Dark matter scattering in dielectrics

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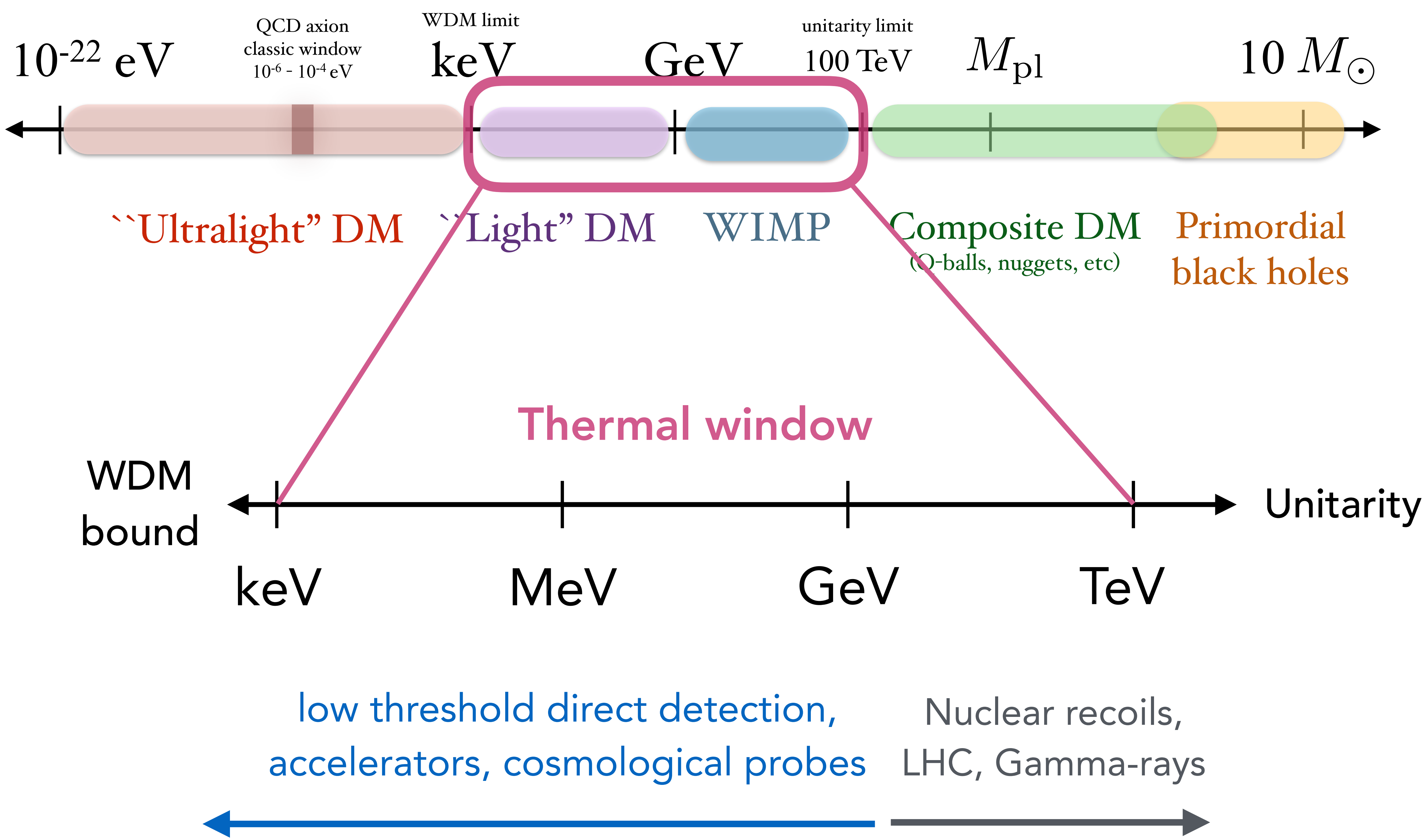
HiDDeN Webinar

Based on work with Simon Knapen and Jonathan Kozaczuk
2003.12077, 2011.09496, 2101.08275

