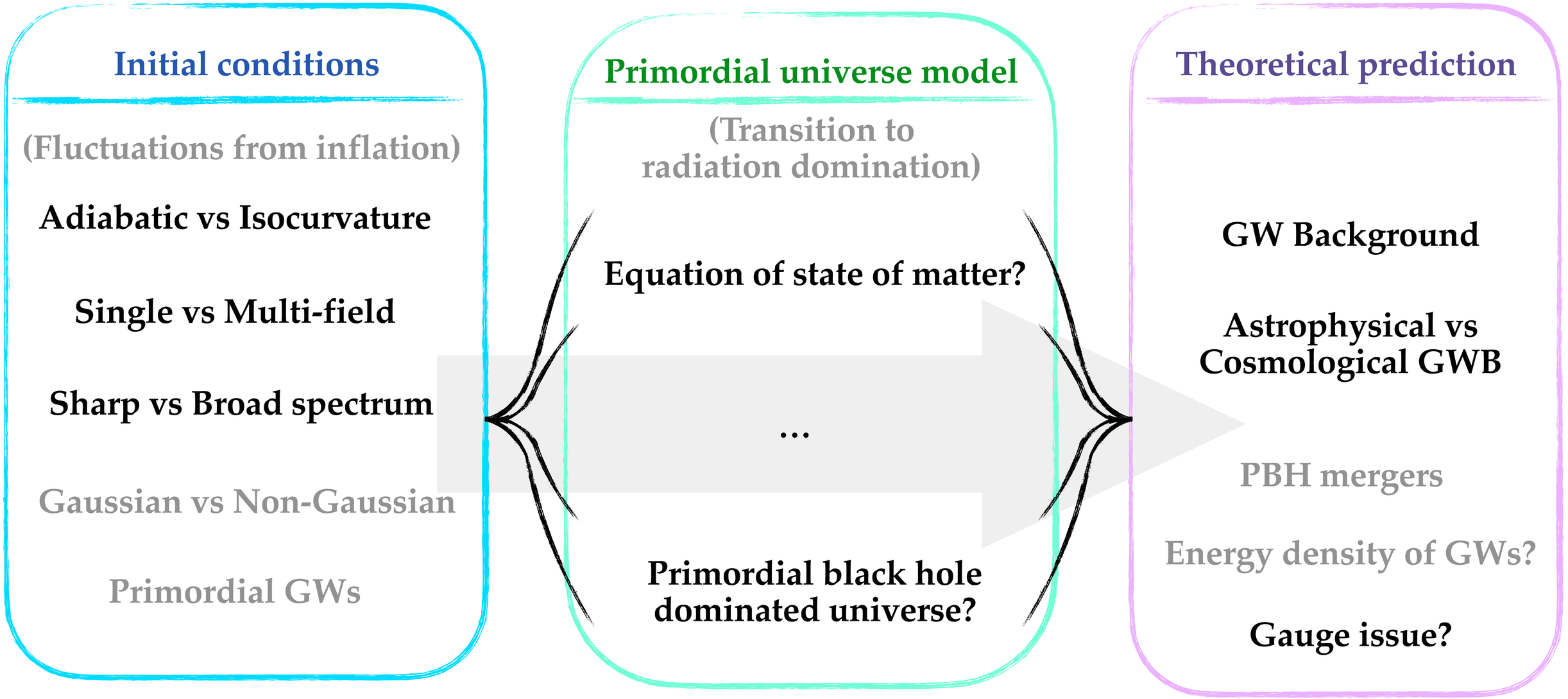
$\mathscr{P}_{\mathscr{R}} > 10^{-4}$ **Detectable GWs**

$\Omega_{\mathrm{GW}}^{\mathrm{inflation}}$

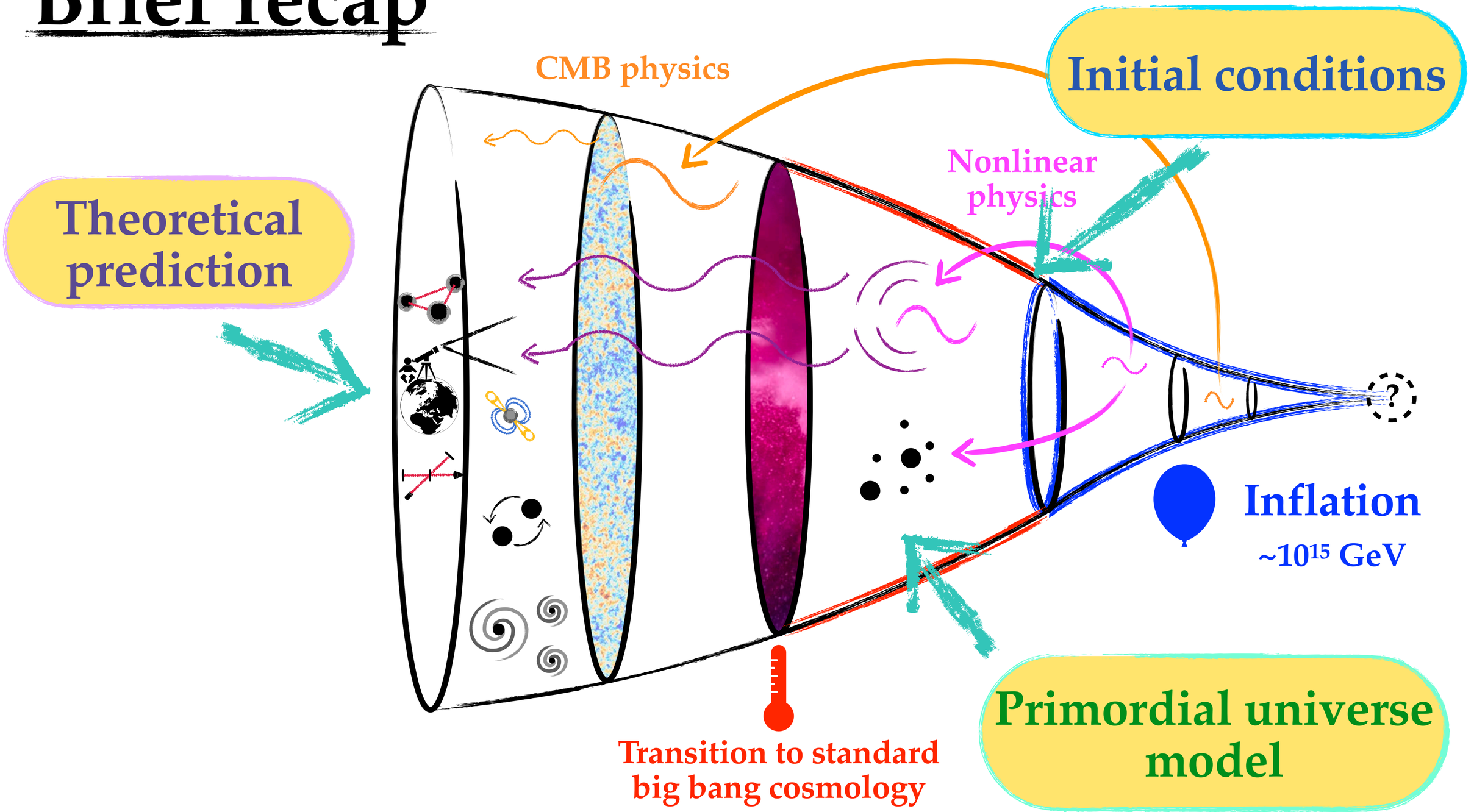
Undetectable?

Power-law integrated sensitivity curves: Thrane & Romano 1310.5300

To derive predictions…



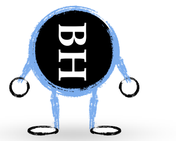
Brief recap


CMB physics
Initial conditions
Nonlinear physics
Theoretical prediction
?
Inflation
~10^15 GeV
Transition to standard big bang cosmology
Primordial universe model

