HIDDeN ESR Webinar 14th June 2021

Self-Consistent Dark Matter Halo from Axion Particles

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This project has received funding /support from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska -Curie grant agreement No 860881-HIDDeN

Outline

- Motivation
- Axion Under Self Gravity
- Self-Consistent Dark Matter Halo
- Outlook

Motivation

- Axion is a very well motivated Standard Model extension.

→ Solves the *Strong CP Problem*

→ Explains *Dark Matter*

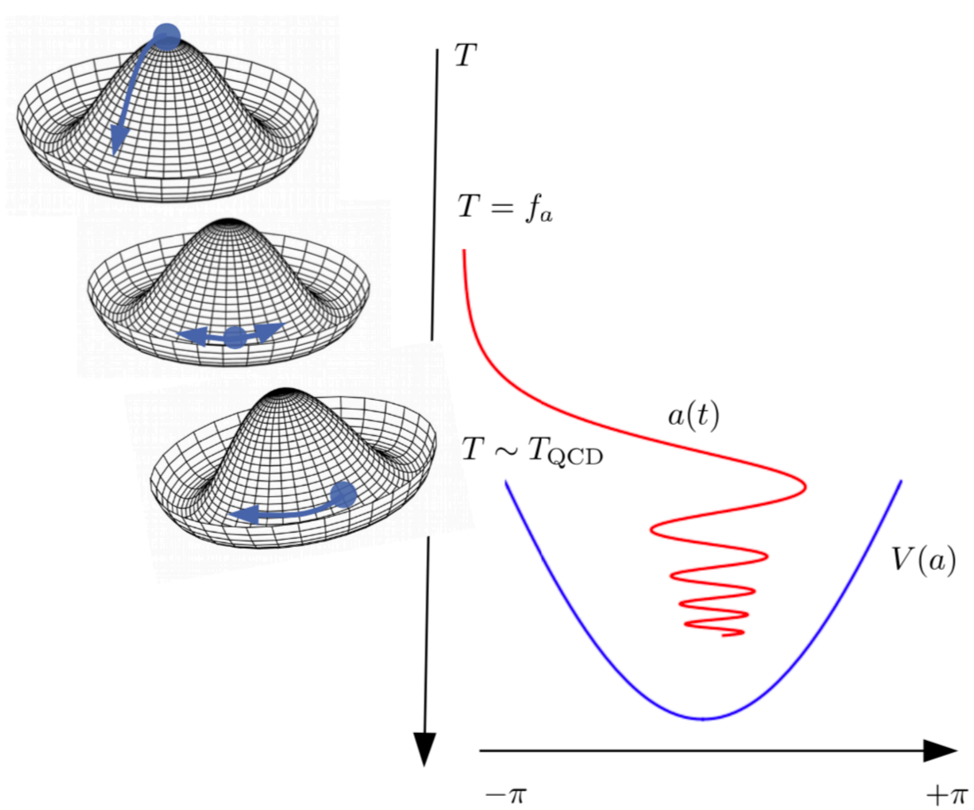


Fig. 1: Axion Misalignment [1]

