Heavy Field Radiative Corrections to the Primordial Power Spectrum during inflation

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The Standard Model of Cosmology present shortcomings related to the initial conditions. Cosmic inflation is a dynamical and attractor solution of these problems.

Key feature: nearly scale-invariant primordial power spectrum.

1. Radiative effects of **heavy fields** on the inflaton two-point function.
2. Dependence of the primordial power spectrum on the **initial time**.

Poincaré symmetry is explicitly broken by the evolving background metric.

Cosmological observables acquire an explicit time dependence.

We are interested in expectation values of the type

$$\langle Q(t)\rangle = \langle in|Q(t)|in\rangle,$$

Bunch Davies vacuum

At sub-horizon scale the spacetime has to be locally approximated by Minkowski. $|0\rangle$ free theory vacuum.

The expectation value we are interested in is

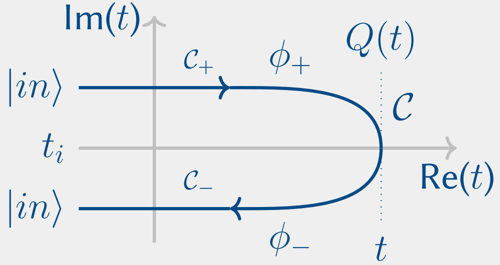
$$\langle Q(t)\rangle = \langle in|Q(t)|in\rangle,$$

In-in master formula

$$\langle Q(t)\rangle = \left\langle 0\left|\tilde{T}e^{i\int_{t_i}^{t} H_{\rm int}^{I}(t')\mathrm{d}t'}Q^{I}(t)Te^{-i\int_{t_i}^{t} H_{\rm int}^{I}(t'')\mathrm{d}t''}\right|0\right\rangle.$$

In-in master formula

$$\langle Q(t)\rangle = \left\langle 0 \left| \tilde{T} e^{i\int_{t_i}^{t} H_{\rm int}^I(t')\mathrm{d}t'} Q^I(t) T e^{-i\int_{t_i}^{t} H_{\rm int}^I(t'')\mathrm{d}t''} \right| 0 \right\rangle .$$



The four propagators are

$$G^{++}(x,y) = i\langle 0|T_{\mathcal{C}}\phi_+(x)\phi_+(y)|0\rangle = i\langle 0|T\phi(x)\phi(y)|0\rangle\ ,$$

$$G^{+-}(x,y) = i\langle 0|T_{\mathcal{C}}\phi_+(x)\phi_-(y)|0\rangle = i\langle 0|\phi(y)\phi(x)|0\rangle\ ,$$

$$G^{-+}(x,y) = i\langle 0|T_{\mathcal{C}}\phi_-(x)\phi_+(y)|0\rangle = i\langle 0|\phi(x)\phi(y)|0\rangle\ ,$$

$$G^{--}(x,y) = i\langle 0|T_{\mathcal{C}}\phi_-(x)\phi_-(y)|0\rangle = i\langle 0|\tilde{T}\phi(x)\phi(y)|0\rangle\ .$$

Rotating the fields to the **Keldysh basis**:

$$\phi_1 = \frac{1}{2}(\phi_+ + \phi_-) \quad \& \quad \phi_2 = (\phi_- - \phi_+),$$

and the propagators:

$$F(x,y) := -i\,G_{11} = -\frac{i}{2}\left[G^{-+}(x,y) + G^{+-}(x,y)\right],$$

$$G^R(x,y) := G_{12} = \theta\,(x_0 - y_0)\left[G^{-+}(x,y) - G^{+-}(x,y)\right],$$

$$G^A(x,y) := G_{21} = \theta\,(y_0 - x_0)\left[G^{+-}(x,y) - G^{-+}(x,y)\right],$$

$$G_{22} = 0.$$

Propagators in the Schwinger-Keldysh formalism

Field	Diagram	Hadamard Propagator
Inflaton		$F_\varphi(k, \tau_1, \tau_2)$
Heavy scalar		$F_\sigma(k, \tau_1, \tau_2)$
Heavy fermion		$F_\psi(k, \tau_1, \tau_2)$

Field	Diagram	Retarded Propagator
Inflaton		$-iG^R_\varphi(k, \tau_1, \tau_2)$
Heavy scalar		$-iG^R_\sigma(k, \tau_1, \tau_2)$
Heavy fermion		$-iG^R_\psi(k, \tau_1, \tau_2)$