Overview

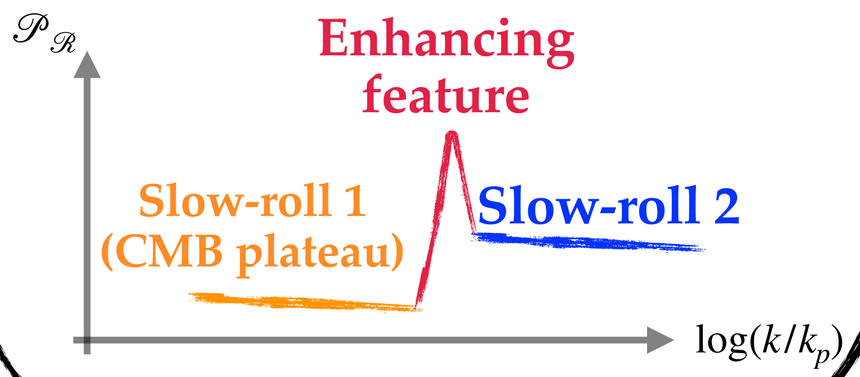
1. Induced GWs in 3 messages

2. General results on induced GWs

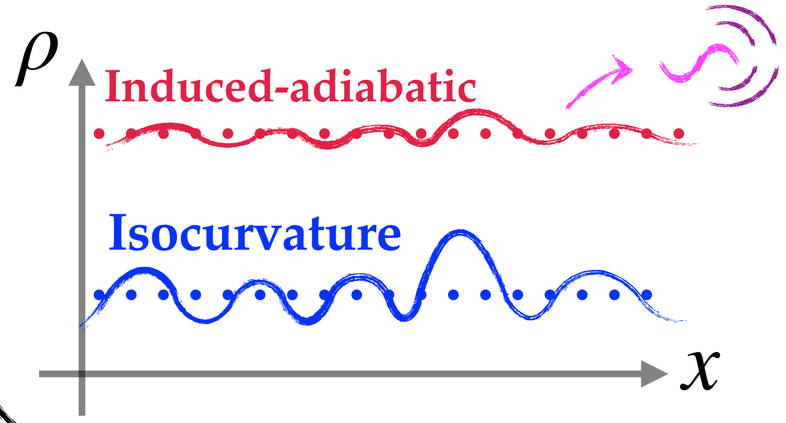


3. GWs from the PBH dominated universe

Cosmic inflation

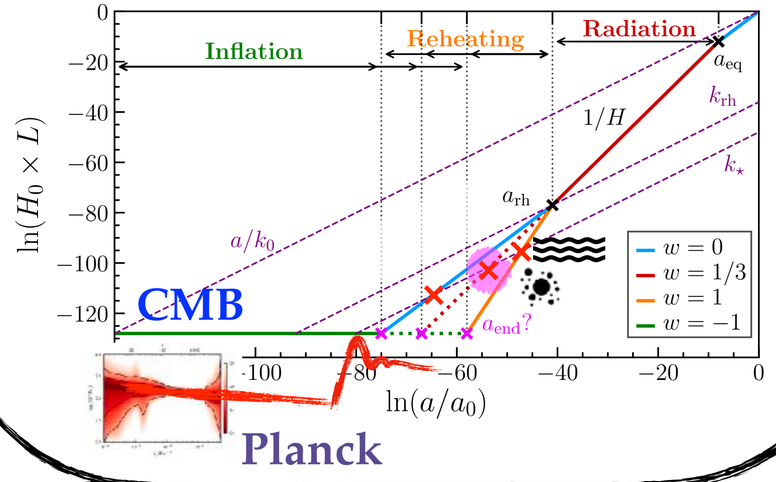


Initial conditions

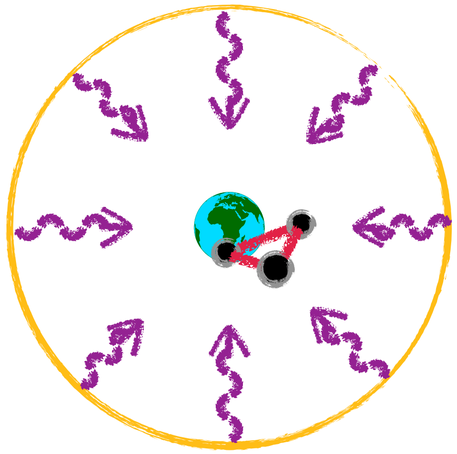


Summary of induced GWs research

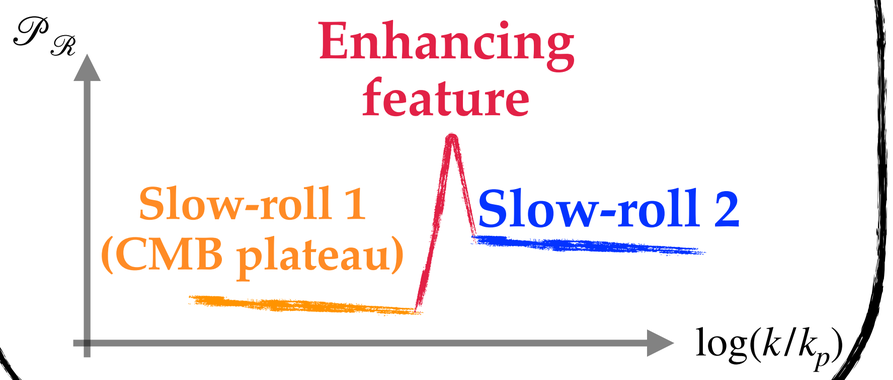
Content of the universe



GWB anisotropies



Cosmic inflation



Initial conditions

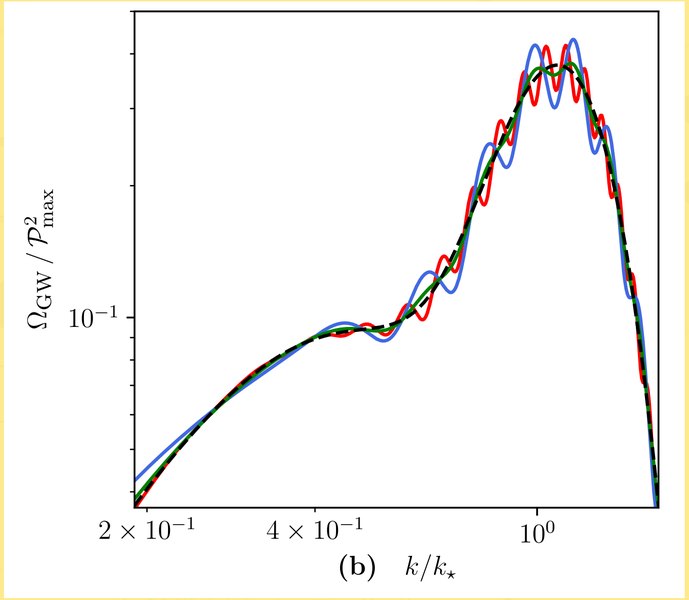
Summary of induced GWs research

Content of the universe

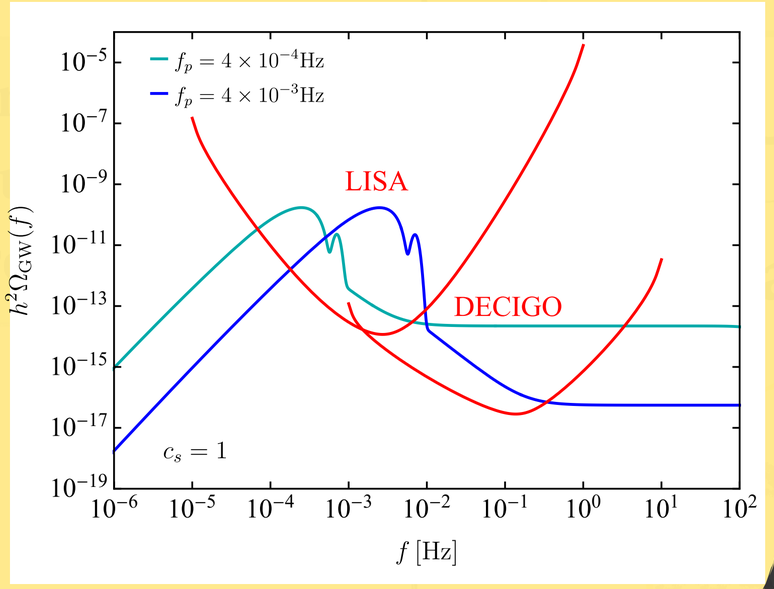
GWB anisotropies

Cosmic inflation

Oscillations from particle production, second phase of inflation, etc.



[2110.09480]



[2205.01696]

Cosmic inflation

Initial conditions

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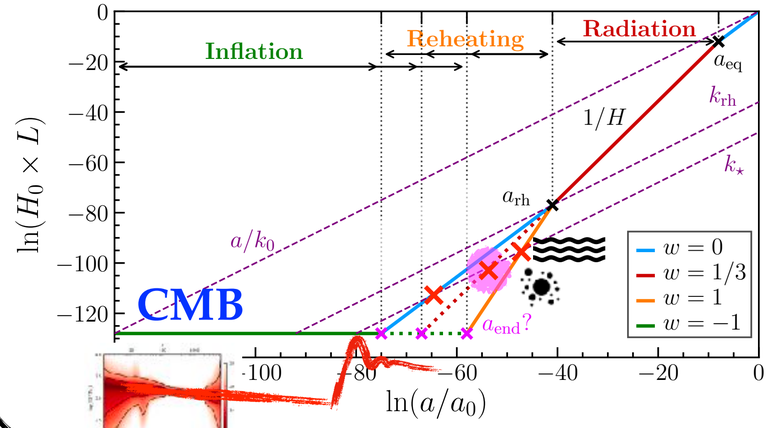
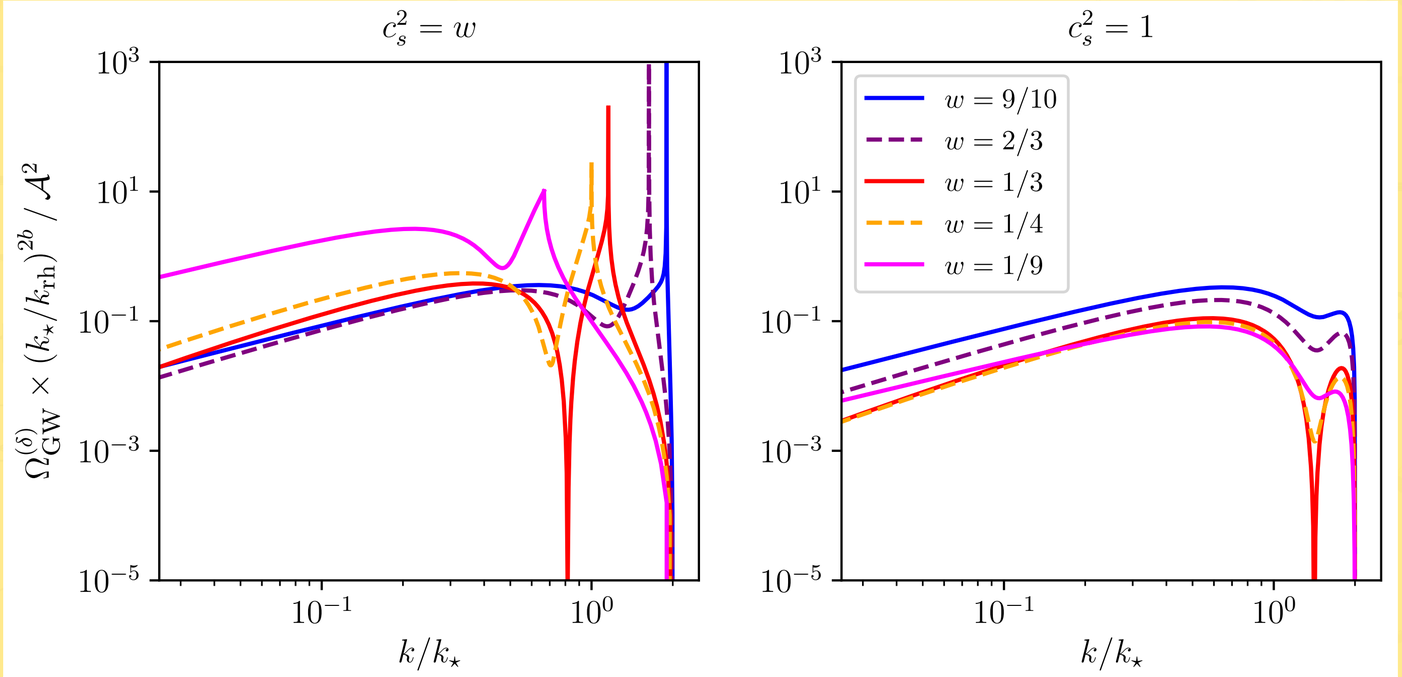


GWB anisotropies

Content of the universe

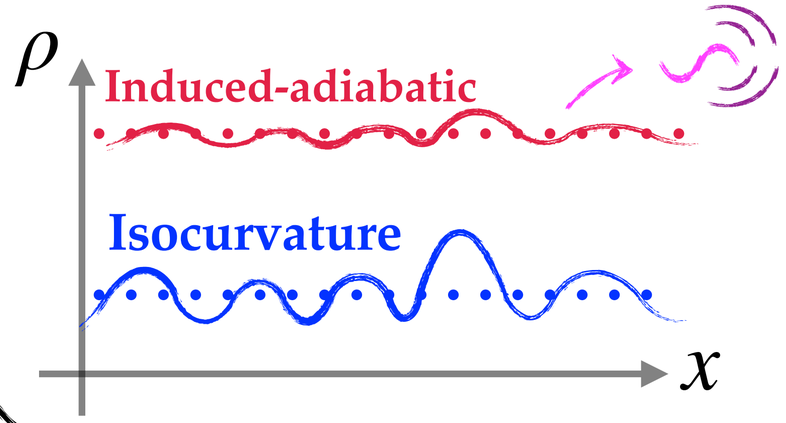
Equation of state of the early universe,
propagation speed of fluctuations, etc.



[1912.05583, 2005.12314]

Cosmic inflation

Initial conditions



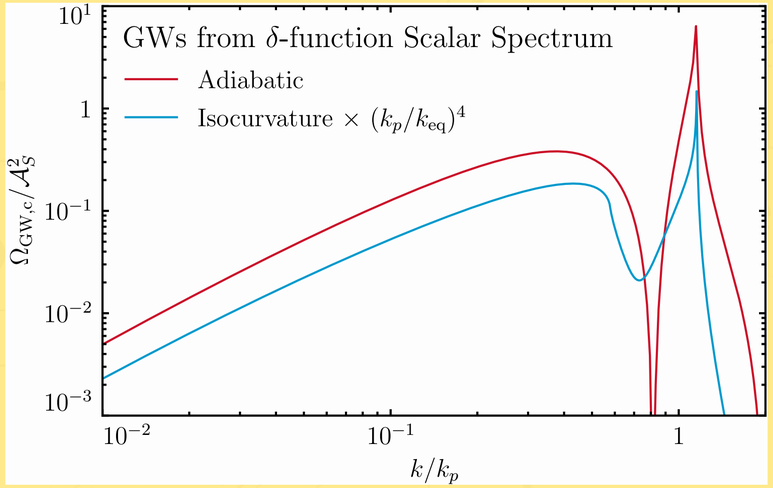
Summary of induced GWs research

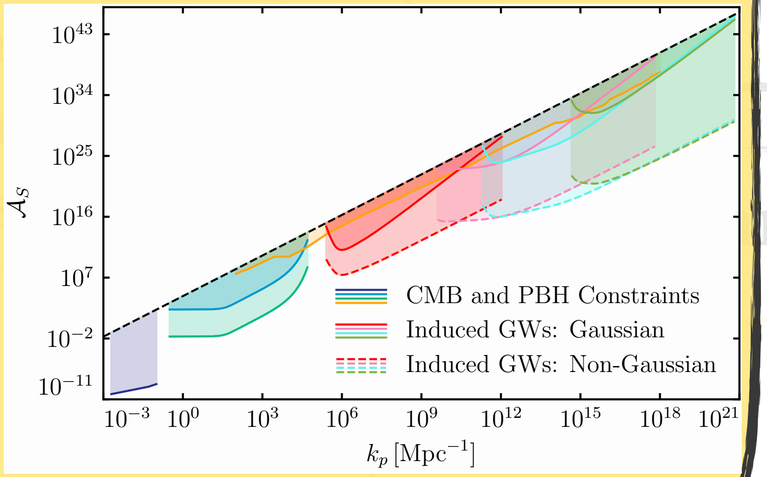
Content of the universe

GWB anisotropies

Initial conditions

Any type of isocurvature, CDM, PBH, solitons, oscillons, etc.





