Post-inflationary Axions:

Gravitational waves and targets for haloscopes

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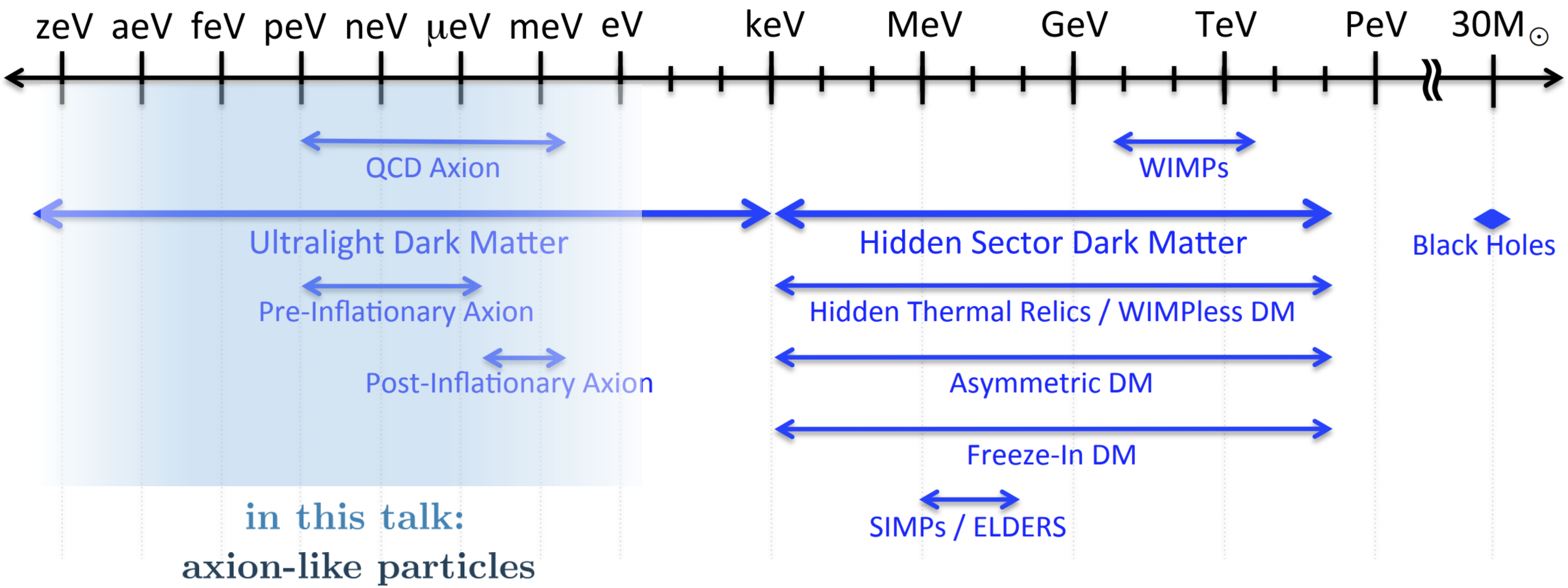
Mostly based on:	[2212.13263]	**MG, E.Hardy**
	[2101.11007]	**MG, E.Hardy, H.Nicoleascu; JCAP**
	[2007.04990]	**MG, E.Hardy, G.Villadoro; SciPost**

Dark Matter Candidates



from 1707.04591

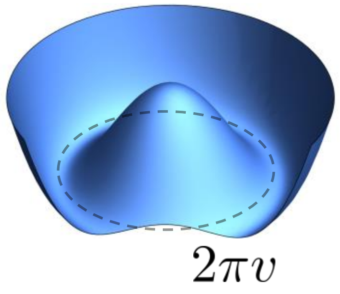
Axion (= ALP)

- PNGB of a new U(1) symmetry, spontaneously broken at a high scale v

$$\phi = \frac{r+v}{\sqrt{2}} e^{i\frac{a}{v}}$$

$$\mathcal{L}_\phi = |\partial_\mu \phi|^2 - \frac{m_r^2}{2v^2}\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$

$$m_r \simeq v$$

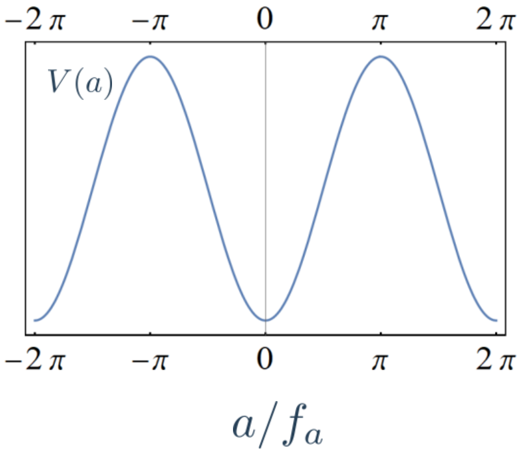


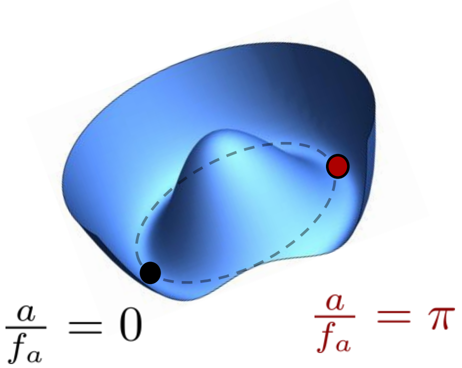
- U(1) broken by $V(a) \implies$ axion mass m_a

  → $V(a)$ invariant under $a \to a + 2\pi f_a$

  → in general $v = N f_a$, N integer

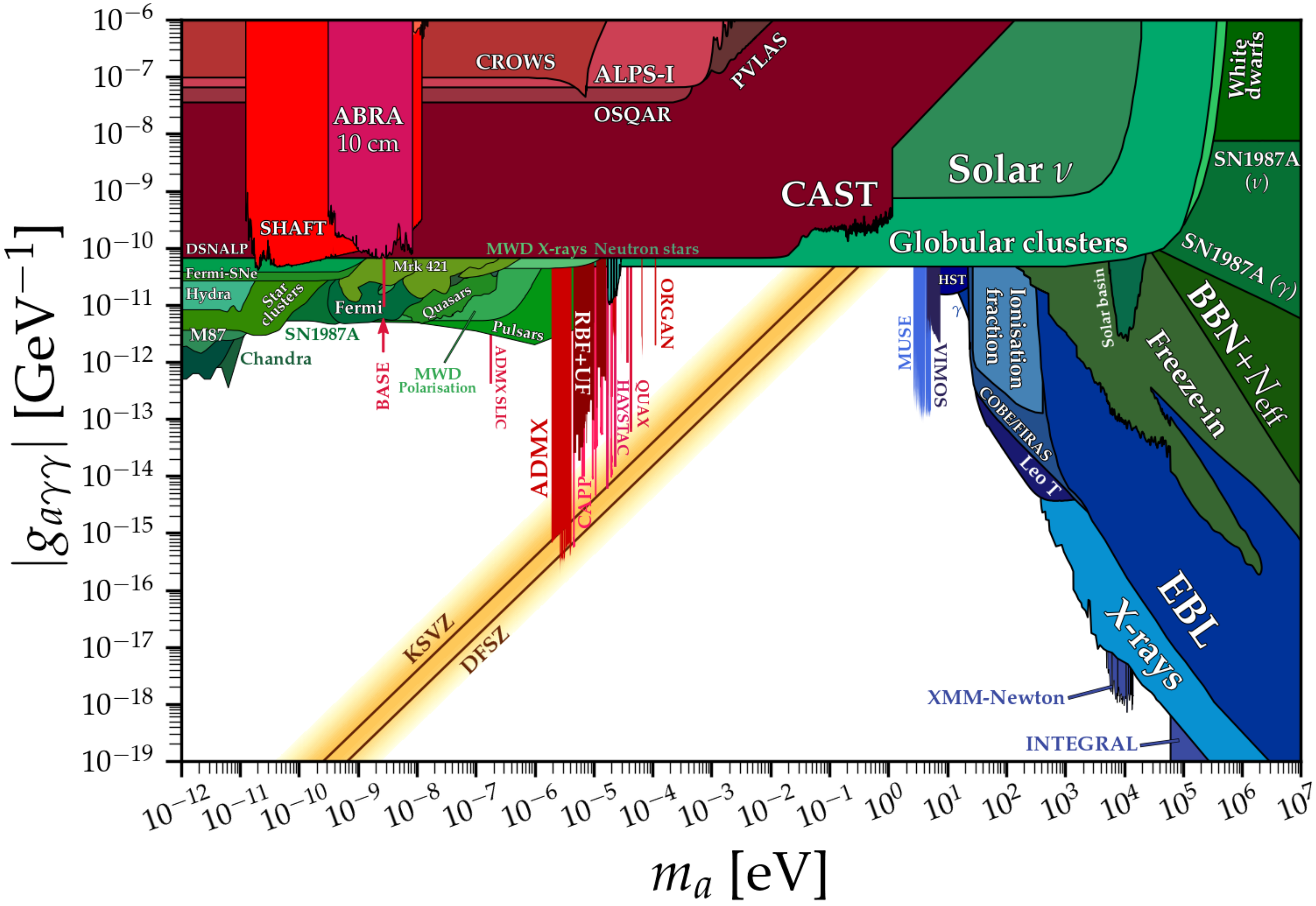
    → for now assume $N = 1$, i.e. $v = f_a$