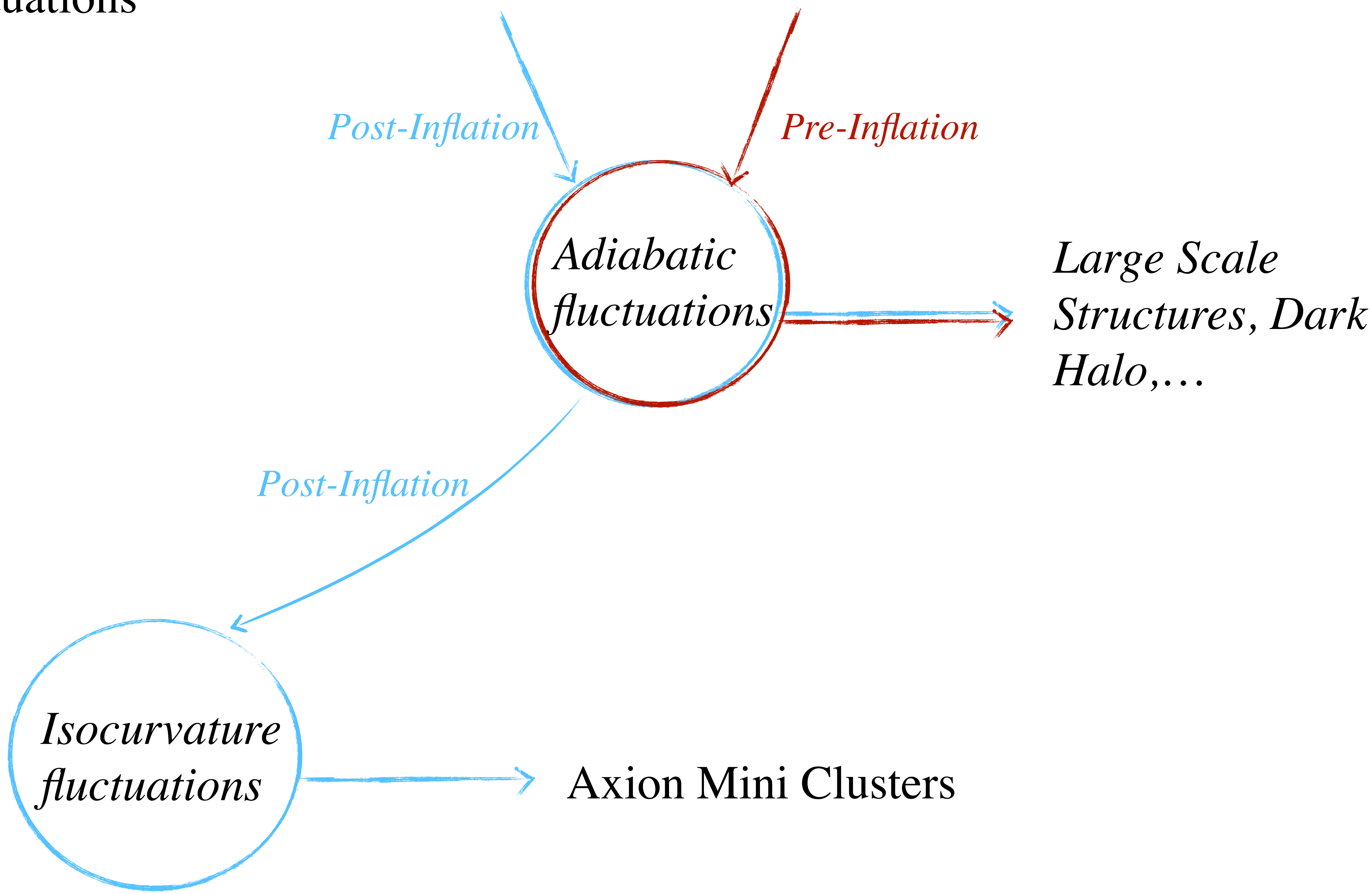
- U(1) Symmetry Breaking: Axion field reaches random initial value

- Cooling down of the Universe: potential for the axion + oscillation of the field

- Dark Matter energy density

Motivation

- As expected the axion will also develop different types of density fluctuations



Motivation

- How could we describe these structures?

Numerical Simulations

Could become computationally challenging

Analytical Treatments

Axion Under Self-Gravity

- When submitted to its own gravity, the axion field is described by the action

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \left[-\frac{1}{2} g_{\mu\nu} \partial^\mu a \partial^\nu a - V(a) + \frac{\mathcal{R}}{16\pi G_N} \right]$$

Eq. of Motion

$$\frac{1}{\sqrt{g}} \partial_\mu \left[\sqrt{-g} g^{\mu\nu} \partial_\nu \right] a + V'(a) = 0 \,,$$ → Klein-Gordon eq.

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} = 8\pi G_N T_a^{\mu\nu} \,,$$ → Einstein eq.

