Asymmetry
Essential Asymmetries of Nature

NYU

(little) BIG AXIONS

Based on **arxiv:2606.05276**
with Marius Kongsore & Neal Weiner

22/07/26

The LOW DOWN

1. New framework for generating **high quality axions** from **theory space non-locality**
2. Captures several existing solutions and reveals **rich algebraic** and **topological features**
3. Yields a high quality **phenomenologically viable QCD axion** in 4D compatible with **post-inflationary cosmology**

The STRONG CP PROBLEM

Expect **CP violation** in the strong sector

$$\mathcal{L} \supset \bar{\theta}\frac{g_s^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

BUT

Measurements of the **neutron EDM** constrain

$$\bar{\theta} \lesssim 10^{-10}$$

+ no anthropic solution ...

Enter THE QCD AXION

Pseudo-goldstone boson associated with the breaking of a global $U(1)_{PQ}$ symmetry that is **anomalous under QCD**

In the canonical PQ mechanism:

$U(1)_{PQ}$ broken by a **scalar field** getting a VEV:

$$\Phi = \frac{f+\rho}{\sqrt{2}} e^{i\frac{a}{f}}$$

axion

Mixed anomaly with QCD generated by coupling scalar to colored fermions with chiral $U(1)_{PQ}$ charges

$$\mathcal{L} \supset \theta_{\text{eff}} \frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

where

$$\theta_{\text{eff}} = \bar{\theta} - \frac{a}{f}$$

θ promoted to a dynamical variable!

