Two Higgs Doublet Solutions to the Strong CP Problem

Based on 2506.13853 and 2407.14585

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Outline

The Strong CP Problem

- Absence of P and C violation in strong interactions
- Quantified by dimensionless parameter $\bar{\theta}$
- Experimental bound from neutron EDM: $\boxed{\bar{\theta} \leq 10^{-10}}$

Two Sources of $\bar{\theta}$ in the Standard Model

1. QCD Lagrangian CP-odd term:

$$\theta_{\mathrm{QCD}} \frac{g_s^2}{32\pi^2} G_a^{\mu\nu} \tilde{G}_{a,\mu\nu}$$

2. Yukawa matrix phase:

$$\bar{\theta} = \theta_{\mathrm{QCD}} + \arg\det\left(y^u y^d\right)$$

The Puzzle

These contributions from different SM sectors have no reason to cancel!

This Work: Multi-Higgs Solution

Framework

Multi-Higgs doublet extensions with CP and flavor symmetry G_{HF}, both softly broken only in scalar potential

Previous work [Hall et al. 2024]:

- Found many G_{HF} giving realistic masses/mixing with $\bar{\theta} = 0$ at tree-level
- Radiative contributions vanish if second Higgs mass $\gg$ EW scale

Question Not Addressed

What is the lower bound on the second Higgs doublet mass scale?

Main Results

1. **Proof:** Large class of 2HDM have *no one-loop corrections to* $\bar{\theta}$, regardless of second Higgs doublet mass
2. **Two-loop corrections:** Present two models with $\bar{\theta}^{(2)}$ well below experimental limit
3. **Surprising result:** In all 2HDM with Abelian flavor symmetry:

$$\boxed{\text{All neutral FCNC are CP conserving}}$$

4. **Phenomenology:**
   - Neutral meson CP bounds: 20 TeV $\rightarrow$ 1 TeV
   - Rich signals at colliders

The Multi-Higgs Mechanism

Additions to SM

N Higgs doublets with CP and flavor symmetry G_{HF}, both softly broken in scalar potential only

How It Works

1. Phases in scalar potential transferred with *opposite signs* to up/down Yukawa
2. Flavor symmetry dictates Yukawa texture with zeros
3. Results in: realistic masses, CKM angles, and $\arg\det\left(y^u y^d\right) = 0$

$$\Rightarrow \boxed{\text{Strong CP problem solved at tree level}}$$

Focus: Two Higgs doublets with discrete $\mathbb{Z}_N$ flavor symmetry

Scalar Potential

Most General with Softly Broken CP

$$V = \mu_{11}\Phi_1^\dagger\Phi_1 + \mu_{22}\Phi_2^\dagger\Phi_2 + \mu_{12}(e^{i\alpha}\Phi_2^\dagger\Phi_1 + \text{h.c.})$$
$$+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$
$$+ 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

All parameters real. CP softly broken by phase $\alpha \neq 0$.

Flavor Symmetry Constraint

Requiring G_{HF} softly broken: $\boxed{\lambda_5 = \lambda_6 = \lambda_7 = 0}$

CP and G_{HF} only explicitly broken by mixed mass term μ_{12}

Vacuum and Higgs Basis

Vacuum

$$\langle\Phi_1\rangle = \frac{v_1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\langle\Phi_2\rangle = \frac{v_2}{\sqrt{2}}\begin{pmatrix}0\\e^{i\theta}\end{pmatrix}$$

Minimization: $\theta = \alpha$

Higgs Basis

Change to basis where H_1 has all vev:

$$\begin{pmatrix}\Phi_1\\\Phi_2\end{pmatrix} = U\begin{pmatrix}H_1\\H_2\end{pmatrix}$$

where U involves β with $\tan\beta = v_2/v_1$

Key Property

Using minimization conditions, scalar potential in Higgs basis is *CP preserving*!