Axion Under Self-Gravity

- Two simplifications could be done in this case:

High occupation number: Classical description of the axion field

Non-relativistic: New field description

$$a = \frac{1}{\sqrt{2m}}\left(\phi \mathrm{e}^{-imt} + \phi^* \mathrm{e}^{imt}\right)$$

$$\rho_a = m\phi\phi^* \quad \Leftrightarrow \quad \phi\phi^* = \rho_a/m \equiv n_a$$

Axion Under Self-Gravity

$$\frac{1}{\sqrt{g}}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\right]a + V'(a) = 0\,,$$

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu} = 8\pi G_N T_a^{\mu\nu}\,,$$

High occupation number

Non-relativistic

$$\mathrm{i}\partial_t\phi = -\frac{\Delta\phi}{2m} + m\Phi_N\phi$$

$$\Delta\Phi_N = 4\pi G_N m\phi\phi^*\,.$$

Schrodinger-Poisson system

Axion Under Self-Gravity

- Simulations for wave dark matter have already been performed and solve two main problems arising in CDM simulations [2]

Cups-like density profile

Too big to fail problem

→ Wave Dark Matter simulations, but other limitations…

→ Analytical construction of the halos

Self-Consistent Dark-Matter Halo

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + m_a\phi\psi,$$

$$\frac{\nabla^2\phi}{4\pi G} = m_a|\psi|^2,$$

→ How could we construct a steady solution for a given Halo?

- How do we characterize a Halo? By its *density*, *gravitational potential* profile and its *distribution function*

$$\rho(r) = m_a\int d^3v\, f(v,\phi(r))$$

→ *Classical Distribution Function*

$$\nabla^2\phi(r) = 4\pi G\rho(r)$$

Self-Consistent Dark-Matter Halo

$$\rho(r) = m_a \int d^3v\, f(v, \phi(r))$$

Classical Distribution Function

$$\nabla^2 \phi(r) = 4\pi G \rho(r)$$

Power Law profile

$$f(E) = \frac{1}{G m_a^4} \frac{9!}{(2\pi)^{3/2}(15/2)!} \left(\frac{3}{64\pi \rho_s R_s^{9/4} G} \right)^9 (-E/m_a)^{-15/2}$$

$$\rho(r) = \rho_s \left(\frac{R_s}{r} \right)^{9/4}$$

$$\phi(r) = -\frac{64\pi}{3} G \rho_s R_s^{9/4} \frac{1}{r^{1/4}}$$

Self consistent system

Self-Consistent Dark-Matter Halo

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + m_a\phi\psi,$$

✔ Potential fixed

$$\frac{\nabla^2\phi}{4\pi G} = m_a|\psi|^2,$$

→ What is the solution for the Axion field?

WKBJ Approximation:

General Solution

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \frac{C_{n,l,m}}{\left(\frac{2m_a}{\hbar^2}(E_n - V_{eff}(r))\right)^{1/4} r} \sin\left[\int_r |p_r(r)|dr + \pi/4\right] Y_{lm}(\theta,\phi).$$

Self-Consistent Dark-Matter Halo

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \frac{C_{n,l,m}}{\left(\frac{2m_a}{\hbar^2}(E_n - V_{eff}(r))\right)^{1/4}_r} \sin\left[\int_r |p_r(r)| dr + \pi/4\right] Y_{lm}(\theta,\phi)$$

Have to be self-consistent with the Poisson-Equation → $m_a|\psi|^2 = \rho(r)$

Random Phase Model [3]

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \frac{C_{n,l,m} e^{i\phi_{nlm}}}{\left(\frac{2m_a}{\hbar^2}(E_n - V_{eff}(r))\right)^{1/4}_r} \sin\left[\int_r |p_r(r)| dr + \pi/4\right] Y_{lm}(\theta,\phi)$$

Self-Consistent Dark-Matter Halo

Random Phase Model

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \frac{C_{n,l,m} e^{i\phi_{nlm}}}{\left(\frac{2m_a}{\hbar^2}(E_n - V_{eff}(r))\right)^{1/4} r} \sin\left[\int_r |p_r(r)| dr + \pi/4\right] Y_{lm}(\theta,\phi)$$