[2112.10163]

Cosmic inflation

Initial conditions

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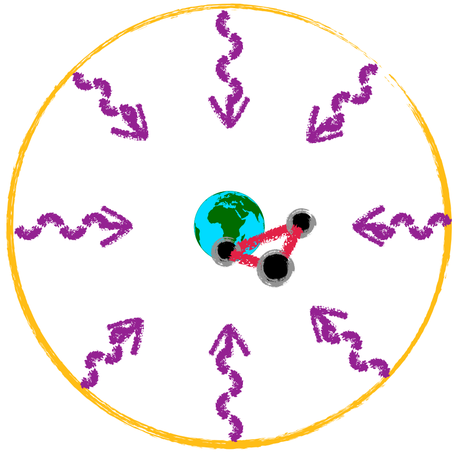
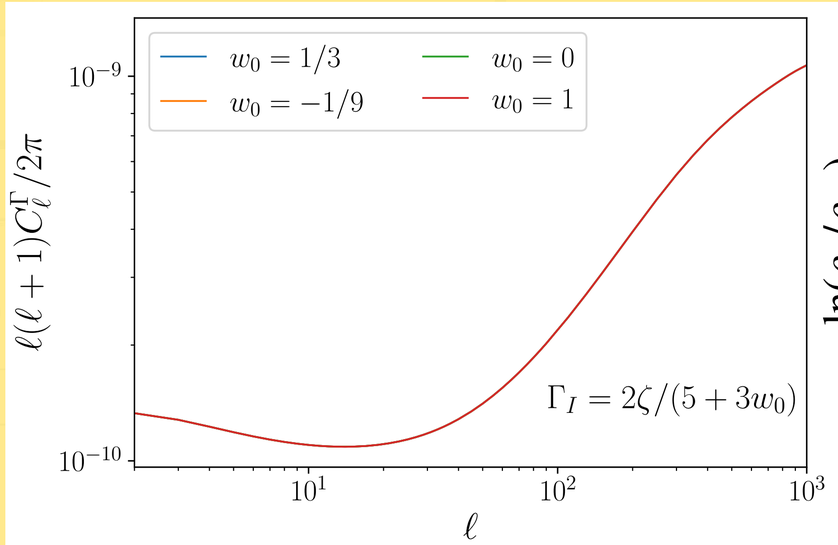
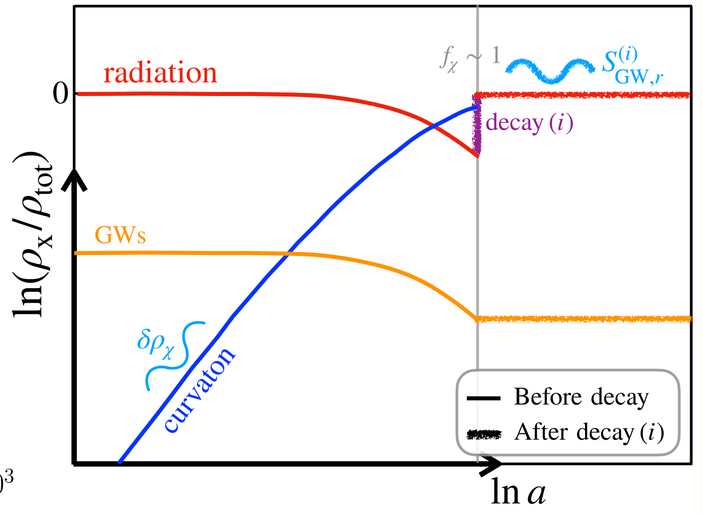
GWB anisotropies



GWB anisotropies

Independence on initial equation of state,
GW isocurvature from curvaton, etc.





[2212.10316]

Overview

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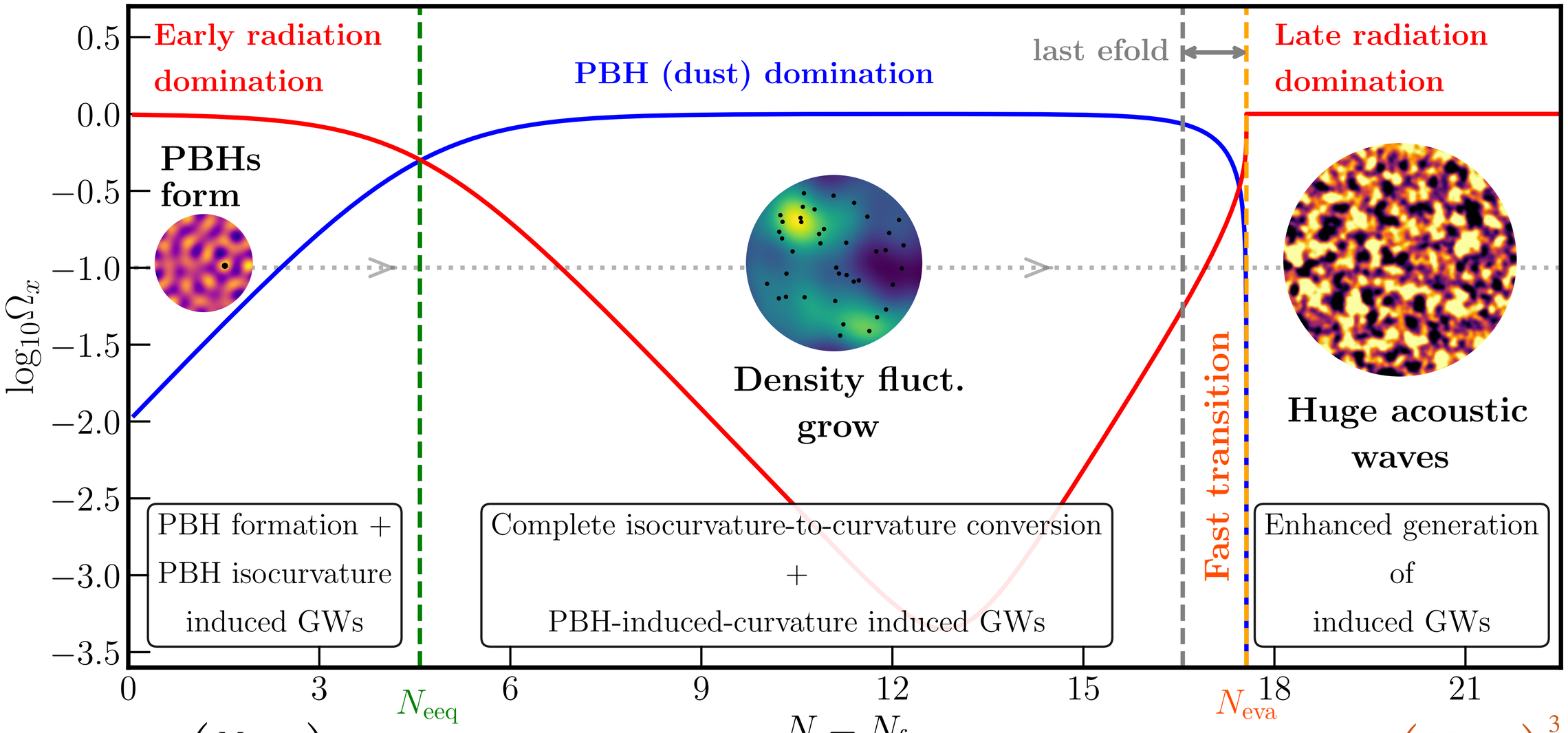
2. General results on induced GWs

3. GWs from the PBH dominated universe

The PBH dominated universe

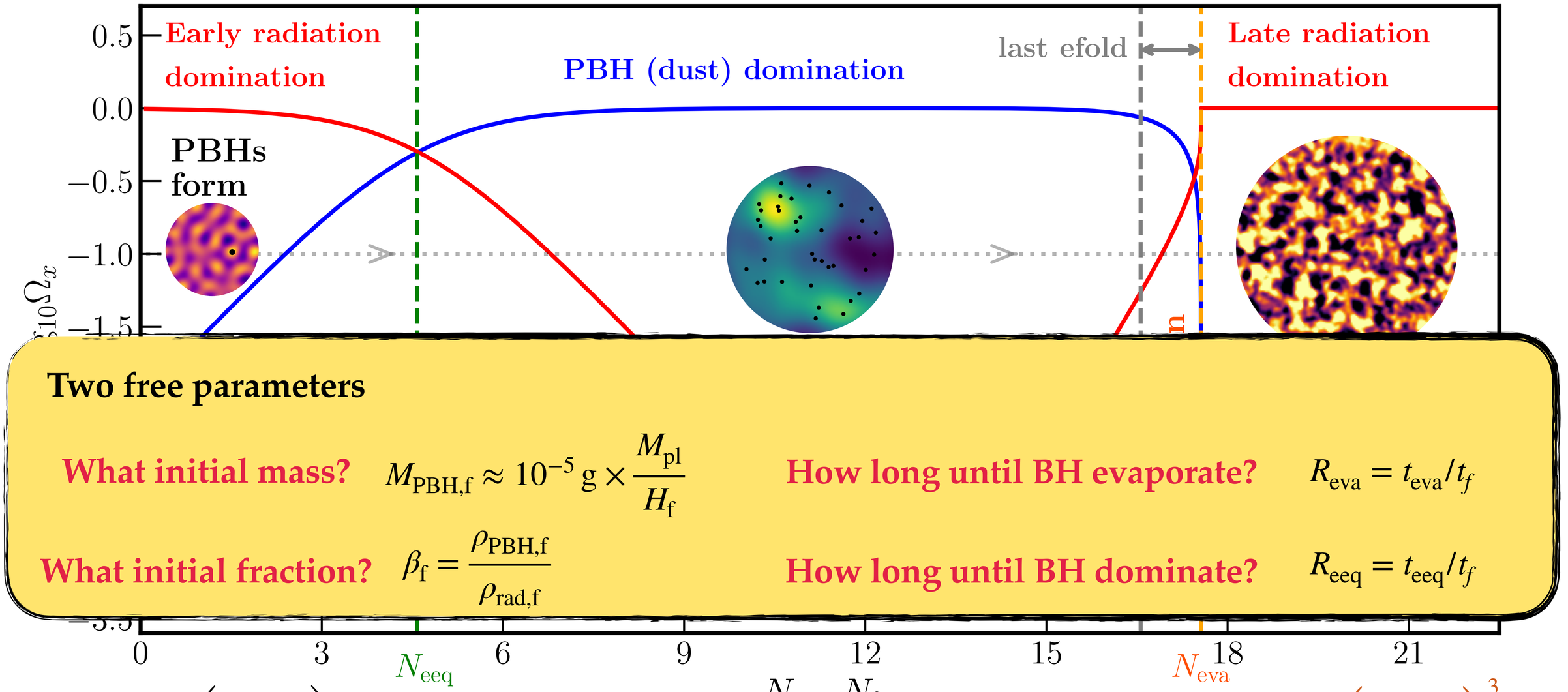


$$t_{\rm f} \approx \frac{1}{2H_{\rm f}} \approx 10^{-8}\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)$$

$$t_{\rm eeq} \sim \beta^{-2} t_f$$

$$t_{\rm eva} \approx 400\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^3$$

The PBH dominated universe



Two free parameters

What initial mass? $M_{\rm PBH,f} \approx 10^{-5}\,{\rm g} \times \dfrac{M_{\rm pl}}{H_{\rm f}}$