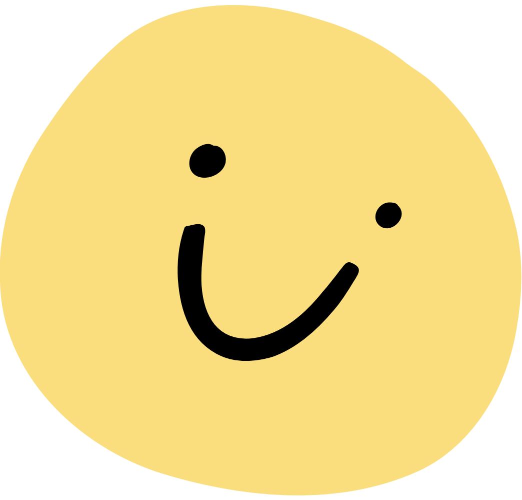
Enter THE QCD AXION

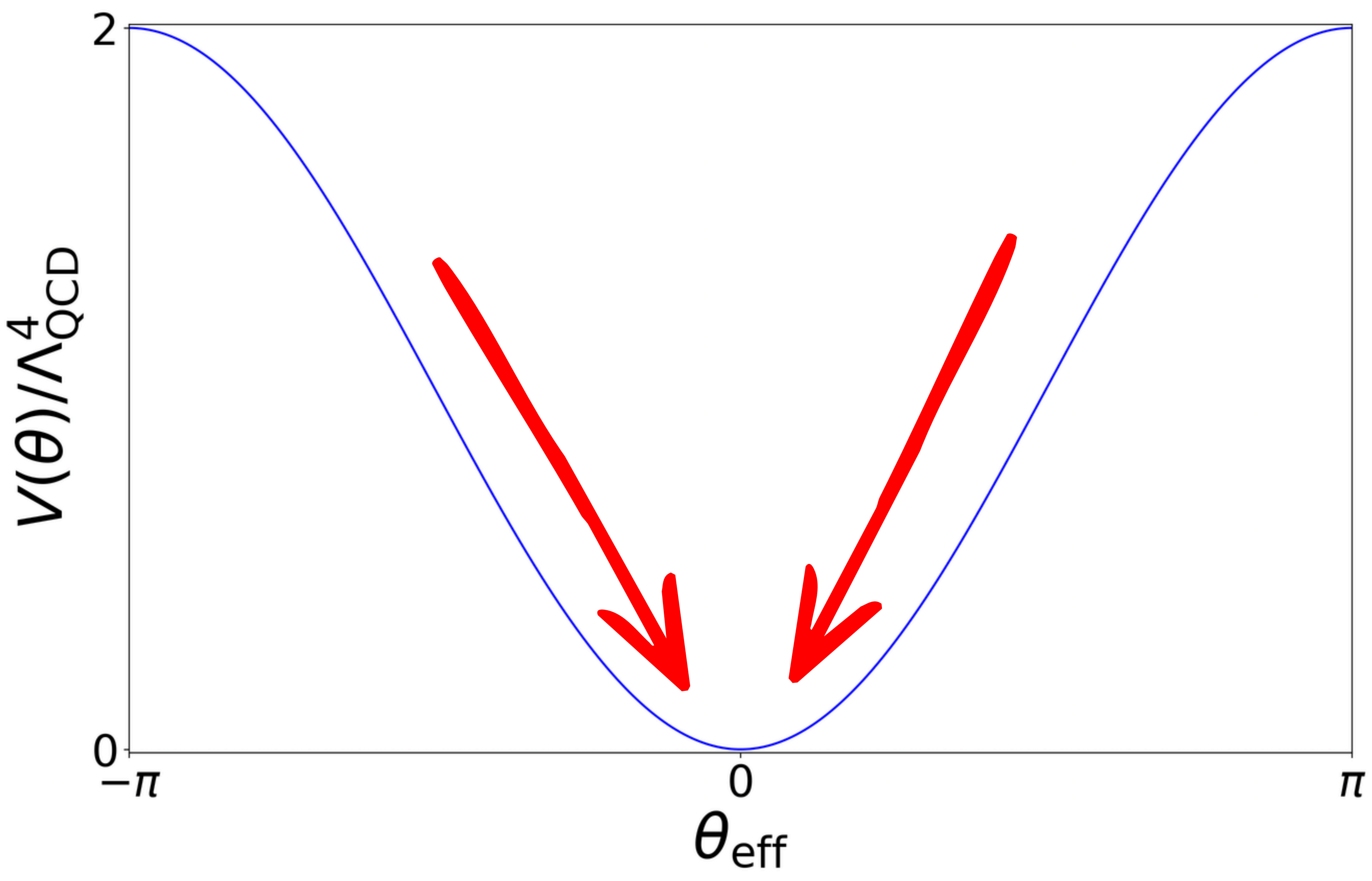
Non-perturbative instanton effects generate a potential $V(\theta_{\text{eff}}) \sim \Lambda_{\text{QCD}}^4 (1 - \cos\theta_{\text{eff}})$

with minimum at $\theta_{\text{eff}} = 0$

Dynamical relaxation: $\langle \theta_{\text{eff}} \rangle = 0$

NO STRONG CP!





The AXION QUALITY PROBLEM

The axion provides an elegant resolution to the strong CP in the IR

BUT...

Quantum gravity expected to break global symmetries via terms of the form:

$$\mathcal{L} \supset c\frac{\Phi^n}{M_{\mathrm{pl}}^{n-4}} + \mathrm{h.c.}$$

Generates additional contributions to $V(\theta_{\mathrm{eff}})$ which **shift the minimum**

from 0 and **re-introduce strong CP...**

For $f_a = 10^{12}$ GeV need $n > 13$

High QUALITY SOLUTIONS

1 Discrete Symmetries

Impose a discrete gauge symmetry:

$$\Phi \to e^{\frac{2\pi i}{N}} \Phi$$

Typically realised by breaking a gauged U(1) with very high charge e.g. in Barr & Seckel

BUT

In tension with post-inflationary cosmology;

- stable fractionally charged states
- domain wall problem

See Lu, Reece, Sun: [2312.07650]

2 Extra-Dimensions

Axion arises as the wilson loop of an **extra-dimensional gauge field** around the **extra dimension**

Embeds the shift symmetry in a **higher form symmetry**

See Craig & Kongsore: [2408.10295]

A FALSE DICHOTOMY

4D Solutions

- Involve a global PQ symmetry
- Can yield pre- or post- inflationary cosmology (i.e. axion strings/defects)
- High quality solutions incompatible with post-inflationary cosmology

Recent work has solved quality whilst retaining strings/walls by combining PQ symmetry with Extra dimensions
[Petrossian-Byrne, Villadoro 2503.16387]
[Loladze, Platschorre, Reig 2503.18707]