Density

$$m_a|\psi|^2 = m_a \sum_{n,l,m} |C_{n,l,m}|^2 |\psi_{n,l,m}|^2 + m_a \sum_{n,l,m} \sum_{n',l',m'} e^{i(\phi_{nlm} - \phi_{nlm'})} C_{n,l,m} C^*_{n',l',m'} \psi_{n,l,m} \psi^*_{n',l',m'}$$

Ensemble Average

Small scales time dependent fluctuations

$$m_a \langle |\psi|^2 \rangle = m_a \sum_{n,l,m} |C_{n,l,m}|^2 |\psi_{n,l,m}|^2$$

Self-Consistent Dark-Matter Halo

Random Phase Model

$$m_a \langle |\psi|^2 \rangle = m_a \sum_{n,l,m} |C_{n,l,m}|^2 |\psi_{n,l,m}|^2 = \rho(r)$$

$$\rho(r) = \int dE dl \frac{4\pi m_a^2 l}{\sqrt{2m_a\left(E - \frac{l^2}{2m_a r^2} - m_a\phi(r)\right) r^2}} f(E)$$

- Use the distribution function to match both sides
- Smooth out the fast oscillations

$$\int dE\, dl \frac{4\pi m_a^2 l}{\sqrt{2m_a\left(E - \frac{l^2}{2m_a r^2} - m_a\phi(r)\right) r^2}} f(E) = \frac{m_a}{4\pi} \sum_{n,l,m} \frac{|C_{n,l,m}|^2}{\sqrt{2m_a\left(E_n - \frac{l^2}{2m_a r^2} - m_a\phi(r)\right) r^2}}$$

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \sqrt{8\pi^2 \frac{m_a}{\hbar}} \sqrt{f(E_n) dE\, dl\, dm}\, e^{i\phi_{nlm}}\, \psi_{nlm}(r)$$

Self-Consistent Dark-Matter Halo

- Dark Matter Halo is now described by the axion wave function

$$\psi(r,\theta,\phi) = \sum_{n,l,m} \sqrt{8\pi^2 \frac{m_a}{\hbar}} \sqrt{f(E_n)dE\, dl\, dm}\, e^{i\phi_{nlm}}\, \psi_{nlm}(r)$$