What initial fraction? $\beta_{\rm f} = \dfrac{\rho_{\rm PBH,f}}{\rho_{\rm rad,f}}$

How long until BH evaporate? $R_{\rm eva} = t_{\rm eva}/t_f$

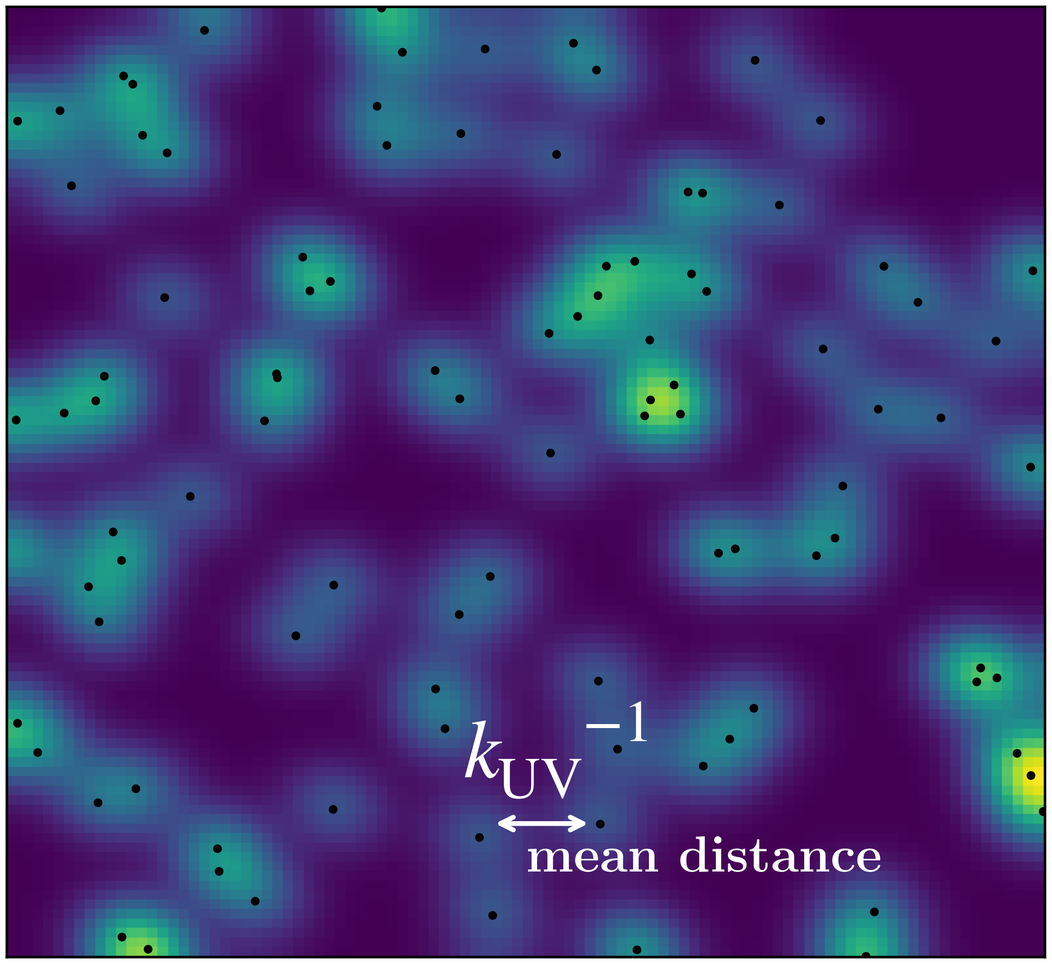
How long until BH dominate? $R_{\rm eeq} = t_{\rm eeq}/t_f$

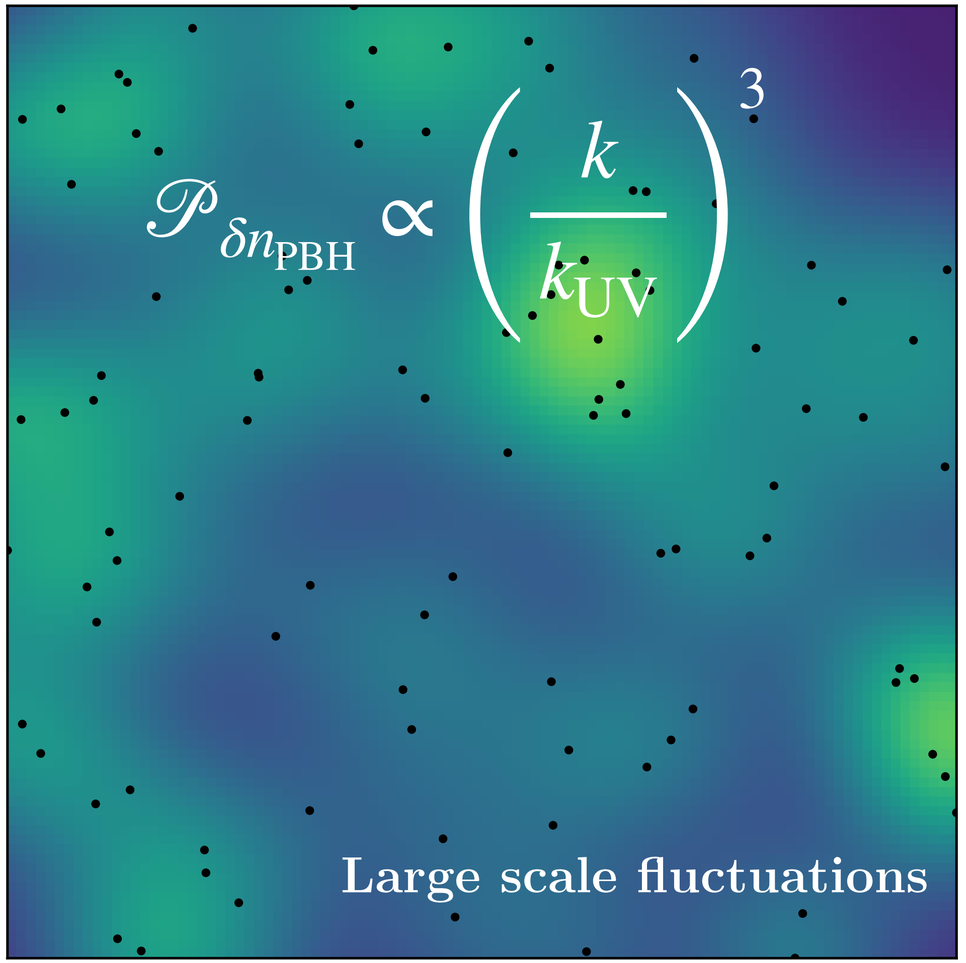
$$t_{\rm f} \approx \frac{1}{2H_{\rm f}} \approx 10^{-8}\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)$$

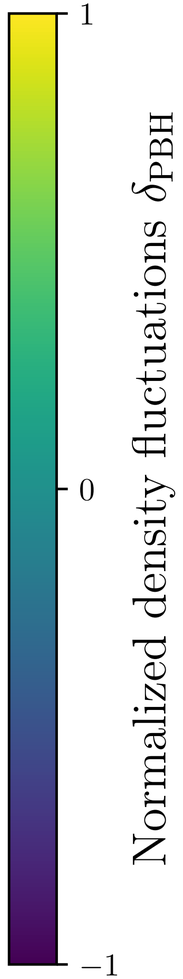
$$t_{\rm eeq} \sim \beta^{-2} t_f$$

$$t_{\rm eva} \approx 400\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^3$$

PBH number density fluctuations





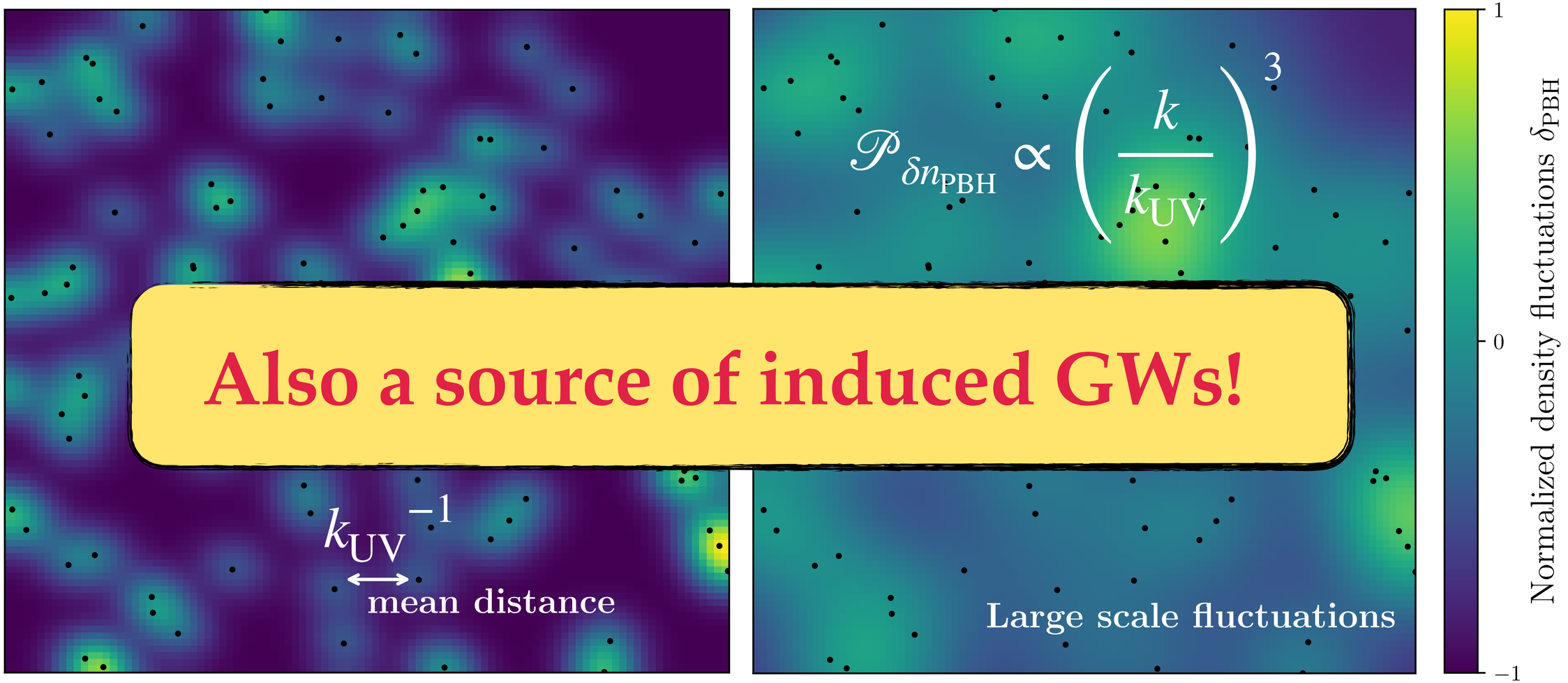


Uniform, rare, random PBH formation

Poisson fluctuations

Initial nature of fluctuations?