Physical Spectrum

Mass Eigenstates

- **Charged:** H^+ (mass eigenstate)
- **CP-odd neutral:** I^0 (mass eigenstate)
- **CP-even neutral:** H^0, R^0 mix $\rightarrow h$ (125 GeV), H (heavy)

No mixing between CP-odd and CP-even (CP is conserved in Higgs basis)

Mixing angle α between CP-even states:

$$\sin\alpha \simeq \cos\beta \sin\beta \,(\text{quartic couplings})\, \frac{v^2}{M^2}$$

Yukawa Couplings in Two Bases

Flavor Basis

$$\mathcal{L}_Y = q^i x^{\alpha}_{ij} \bar{u}^j \Phi_\alpha + q^i \tilde{x}_{\alpha ik} \bar{d}^k \Phi^{*\alpha} + \text{h.c.}$$

Flavor symmetry: entries with non-zero charges vanish, zero-charge entries are real

Higgs Basis

H_1 **couplings:**

$$y^{u,d} = x^{u,d} U_{*1}$$

Give quark masses

H_2 **couplings:**

$$z^{u,d} = x^{u,d} U_{*2}$$

FCNC interactions

Flavor symmetry ensures: $\boxed{\arg\det\left(y^u y^d\right) = 0}$ Solves strong CP at tree-level

Example: Yukawa Texture

$$y^u = \begin{pmatrix} x_{11} U_{11} & x_{12} U_{11} & x_{13} U_{21} \\ x_{21} U_{21} & x_{22} U_{21} & 0 \\ 0 & 0 & x_{33} U_{11} \end{pmatrix} \qquad y^d = \begin{pmatrix} \tilde{x}_{11} U_{11}^* & 0 & 0 \\ 0 & \tilde{x}_{22} U_{21}^* & \tilde{x}_{23} U_{21}^* \\ \tilde{x}_{31} U_{21}^* & \tilde{x}_{32} U_{11}^* & \tilde{x}_{33} U_{11}^* \end{pmatrix}$$

$$\det(y^u) = (x_{11}x_{22} - x_{12}x_{21})x_{33} U_{11}^2 U_{21} \qquad \det(y^d) = (\tilde{x}_{11}\tilde{x}_{22}\tilde{x}_{33} - \tilde{x}_{11}\tilde{x}_{23}\tilde{x}_{32}) \times U_{11}^{*2} U_{21}^*$$

$$\Rightarrow \boxed{\arg\det\left(y^u y^d\right) = 0}$$

CP Conservation in Neutral Higgs Interactions

Main Result

For discrete $\mathbb{Z}_N$ flavor symmetry: quark field rephasing allows Yukawa interactions of *all neutral scalars* to conserve CP

The Phase Basis

There exists a basis where all $y^{u,d}$, $z^{u,d}$ are real:

$$\bar{y}^{u,d} = P^{u,d} y^{u,d} P^{\bar{u},\bar{d}}, \quad \bar{z}^{u,d} = P^{u,d} z^{u,d} P^{\bar{u},\bar{d}}$$

where P are diagonal phase matrices

CP violation sources: (1) CKM matrix, (2) Charged Higgs couplings only (CKM phase permitted by $P^u \neq P^d$)

Proof Sketch

Key observation: $y_{ij}^{u,d}$ and $z_{ij}^{u,d}$ have same phase, same zero pattern

1. All neutral Higgs couplings can be made real **iff** $y^{u,d}$ can be made real
2. If not possible, must have rephasing-invariant with imaginary part
3. Relevant invariants: $y_{11}y_{22}y_{12}^*y_{21}^*$ and $y_{11}y_{22}y_{33}y_{12}^*y_{23}^*y_{31}^*$
4. **Proven:** If either $\mathrm{Im} \neq 0$, then $\det(y) \neq 0$ requires multiple monomials of $\Phi_\alpha \Rightarrow \bar{\theta} \neq 0$ at tree-level
5. Contradicts G_{HF} restrictions

$\therefore$ Phase basis exists where all neutral couplings real

Phenomenological Impact

Neutral Meson Mixing

Naive expectation: CP violation bound from $\epsilon_K \sim 10^5$ TeV

With approximate $U(1)^9$ flavor: Bound ~ 20 TeV

Our result: All neutral scalars conserve CP

- No tree-level contribution to ϵ_K!
- Bounds only from CP-conserving mixing: ~ 1 TeV

Bottom Line

TeV-scale second Higgs doublet is phenomenologically viable and testable!