$$m_a \langle|\psi|^2\rangle = \rho(r)$$

$$\nabla^2 \phi(r) = 4\pi G m_a \langle|\psi|^2\rangle$$

→ *Self-Consistent Solution for a given Dark Halo*

- We recover on average the properties of the Halo

Self-Consistent Dark-Matter Halo

- Dark Matter Halo is now fully described by the axion wave function

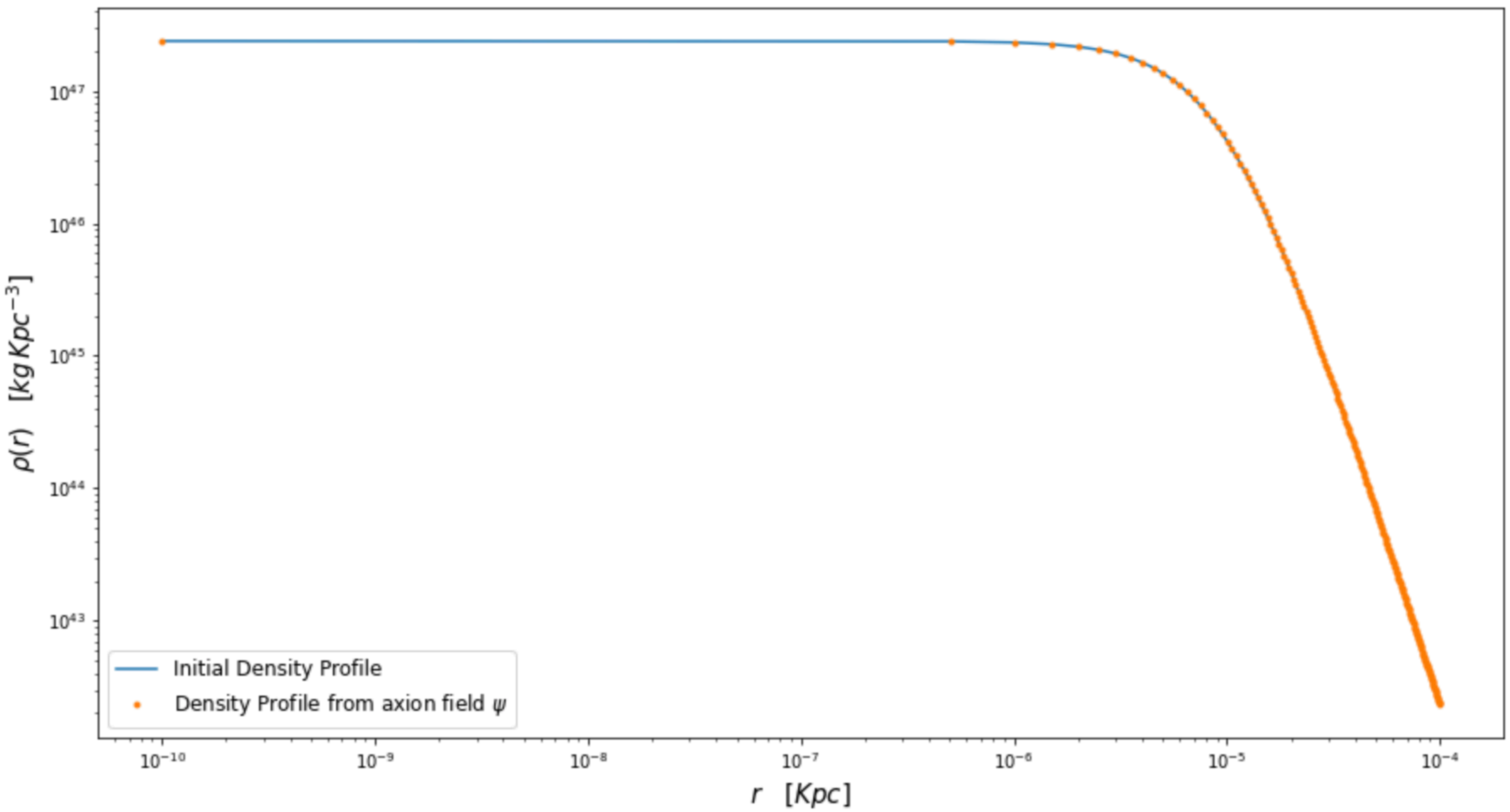


Fig. 2: Self consistent density profile for a power law dark halo with n=5.

- What about the deviations from the ensemble average?

Self-Consistent Dark-Matter Halo

$$m_a|\psi|^2 = m_a \sum_{n,l,m} |C_{n,l,m}|^2 |\psi_{n,l,m}|^2 + m_a \sum_{n,l,m} \sum_{n',l',m'} e^{i(\phi_{nlm}-\phi_{nlm'})} C_{n,l,m} C^*_{n',l',m'} \psi_{n,l,m} \psi^*_{n',l',m'}$$

- Interferences between different modes: *small scales time-dependent 'granules'* .