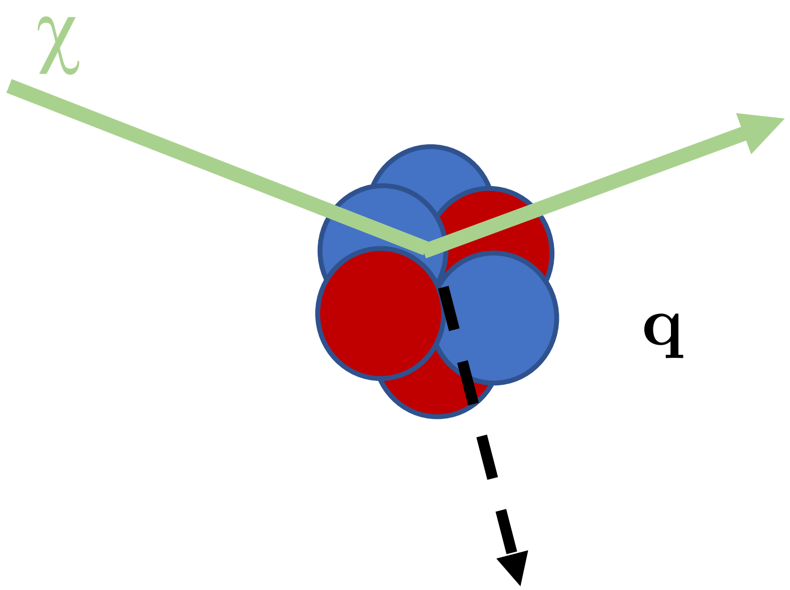
Mass scale of dark matter

(not to scale)