BIG AXIONS

High quality & post-inflationary cosmology in 4D

Extra-Dimensions

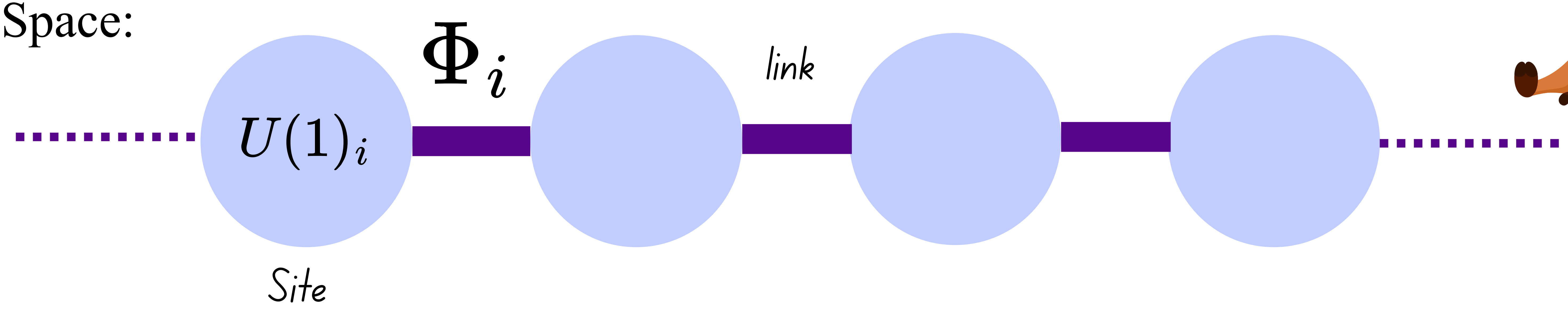
- Higher form symmetry
- Strictly pre-inflationary cosmology (no PQ strings)

Dimensional DECONSTRUCTION

Attractive features of extra-dimensions captured in 4D via **theory space non-locality**

Organised using **Moose** or **Quiver** diagrams:

Theory Space:



e.g. 1d Deconstructed Moose, gauged clockwork

Each scalar is **charged** under **exactly 2** (nearest neighbor) groups

Gauge invariant operators have to be built from **multiple different fields**

High Dimension = Symmetry Protection

LESSONS

1. **Theory space non-locality** is a powerful protector of quality
2. **Moose diagrams** provide an **intuitive graphical representation** BUT are **limited to highly restricted theory space structures** (each field is charged under exactly 2 groups)

GOALS

1. **Elevate theory space structure** from a tool for discretizing extra dimensions to an **organising principle** for protected low energy physics
2. Allow for **more general theory space topologies** which go beyond the linear 1d moose

Big Axion THEORY SPACES

The ingredients:

N_Φ scalar
'network Higgs' fields
$\Phi_i \quad i \in 1 \ldots N_\Phi$

$N_A \;\; U(1)_\alpha$
gauge fields
$A_\alpha \quad \alpha \in 1 \ldots N_A$

Each with a U(1) phase rotation symmetry → **COUPLE TOGETHER** → Combinations of the U(1) phase symmetries become gauged

The CHARGE MATRIX

Characterises the **gauge structure** of network Higgs fields

Central organizing object of the Big Axion framework

1D Moose corresponds to square matrices with 2 non-zero entries in every row and column

$$\hat{\mathbf{Q}} = \overbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}^{N_\Phi} \left.\vphantom{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}}\right\} N_A$$

$$\hat{\mathbf{Q}} : \mathbb{Z}^{N_\Phi} \to \mathbb{Z}^{N_A}$$

$\hat{\mathbf{Q}}_{\alpha i} =$ Charge of i^{th} network Higgs under the α^{th} $U(1)$

Under a gauge transform:

$$\Phi_i \to \exp\left(i \sum_\alpha Q_{\alpha i} \lambda_\alpha(x) \right) \Phi_i$$