The field content:

inflaton ϕ & massive scalar Σ & massive fermion χ

The fluctuations of the fields around their background:

$$\phi = \phi_0 + \varphi, \quad \Sigma = 0 + \sigma, \quad \chi = 0 + \psi,$$

The interacting potential:

$$V(\phi_0, \varphi, \sigma, \psi) = V_{\rm inf}(\phi) + \frac{1}{2}M_s^2\sigma^2 + \frac{\lambda_\Sigma}{4!}\sigma^4 + \mu\,\varphi\sigma^2 + \lambda\,\varphi^2\sigma^2 + M_f\bar{\psi}\psi + Y\varphi\bar{\psi}\psi.$$

Inflaton 2-point function - tree level

$$\underline{\tau \qquad\qquad \tau} \qquad : \qquad F_\varphi(k, \tau, \tau).$$

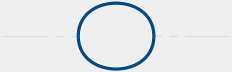


diagram (a)

diagram (b)

diagram (c)

diagram (d)

diagram (a)

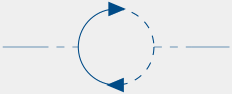


diagram (b)

diagram (c)

One loop amplitude

$$\begin{aligned} A^{1\,\mathrm{loop}}(k) \;&=\; \int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \left[-iG_{\varphi}^{R}(k,\tau,\tau_1)\right]\left[-iG_{\varphi}^{R}(k,\tau,\tau_2)\right] A_{GG} \\ &+ \int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \left[-iG_{\varphi}^{R}(k,\tau,\tau_1)\right]\left[F_{\varphi}(k,\tau,\tau_2)\right] A_{GF}\,, \end{aligned}$$

where

$$\begin{aligned} A_{GG} &:= A_{\mathrm{scalar}}^{(a+b+b')} + A_{\mathrm{fermion}}^{(a+b+b')}\,, \\ A_{GF} &:= A_{\mathrm{scalar}}^{(c+c'+d')} + A_{\mathrm{fermion}}^{(c+c')}\,. \end{aligned}$$

Ultraviolet divergences

UV regulator: Λ

The ultraviolet divergences in the one-loop two point function are

$$\boxed{A_{UV} = \frac{1}{2\pi^2 H^4 \tau_1^4}\left[\left(Y^2 - \frac{\lambda}{2}\right)\Lambda^2 + \left(\frac{\mu^2}{2} + \frac{\lambda M_s^2}{2}\right)\ln\frac{\Lambda}{M_s} - 3Y^2 M_f^2 \ln\frac{\Lambda}{M_f}\right]}$$

The UV divergences are the same as in the Minkowski spacetime. **Constant and covariant counterterms** can reabsorb the divergences.

The ultraviolet divergences in the one-loop two point function are

UV regulator: Λ

$$A_{UV} = \frac{1}{2\pi^2 H^4 \tau_1^4}\left[\left(Y^2 - \frac{\lambda}{2}\right)\Lambda^2 + \frac{1}{2}\left(\mu^2 + \lambda M_s^2\right)\ln\frac{\Lambda}{M_s} - 3Y^2 M_f^2 \ln\frac{\Lambda}{M_f}\right]$$

Having d_s scalar and d_f fermion degrees of freedom which independently contribute to the one-loop diagrams

$$A_{UV} = \frac{1}{2\pi^2 H^4 \tau_1^4}\left[\left(Y^2 d_f - \frac{\lambda}{2} d_s\right)\Lambda^2 + d_s\left(\frac{\mu^2}{2} + \frac{\lambda M_s^2}{2}\right)\ln\frac{\Lambda}{M_s} - 3Y^2 M_f^2 d_f \ln\frac{\Lambda}{M_f}\right].$$

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$$\lambda = \frac{Y^2}{2} \qquad \& \qquad \boxed{Y^2 d_f - \frac{\lambda d_s}{2} = 0}$$

The statistical properties of a random field $\delta(t, x)$ are determined by the correlation functions.

Power Spectrum

$$\langle \delta_{k_1}(t)\delta_{k_2}(t)\rangle = (2\pi)^3\delta^{(3)}\left(k_1 + k_2\right) P_\delta(k),$$

Dimensionless Power Spectrum

$$\Delta(k) := \frac{k^3}{2\pi^2} P(k),$$

Power Spectrum

Dimensionless Power Spectrum

$$\Delta(k) := \frac{k^3}{2\pi^2} P(k),$$

Perturbative Expansion

$$\Delta_\varphi(k) = \frac{k^3}{2\pi^2} \left(P_\varphi^{\text{tree}}(k) + P_\varphi^{1-\text{loop}}(k) + \dots \right) .$$