PBH number density fluctuations

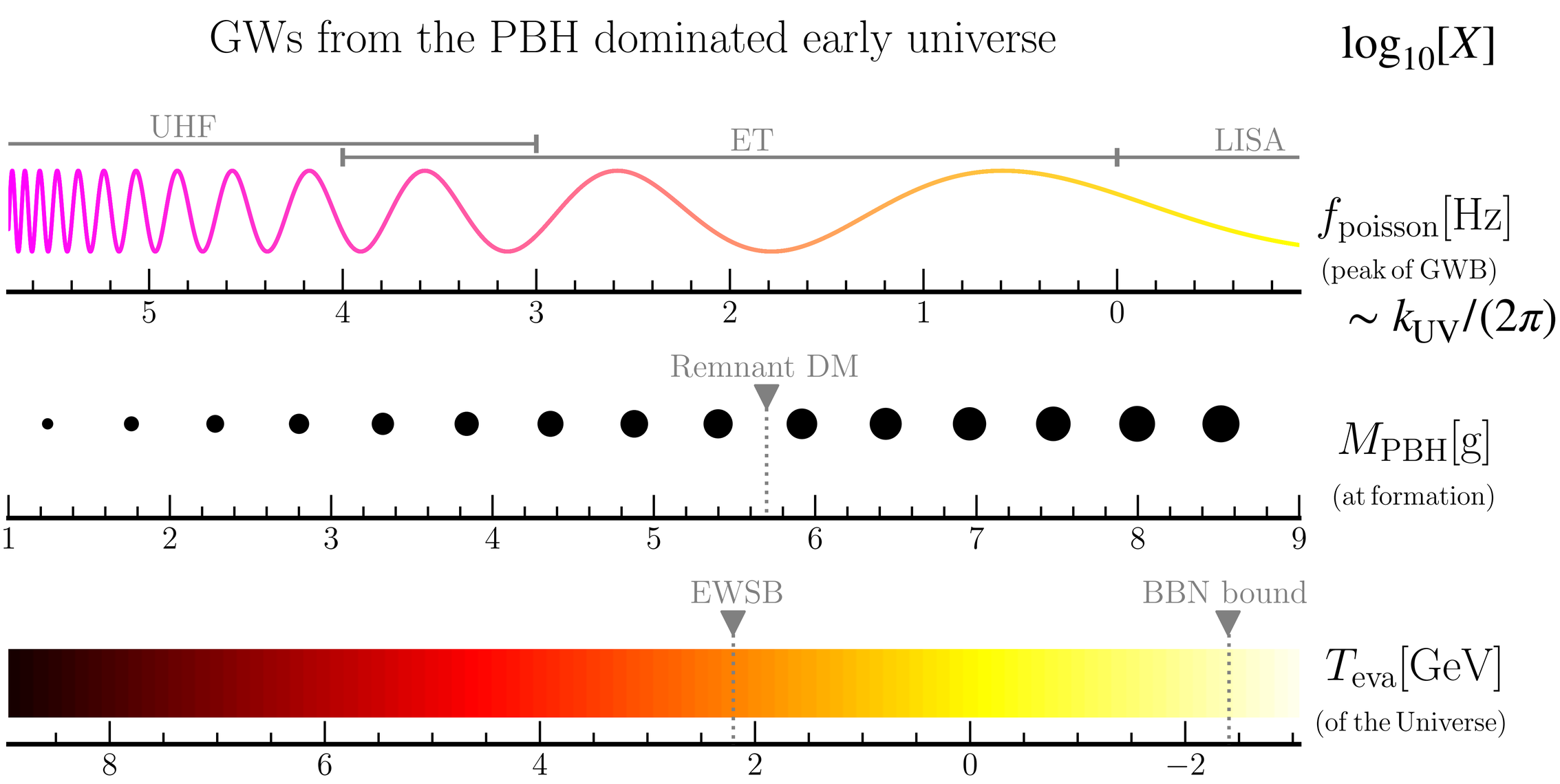


Also a source of induced GWs!

Uniform, rare, random PBH formation

Poisson fluctuations

Initial nature of fluctuations?


GWs from the PBH dominated early universe
$\log_{10}[X]$
UHF
ET
LISA
$f_{\rm poisson}[{\rm Hz}]$
(peak of GWB)
$\sim k_{\rm UV}/(2\pi)$
5
4
3
2
1
0
Remnant DM
$M_{\rm PBH}[{\rm g}]$
(at formation)
1
2
3
4
5
6
7
8
9
EWSB
BBN bound
$T_{\rm eva}[{\rm GeV}]$
(of the Universe)
8
6
4
2
0
−2

Initial conditions (review)

Definition: **No initial curvature => Relative number density fluctuations**

$$\Phi(k \ll \mathscr{H}) \sim \frac{\delta\rho_{\text{total}}}{\rho_{\text{total}}}$$

00-component Einstein Equations

$$\delta\rho_{\text{DM}} + \delta\rho_{\text{rad}} = 0 \qquad S = \frac{\delta\rho_{DM}}{\rho_{DM}} - \frac{3}{4}\frac{\delta\rho_{\text{rad}}}{\rho_{\text{rad}}}$$

E.g. initial CDM isocurvature $\Phi_i = 0$

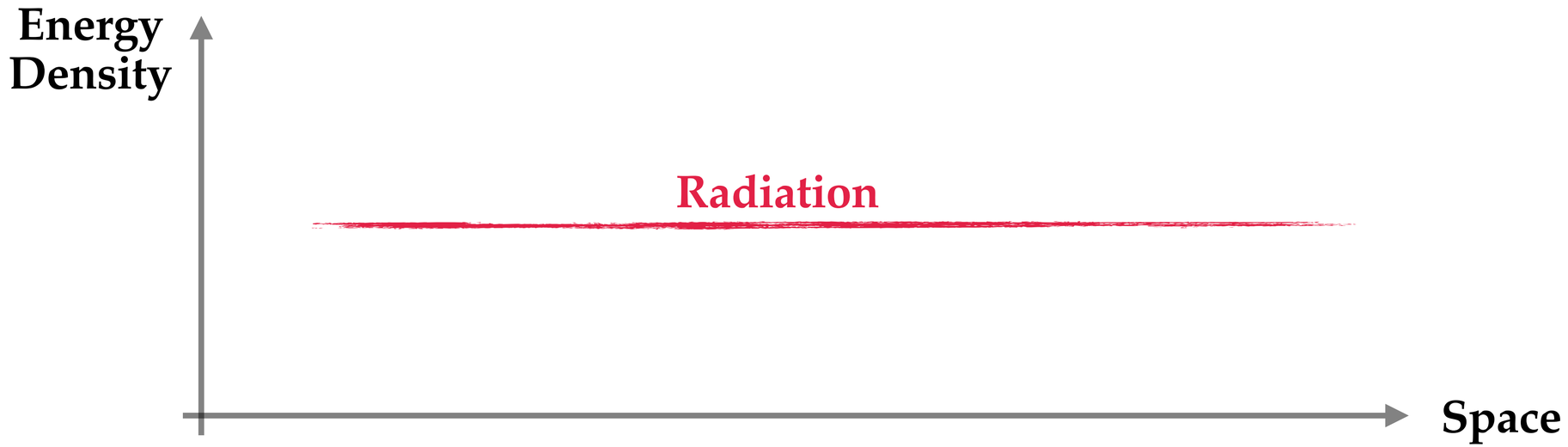
E.g. adiabatic initial conditions S=0

Due to, e.g., extra fields during inflation or Primordial black hole formation

Constrained to be <10% on CMB scales but unconstrained on smaller scales

Initial isocurvature from PBHs

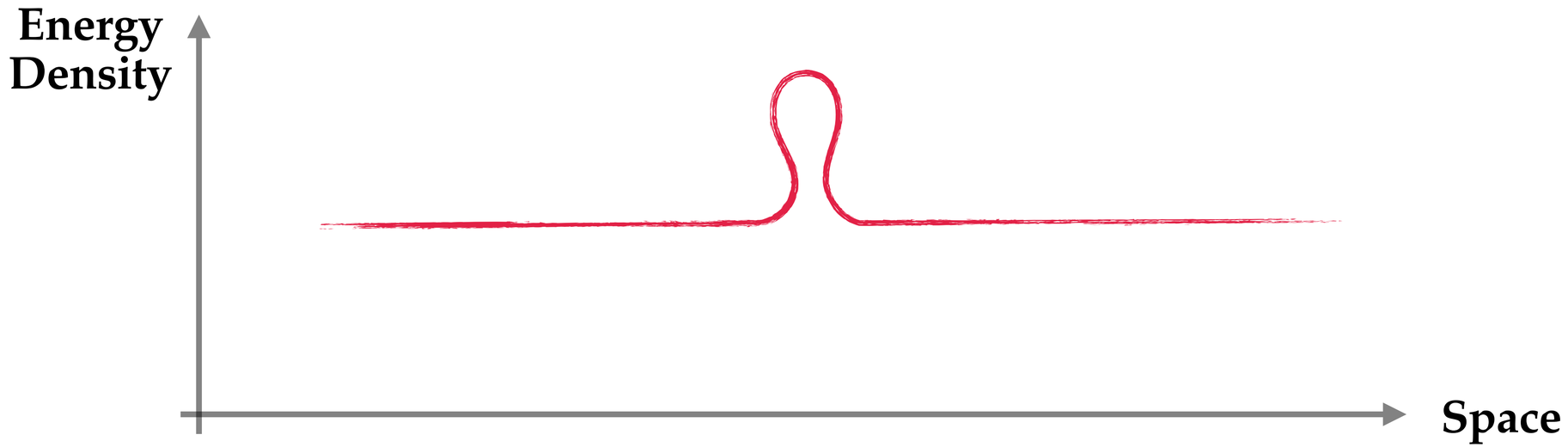
Imagine a radiation dominated homogeneous universe…