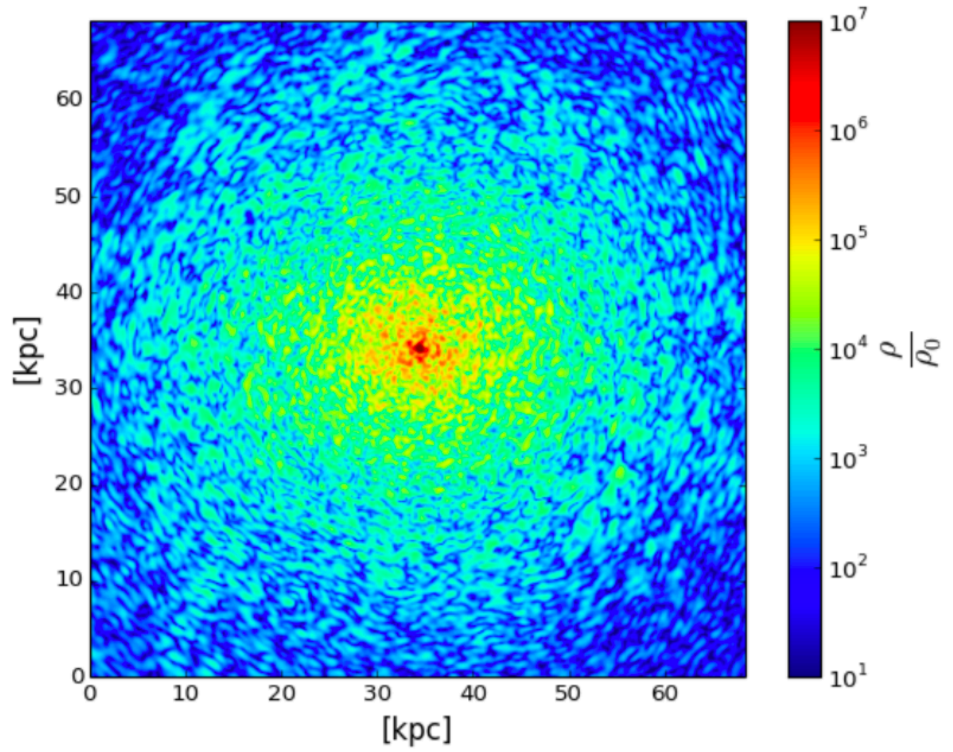


Fig. 3: Self-consistent halo for a given draw of the random phases [2]

Self-Consistent Dark-Matter Halo

$$m_a|\psi|^2 = m_a \sum_{n,l,m} |C_{n,l,m}|^2 |\psi_{n,l,m}|^2 + m_a \sum_{n,l,m} \sum_{n',l',m'} e^{i(\phi_{nlm} - \phi_{nlm'})} C_{n,l,m} C^*_{n',l',m'} \psi_{n,l,m} \psi^*_{n',l',m'}$$

- Interferences between different modes: *small scales time-dependent 'granules'* .

→ Observed in high resolution simulations and explained here with a random distribution

→ How are they statistically described?

Self-Consistent Dark-Matter Halo

Fluctuation Statistics on different scales

- Deviation from the density on scales W:

$$C = \left(\frac{h^2}{\pi^3 G}\right)^{1/2}$$

$$m_a^2\left(\langle\psi_W^2(r)\psi_W^2(r)\rangle - (\langle\psi_W^2(r)\rangle)^2\right) = \sigma_W^2 = \rho^2(r)\frac{C}{R_s^2(\rho_s m_a^2 W^2)^{1/2}}$$

- As we increase the scale W, the deviation decreases: average density profile recovered at large scales

Conclusion and Outlook

- Axion bounded structures are described by the Schrodinger-Poisson system
- Simulations of this wave dark matter already solved some problems arising in the cold dark matter simulations
- It is however possible to create some analytical solutions for these structures: on average, the large scale density profile is recovered and on small scales, some granules appear
- The next step is to use this analytical solution on bounded axion structures (ex: mini cluster) to understand their properties.

Thank you !

Sources

[1] Pargner, Andreas. ‘ *Phenomenology of Axion Dark Matter* ‘, KIT, Karlsruhe, IKP, 2019

[2] S.C Lin et al. ‘*Self-consistent construction of virialized wave dark matter halos*’, ArXiv: 1801.02320, 2018

[3] M.Widrow, N.Kaiser. ‘*Using the Schrodinger Equation to Simulate Collisionless Dark Matter*’, The Astrophysics Journal, 1993

[4] N. Dalal, J. Bovy, L. Hui and Xinyu Li, ‘ *Don't cross the streams: caustics from fuzzy dark matter*’, JCAP03(2021)076, 2021