The Challenge

- Tree-level: $\bar{\theta} = 0$
- Experimental: $\bar{\theta} \leq 10^{-10}$
- Radiative corrections could spoil solution!

What to Check

Corrections to quark mass matrices:

$$\begin{aligned}\bar{\theta} \approx \arg\det(m_u m_d) \\ + \operatorname{Im}\,\mathrm{Tr}(m_u^{-1}\delta m_u \\ + m_d^{-1}\delta m_d)\end{aligned}$$

Phase Basis Advantage

Phases only in:

- CKM matrix
- Charged Higgs couplings

$\Rightarrow$ Vastly reduces diagrams to check!

SM Baseline

Ellis-Gaillard (1978): corrections $\geq$ 3-loops, $\bar{\theta} \sim 10^{-16}$

One-Loop: Charged Boson Exchange

Result After Using Unitarity

$$\delta y^u = \frac{y^d y^{d\dagger} y^u}{16\pi^2} \log\left(\frac{m_2^2}{m_1^2}\right) + \frac{\tilde{x}_\alpha \tilde{x}^{\dagger\beta} x^\alpha}{16\pi^2} U_{\beta 1}\left[1 + \frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_2^2}\right)\right]$$

Why It Vanishes

First term: $y^d y^{d\dagger} y^u$ = Hermitian $\times y^u \Rightarrow$ no contribution

Second term: $\tilde{x}_\alpha \tilde{x}^{\dagger\beta} x^\alpha$ has:

- Same flavor charges
- Same pattern of zeros
- Real non-zero entries

Pattern for tree-level $\arg\det = 0$ survives!

One-Loop: The Complete Result

Main Result

No one-loop contributions to $\bar{\theta}$ from charged boson exchange!

Holds for any Abelian G_{HF} ensuring $\arg\det\left(y^u y^d\right) = 0$ at tree-level

Neutral Scalars

- Scalar potential in Higgs basis (= phase basis) is CP-conserving
- All quartic couplings real
- All neutral scalar Yukawa couplings real in phase basis
- $\Rightarrow$ One-loop diagrams with neutral scalar exchange cannot induce CP violation

Key Point

Vanishing holds **independently of mass scale** of second Higgs doublet!

Two-Loop Contributions

- Many diagrams at two loops
- Phases from CKM and charged Higgs
- Work in phase basis

General Form

$$\mathcal{M} = \frac{\mathcal{A}}{(16\pi^2)^2}(1 + \tfrac{1}{\epsilon} + f)$$

$\mathcal{A}$: flavor (5 Yukawas)
$f(m_1, m_2)$: mass function

Result Using Unitarity

Finite and divergent pieces don't change det phases

Mass-Dependent Part

- Estimate assuming $f \sim 1$
- Check in specific models

Finding

Contributions naturally small: $\bar{\theta} < 10^{-10}$

Model I with $\mathbb{Z}_3$

	Φ_1	Φ_2	q_1	q_2	q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$\mathbb{Z}_3$	0	2	0	1	2	0	0	1	0	1	1

Features:

- Realistic quark masses and CKM
- $\bar{\theta} = 0$ at tree-level
- No 1-loop corrections (proven)

Requirements for realistic dets:

$$|x_{21} U_{21}| \lesssim y_u/|V_{us}| \approx 5 \times 10^{-5}$$
$$|\tilde{x}_{32} U_{11}| \lesssim y_s/|V_{cb}| \approx 0.01$$

Two-Loop

$$\bar{\theta} \simeq 10^{-10} \tilde{x}_{31} c \sin 3\theta$$

$c = 3 \sin\beta$ or $-2\cos\beta$

From hierarchies:
$\tilde{x}_{31} \ll y_b \sim 10^{-2}$

Well below limit!

	Φ_1	Φ_2	q_1	q_2	q_3	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{d}_1$	$\bar{d}_2$	$\bar{d}_3$
$\mathbb{Z}_3$	0	1	0	1	2	1	2	1	1	0	1

Features:

- Different Yukawa texture
- Realistic masses and CKM
- $\bar{\theta} = 0$ at tree-level
- No 1-loop corrections

Naturalness expectations:

$$|x_{21} U_{21}| \sim y_u / |V_{us}|$$
$$|x_{31} U_{11}| \sim y_u / |V_{us} V_{cb}|$$

Two-Loop

$$\bar{\theta} \simeq 10^{-12} \times c$$

with $c = 1$ or $\cot^2 \beta$

Exciting!