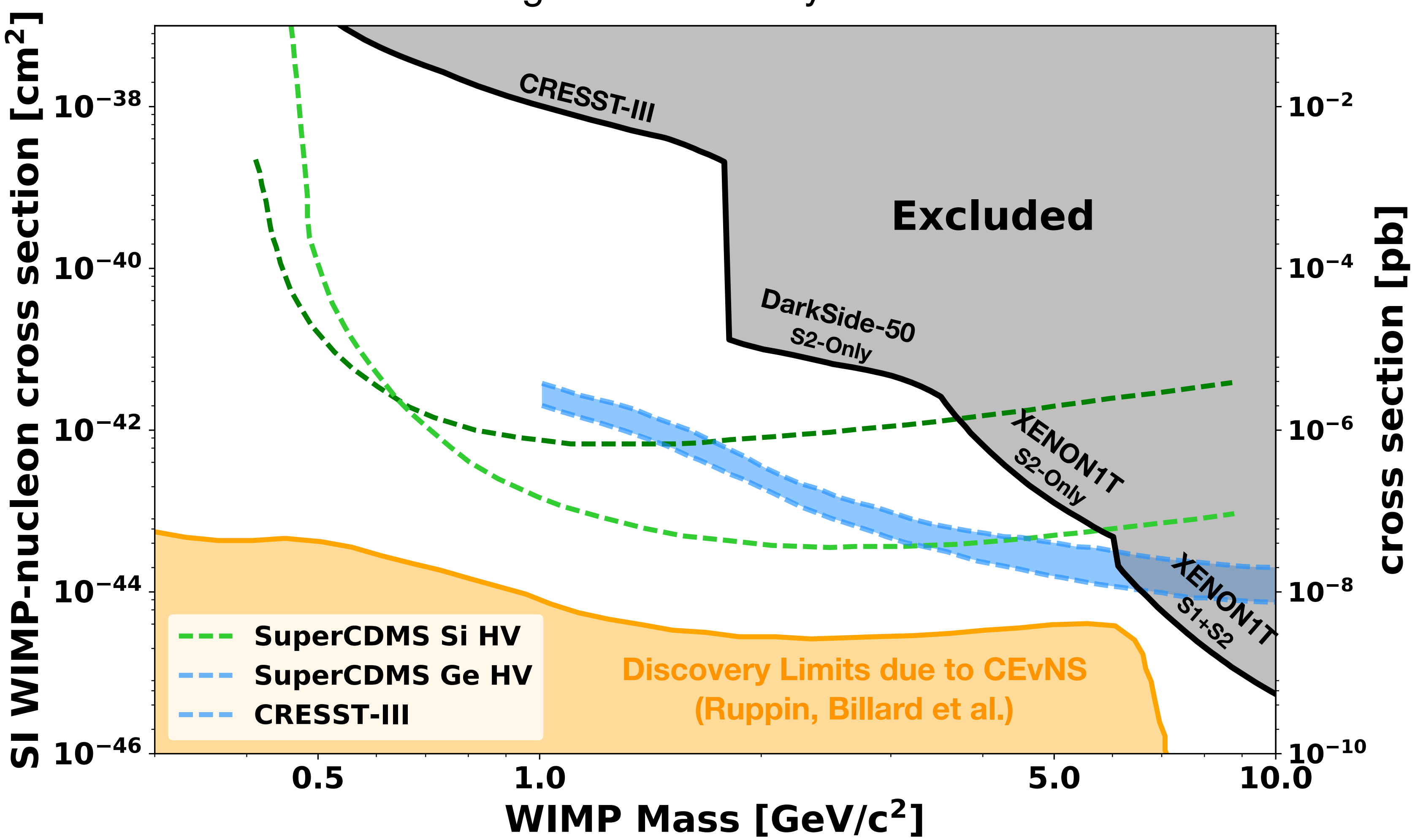
Detecting sub-GeV dark matter

Traditional approach to direct detection of dark matter: DM-nucleus scattering

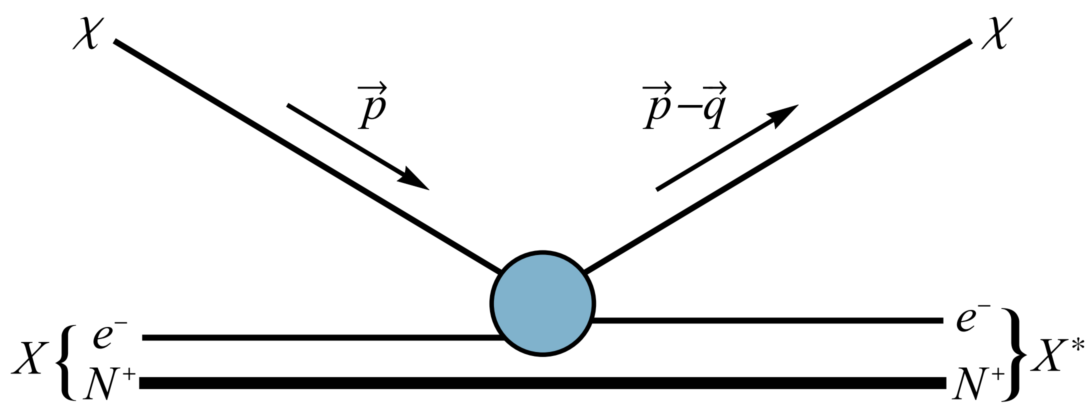


$$E_{NR} \leq \frac{2m_\chi^2 v^2}{m_N} \text{ for sub-GeV DM}$$