Adiabatic and isocurvature fluctuations are evolve independently

Initial isocurvature from PBHs

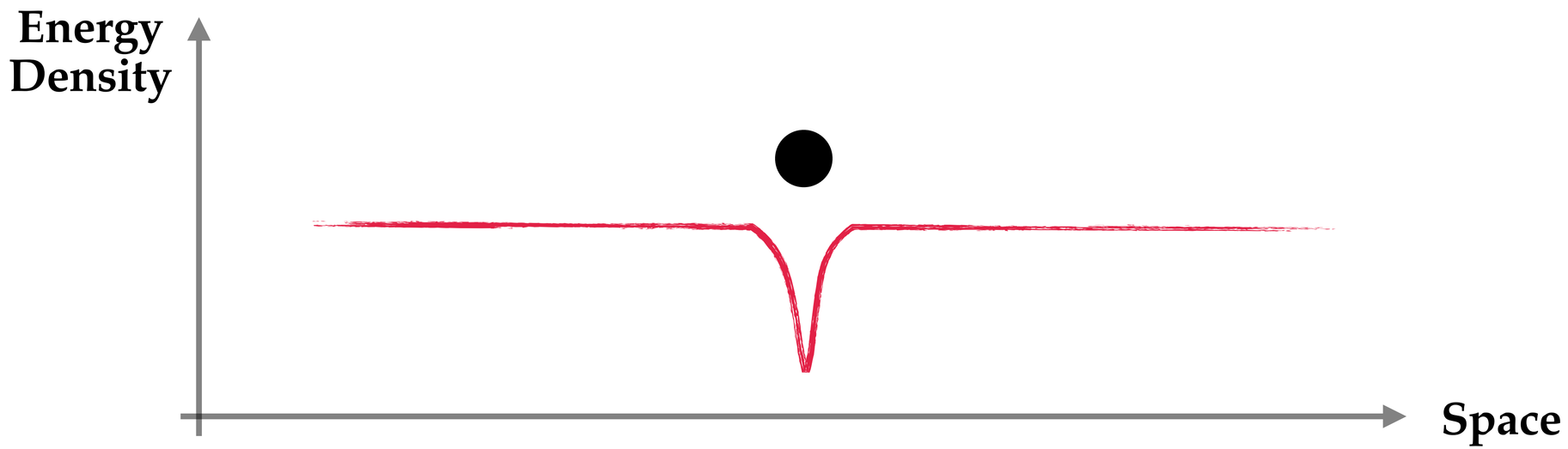
One Hubble patch collapses…



Collapse due to large adiabatic fluctuations

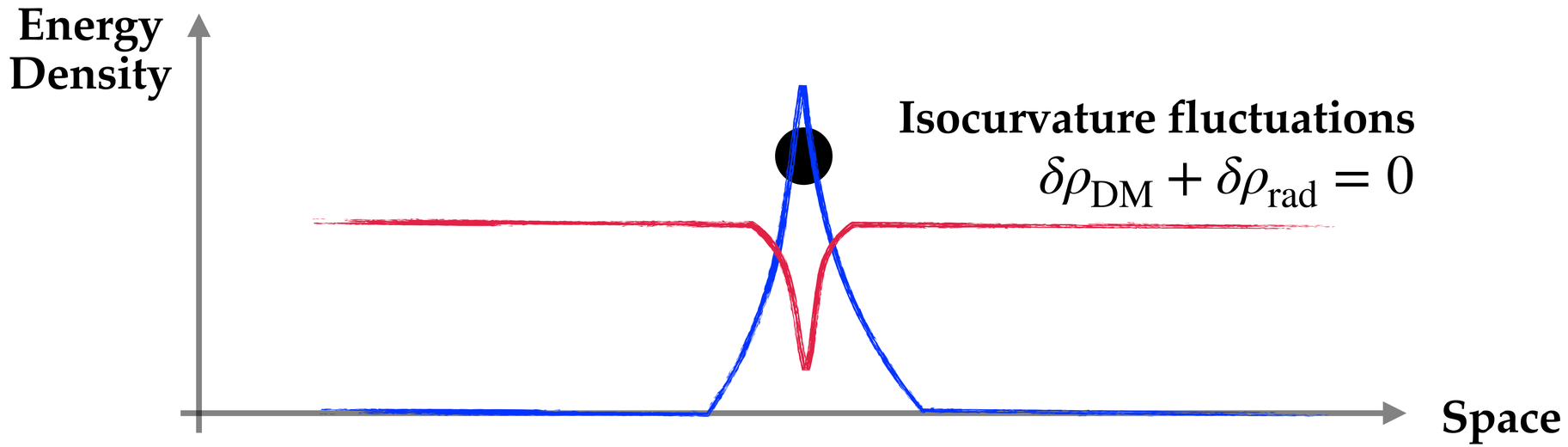
Initial isocurvature from PBHs

And a black hole forms…



Initial isocurvature from PBHs

If we look at fluctuations on coarse grained scales…



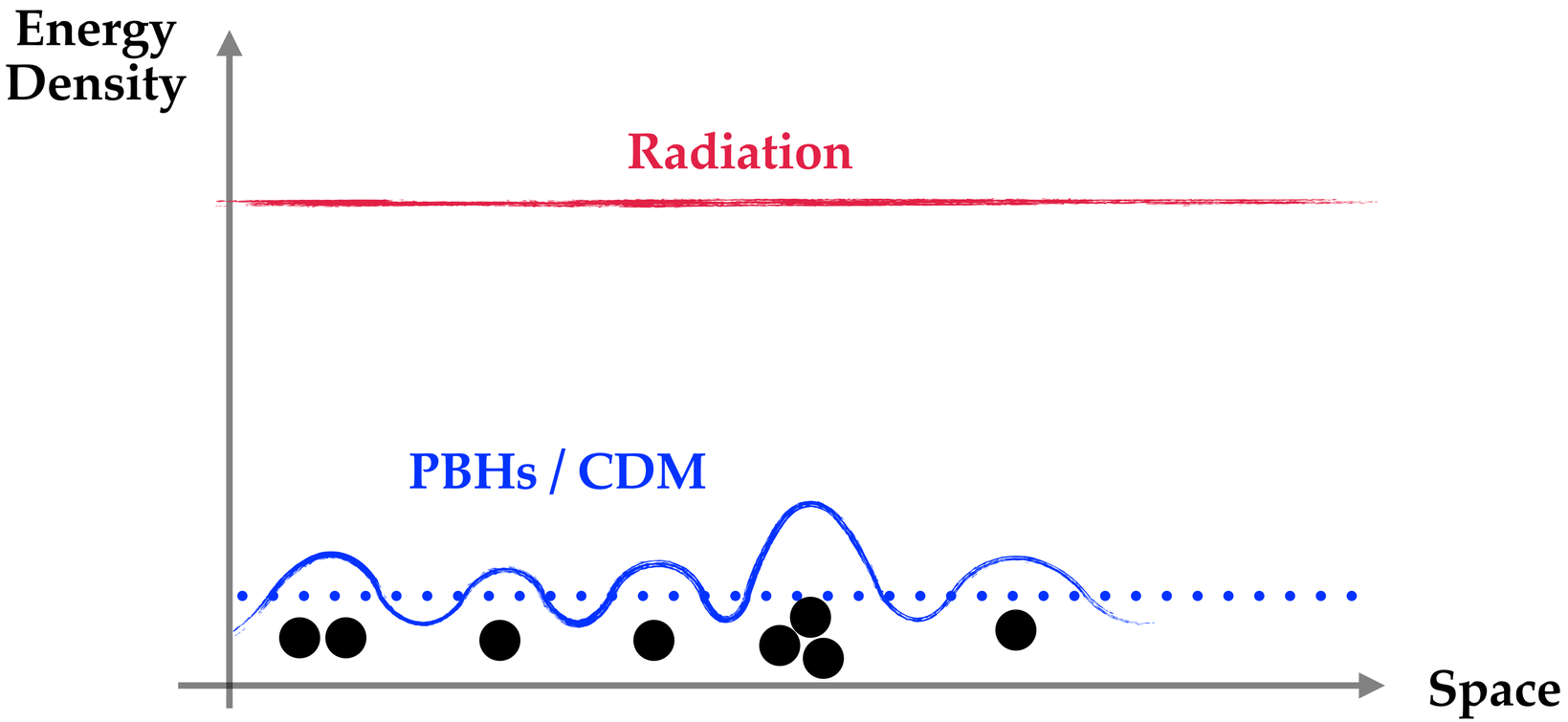
PBHs form randomly in space and yield density fluctuations

Transfer to curvature

[T.Papanikolaou, V.Vennin, D.Langlois, 2010.11573]

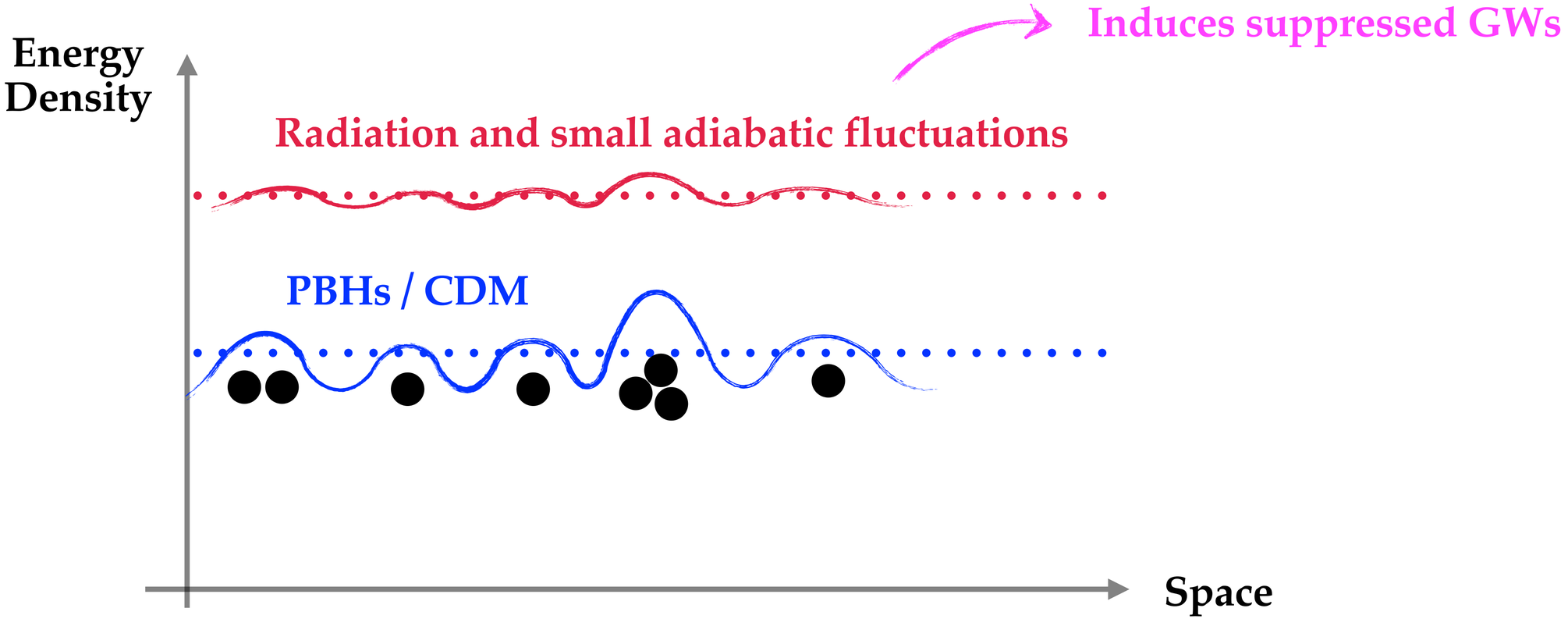
Initial radiation domination with random PBHs

[GD, C.Lin, M.Sasaki, 2012.08151]



Transfer to curvature

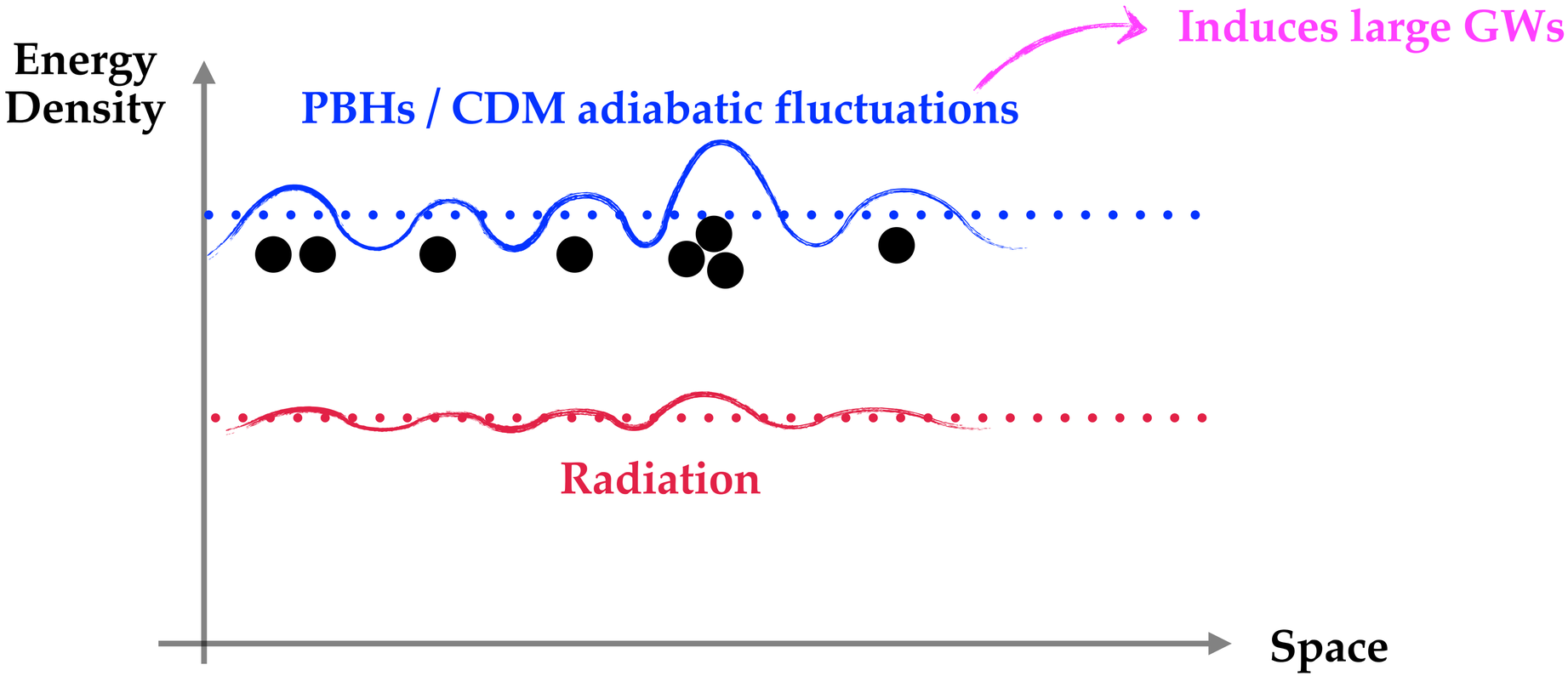
PBH energy density decays slower than radiation