Since $\cot \beta$ unknown, $\bar{\theta}$ could be $\sim 10^{-10}$

Next-gen nEDM experiments could discover signal!

Model Comparison

	Model I	**Model II**
$\mathbb{Z}_3$ charges	$(\Phi_1, \Phi_2) = (0, 2)$	$(0, 1)$
Tree-level $\bar{\theta}$	0	0
One-loop $\bar{\theta}$	0	0
Two-loop estimate	$\sim 10^{-10} \tilde{x}_{31} c \sin 3\theta$ $(\tilde{x}_{31} \ll y_b)$	$\sim 10^{-12} c$ $(c = 1 \text{ or } \cot^2 \beta)$
Experimental prospect in EDMs	Likely undetectable	Potentially observable

Key Conclusions

- Both phenomenologically viable
- Two-loop corrections calculable and controlled
- Strong CP problem unambiguously solved
- Model II: discovery potential in nEDM experiments

Three Main Probes

Our theories solve strong CP for *any* mass scale of second Higgs
$\Rightarrow$ How light can it be? What signals?

1. **Neutron EDM**
   - Direct contributions (quark EDMs, Weinberg operator)
   - Indirect via $\bar{\theta}$
2. **Neutral Meson Mixing**
   - Tree-level FCNC from neutral scalars
   - Most stringent constraints
3. **Higgs Coupling Deviations**
   - Mixing of 125 GeV Higgs with heavy state
   - Observable at HL-LHC and future colliders

Neutron EDM

Direct Contributions

1-loop quark EDMs:

- Vanish in Models I, II
- Neutral Higgses don't mediate CP violation

2-loop quark EDMs:

- Negligibly small
- Same flavor structure as $\bar{\theta}$

Weinberg operator:

- Not generated at 1- or 2-loop
- CP-conserving neutral couplings

Indirect via $\bar{\theta}$

Model II: $\bar{\theta}$ could be $\sim 10^{-10}$ for large $\cot\beta$

Next-gen experiments aim for 2 orders of magnitude improvement

Discovery Potential

Could observe non-zero nEDM in coming decade!

Neutral Meson Mixing: Setup

Scalars

- Neutral CP-even (H)
- Neutral CP-odd (I^0)
- Charged ($H^\pm$)

Key Advantage

All neutral scalars have *real* couplings

Much less constrained than typical models

No ϵ_K bound!

$\Delta F = 2$ Processes

K, B_d, B_s, D mixing mediated by neutral scalars at tree-level

Most stringent: D mixing

Comparison

- No flavor sym: $\gg$ TeV
- Approx $U(1)^9$: ~ 20 TeV
- Our theories: ~ 1 TeV

Wilson Coefficients

Scalar Exchange Generates

$$C_2 \simeq -\frac{(\hat{z}_{ij}^{u,d} s_\alpha)^2}{4}\left(\frac{1}{m_h^2} - \frac{1}{m_H^2}\right)$$

$$C_4 \simeq -\frac{\hat{z}_{ij}^{u,d}\,\hat{z}_{ji}^{u,d}}{2}\left(\frac{s_\alpha^2}{m_h^2} + \frac{2 - s_\alpha^2}{m_H^2}\right)$$

$$\tilde{C}_2 \simeq -\frac{(\hat{z}_{ji}^{u,d} s_\alpha)^2}{4}\left(\frac{1}{m_h^2} - \frac{1}{m_H^2}\right)$$

$(i, j) = (12), (23), (13)$ for K/D, B_s, B_d mixing
$M = m_H \simeq m_I$ (approximately degenerate)

Scaled to relevant scale using matching, RG evolution, lattice matrix elements

D Mixing Constraints

Model I

$$\hat{z}^u_{12} \simeq (\cot\beta + \tan\beta) V_{us} y_c$$
$$\hat{z}^u_{21} \simeq (\cot\beta + \tan\beta) V_{us} y_u$$