Inflaton Potential

$$V_{\rm inf} \simeq \lambda_h^2 M_G^4 + \frac{\lambda_h^4 M_G^4}{64\pi^2}\left[\ln\left(\frac{\lambda_h^2\phi_0^2}{2\mu_\Lambda^2}\right) + \mathcal{O}\left(\frac{M_G^4}{\phi_0^4}\right)\right] ,$$

The interaction between the inflaton and the heavy fields are

$$\mathcal{L}_{\rm int} \ni -\frac{\lambda_h^2}{4}\phi^2 \sum_{i=1}^{4} \Sigma_i^2 + \frac{\lambda_h}{\sqrt{2}}\phi\bar{\chi}\chi \,.$$

Radiative Correction to the Primordial Power Spectrum

In the supersymmetric setting we have **analytical cancellation** of the divergences, leaving a **finite contribution**:

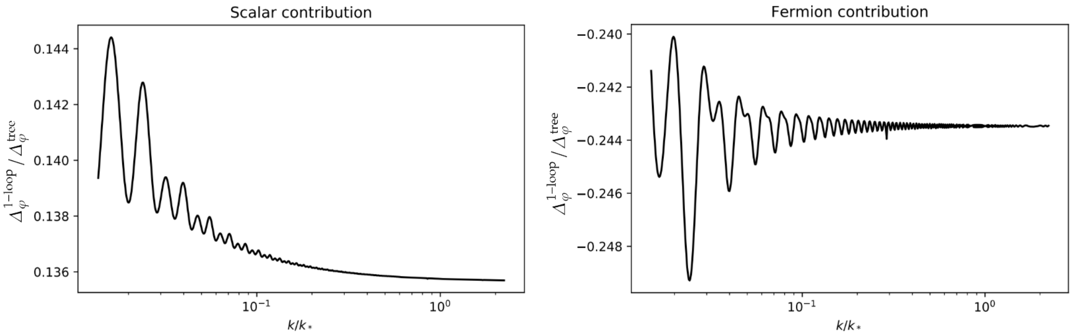
$$A_{UV} = \frac{1}{2\pi^2 H^4 \tau_1^4}\left[-\frac{3}{16}\lambda_h^4\phi_0^2 \log\left(1-\frac{M_G^4}{\phi_0^4}\right)+\frac{1}{16}\lambda_h^4 M_G^2 \ln\left(\frac{\phi_0^2-M_G^2}{\phi_0^2+M_G^2}\right)\right] .$$

Due to spontaneous SUSY breaking both the divergences cancel exactly.

We attach the external propagators to the amputated amplitude and integrate over τ_1 and τ_2.

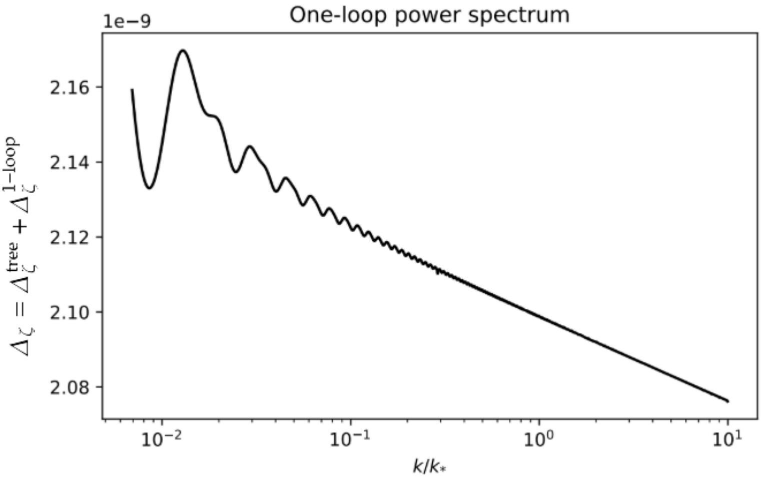
The **time dependence**: initial vacuum state chosen at a finite τ_{in}.

Radiative Correction to the Primordial Power Spectrum



One-loop contributions:

$$\left| \frac{\Delta_\varphi^{1-\text{loop}}}{\Delta_\varphi^{\text{tree}}} \right| \sim \mathcal{O}(0.1)$$


One-loop power spectrum
1e−9
2.16
2.14
2.12
2.10
2.08
$\Delta_\zeta = \Delta_\zeta^{\rm tree} + \Delta_\zeta^{\rm 1-loop}$
10^{-2}
10^{-1}
10^{0}
10^{1}
k/k_*

- Inflationary one-loop two-point function: isolation of the **quadratic** and **logarithmic divergences**. The divergences be cancelled by **covariant counterterms**.
- Fixing the initial conditions at finite time τ_{in}: **oscillatory behavior**.
- SUSY hybrid inflation: **ultraviolet divergences cancellation**; finite one-loop corrections $\sim \mathcal{O}(0.1)$.