Axion searches

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$



https://cajohare.github.io/AxionLimits

Axion searches

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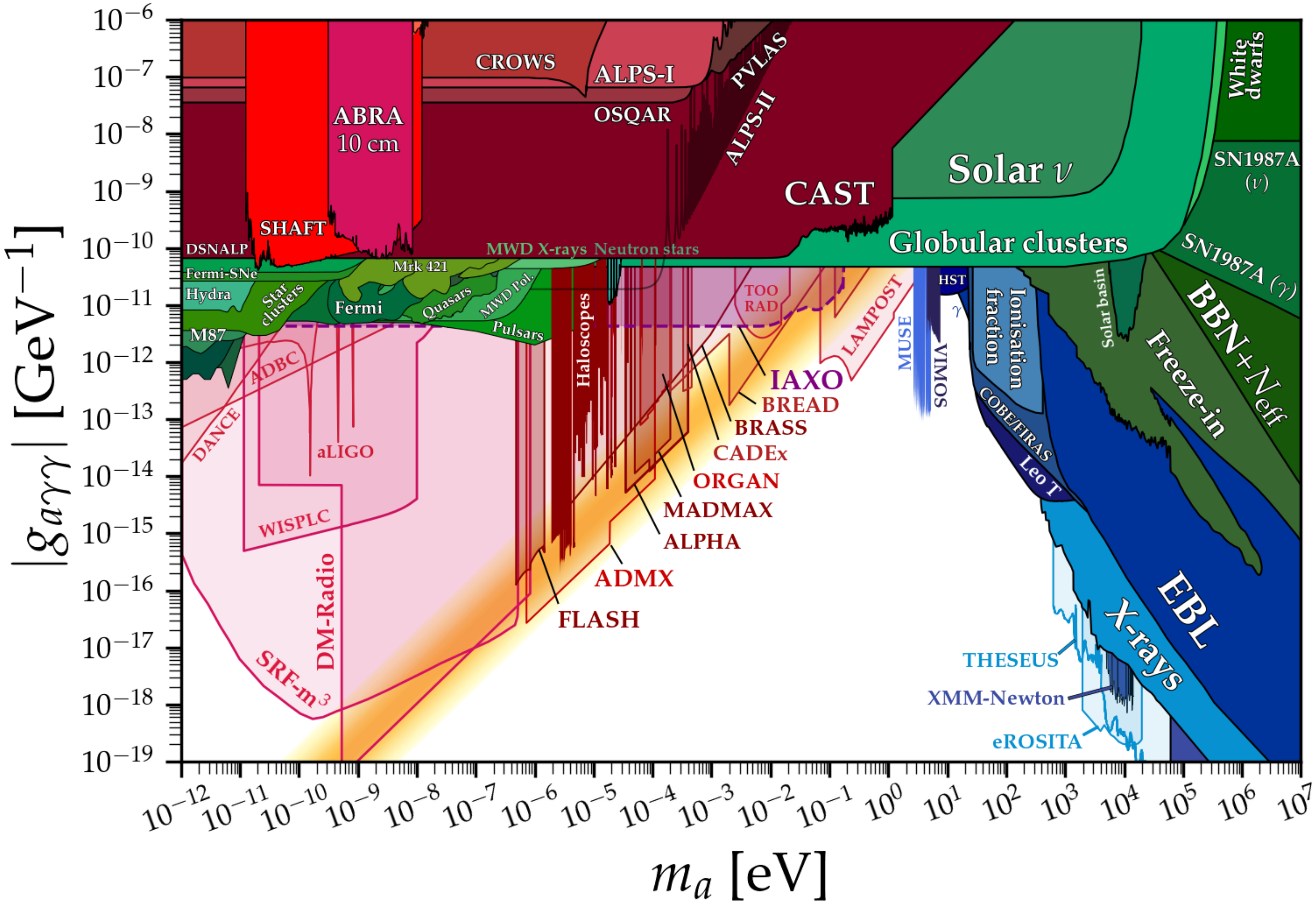
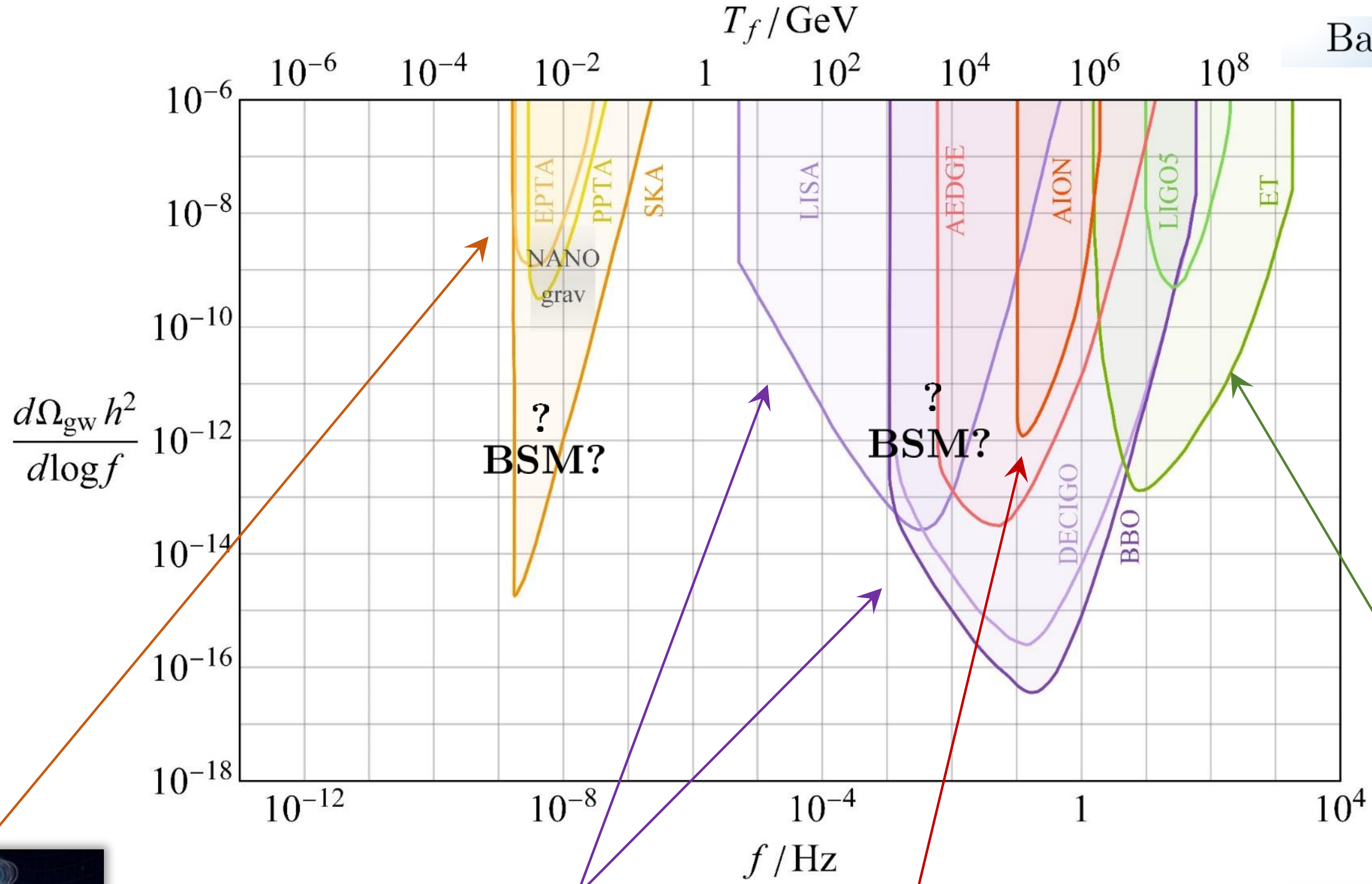


https://cajohare.github.io/AxionLimits

GW searches

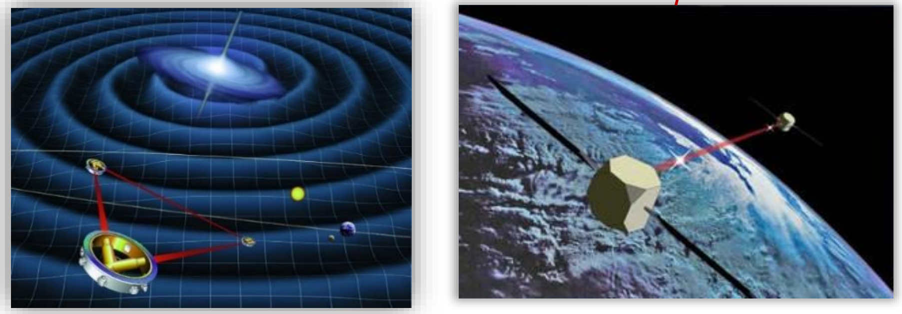
Back to the past



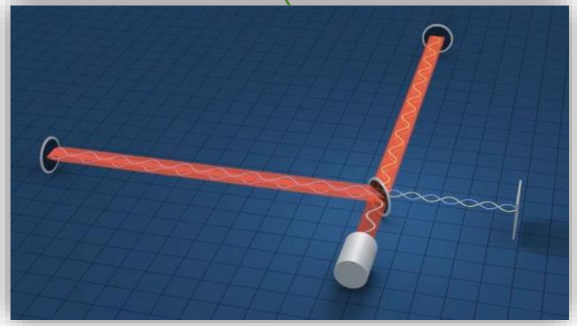
Caution:

Many experimental challenges to overcome

Pulsar Timing Arrays



Space-based Light/Atom Interferometers



Ground-based Interferometers

Outline

- **Axion post-inflationary scenario**

- **Gravitational waves from axion strings**

- $N > 1$**: minimal targets for haloscopes & substructure**

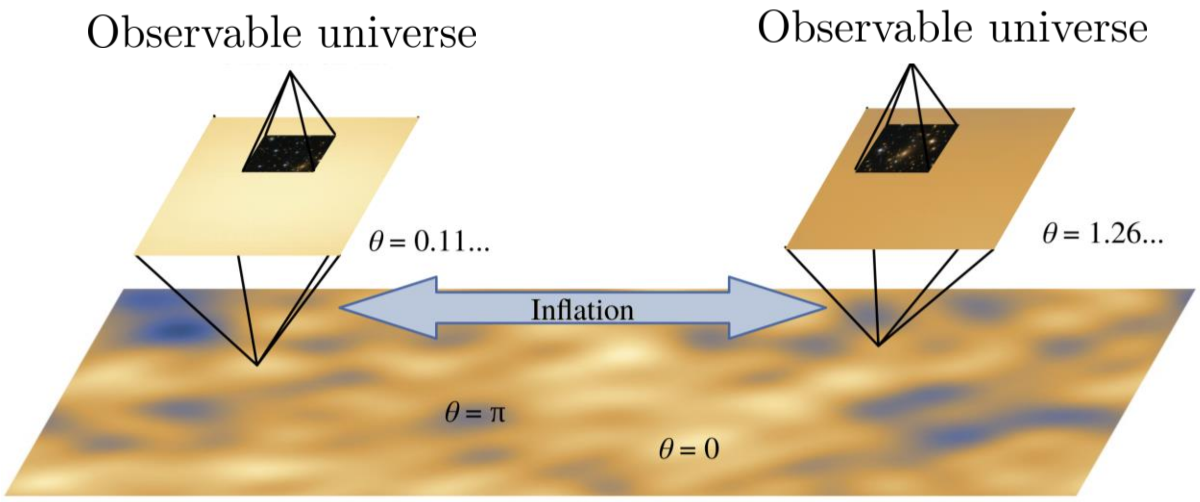
Cosmological Initial Conditions

Pre-inflationary scenario

$T_R \lesssim f_a$ and $H_I \lesssim f_a$

Reheating temperature

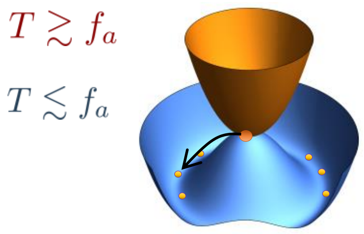
Hubble parameter during inflation

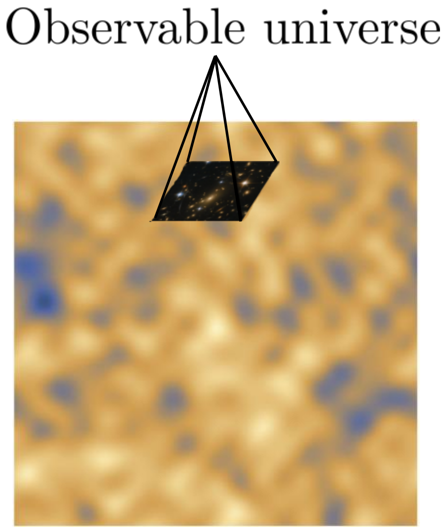


$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

Post-inflationary scenario

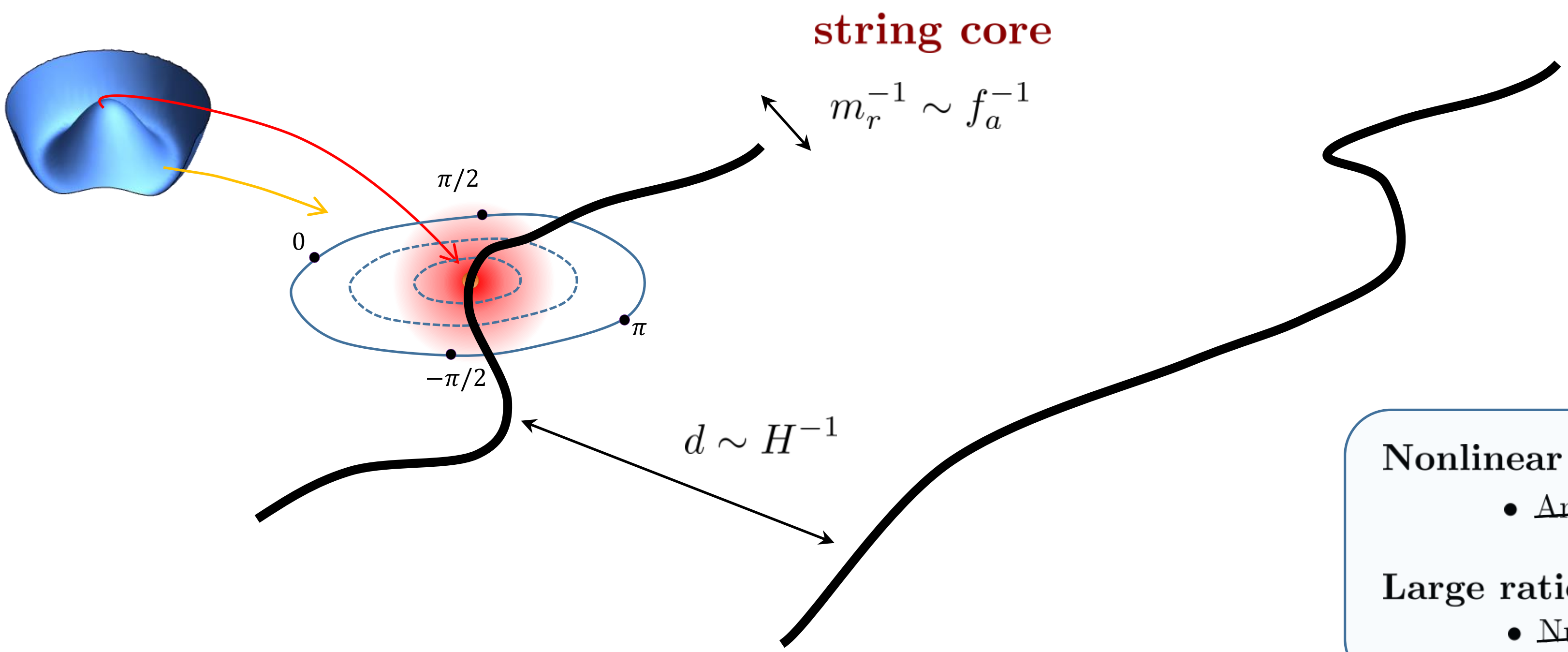
$T_R \gtrsim f_a$ or $H_I \gtrsim f_a$





focus of this talk

Strings



Nonlinear dynamics:
- ~~Analytical approach~~ ☹

Large ratio of scales:
- ~~Numerical approach~~ ☹

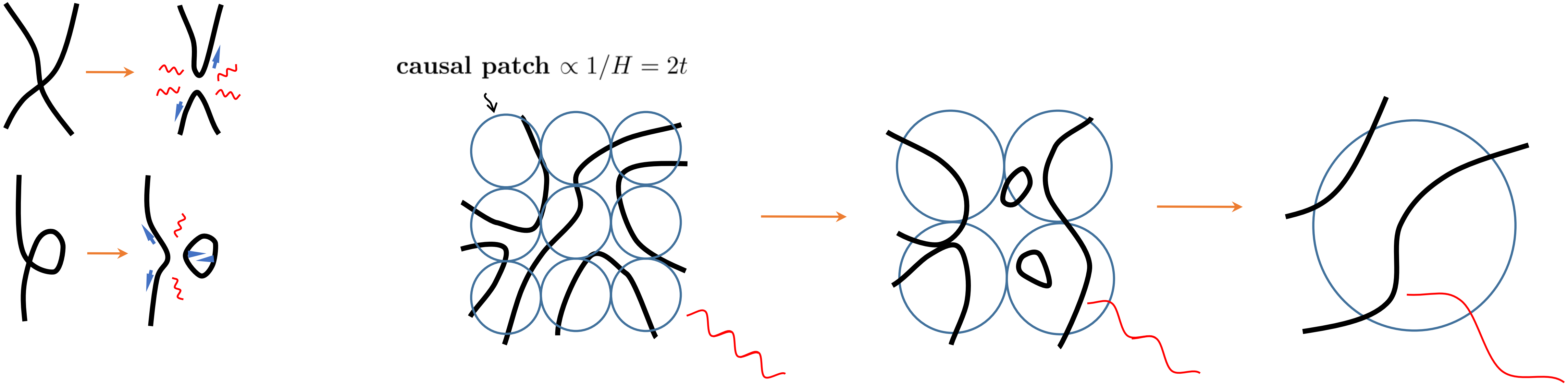
string tension

$$\mu = \frac{E}{L} \sim \underset{\text{core}}{\boxed{\pi f_a^2}}\,\underset{\text{axion gradient}}{\boxed{\log \frac{d}{m_r^{-1}}}} \sim \pi f_a^2 \log \frac{m_r}{\boxed{H}}$$