$\tilde{C}_2$ negligible

C_2 constraint often stronger

Model II

$$\hat{z}^u_{12} \simeq (\cot\beta + \tan\beta) \sqrt{y_u y_c}$$
$$\hat{z}^u_{21} \simeq (\cot\beta + \tan\beta) \sqrt{y_u y_c}$$

$\tilde{C}_2$ contributes as much as C_2

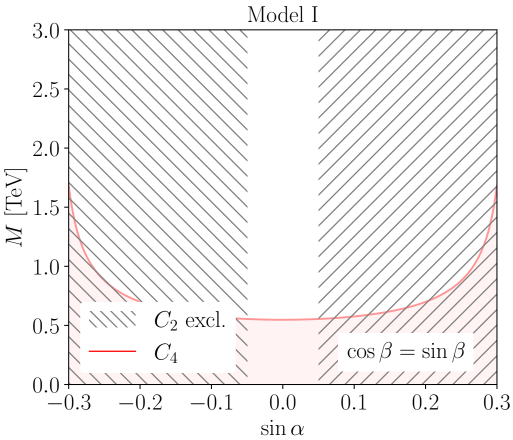
C_4 constraint dominant

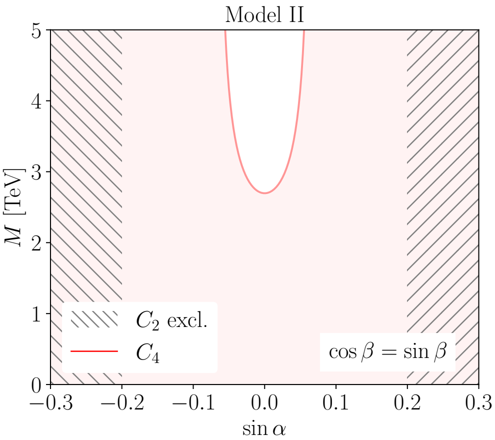
General Expectation

Lower bound on M from FCNC: ~ 1 TeV (model-dependent)

Without 20 TeV ϵ_K bound! TeV-scale phenomenology viable.

Meson Mixing Constraints





Higgs Coupling Deviations

Effective Couplings

$$\kappa_i^{u,d} \simeq \cos\alpha + \sin\alpha\,\frac{\hat{z}_{ii}^{u,d}}{\hat{y}_{ii}^{u,d}}$$

Ratio is model-dependent: $\tan\beta$, $\cot\beta$, $(2\tan\beta + \cot\beta)$, $(-2\cot\beta - \tan\beta)$

Characteristic Features

- Pattern constrained by flavor structure
- Can extract $\tan\alpha$ and $\tan\beta$ from multiple measurements

Characteristic Features (cont.)

- Distinctive signatures identifying the model

Example (Model II, small α)

$$\begin{aligned} |\kappa^u_{2,3} - 1| \simeq |\kappa^d_3 - 1| &\\ \simeq |\sin\alpha\tan\beta| &\\ |\kappa^d_{1,2} - 1| \simeq |\sin\alpha\cot\beta| & \end{aligned}$$