Thank You!

Propagators

Mode functions solution

$$u(k,\tau) = a(\tau)\frac{H}{\sqrt{2k^3}}(1+ik\tau)e^{-ik\tau},$$

$$v(k,\tau) = -ie^{-\frac{\pi}{2}\nu+i\frac{\pi}{4}}\frac{\sqrt{\pi}}{2a(\tau)}H(-\tau)^{3/2}H^{(1)}_{i\nu}(-k\tau).$$

WKB approximated solution

$$v = \frac{1}{\sqrt{2\omega(\tau)}}\exp\left(\pm i\int_{\tau_{in}}^{\tau} \mathrm{d}\tau'\,\omega(\tau')\right) \quad \& \quad \omega_k(\tau) = \sqrt{k^2 + M_s^2 a^2(\tau)}.$$

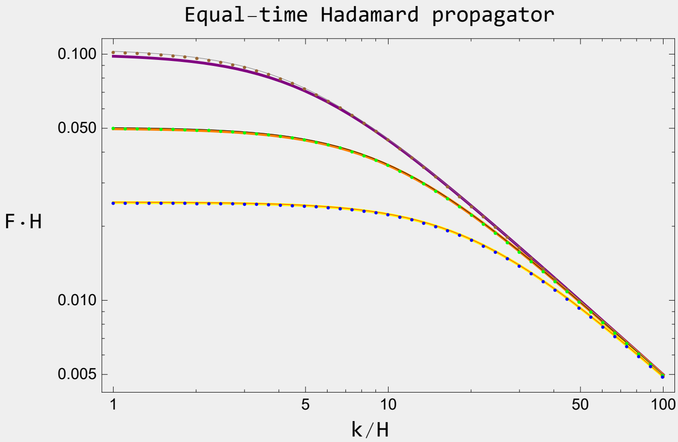


Figure: Comparison of the equal time Hadamard propagator constructed from the *full* and approximated mode functions.

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a^4(\tau_1) a^4(\tau_2)}{2} \int F_\sigma(p, \tau_1, \tau_2) F_\sigma(p, \tau_1, \tau_2) \, ,$$

Analytical Approximations

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a^4(\tau_1) a^4(\tau_2)}{2} \int F_\sigma(p,\tau_1,\tau_2) F_\sigma(p,\tau_1,\tau_2)\,,$$

Split of the momentum integral

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a_1^4 a_2^4}{4\pi^2} \left\{ \int_0^{p_c} p^2 \left[F_\sigma(p,\tau_1,\tau_2)\right]^2 + \int_{p_c}^{p_{UV}} p^2 \left[F_\sigma(p,\tau_1,\tau_2)\right]^2 \right\}\,.$$

Analytical Approximations

Split of the momentum integral

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a_1^4 a_2^4}{4\pi^2} \left\{ \int_0^{p_c} p^2 \left[F_\sigma(p, \tau_1, \tau_2)\right]^2 + \int_{p_c}^{p_{UV}} p^2 \left[F_\sigma(p, \tau_1, \tau_2)\right]^2 \right\} .$$

Neglect the external momentum

$$\left|\mathbf{k} + \boldsymbol{p}_{UV}\right| \approx \left|\boldsymbol{p}_{UV}\right| \equiv p.$$

Ultraviolet Cut-Off

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a_1^4 a_2^4}{4\pi^2} \left\{ \int_0^{p_c} p^2 \left[F_\sigma(p, \tau_1, \tau_2)\right]^2 + \int_{p_c}^{p_{UV}} p^2 \left[F_\sigma(p, \tau_1, \tau_2)\right]^2 \right\} .$$

Regularization

$$A^{(a)}_{\text{amp, scalar}} = \frac{\mu^2 a_1^4 a_2^4}{4\pi^2} \left\{ \int_0^{p_c} p^2 \left[F_\sigma(p,\tau_1,\tau_2)\right]^2 + \int_{p_c}^{p_{UV}} p^2 \left[F_\sigma(p,\tau_1,\tau_2)\right]^2 \right\} .$$