Big AXIONS

Big axions correspond to the phases of **gauge invariant** operators

For every vector in the kernel of the charge matrix i.e. $\hat{Q}\mathbf{n} = 0$

the operator:

$$\mathcal{O}_{\mathbf{n}} = \prod_{i=1}^{N_\Phi} \Phi_i^{[n_i]} \qquad [n_i] = \begin{cases} \Phi_i^{|n_i|} & \text{if} \quad n_i > 0 \\ (\Phi_i^*)^{|n_i|} & \text{if} \quad n_i < 0 \end{cases}$$

is

GAUGE INVARIANT

Big AXIONS

Parametrize Network Higgs fields as: $\Phi_i = \frac{f_i}{\sqrt{2}} e^{i\theta_i}$ where $\theta_i = \frac{a_i}{f_i}$

such that under gauge transformations: $\theta_i \to \theta_i + Q_{\alpha i}\lambda_\alpha$

Thus:

BIG AXION

$$\Theta_{\mathbf{n}} = \arg\left(\prod_{i=1}^{N_\Phi} \Phi_i^{[n_i]}\right) = \sum_{i=1}^{N_\Phi} n_i\theta_i$$

is **gauge invariant**

and has an associated **shift symmetry**

All other phase combinations are eaten by gauge bosons

A HIGH QUALITY SOLUTION

Quantum gravity effects expected to **break the shift symmetry** at the Planck scale do so at **lowest order via terms** of the form:

$$\mathcal{L} \supset \frac{c \prod_i \Phi_i^{[n_i]}}{M_{\rm Pl}^{(\sum_i |n_i|)-4}} + \text{h.c.}$$

Accidental global PQ symmetry which is **protected** up to operators of dimension

$$D_{\rm quality} = \sum_i |n_i|$$

Due to **delocalisation** of the axion **across theory space**, this dimension can be **naturally large**, and the shift symmetry high quality

Big AXION GRAPHS

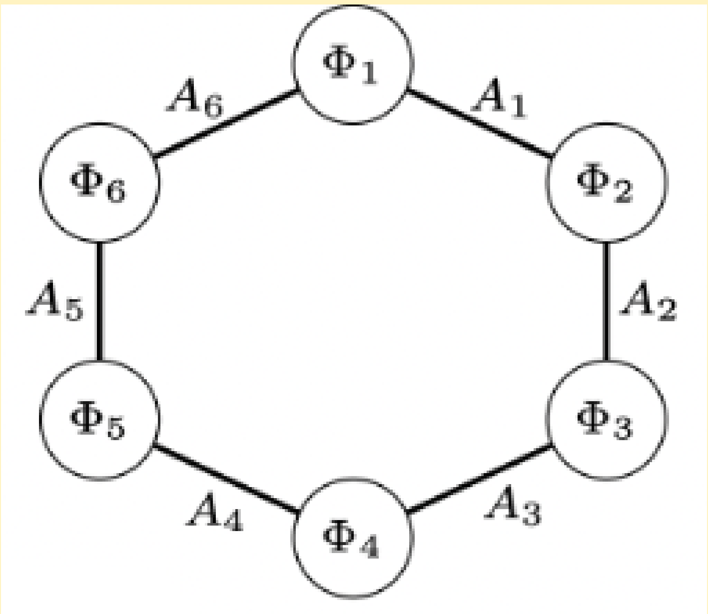
The conventional Moose provided a convenient graphical representation **BUT** **too restrictive** to capture full landscape of possible charge matrices

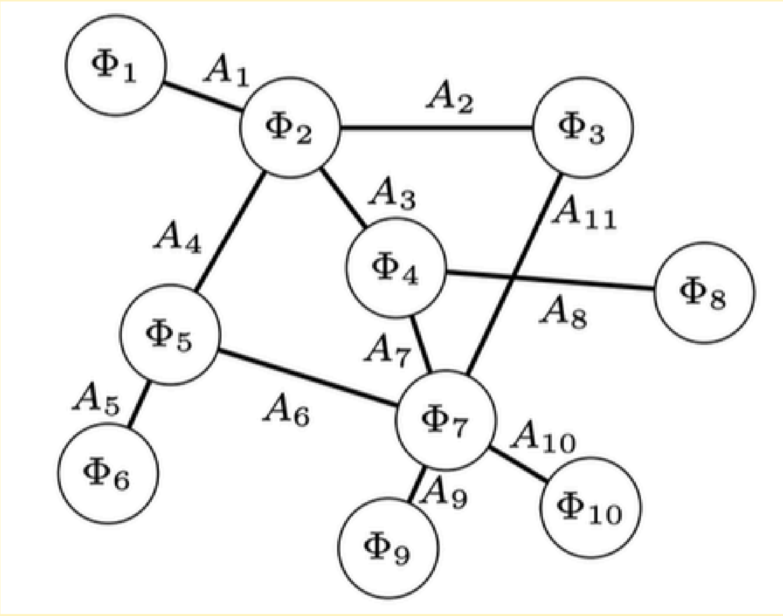
ENTER THE DUAL MOOSE:

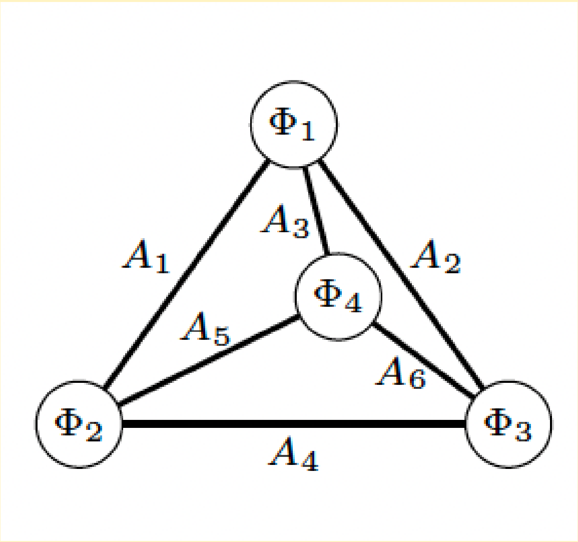
SITES ⟷ LINKS

Network Higgs fields (sites) can now be charged under **arbitrary number** of gauge groups (links) $\rightarrow$ **significantly richer** theory space structures and topology possible

e.g.



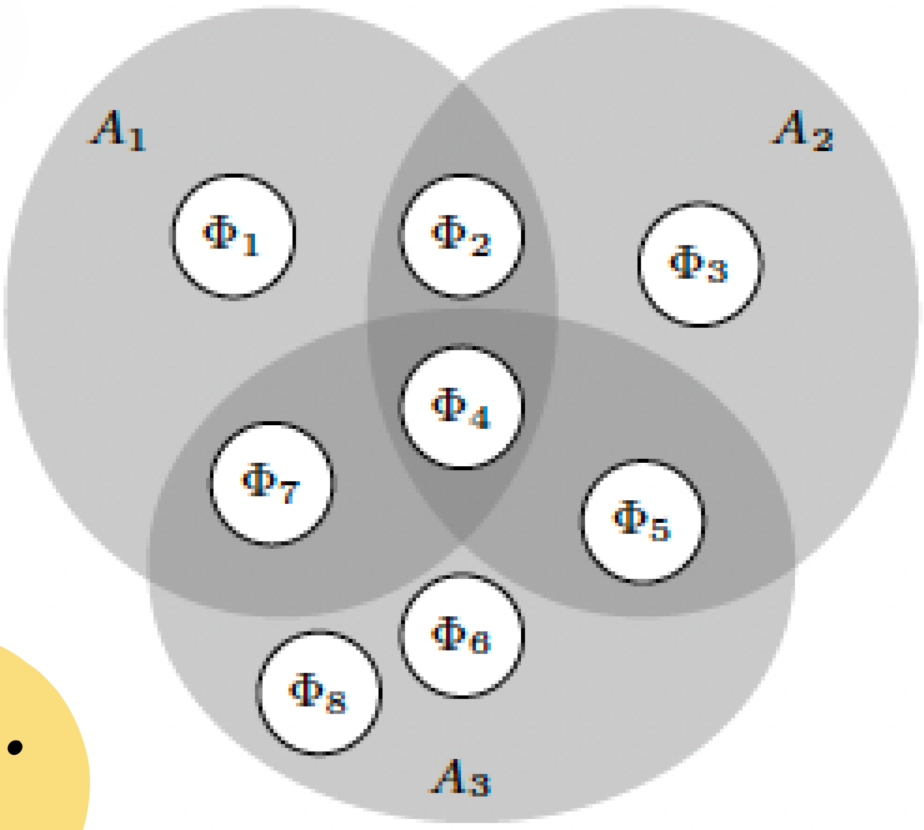




Big AXION GRAPHS

Still not fully general - each gauge group can couple to at most 2 network Higgs fields