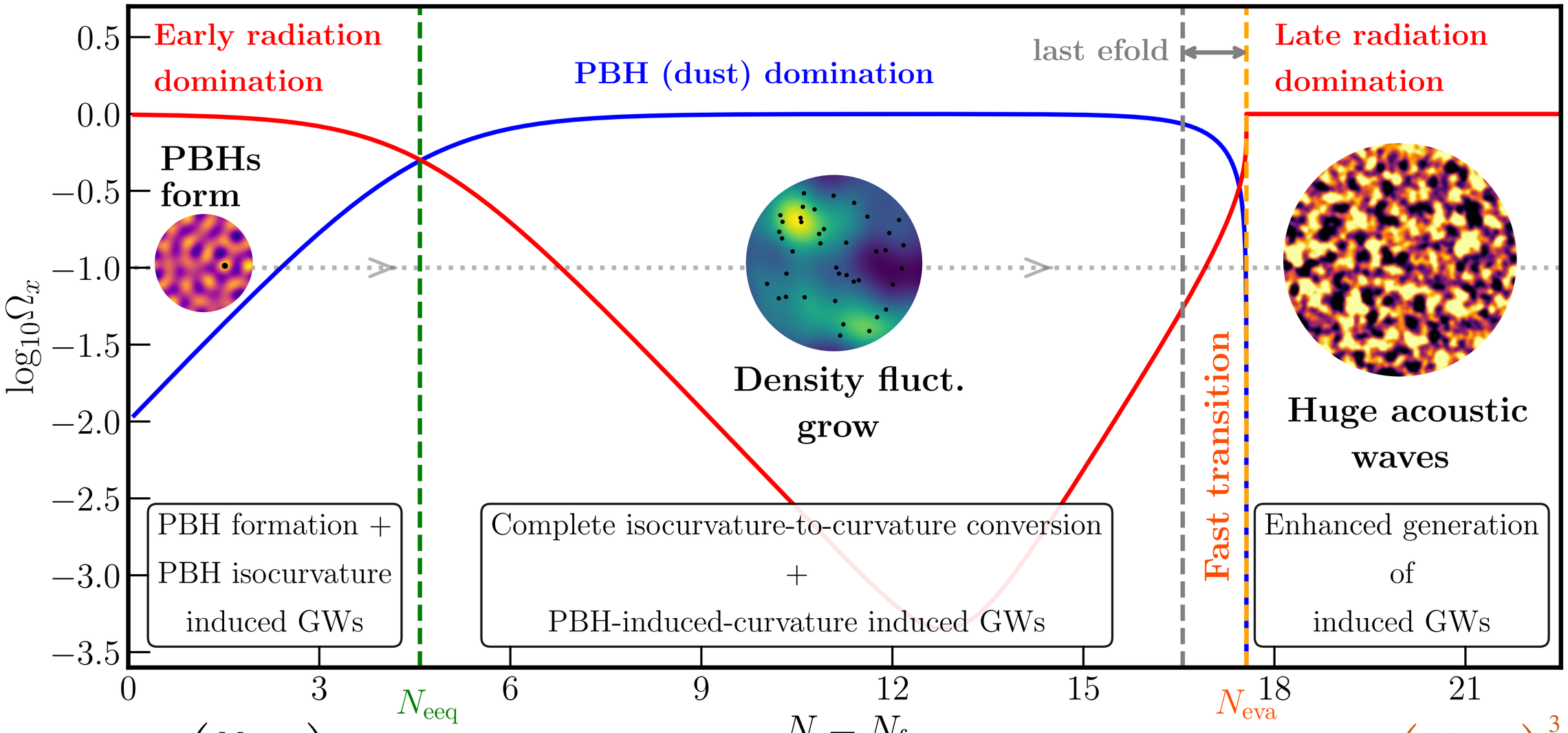
NOTE: The later a mode enters the horizon, the larger the curvature fluctuation induced

Transfer to curvature

We get a PBH dominated universe!



The PBH dominated universe



$$t_{\rm f} \approx \frac{1}{2H_{\rm f}} \approx 10^{-8}\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)$$

$$t_{\rm eeq} \sim \beta^{-2} t_f$$

$$t_{\rm eva} \approx 400\,{\rm qs}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^3$$

GWs from PBH density fluctuations

[K.Inomata+2003.10455]
[T.Papanikolaou+ 2010.11573]
[GD+2012.08151]

Sudden evaporation of all PBHs, creates huge velocity fluctuations in the radiation fluid

and a loud GW signal! $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab}(\partial_a V \partial_b V)$ $V \propto \Phi'/\mathscr{H}$

GWs induced in an early matter era get greatly enhanced for sudden reheating!

[Inomata+1904.12878 ,1904.12879]

There also induced GWs during the PBH dominated era [T.Papanikolaou+ 2010.11573]

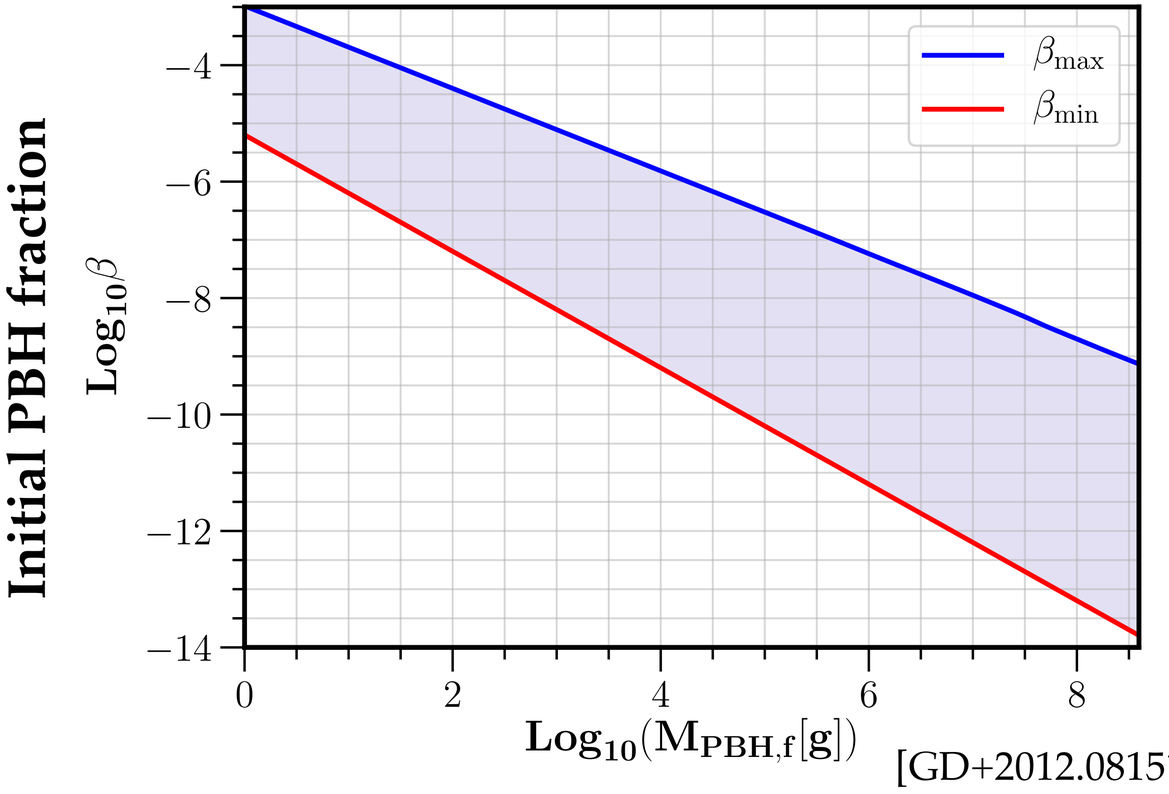
GWs from PBH density fluctuations

[K.Inomata+2003.10455]
[T.Papanikolaou+ 2010.11573]
[GD+2012.08151]

Sudden evaporation of all PBHs, creates huge velocity fluctuations in the radiation fluid

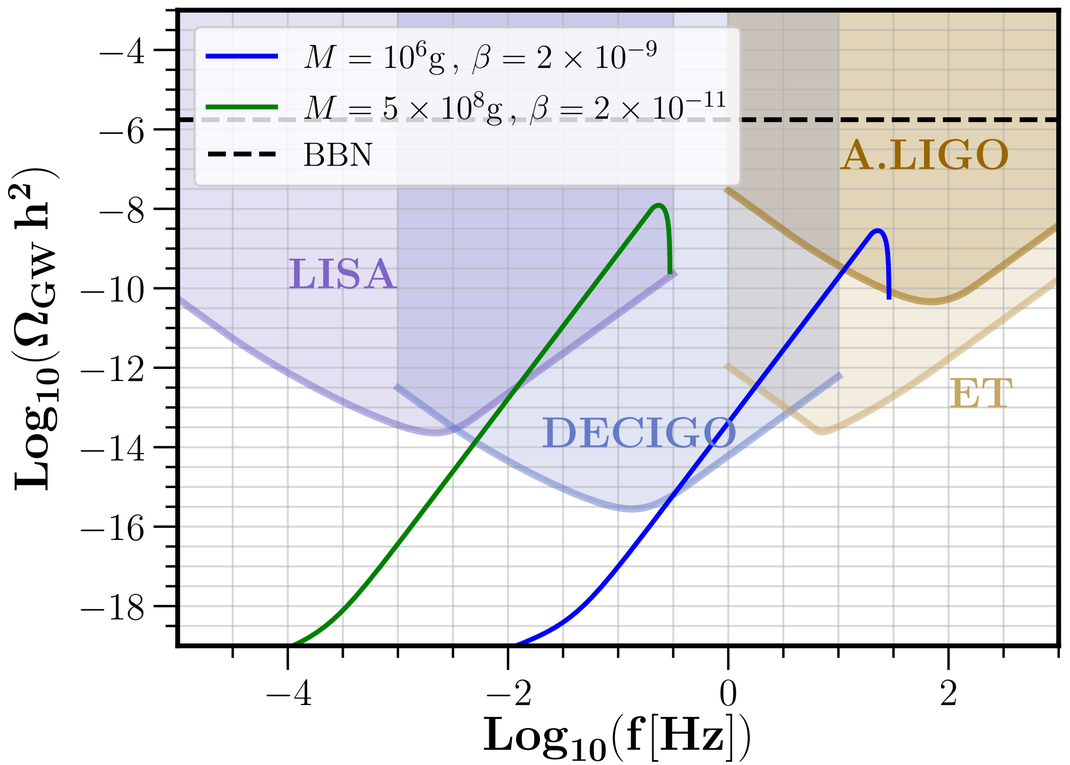
and a loud GW signal! $\Omega_{\rm GW} \approx 10^{49}\beta^{16/3}\left(\frac{M_{\rm PBH}}{10^9 {\rm g}}\right)^{34/9}$ $f_{\rm uv} \approx 4.4\times10^6\,{\rm Hz}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-5/6}$

We can use it to test the PBH dominated universe:



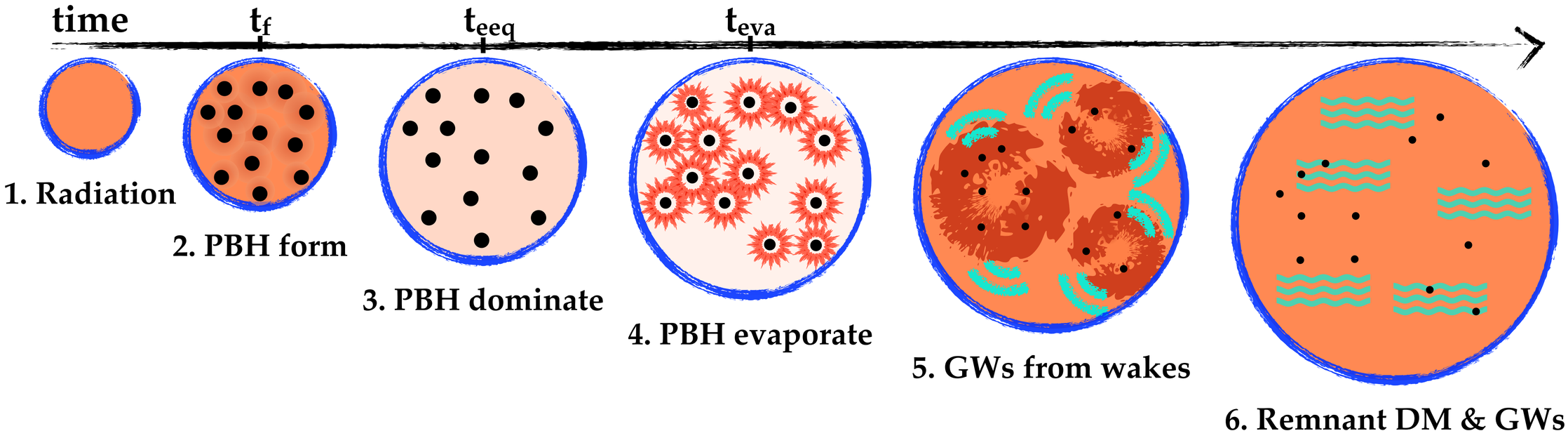
[GD+2012.08151]

(If PBH spin, we can combine info on Neff [GD+2105.06816])



Bonus: PBH Remnants

What if Planck remnants remain?