T^2/M_p

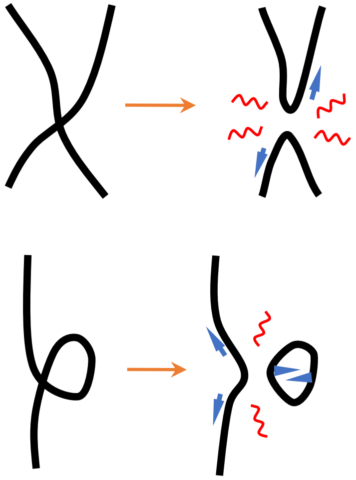
grows logarithmically in time

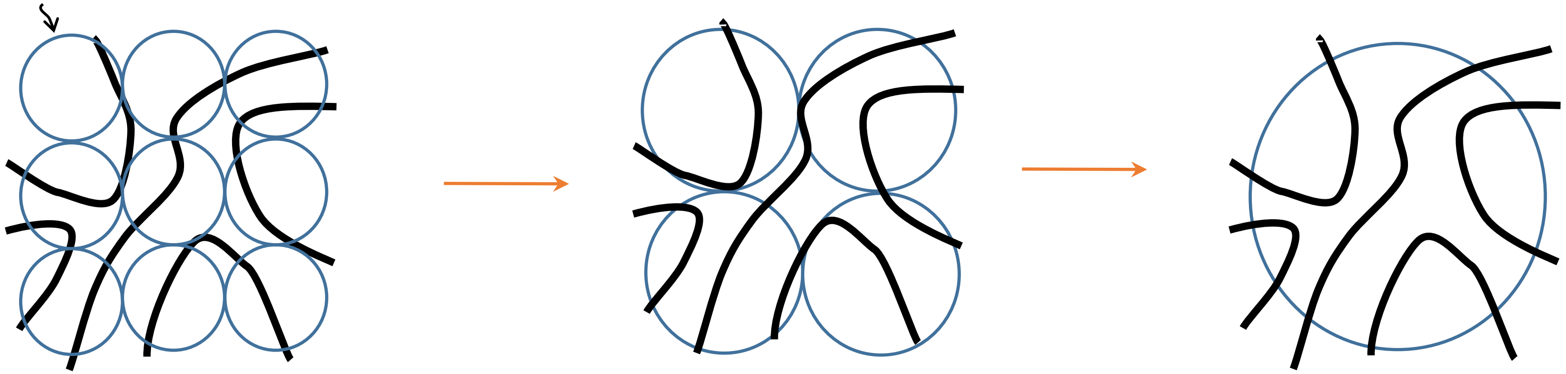
The Scaling Regime



causal patch $\propto 1/H = 2t$

The Scaling Regime

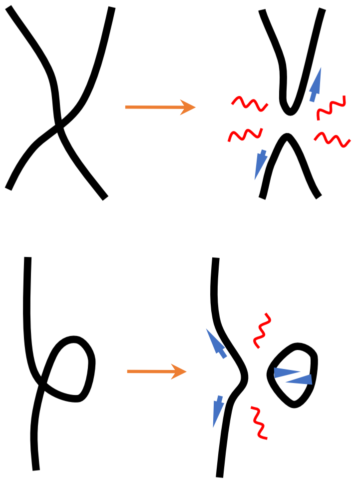


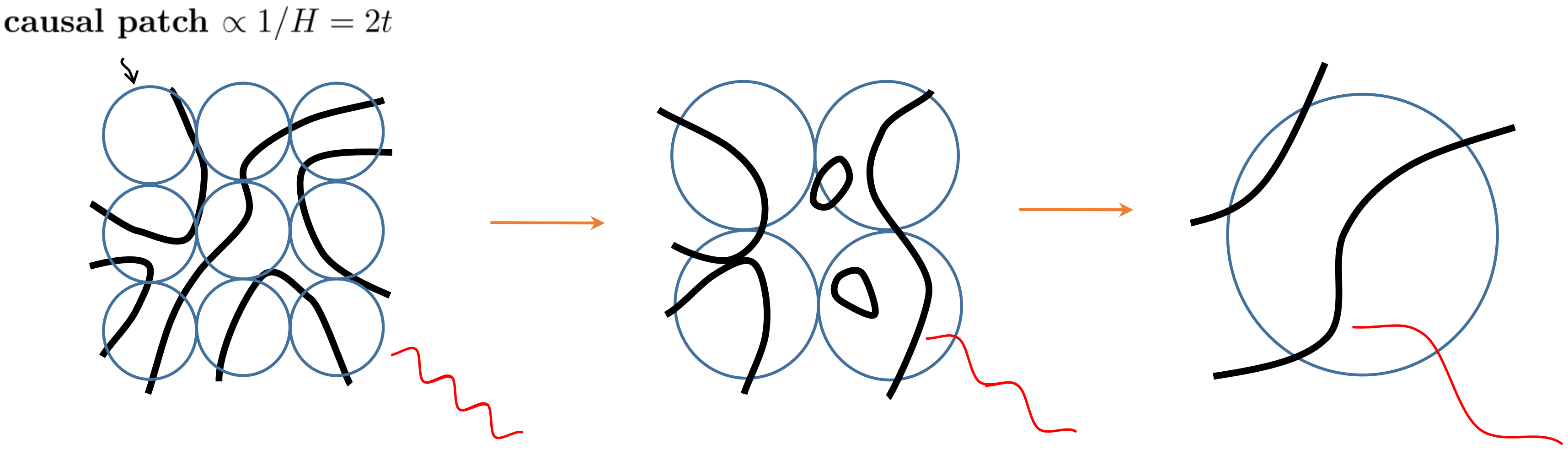


causal patch $\propto 1/H = 2t$

free strings: $\rho^{\text{free}} \propto \frac{1}{R^2} \propto \frac{1}{t}$

The Scaling Regime

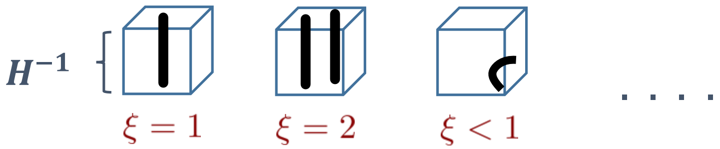


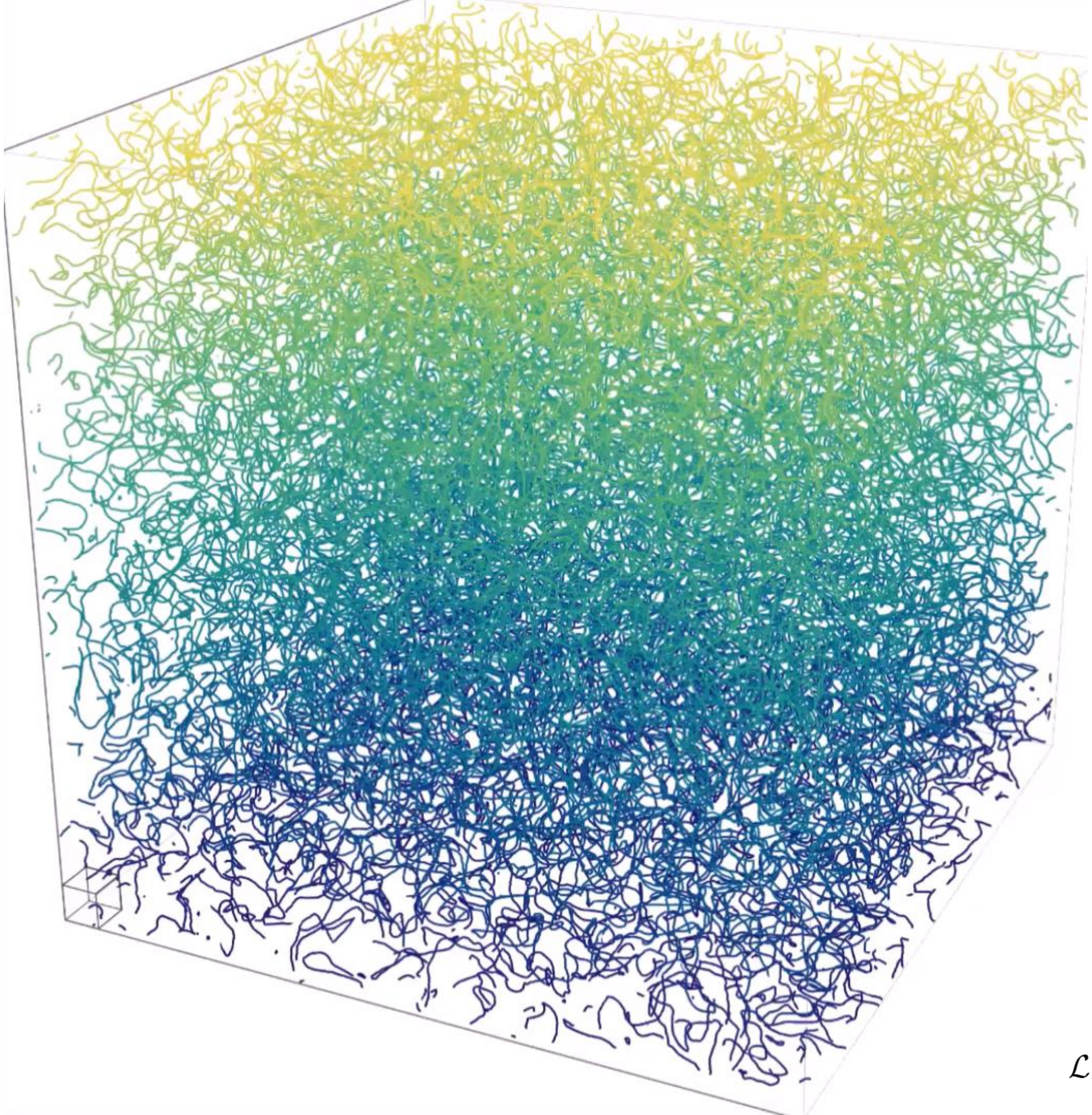


rate of energy loss: $$\Gamma \equiv \frac{d}{dt}\left[\rho^{\text{free}} - \rho^{\text{scal}}\right] \simeq \frac{\xi\mu}{t^3}$$

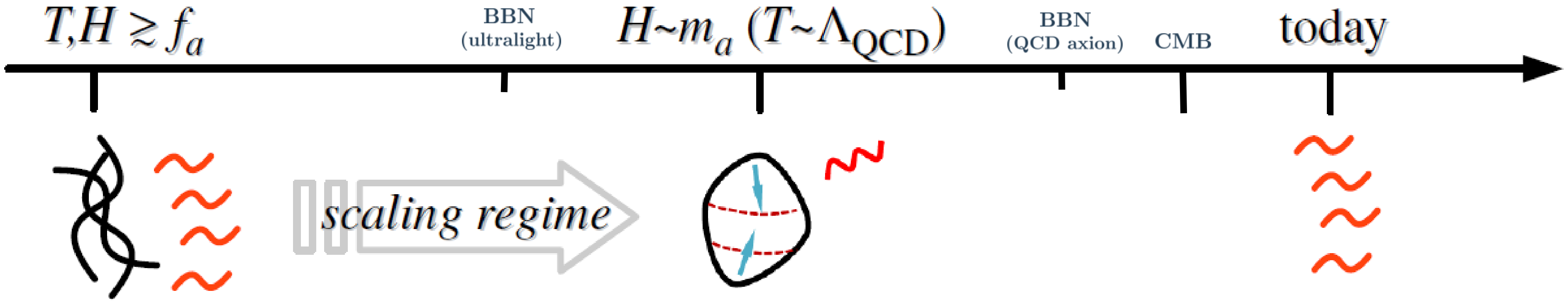
$$\propto \frac{1}{R^2} \propto \frac{1}{t} \qquad \frac{\boxed{\xi}\mu}{t^2}$$

number of strings per Hubble patch





$$\mathcal{L}_\phi = |\partial_\mu \phi|^2 - \frac{m_r^2}{2v^2}\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$



$T,H \gtrsim f_a$ BBN (ultralight) $H\sim m_a$ $(T\sim\Lambda_{QCD})$ BBN (QCD axion) CMB today

scaling regime

simulation extrapolation

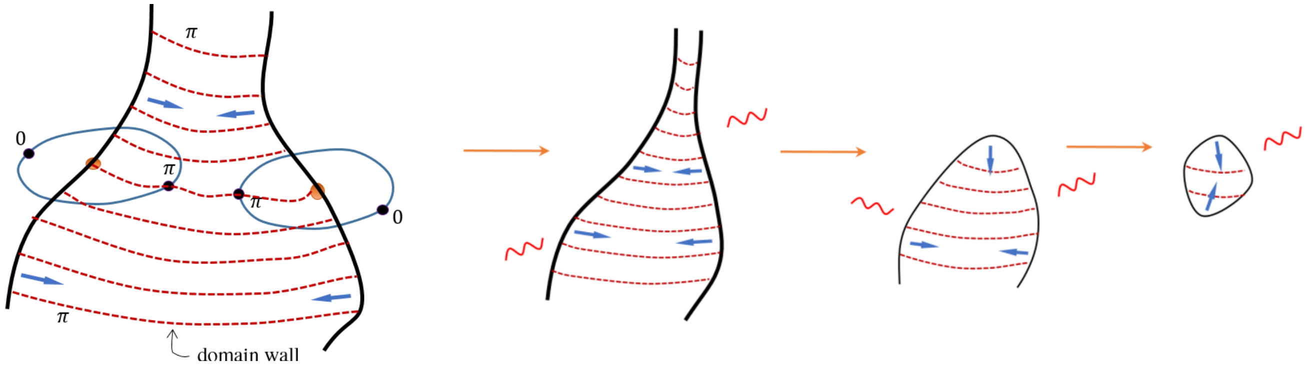
$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ 8 $\sim \mathbf{70 \div 100}$

strings form

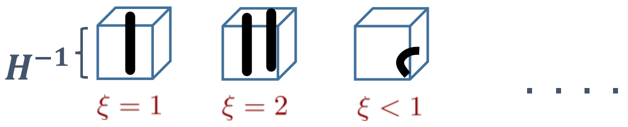
domain walls form and annihilate

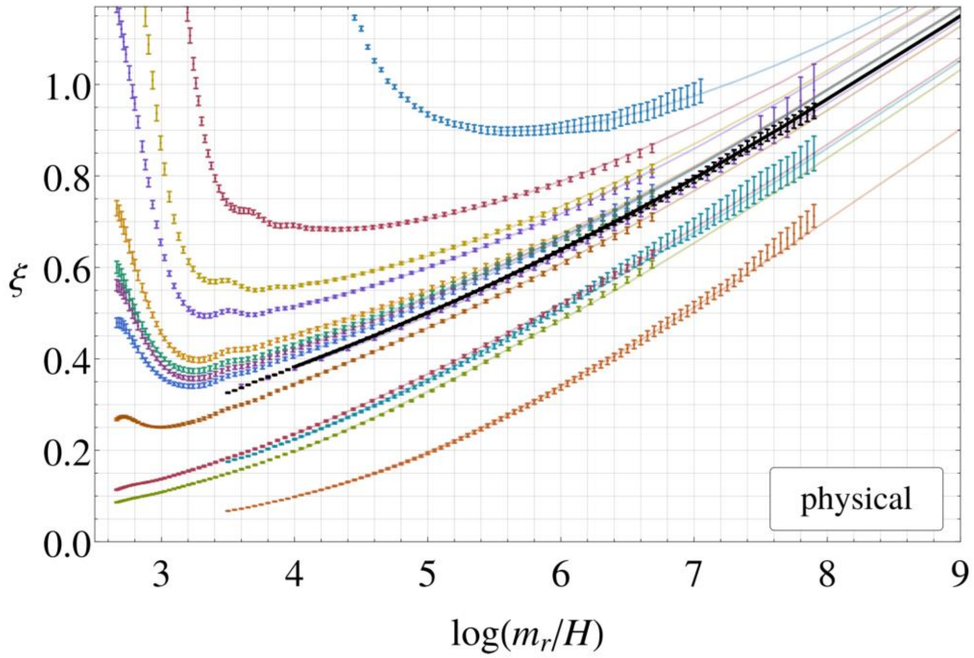
relic axions and gravitational waves

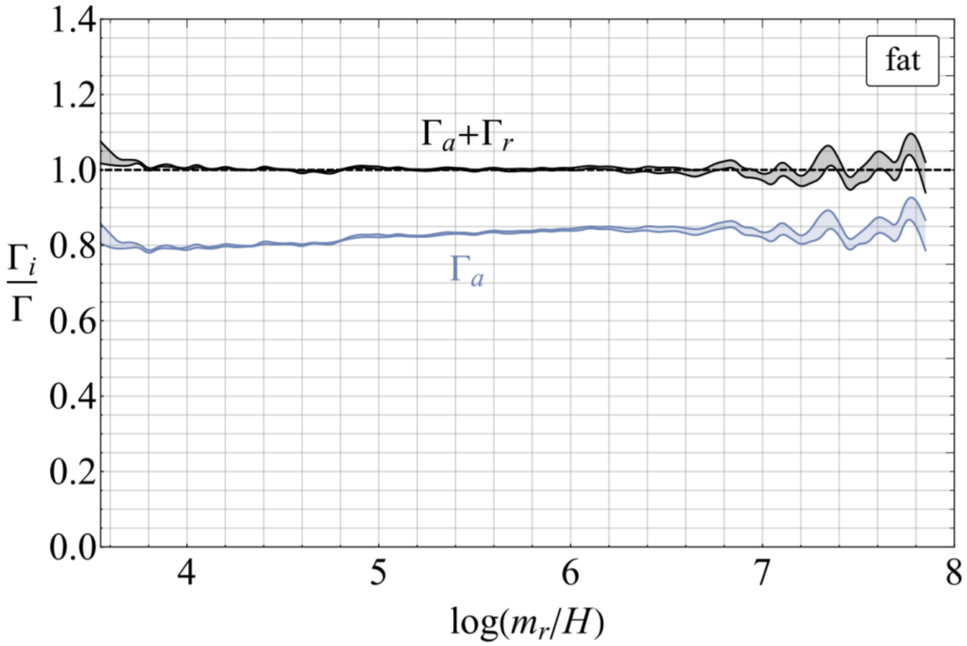
$$\underbrace{\hspace{6cm}}_{\textbf{strings}} \quad \underbrace{\hspace{2cm}}_{\textbf{domain walls}}$$