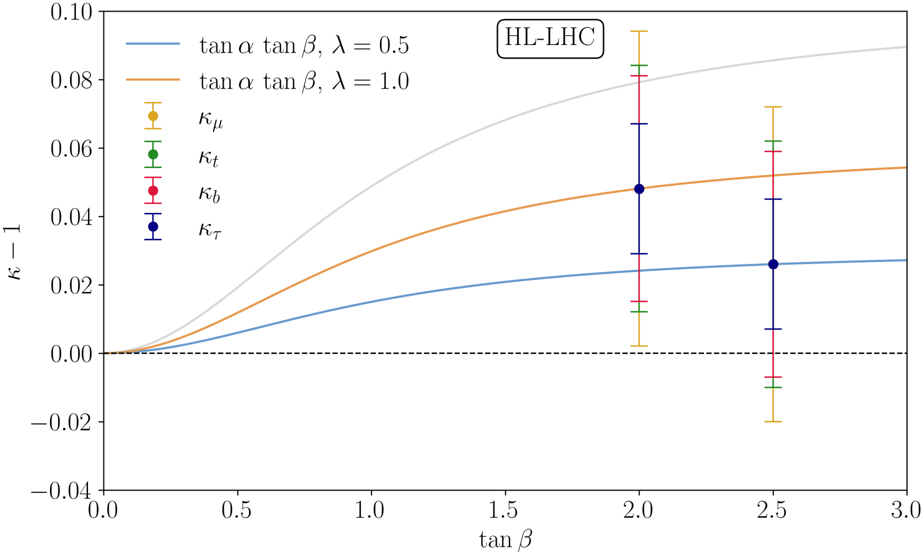
Collider Prospects

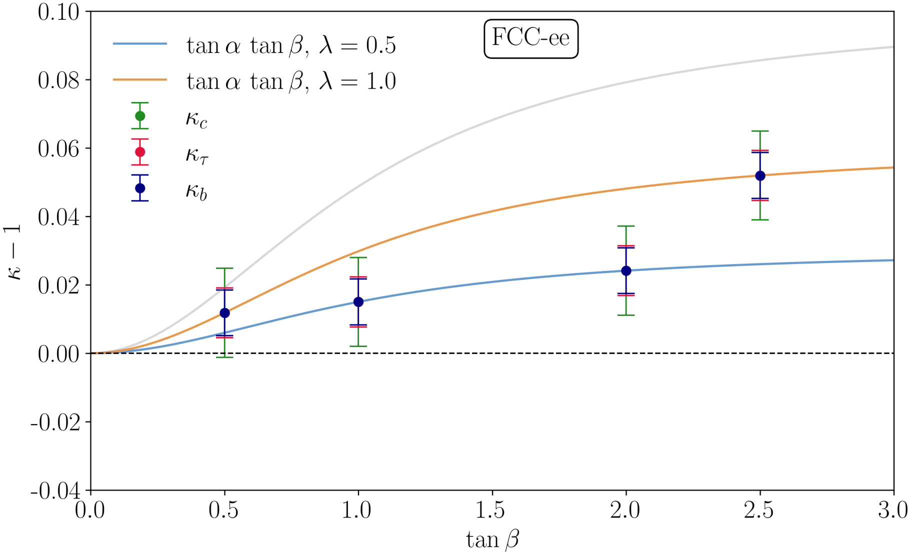
Current and Future Sensitivity

- **Current LHC:** $\sim 10 - 30\%$ precision, not sensitive enough
- **HL-LHC:** 3-5% precision on $\kappa_t, \kappa_b, \kappa_\tau$
  - 2σ signal for $\lambda_a \sim 1$, $M \sim 1$ TeV, most $\tan\beta$
- **FCC-ee:** 1% precision on $\kappa_b, \kappa_c, \kappa_\tau$
  - 5σ discovery for $M \sim 1$ TeV, large $\tan\beta$ range

Flavor-Changing Decays

ATLAS/CMS bounds on $t \to hc$, $t \to hu$: not leading constraints
Future colliders: few orders of magnitude improvement, potentially competitive with $\Delta F = 2$


HL-LHC
tan α tan β, λ = 0.5
tan α tan β, λ = 1.0
κ_μ
κ_t
κ_b
κ_τ
κ − 1
tan β
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
0.0
0.5
1.0
1.5
2.0
2.5
3.0



0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\kappa - 1$
$\tan\beta$
$\tan\alpha\ \tan\beta,\ \lambda = 0.5$
$\tan\alpha\ \tan\beta,\ \lambda = 1.0$
κ_c
κ_τ
κ_b
FCC-ee

Summary: Theoretical Achievement

Broad Class of Solutions

Two-Higgs-doublet models with softly broken CP and Abelian flavor symmetries solve strong CP problem:

1. **Tree-level:** Constructed so $\bar{\theta} = 0$
2. **One-loop:** Corrections necessarily vanish, independent of heavy Higgs mass
3. **Two-loop:** Explicitly estimated in two models, below experimental bounds

Key Discovery

All neutral scalar interactions conserve CP
$\Rightarrow$ Dramatically weakens meson mixing bounds: 20 TeV $\rightarrow$ 1 TeV

Summary: Phenomenology

Rich Experimental Signatures

1. **Neutron EDM:**
   - Model II: potentially observable $\bar{\theta}$ in next-gen experiments
2. **Meson Mixing:**
   - Strongest current constraints: $M \gtrsim 1$ TeV (model-dependent)
   - No ϵ_K bound due to CP conservation
3. **Higgs Couplings:**
   - Characteristic deviation patterns at HL-LHC and future colliders
   - 5σ discovery at FCC-ee for TeV-scale heavy Higgs

Bottom Line

TeV-scale phenomenology with discovery potential in coming decades!