Figure from talk by Kaixuan Ni at DPF 2019



Electron recoils



e- in materials are not free or isolated particles

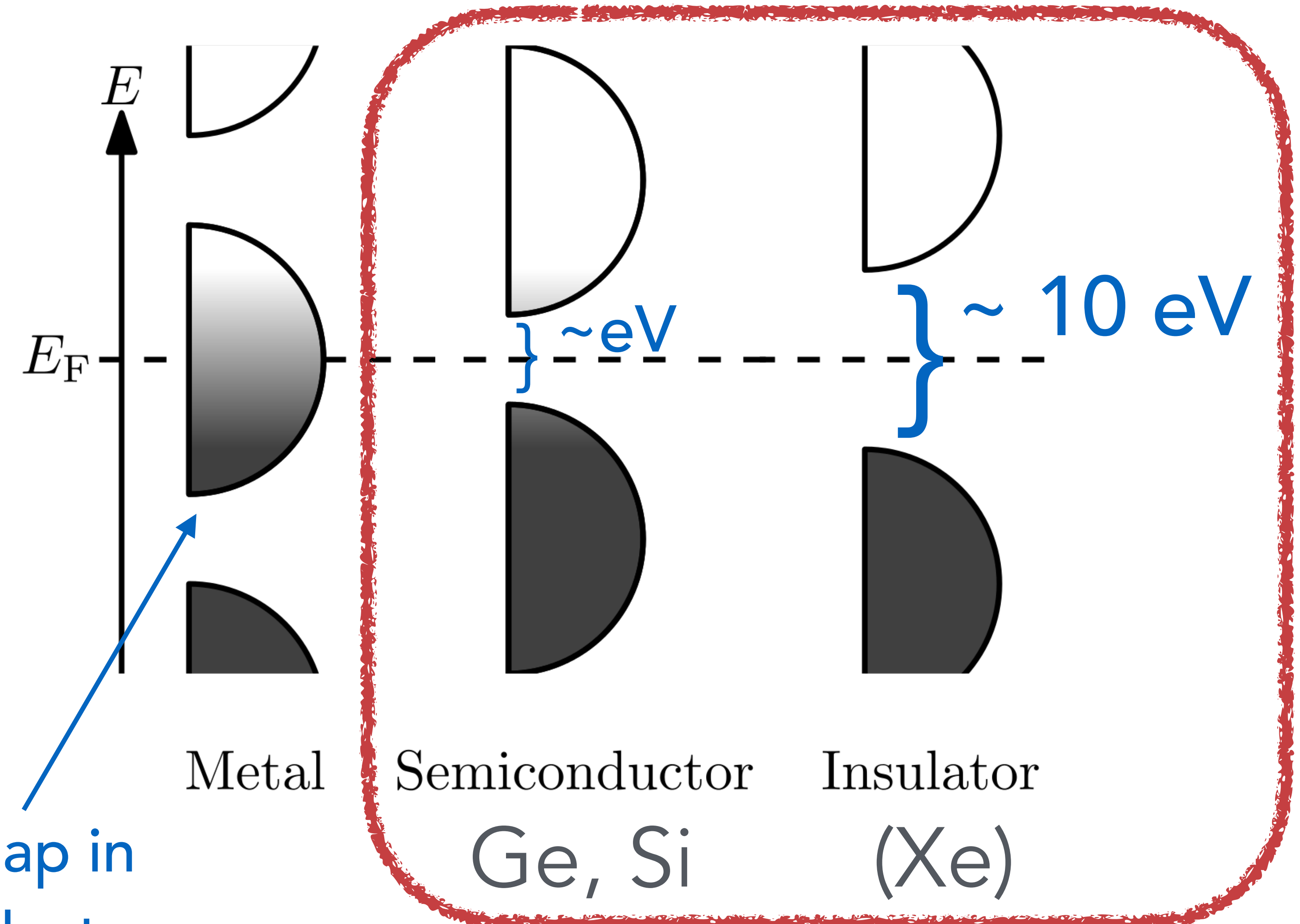
Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