To capture charge matrix in full generality require a **hypergraph**



Dual Moose models sufficient to illustrate novel pheno

An example BIG AXION

$$\hat{\mathbf{Q}}_{\text{star}} = \begin{pmatrix} -1 & q & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

KERNEL

$$\mathbf{n} = \begin{pmatrix} q \\ 1 \\ q \\ q \end{pmatrix}$$

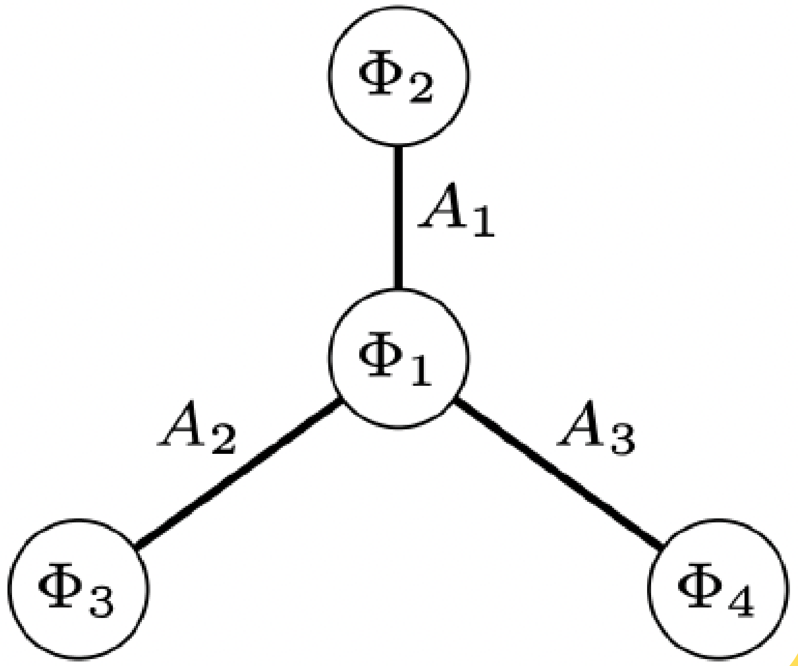
BIG AXION:

$$\begin{aligned} \Theta_n &= \arg\left(\Phi_1^q \Phi_2 \Phi_3^q \Phi_4^q\right) \\ &= q\theta_1 + \theta_2 + q\theta_3 + q\theta_4 \end{aligned}$$