Attractor and Energy Emission







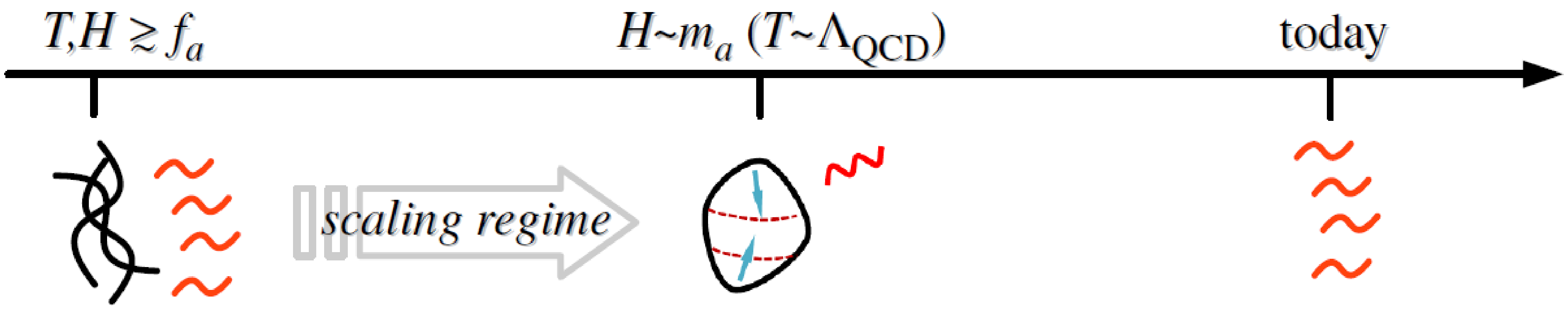
Scaling Violation

$$\boxed{\xi = c_1 \log} + c_0 + \frac{c_{-1}}{\log} + \frac{c_{-2}}{\log^2}$$

→

$$\Gamma_a = \frac{\xi\mu}{t^3} \propto \frac{f_a^2 \log^2}{t^3}$$

Gravitational Waves



$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70 \div 100}$

GWs from the scaling regime

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$

Nambu–Goto EFT

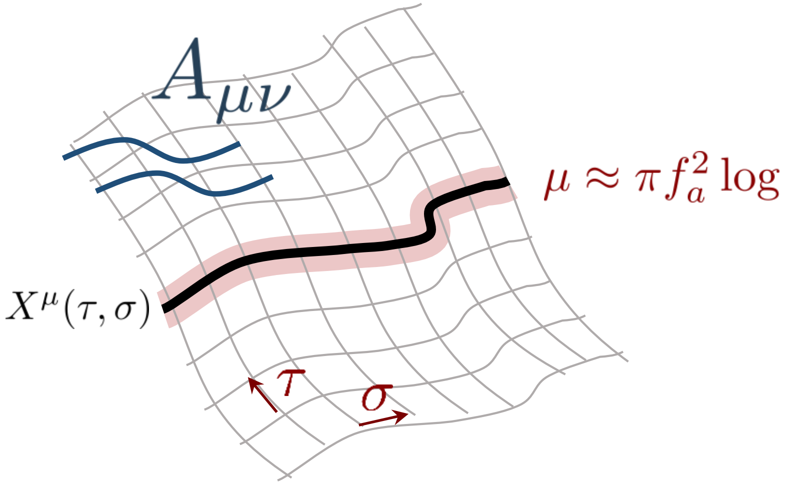
$\Gamma_g \longleftrightarrow \Gamma_a$

String Effective Theory

Degrees of freedom:

- $a \quad \longleftrightarrow \quad A_{\mu\nu}$
- $X^\mu(\tau,\sigma)$

$$\partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma}\partial_\sigma a$$



$$S_\phi[\phi]$$

$$\Big\downarrow$$

axion-to-string coupling

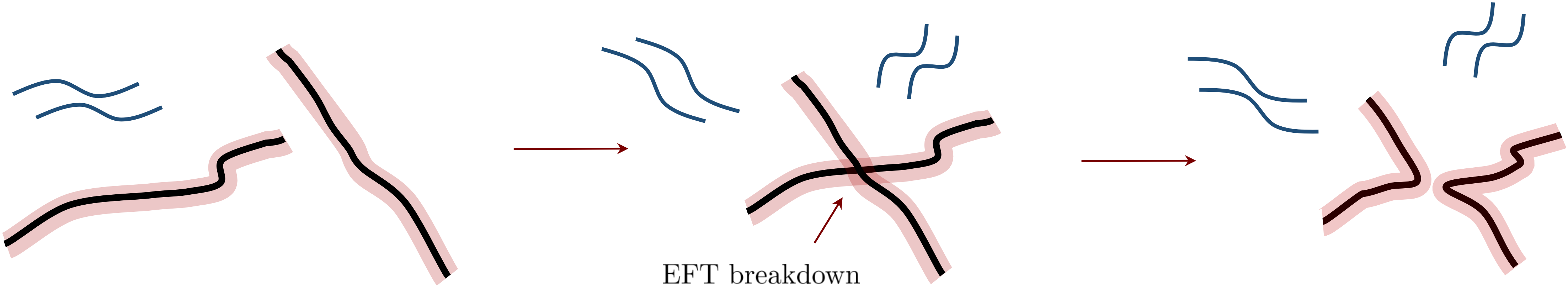
$$S_{\rm EFT}[X,A] = \underbrace{-\mu\int d\tau d\sigma\,\sqrt{-\gamma}}_{\text{Nambu–Goto action}}\;\underbrace{-\tfrac{1}{6}\int d^4x\,(\partial A)^2}_{\text{Axion kinetic term}}\;\underbrace{+\boxed{2\pi f_a}\int d\tau d\sigma\;\partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}}_{\text{Axion-string interaction}}$$

(Kalb–Ramond action)

$$\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$$

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



Gravitational Wave Emission

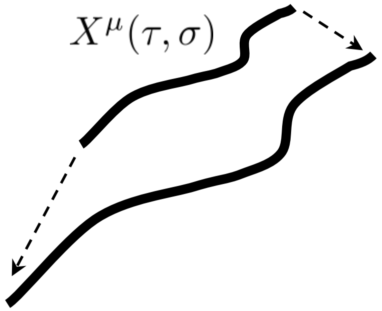
EoM: $\Box_x A^{\mu\nu} = 2\pi \boxed{f_a} \int d\sigma \, \dot{X}^{[\mu} X'^{\nu]} \delta^3(\vec{x} - \vec{X})$

Einstein Eq: $\Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T^{\lambda}_{s\,\lambda}\right)$

$$T_s^{\mu\nu} = \boxed{\mu} \int d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu\right) \delta^3(\vec{x} - \vec{X})$$

$$\frac{dE_a}{dt} = \underbrace{r_a[X]} \boxed{f_a^2} \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]} G\boxed{\mu^2}$$

dimensionless functionals of the shape of the string trajectory X^μ



$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2}$$

$= \text{const} = \mathcal{O}(1)$

$$\boxed{\Gamma_g = r \frac{G\mu^2}{f_a^2} \Gamma_a} \propto \frac{\log^4}{t^3}$$

$\frac{\xi\mu}{t^3}$

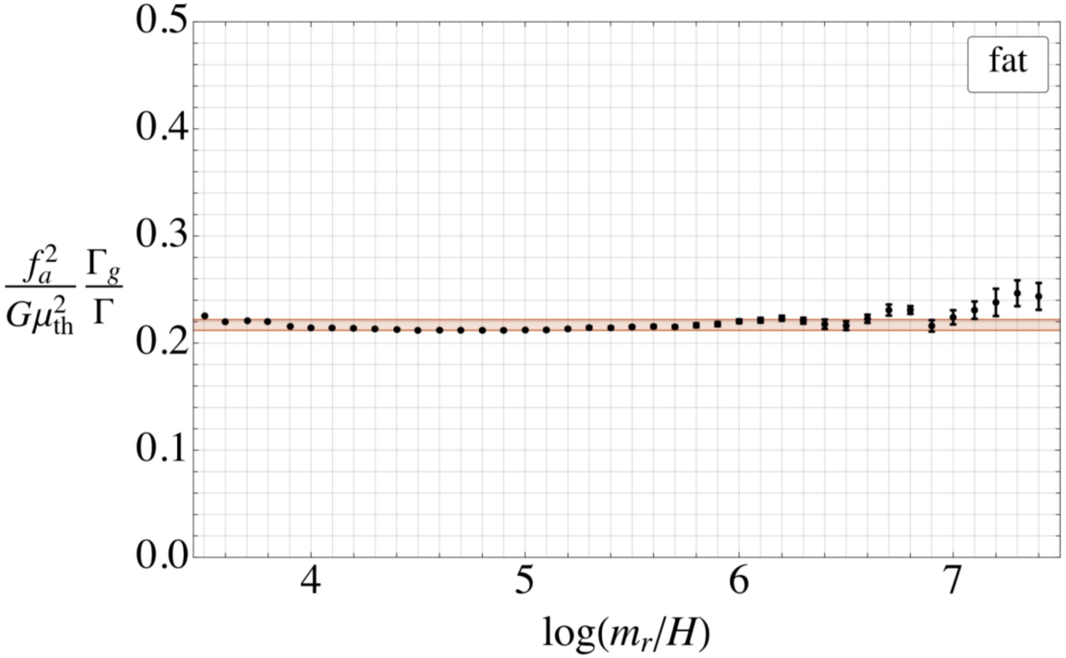
Comparison with field theory simulations