Complication: need to know eigenstates and wavefunctions in a many-body system.

Essig, Mardon, Volansky 2011;
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu 2015

Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

Electronic band structure



sub-eV gap in superconductor, Dirac material, etc

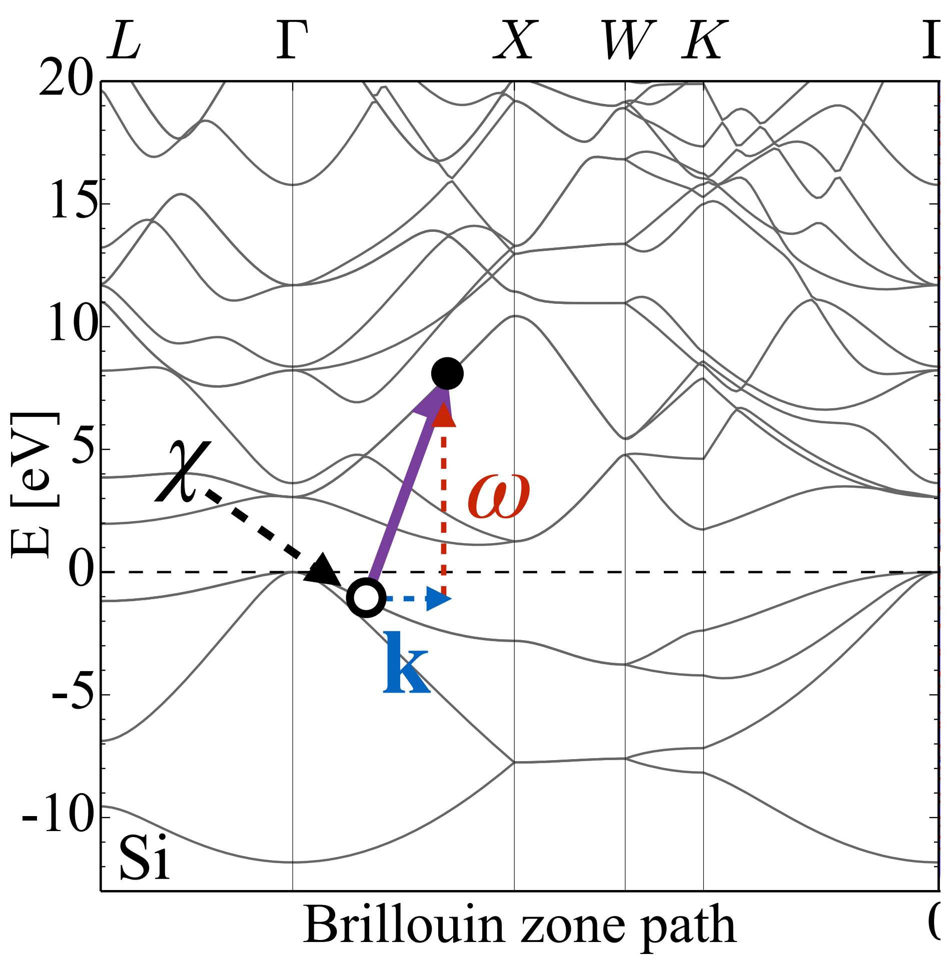
[Hochberg, Pyle, Zhao, Zurek 2015
Dirac: 1708.08929, 1910.02091, etc]

Single or few e- thresholds achieved by a number of experiments!