QUALITY

$$D_{\text{quality}} = 3q + 1$$

DUAL MOOSE

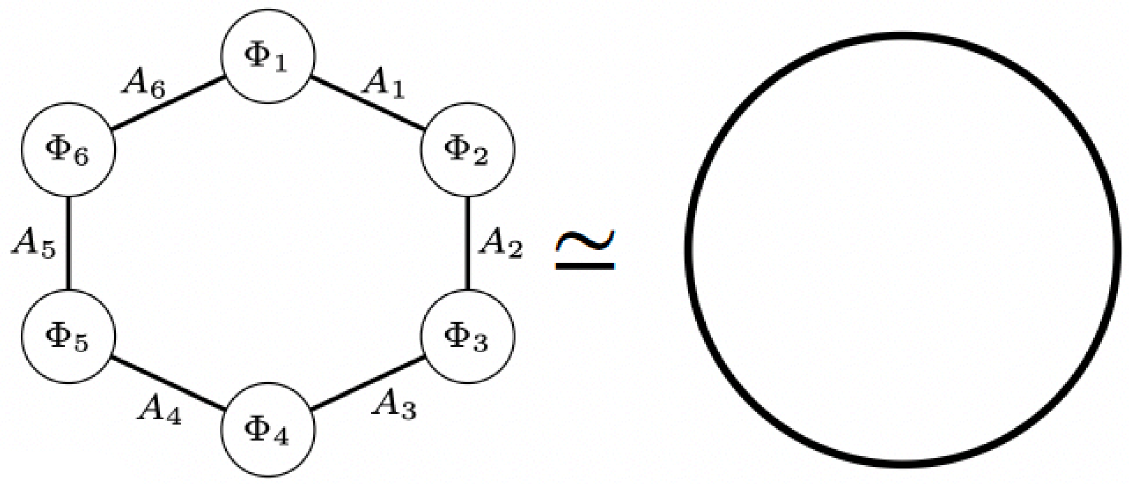


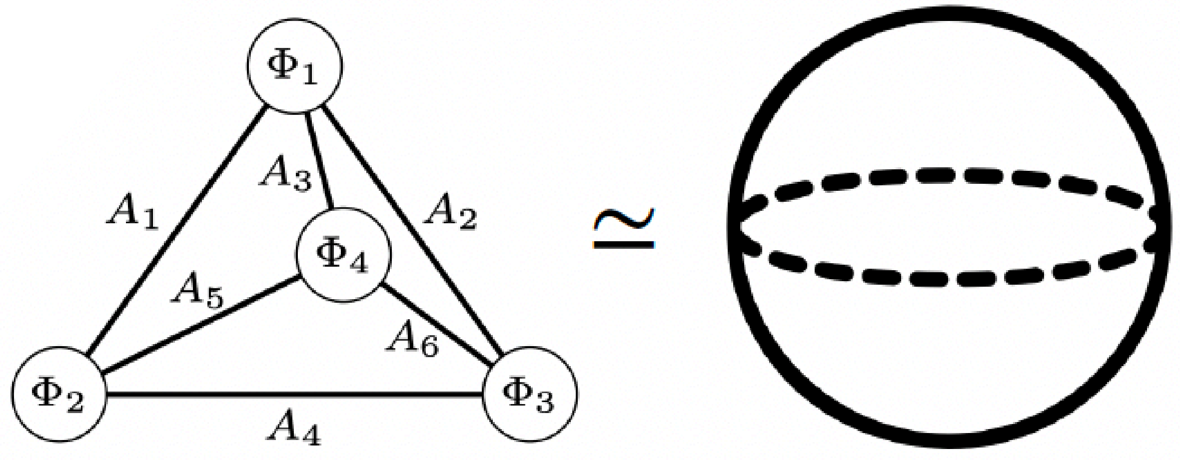
A RICH STRUCTURE

Framework is **entirely general** and captures a **rich** variety of theory space structures

When the charge matrix is an **incidence matrix of a cell complex**, big axion corresponds to a deconstruction of an extra dimension

In this case: Big Axion = lattice version of an extra-dimensional gauge field





Can be made precise using homology of cell complexes

Big QCD AXIONS

To solve strong CP need to **couple the big axion anomalously** to colored quarks

DFSZ
SM quarks

KSVZ
BSM colored fermions

BUT

Must not generate $U(1)_\alpha \times SU(3)_c \times SU(3)_c$ gauge anomalies

→ Need to **distribute colored fermions about the network** a way that mirrors the multiplicity of the network Higgs in the Big Axion parent operator

→ # colored quarks $\geq D_{\text{quality}}$

i.e. higher quality requires more colored fermions

little BIG QCD AXIONS

1-loop QCD beta function: $\beta(\alpha_s) = -\frac{1}{12\pi}(33 - 2n_f)\alpha_s^2$

For asymptotic freedom require $n_f \leq 16.5$

i.e. max 10.5 NEW colored fermions

Requires a *little* Big Axion!