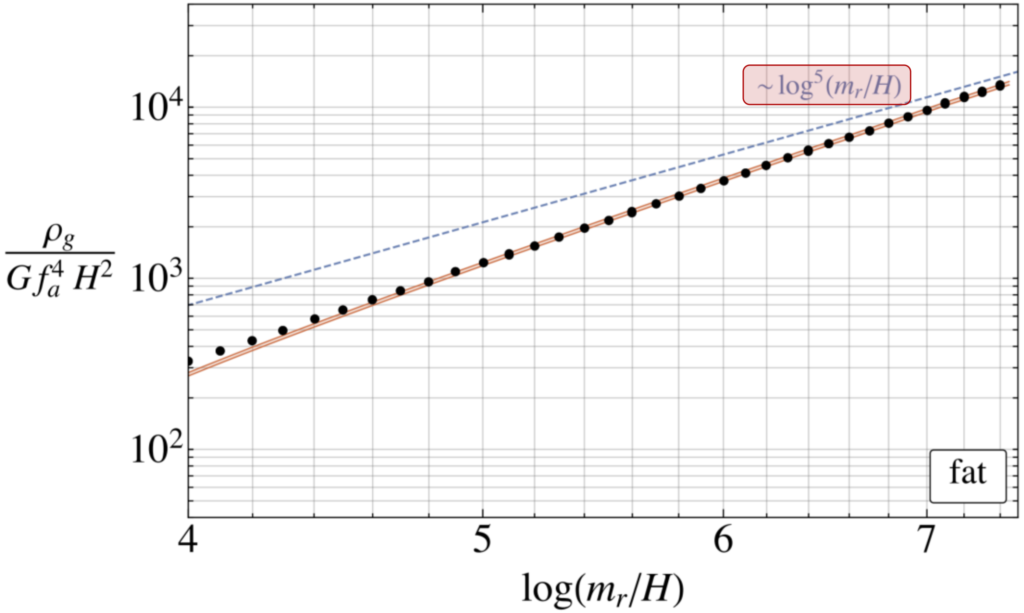
1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial\Gamma_g}{\partial k}$

$$\boxed{\Gamma_g = r\,\frac{G\mu^2}{f_a^2}\,\Gamma_a}$$

$\frac{\xi\mu}{t^3}$



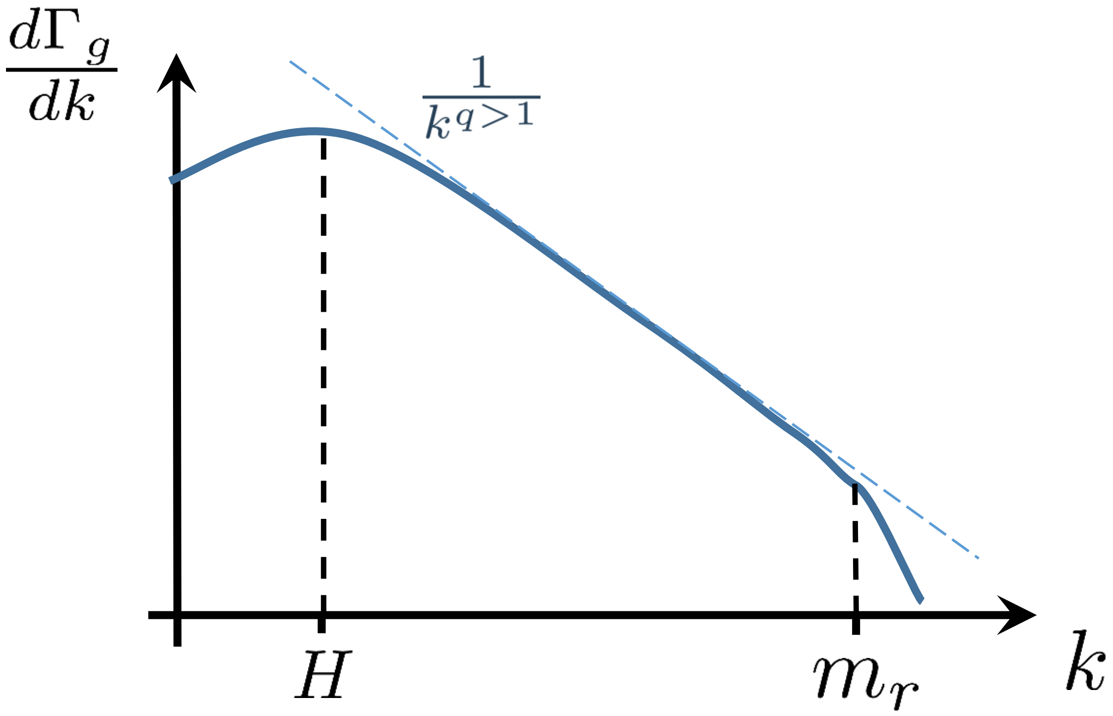


total GW energy: $\rho_g = \int dt'\,\left(\frac{R'}{R}\right)^4\Gamma'_g \propto \frac{\boxed{\log^5}}{t^2}$

The Gravitational Wave Spectrum

1) GW energy emission rate: $\Gamma_g(t)$

2) momentum distribution: $\frac{\partial \Gamma_g}{\partial k}$



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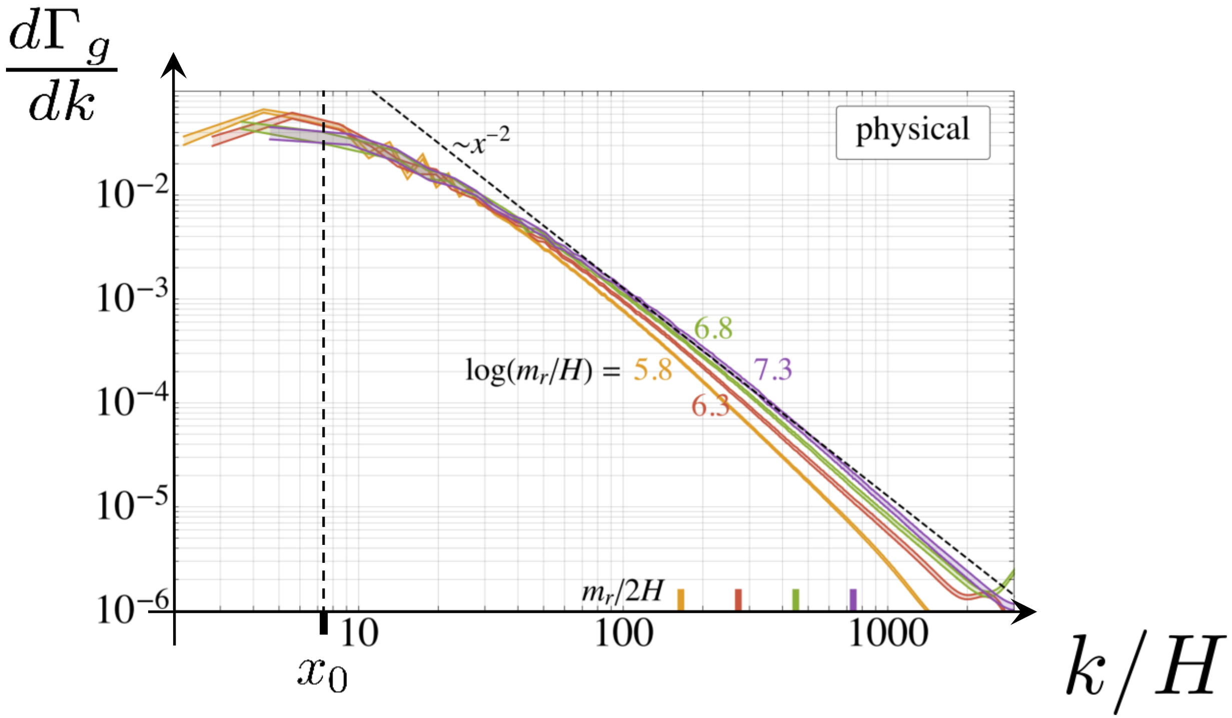
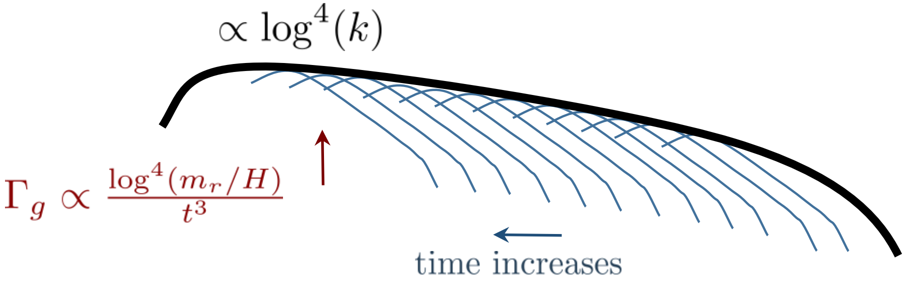


$$\frac{\partial \rho_g}{\partial \log k} \equiv \int dt'\, \frac{d\overset{\propto \frac{\log^4}{t^3}}{\boxed{\Gamma'_g}}}{d\log k}\left(\frac{R'}{R}\right)^4 \qquad =$$