Same picture as before + CDM component after evaporation!

PBH remnants = Reheating + CDM

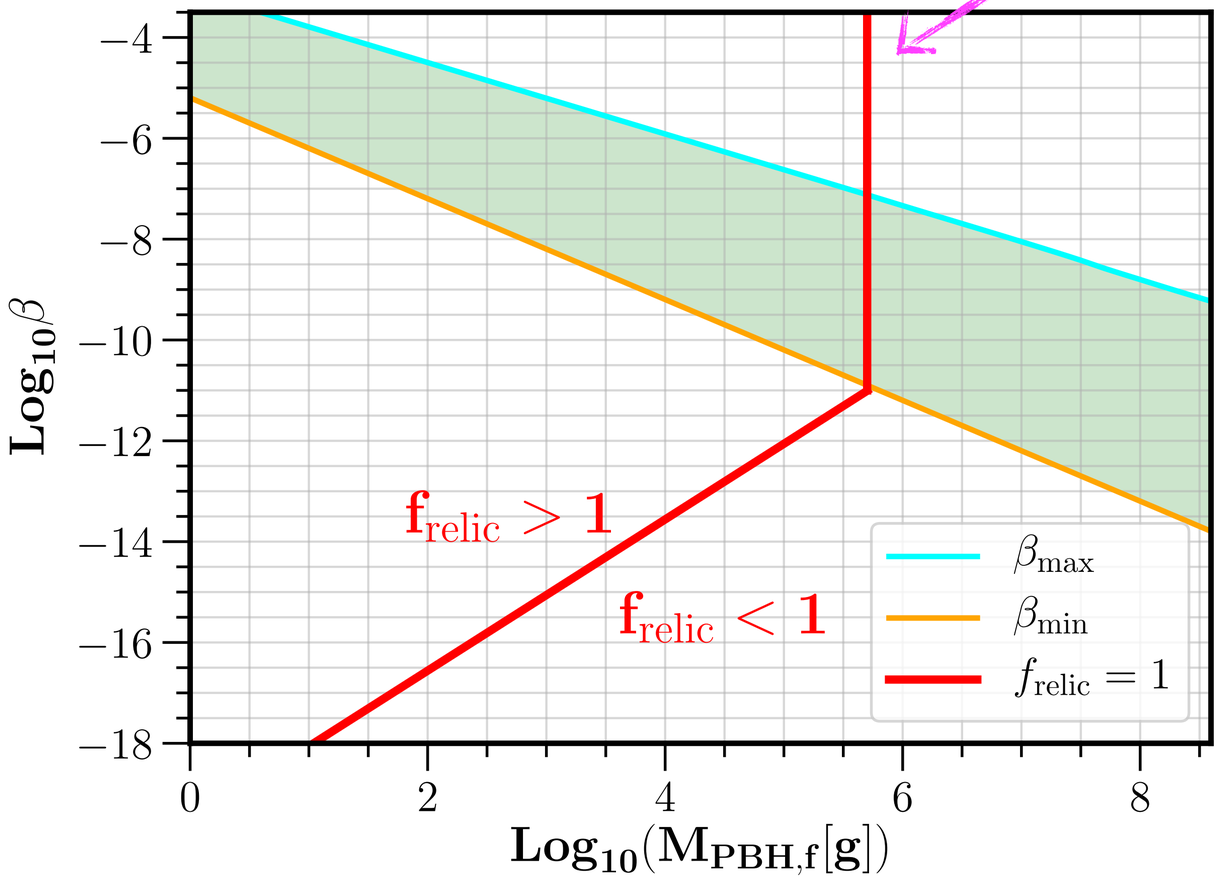
$$M_{\mathrm{PBH,f}} \approx 5 \times 10^5 \ \mathrm{g}$$

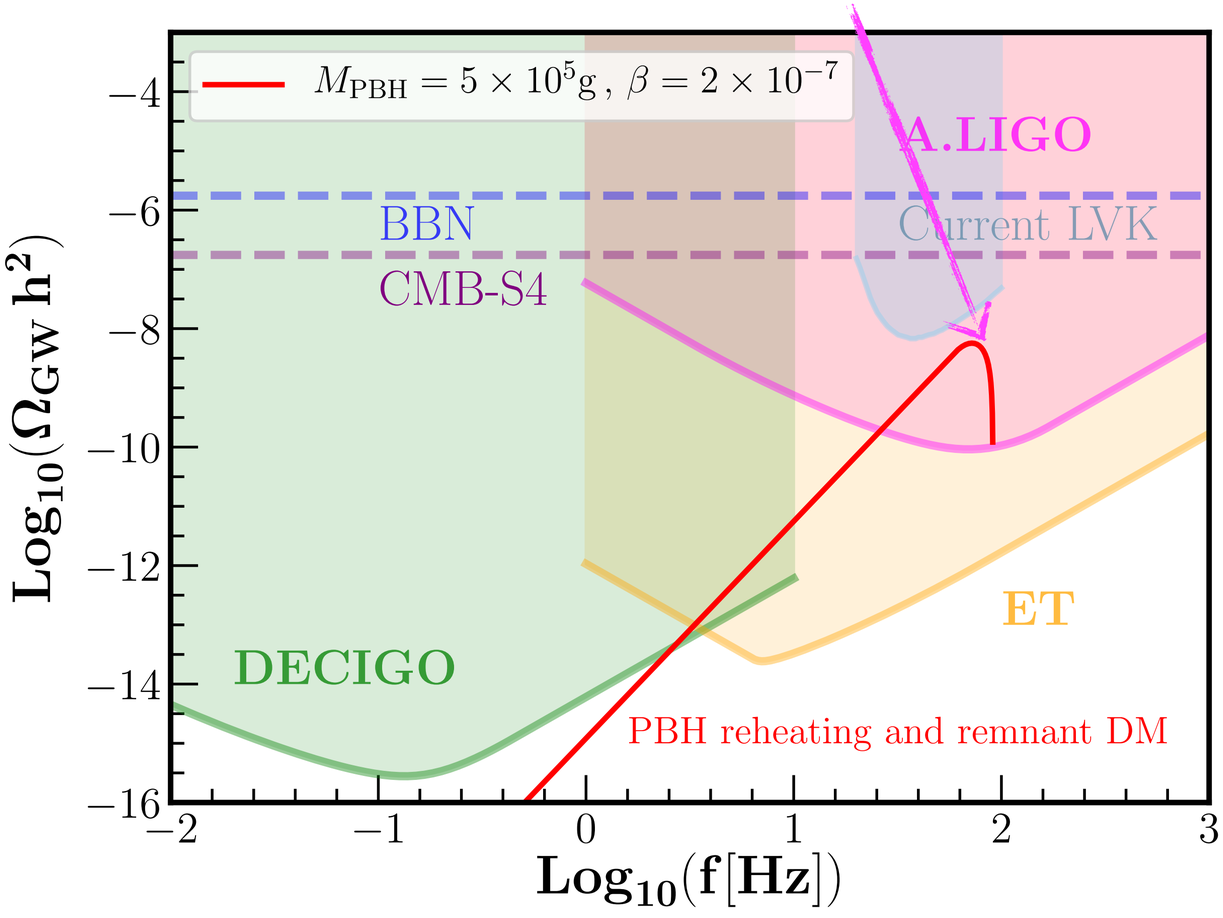
What if Planck remnants remain?

$f_{\rm relic} = \rho_{\rm remnants}/\rho_{\rm cdm}$

$M_{\rm PBH,f} \approx 5 \times 10^5\,{\rm g}$

$f_{\rm uv} \approx 70{\rm Hz}$





GWs from PBH density fluctuations (future work)

Most of PBH density fluctuations reach the non-linear regime during PBH domination

$$0.5 < \delta < 10^4 \left(\frac{M_{\rm PBH}}{10^4 {\rm g}} \right)^{7/18}$$

This means that there will be BH mergers.

However, production of GWs will not stop and we still have Φ<<1

(so our estimate might be on the right track in orders of magnitude)

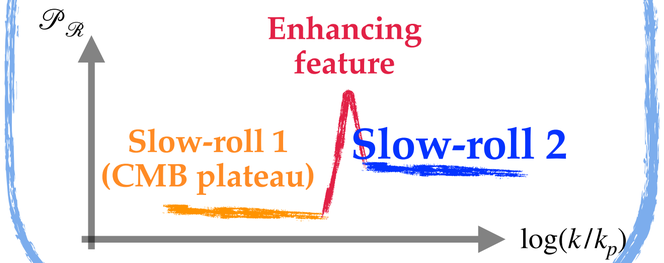
How the system behaves in the non-linear regime with PBH evaporation needs numerical work.

(maybe it generates turbulences? Kozaczuk et al. 2108.12475)

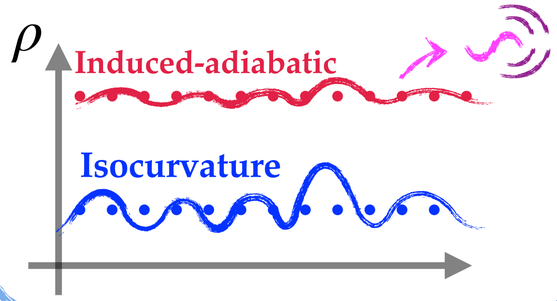
Summary

"Induced GWs and PBHs provide a unique opportunity to test the unexplored primordial universe"

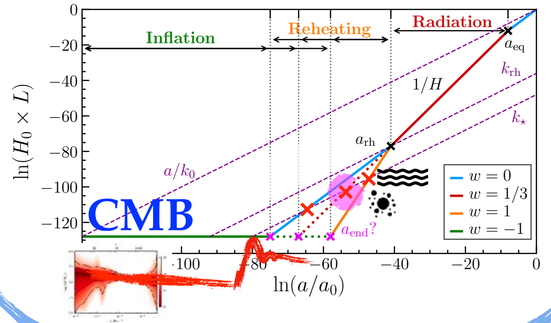
Cosmic inflation



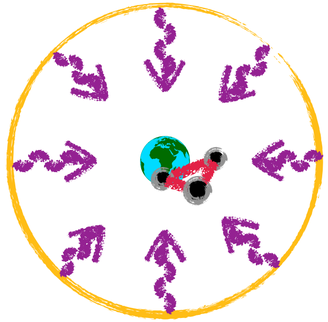
Initial conditions



Content of the universe



GWB anisotropies



+ We can test the PBH dominated universe and the PBH remnant scenario!

$M_{\rm PBH,f} \approx 5 \times 10^5$ g

$f_{\rm uv} \approx 70$Hz