Why These Models Matter

Theoretical Virtues

- Simple: one extra Higgs doublet
- Calculable loop corrections
- Natural connection to flavor
- Less fine-tuning (low-scale UV completion)

Experimental Appeal

- TeV-scale masses achievable
- Multiple complementary probes
- Distinctive signatures
- Testable predictions

The prospect of discovering these scenarios in the coming decade underscores the importance of continued exploration!

Thank You!

Questions?

Backup: Vacuum Minimization Details

Minimization Conditions (for $\mu_{12} < 0$, $v_{1,2} \neq 0$)

$$\theta = \alpha$$
$$\mu_{11} = -\frac{v_2}{v_1}\mu_{12} - v_1^2\lambda_1 - v_2^2(\lambda_3 + \lambda_4)$$
$$\mu_{22} = -\frac{v_1}{v_2}\mu_{12} - v_2^2\lambda_2 - v_1^2(\lambda_3 + \lambda_4)$$

Given μ_{12} and $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV, last two relations constrain μ_{11} and μ_{22}

Higgs Basis Transformation

$$U = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta\, e^{i\theta} & \cos\beta\, e^{i\theta} \end{pmatrix}, \quad H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix}$$

Backup: Full CP Conservation Proof

Step 1: From transformation $y^{u,d} = x^{u,d} U_{*1}$ and $z^{u,d} = x^{u,d} U_{*2}$:

- Each entry $y_{ij}^{u,d}$ and $z_{ij}^{u,d}$ differ only by column of U
- Same phase structure (up to overall U factor)
- Same pattern of zeros

Step 2: All neutral couplings real $\Leftrightarrow$ $y^{u,d}$ can be made real by quark rephasing

Step 3: If not possible, rephasing-invariant has Im part:

$$I_4 = y_{11} y_{22} y_{12}^* y_{21}^*, \quad I_6 = y_{11} y_{22} y_{33} y_{12}^* y_{23}^* y_{31}^*$$

Step 4: Proven: If $\mathrm{Im}(I_4) \neq 0$ or $\mathrm{Im}(I_6) \neq 0$ and $\det(y) \neq 0$, need multiple monomials $\Rightarrow \bar{\theta} \neq 0$ at tree-level, contradicting G_{HF}

$\therefore$ Phase basis exists. Full proof in paper's appendix C.

Backup: One-Loop Details

Before Using Unitarity

$$\delta y^u = \sum_\delta \frac{\tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\gamma}{16\pi^2} U^*_{\alpha\delta} U_{\beta 1} U_{\gamma\delta} \left[1 + \frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_\delta^2}\right)\right]$$

where m_δ are masses of two charged bosons

After Unitarity: $\sum_\delta U^*_{\alpha\delta} U_{\gamma\delta} = \delta_{\alpha\gamma}$

$$\delta y^u = \frac{\tilde{x}_\alpha \tilde{x}^{\dagger\beta} x^\alpha}{16\pi^2} U_{\beta 1} \left[1 + \frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_2^2}\right)\right] + \frac{y^d y^{d\dagger} y^u}{16\pi^2} \log\left(\frac{m_2^2}{m_1^2}\right)$$

Second term real contribution from charged Goldstone vs physical H^+

Backup: Two-Loop Diagrams

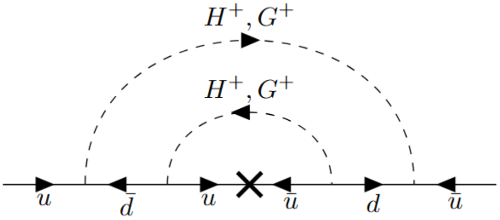


Figure: Example of a two-loop diagram that could contribute to $\bar{\theta}$.