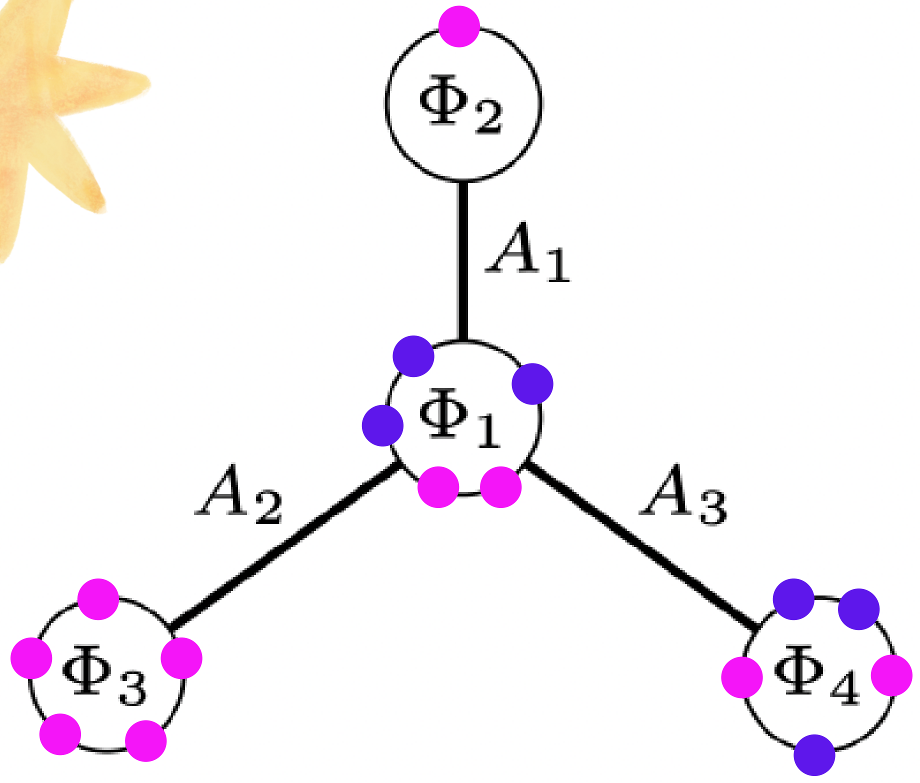
Need to use a **hybrid** of KSVZ & DFSZ mechanisms

A viable LITTLE BIG QCD AXION ?

RETURN OF THE STAR BIG AXION: Take q = 5

$$\Theta_n = 5\theta_1 + \theta_2 + 5\theta_3 + 5\theta_4$$

$D_{\text{quality}} = 16$ sufficient for $f_a = 10^{12}$ GeV



To couple to QCD without $U(1)_\alpha \times SU(3)_c \times SU(3)_c$ gauge anomalies require **16 colored fermions** for $N_{\text{DW}} = 1$

To retain asymptotic freedom:

10 new BSM colored fermions **(KSVZ-like)**

6 SM quarks coupled via a pair of Higgs doublets **(DFSZ-like)**

Post-INFLATIONARY COSMOLOGY

To prevent stable domain-wall networks which would dominate the energy density of the Universe require $N_{\rm DW} = 1$

BUT this alone is **not sufficient** when axion formed from multiple phases :

e.g. if $\Theta_n = \sum_i n_i \theta_i$ then if $\Delta\theta_i = 2\pi \rightarrow \Delta\Theta_n = 2\pi n_i$

Each string has an effective domain wall number $N_{\rm DW}^{\rm eff} = n_i N_{\rm DW}$

Need at least 1 field to have $n_i = 1$ for there to be a chance the network decays

Satisfied by Big Star Axion $\Theta_n = 5\theta_1 + \theta_2 + 5\theta_3 + 5\theta_4$ ✓

CAVEAT: Mixed **gauge-global** strings with different winding numbers - **non-trivial** and unstudied **dynamics**.

Instead may need to impose $f_2 \lesssim T_{\rm RH} \lesssim f_1, f_3, f_4$

Additional REQUIREMENTS

Need to add additional **colorless matter** to cancel

$U(1)_\alpha \times U(1)_\beta \times U(1)_Y$ and $U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$

gauge anomalies

Considerations:

- States carrying **hypercharge** contribute to **running** of α_Y
- In post-inflationary cosmologies: if you have KSVZ quarks that are neutral under both $SU(2)_L$ and $U(1)_Y$ they can combine with SM quarks to form **fractionally charged hadrons**: must **ensure that these can decay efficiently.**

ALL *in* ALL ...

Big Star Axion is a 4D, **phenomenologically viable** QCD axion compatible with **post-inflationary** cosmology

- ✓ $N_{\rm DW} = 1$
- ✓ A string with $N_{\rm DW}^{\rm eff} = 1$
- ✓ Gauge anomaly free
- ✓ Sufficient quality for $f_a = 10^{12}$ GeV
- ✓ All Landau poles above $M_{\rm pl}$
- ✓ No stable fractionally charged states

SUMMARY

New class of axion models with **high quality** from theory space non-locality

Bridges ordinary PQ and extra dimensional axions

Rich landscape of theory space topologies with diverse phenomenology

Can provide a 4D realisation of a post-inflationary, high quality QCD axion