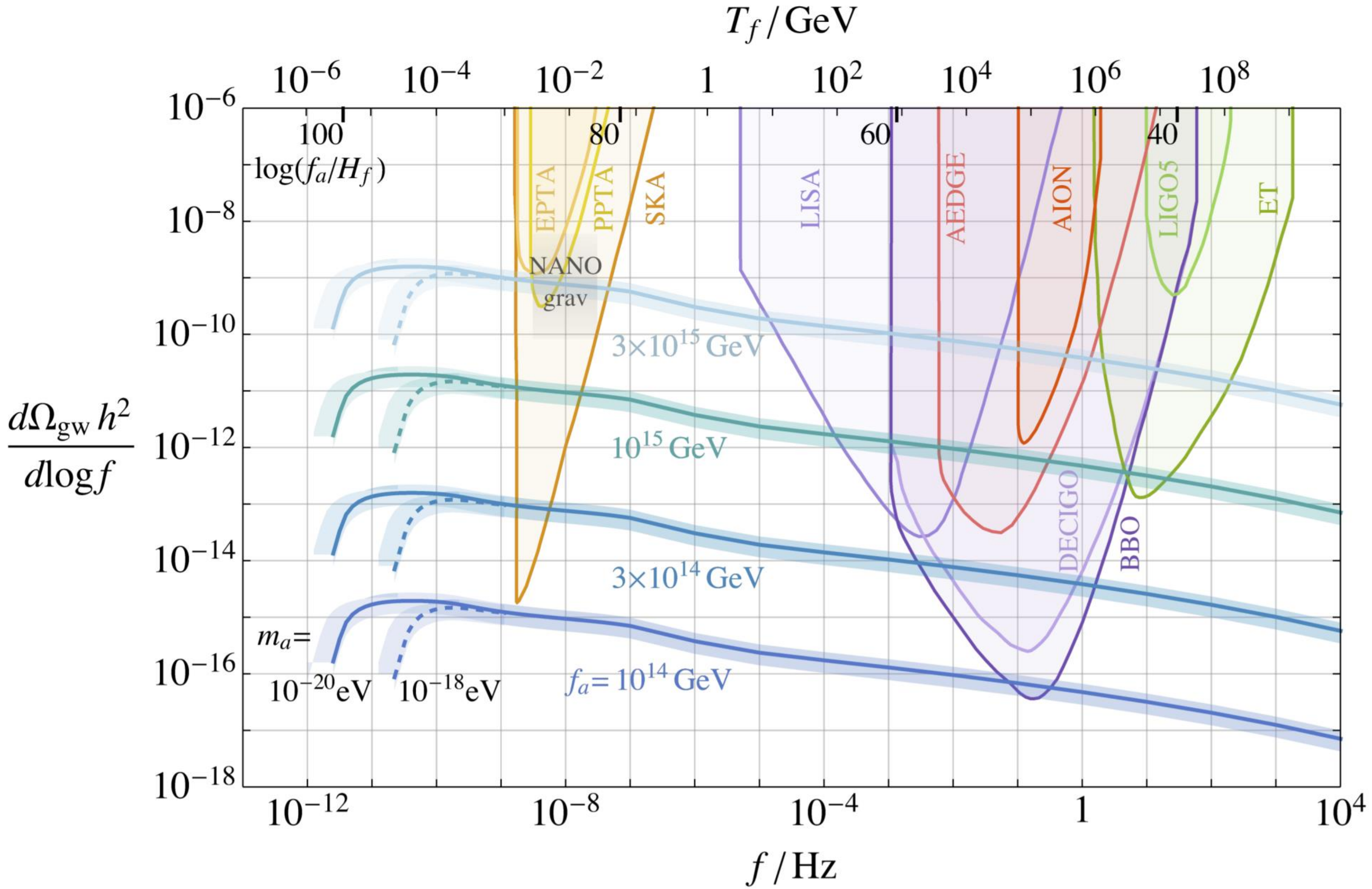


$\propto \log^4(k)$

$\Gamma_g \propto \frac{\log^4(m_r/H)}{t^3}$

time increases

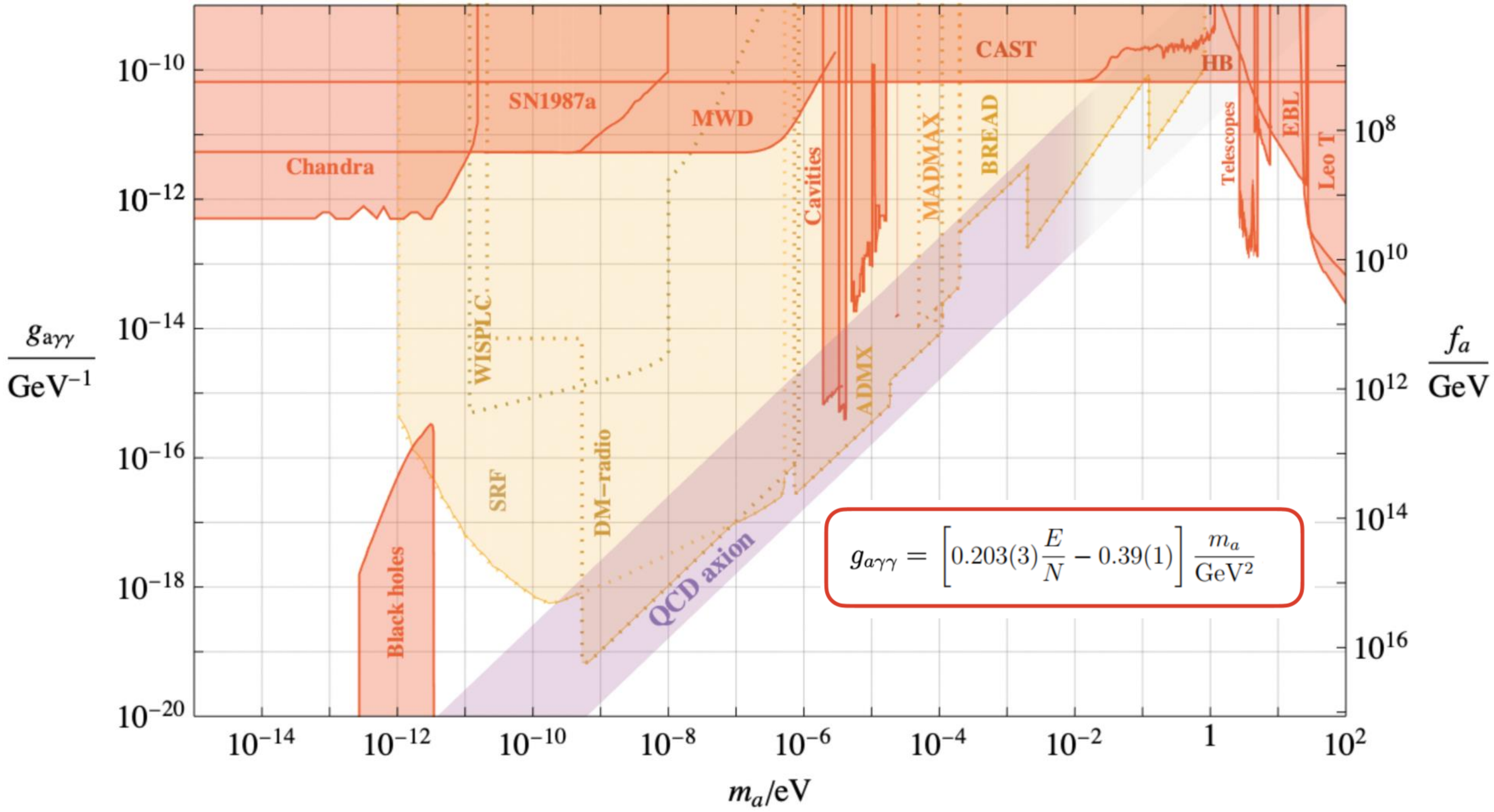
- approximately scale invariant
- $\log^4$ enhancement



$$\begin{cases} f_a \lesssim 10^{15}\ \text{GeV} \\ m_a \lesssim 10^{-18}\ \text{eV} \end{cases}$$

$$\frac{d\Omega_{\rm gw}h^2}{d\log f} \simeq 10^{-15}\left(\frac{r}{0.26}\right)\left(\frac{f_a}{10^{14}\text{GeV}}\right)^4\left(\frac{10}{g_f}\right)^{\frac{1}{3}}\left\{1+0.12\log\left[\left(\frac{m_r}{10^{14}\text{GeV}}\right)\left(\frac{10^{-8}\text{Hz}}{f}\right)^2\right]\right\}^4$$

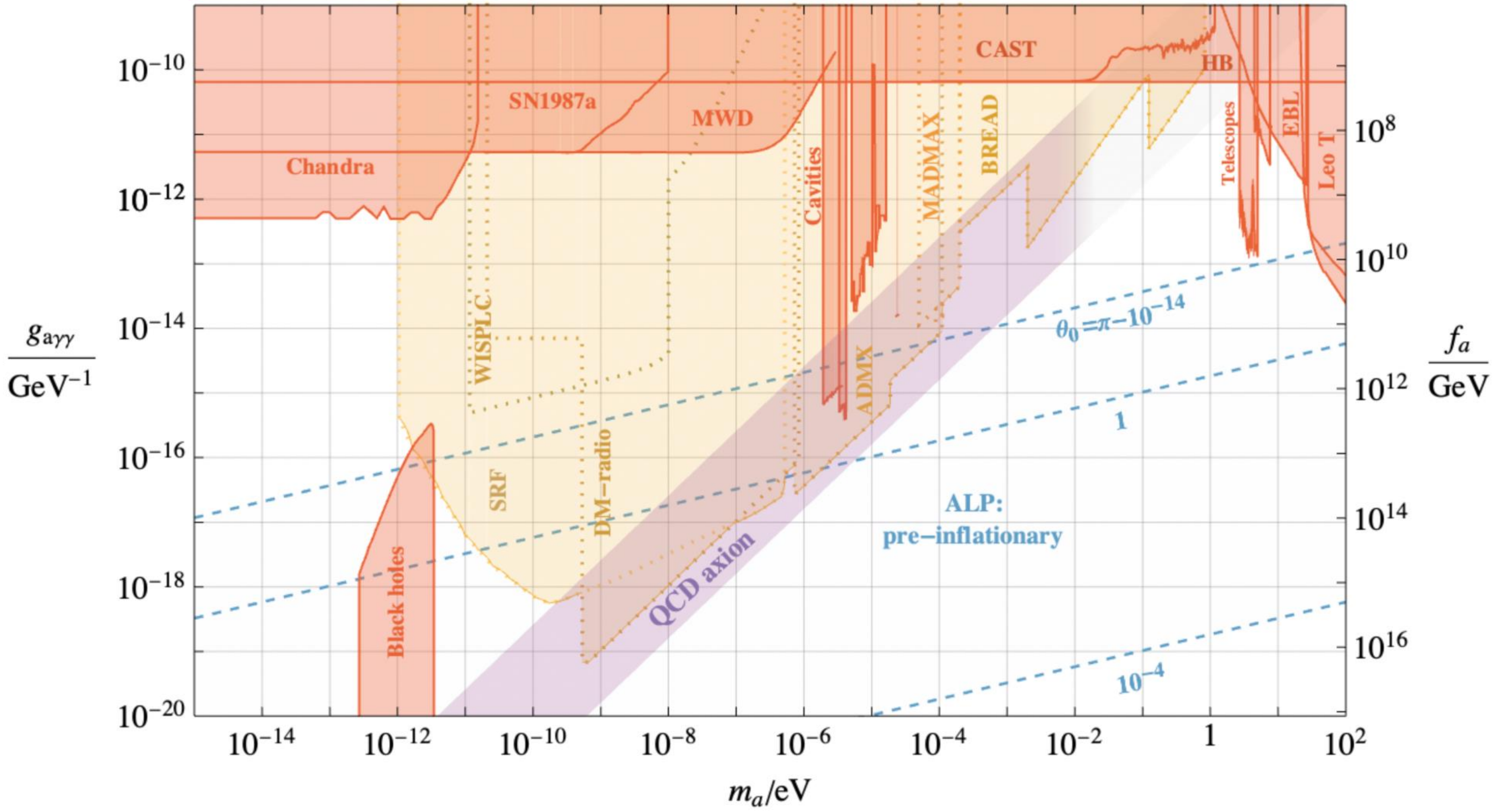
Targets for haloscopes



$$g_{a\gamma\gamma} = \left[0.203(3)\frac{E}{N} - 0.39(1)\right]\frac{m_a}{\mathrm{GeV}^2}$$

$$g_{a\gamma\gamma} \simeq \frac{\alpha_{em}}{2\pi f_a}$$

Targets for haloscopes

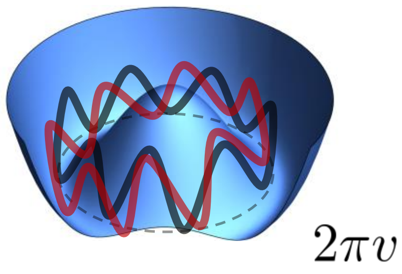


$$\frac{\Omega_a^{\rm mis}}{\Omega_{\rm DM}} \simeq 2.2 \cdot 10^{-3} \left(\frac{f_a}{10^{12}\ \ {\rm GeV}}\right)^2 \left(\frac{m_a}{10^{-6}\ \ {\rm eV}}\right)^{1/2} h(\theta_0)\,\theta_0^2$$

$$g_{a\gamma\gamma} \simeq \frac{\alpha_{em}}{2\pi f_a}$$

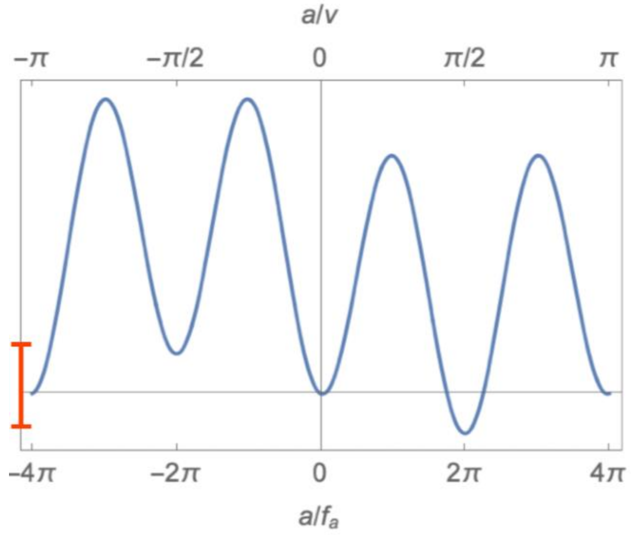
Domain wall number N>1

$$\phi = \frac{r+v}{\sqrt{2}} e^{i\frac{a}{v}}$$

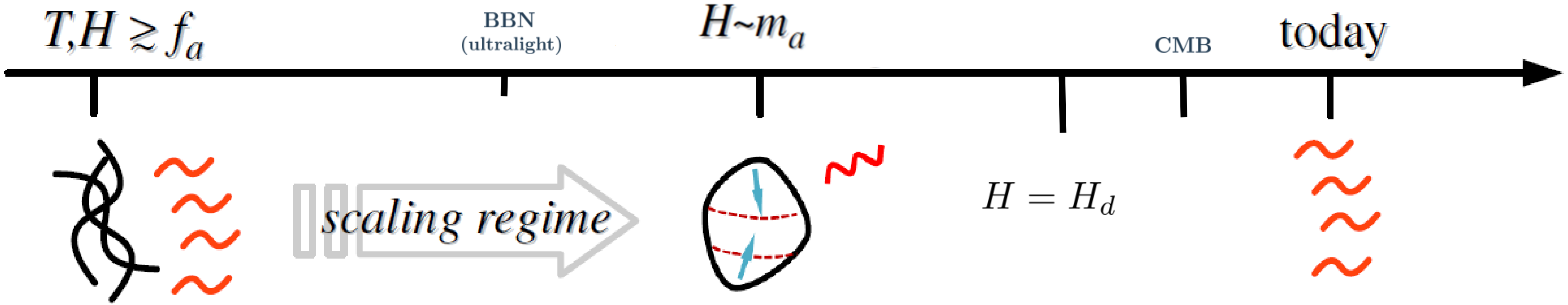


$$\boxed{v = N f_a}$$

$$V(a) = \frac{m_a^2 v^2}{N^2}\left(1-\cos\left(N\frac{a}{v}\right)\right) + \delta V_{PQV}$$