Structure

$$\mathcal{M} = \frac{\mathcal{A}}{(16\pi^2)^2}\left(1 + \frac{1}{\epsilon} + f(m_1, m_2)\right)$$

- $\mathcal{A}$: product of 5 Yukawa matrices (flavor structure)
- $f(m_1, m_2)$: dimensionless function of loop masses
- Finite and $1/\epsilon$ pieces: don't change det phases (unitarity)
- Estimate $f \sim \mathcal{O}(1)$ for mass-dependent part

Backup: Quark Masses and CKM in Models

Model I

$$y_u \approx |x_{11} U_{11}|, \quad y_c \approx |x_{22} U_{21}|, \quad y_t \approx |x_{33} U_{11}|$$
$$y_d \approx |\tilde{x}_{11} U_{11}^*|, \quad y_s \approx |\tilde{x}_{22} U_{21}^*|, \quad y_b \approx |\tilde{x}_{33} U_{11}^*|$$
$$|V_{us}| \approx \frac{|x_{12} U_{11}|}{y_c}, \quad |V_{cb}| \approx \frac{|\tilde{x}_{23} U_{21}^*|}{y_b}, \quad |V_{ub}| \approx \frac{|x_{13} U_{21}|}{y_t}$$

Model II

$$y_u \approx \left| \frac{x_{12} x_{21}}{x_{22}} \frac{U_{21}^2}{U_{11}} \right|, \quad y_c \approx |x_{22} U_{11}|, \quad y_t \approx x_{33} U_{11}$$
$$y_d \approx |\tilde{x}_{11} U_{21}^*|, \quad y_s \approx |\tilde{x}_{22} U_{21}^*|, \quad y_b \approx |\tilde{x}_{33} U_{11}^*|$$

Backup: Detailed Meson Mixing Bounds

Constraints from C_2

$$\text{Model I}: \quad |s_\alpha(\tan\beta + \cot\beta)| \lesssim 0.1$$
$$\text{Model II}: \quad |s_\alpha(\cot\beta + \tan\beta)| \lesssim 0.4$$

Constraints from C_4

$$\text{Model I}: \quad (\tan\beta + \cot\beta)^2 \frac{(M/\text{TeV})^2 s_\alpha^2 + 0.03}{(M/\text{TeV})^2} \lesssim 0.4$$
$$\text{Model II}: \quad (\cot\beta + \tan\beta)^2 \frac{64(M/\text{TeV})^2 s_\alpha^2 + 2}{(M/\text{TeV})^2} \lesssim 1.1$$

For Model **II**, C_4 bound dominant. Bounds from K, B_d, B_s are weaker.

Backup: Higgs Coupling Table

	LHC ATLAS current	HL-LHC %	ILC 500 %	CLIC 3000 %	CEPC %	FCC-ee 240 %
κ_c	$0.03^{+3.02}_{-0.03}$	–	1.3	1.4	2.2	1.8
κ_t	$0.93^{+0.13}_{-0.06}$	3.3	6.9	2.7	–	–
κ_b	$0.89^{+0.14}_{-0.11}$	3.6	0.58	0.37	1.2	1.3
κ_μ	$1.06^{+0.27}_{-0.30}$	4.6	9.4	5.8	8.9	10
κ_τ	$0.92^{+0.13}_{-0.07}$	1.9	0.70	0.88	1.3	1.4

- Current: not sensitive to TeV-scale deviations
- HL-LHC: 2σ for $M \sim 1$ TeV, $\lambda \sim 1$
- FCC-ee: 5σ discovery with 1% precision

Backup: Future Directions

Theoretical

- Origins of soft CP and flavor breaking
- Connection to electroweak hierarchy problem
- UV completion and string embeddings
- Extension to lepton sector (neutrino mixing challenge)

Experimental

- nEDM: 2 orders of magnitude improvement planned
- Precision Higgs: HL-LHC, ILC, FCC-ee, CLIC, muon collider
- Flavor: D, K, B mixing improvements, LFV searches
- Direct searches: production of H, I^0, $H^\pm$