[DAMIC, SENSEI, SuperCDMS]

Complication: need to know eigenstates and wavefunctions in a many-body system.

Semiconductor target



Rate to create electron-hole pairs:

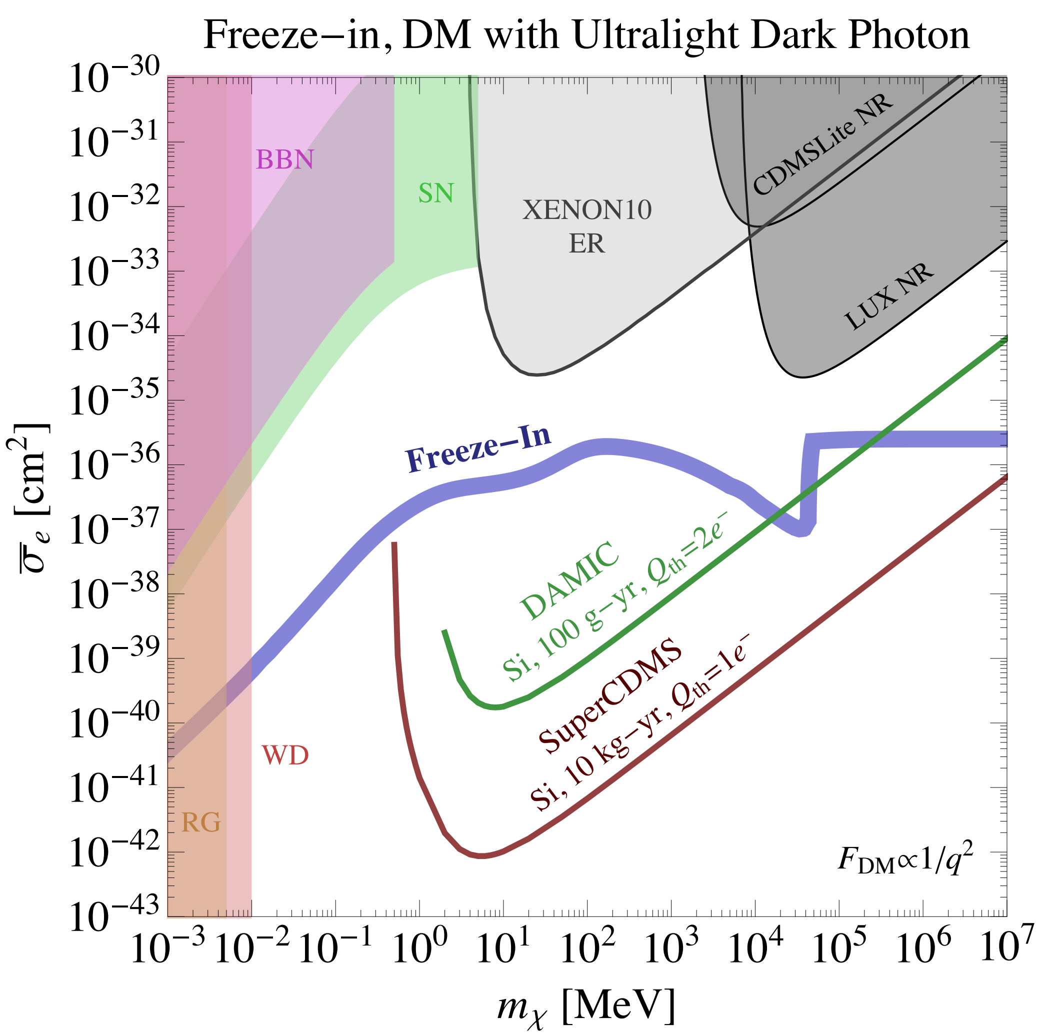
Wavefunction overlap

$$\frac{d\sigma}{d^3\mathbf{k}\,d\omega} \propto \bar{\sigma}_e F^2_{\rm med}(k) \sum_{\ell,\ell'} \sum_{\mathbf{p},\mathbf{p}'} |\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2$$
$$\times f^0(\omega_{\mathbf{p},\ell})\big(1 - f^0(\omega_{\mathbf{p}',\ell'})\big)\ \delta(\omega + \omega_{\mathbf{p},\ell} - \omega_{\mathbf{p}',\ell'})$$