Physical cut-off

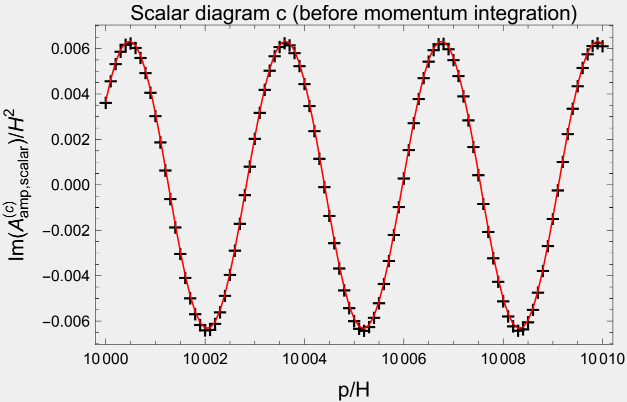
We are imposing cut-off on the comoving momenta

$$p_{UV} := \Lambda \, a(\tau_\beta),$$

with

$$a_\beta := a(\tau_\beta).$$

Using the *full* solution of the mode function we performed a numerical check of the calculation done with the WKB and other approximations.



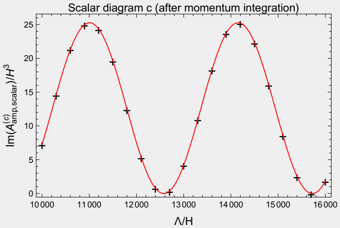
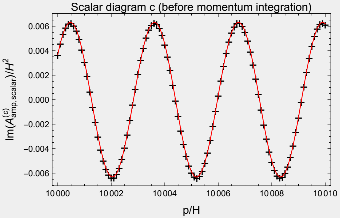


Figure: Integrand of the amputated amplitude of diagram (c), scalar contribution.

Numerical Analysis



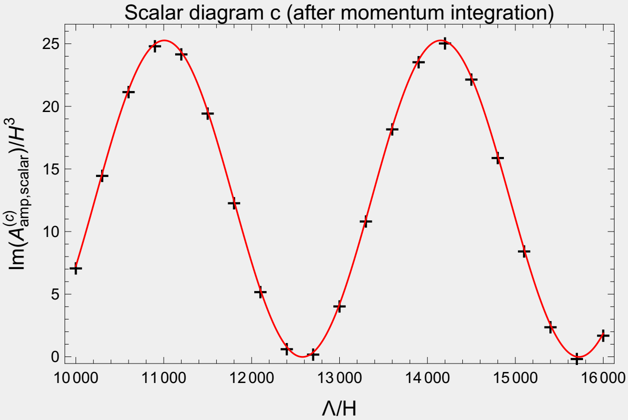
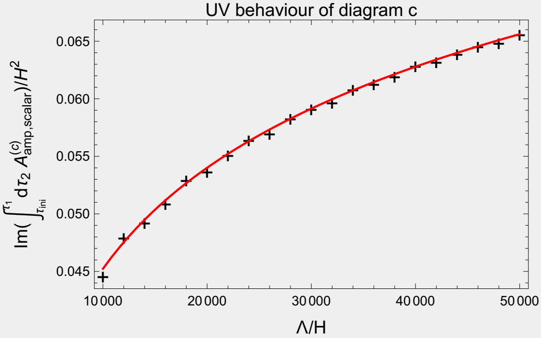
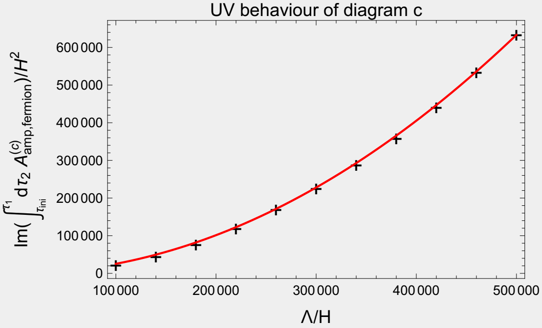


Figure: Result of the integration for the amputated amplitude of diagram (c), scalar contribution.

We can conclude that in both diagrams the WKB approximation and the split of the momentum integrations are good approximations of the *full* result.

Numerical Analysis





Numerical Analysis

We performed the numerical calculations without resorting to the adiabatic expansion approximation. We computed propagators, amputated amplitudes and UV behaviours fully numerically and compared with the analytical expressions. It shows that the use of the WKB approximation is perfectly sufficient to reproduce the behaviour of the exact numerical amplitude, both regarding the leading and sub-leading UV behaviour.

CMB temperature power spectrum

Due to the smoothing around a particular angular scale, the oscillations are not directly visible in the predicted power spectrum. In the optimistic scenario of large coupling, we were expecting to see the imprint of the primordial power spectrum oscillations in the CMB and be able to constrain the model parameters and τ_{in}.