$N = 2$



$T,H \gtrsim f_a$

BBN
(ultralight)

$H{\sim}m_a$

CMB

today

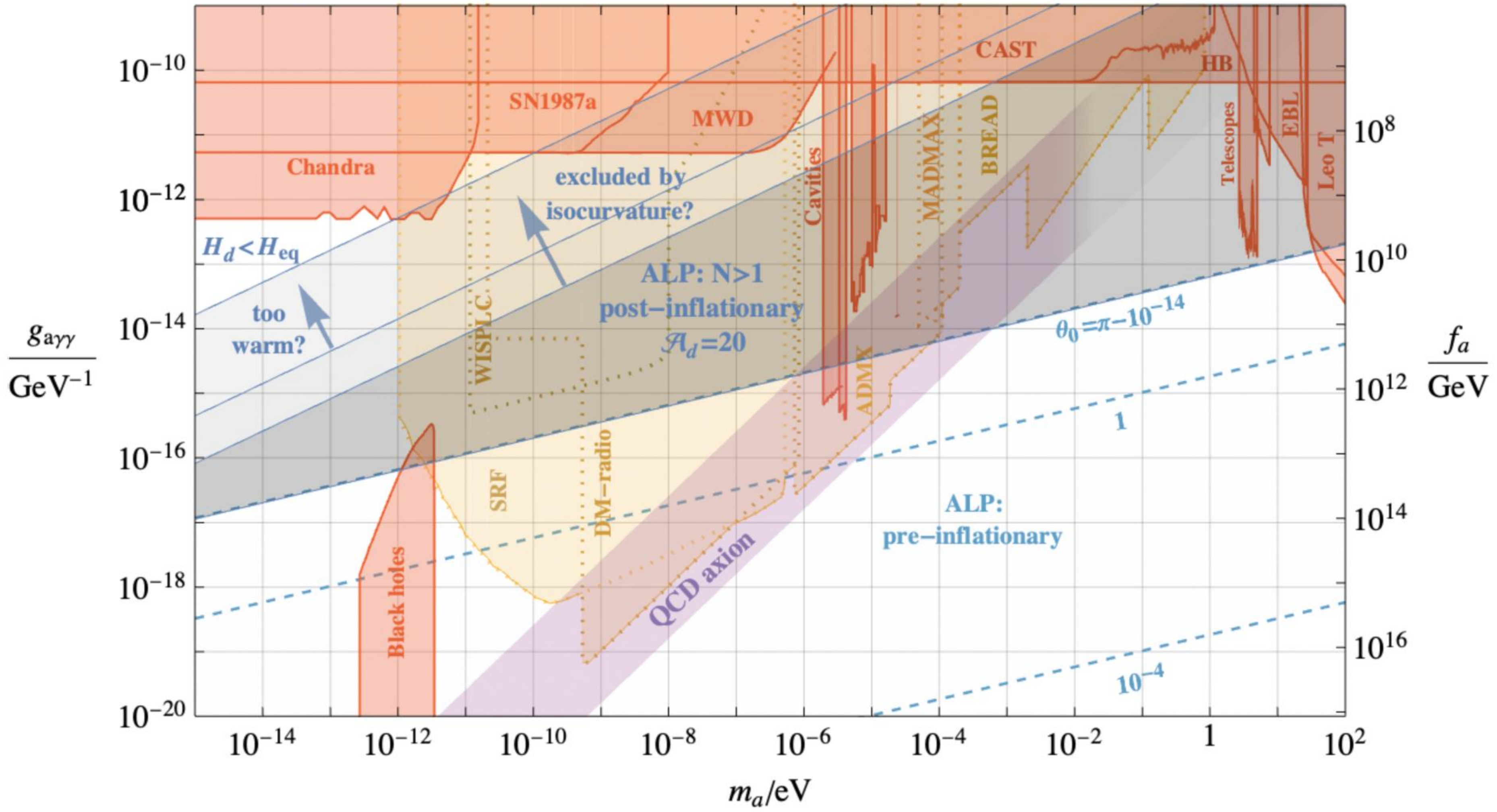
scaling regime

$H = H_d$

domain walls
annihilate

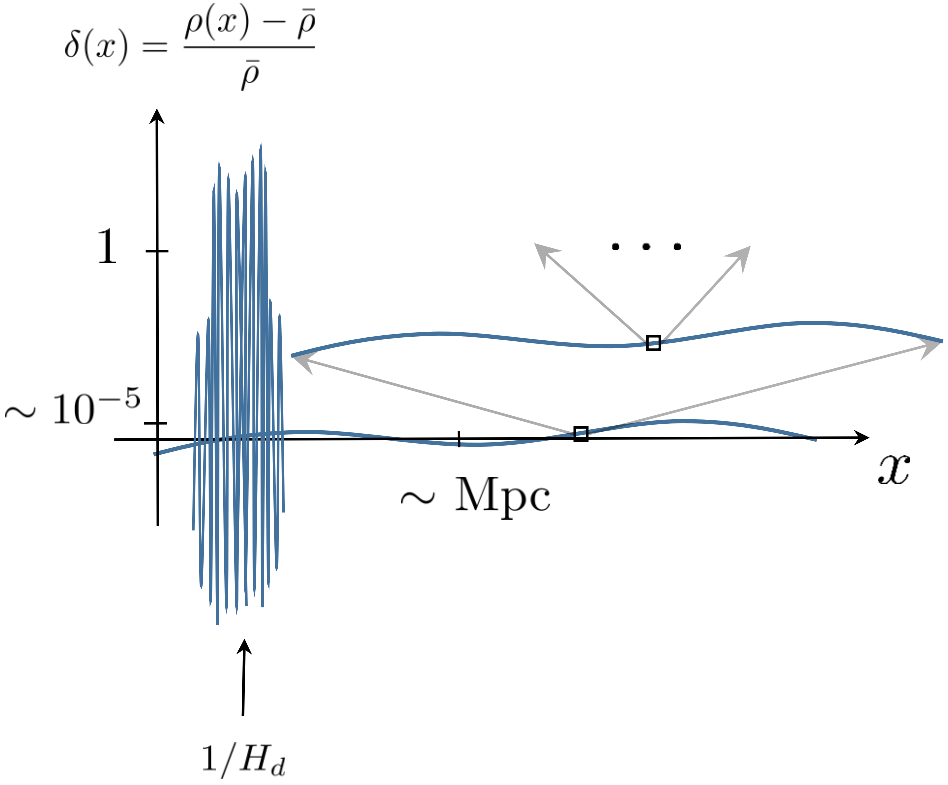
relic axions
and gravitational waves

Targets for haloscopes

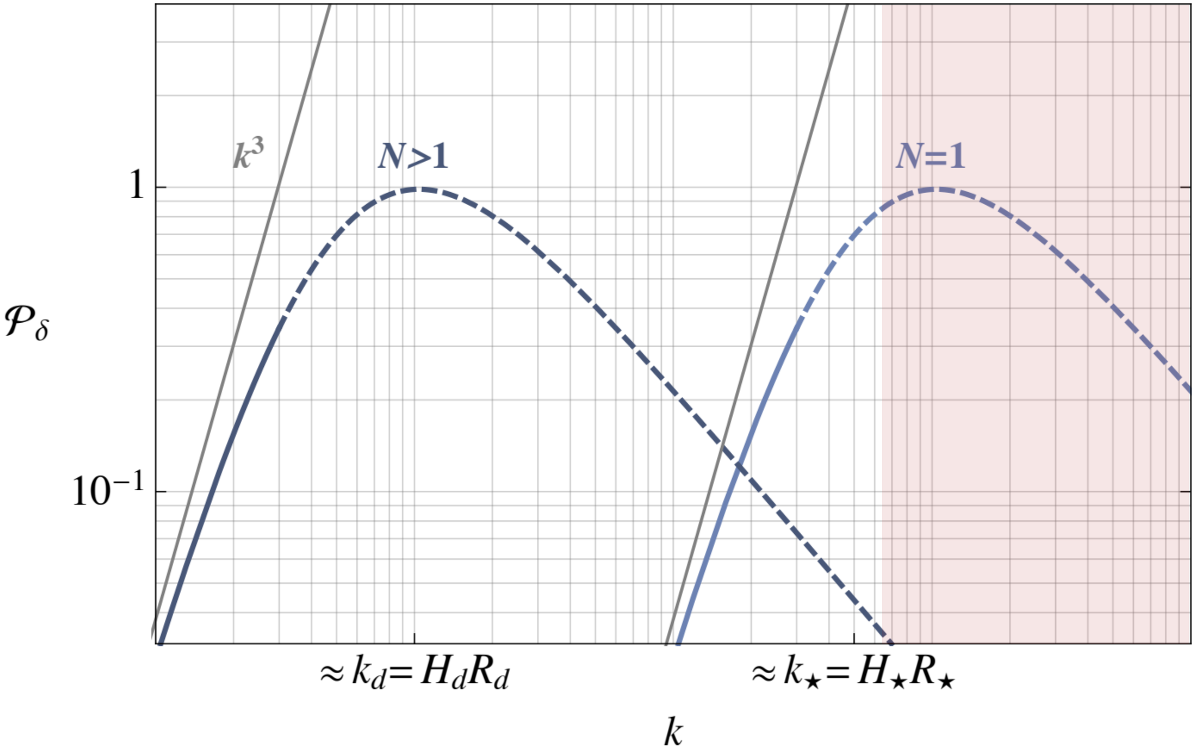


$$\frac{\Omega_\mathrm{a}}{\Omega_\mathrm{DM}} \simeq 2\left(\frac{\mathcal{A}_d}{20}\right)\left(\frac{m_a}{H_d}\right)^{1/2}\left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^2\left(\frac{m_a}{10^{-6}\ \mathrm{eV}}\right)^{1/2}$$

Overdensities at small scales



affected by
quantum pressure/self-interactions?



work in progress

standard lore: collapse at around
matter-radiation equality into miniclusters

Conclusions

- **The scaling regime produces an approximately scale invariant GW spectrum**

  → $\Gamma_g \propto \log^4$ (from the increase in $\mu \propto \log$ and $\xi \propto \log$)

  → enhances the spectrum at low frequencies

- **The spectrum is visible by multiple experiments for $f_a > 10^{14}$ GeV**

  → best prospects in PTAs and LISA

- **ALPs with $N > 1$ in the post-inflationary scenario have a potentially large coupling to photons**

Much more to do

- Local strings?
- QCD axion dark matter substructure
- Similar dynamics in other theories of light dark matter (e.g. dark photons)

Thank You