Sum over occupied bands ℓ and Bloch momentum $\mathbf{p}$ to excited state $|\mathbf{p}', \ell'\rangle$

Essig, Mardon, Volansky 2011;
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu 2015

Semiconductor target



Plot from Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu 2015

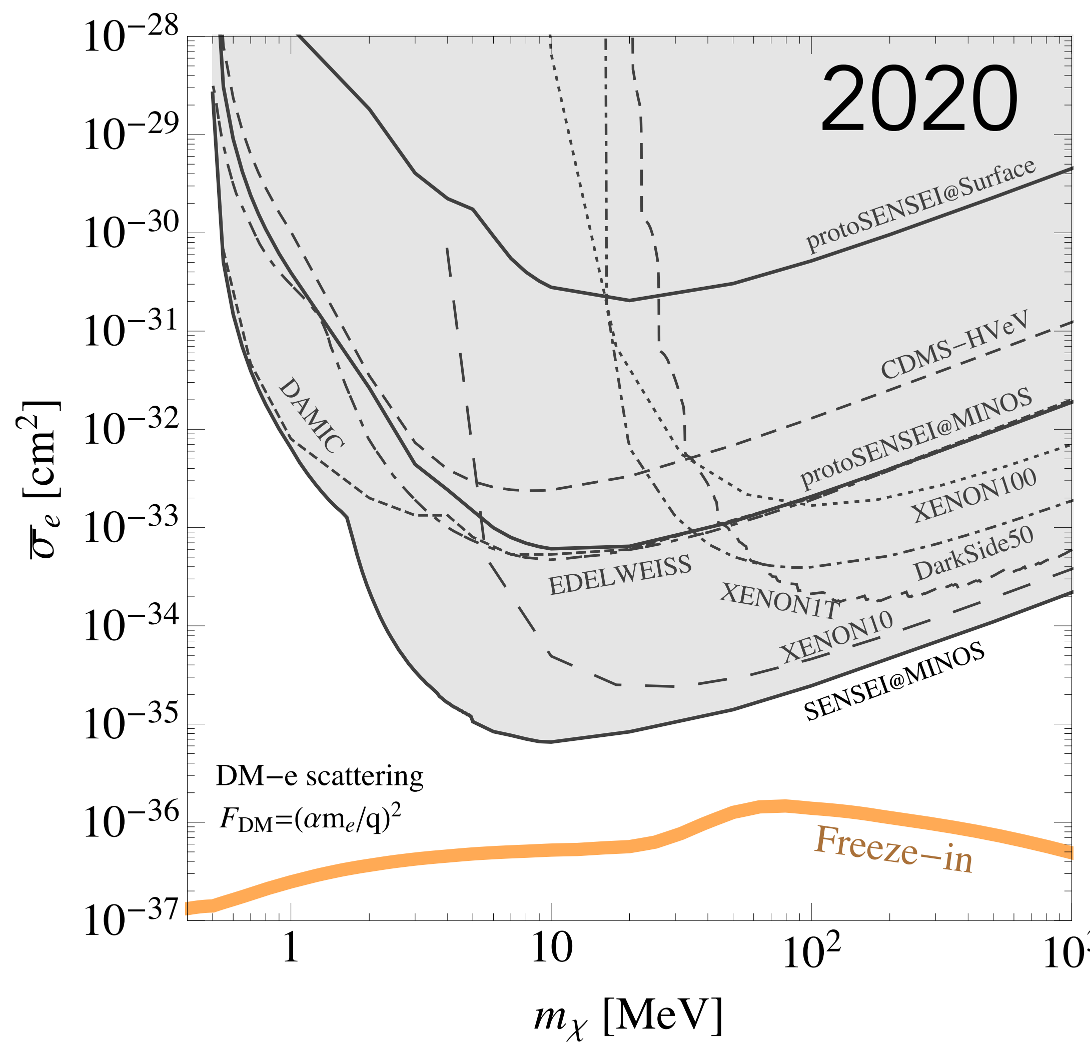
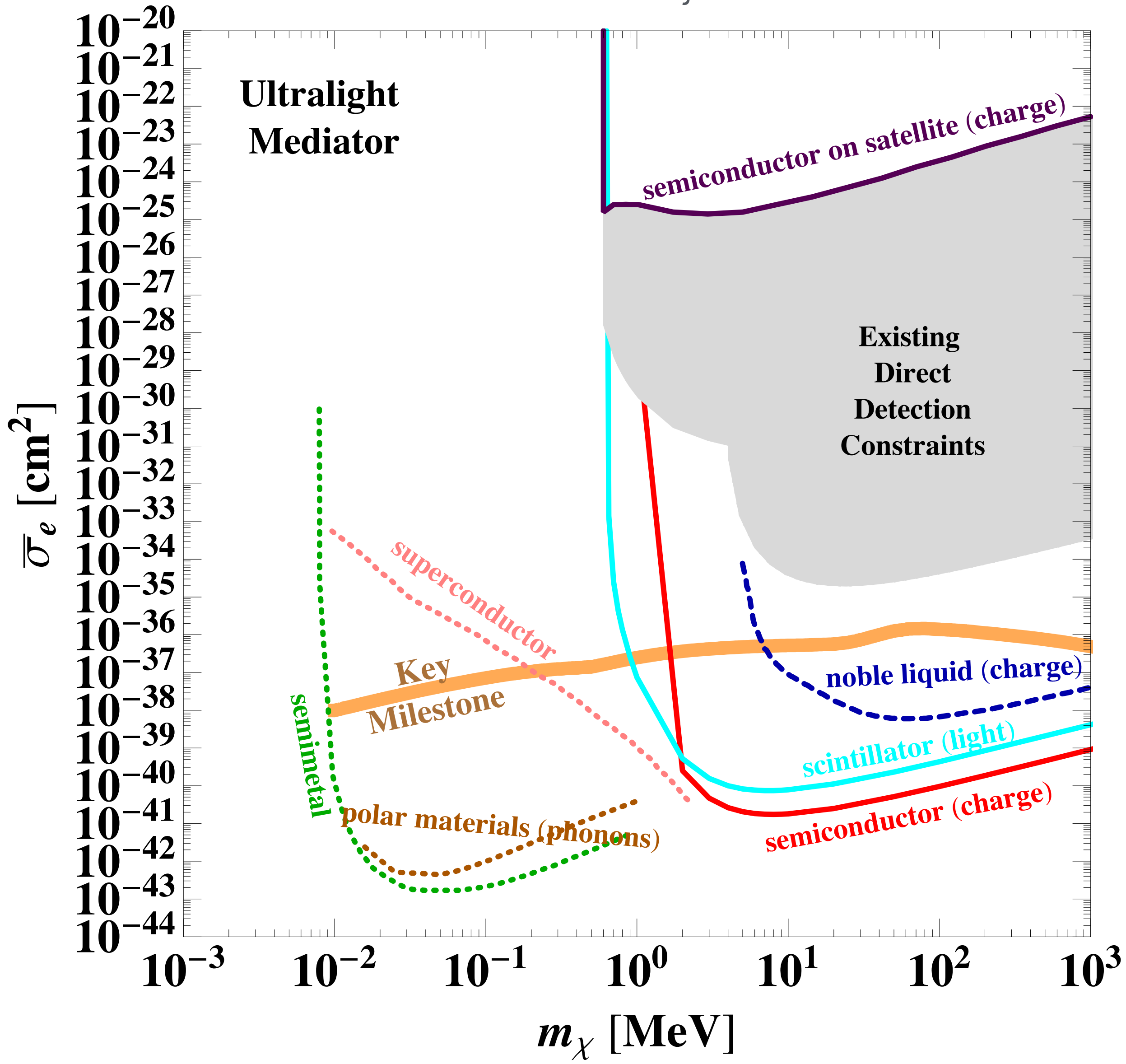


Figure from talk by Rouven Essig

From Basic Research Needs Report:
"Dark Matter Small Projects New Initiatives"



Now many papers studying different targets, proposed experiments, and new experiments in development.

All dielectrics

Today: how to think about DM scattering in all these materials in terms of dielectric response, and how that led us to identify and calculate new effects.

Outline

Dark matter scattering as dielectric response

Implications for DM-electron scattering

The Migdal effect in semiconductors

Linear response

Dielectric response

$\epsilon^{-1}(\omega, \mathbf{k})$ — response of E fields*

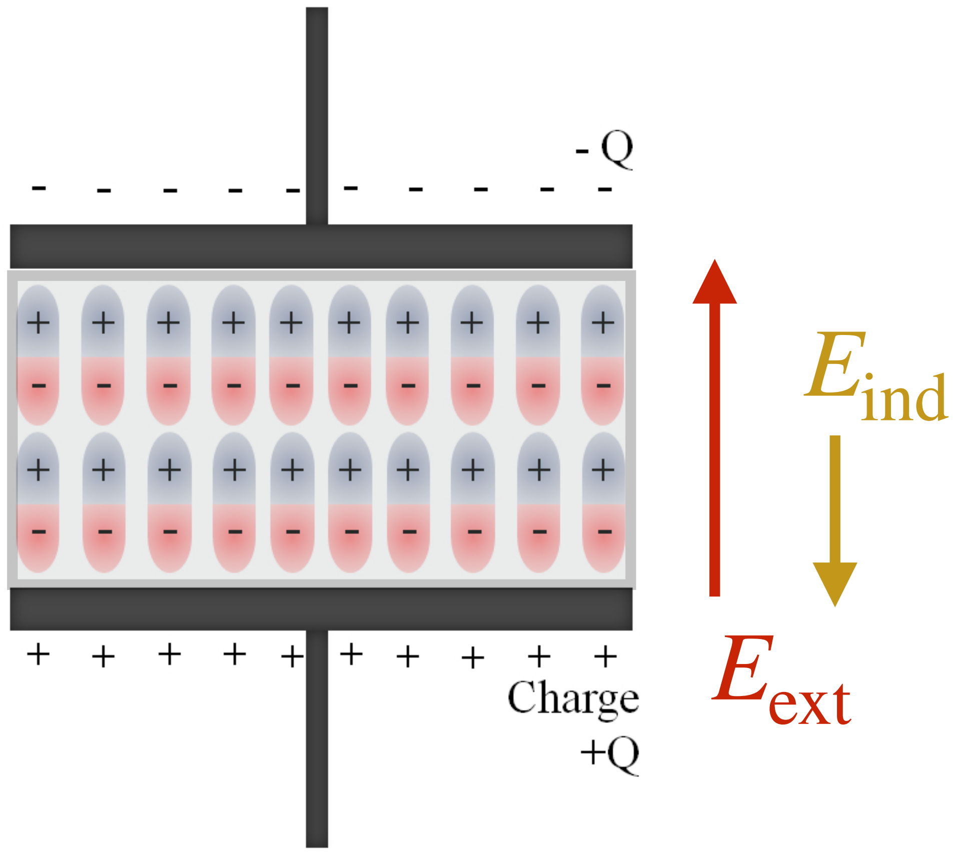
Susceptibility

$\chi(\omega, \mathbf{k})$ — response of electron number density

* Some technicalities: consider only longitudinal response; neglect crystal periodicity

Pines and Nozieres, Theory of Quantum Liquids;
Girvin and Yang, Modern Condensed Matter Physics

Dielectric response



$$\nabla \cdot \mathbf{E} = \frac{4\pi \rho_{\rm ext}}{\epsilon}$$

$$\mathbf{E} = \frac{\mathbf{E}_{\rm ext}}{\epsilon}$$

More generally:

$$\mathbf{E}(\mathbf{r}, \omega) = \int d^3\mathbf{r}' \, \epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \, \mathbf{E}_{\rm ext}(\mathbf{r}', \omega)$$

$$\mathbf{E}(\omega, \mathbf{k}) = \epsilon^{-1}(\omega, \mathbf{k}) \, \mathbf{E}_{\rm ext}(\omega, \mathbf{k})$$

Susceptibility

Induced charge density*: $\rho_{\text{ind}} = \frac{\rho_{\text{ext}}}{\epsilon} - \rho_{\text{ext}}$

Linear response to a perturbation:

$$H = -en(\mathbf{r})\,\Phi_{\text{ext}}(\mathbf{r}) = -e\int d^3\mathbf{k}\; n_{-\mathbf{k}}\,\frac{4\pi\rho_{\text{ext}}(\mathbf{k})}{k^2}$$

$$\rho_{\text{ind}} = -en_{\text{ind}} = \chi\frac{4\pi e^2}{k^2}\rho_{\text{ext}}$$

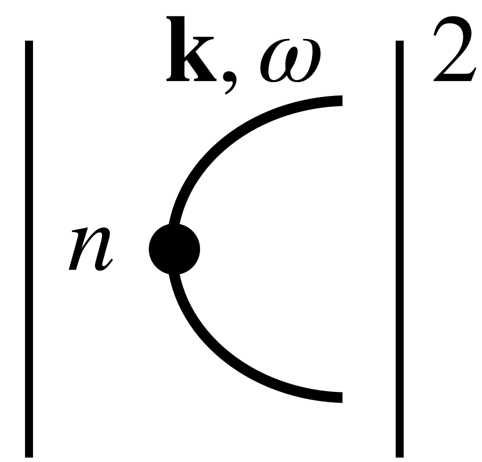
$$\chi(\omega,\mathbf{k}) = \frac{-i}{V}\int_0^\infty dt\; e^{i\omega t}\;\langle[n_{\mathbf{k}}(t), n_{-\mathbf{k}}(0)]\rangle$$

* Assume dominated by electrons

Induced charge is related to amount of screening:

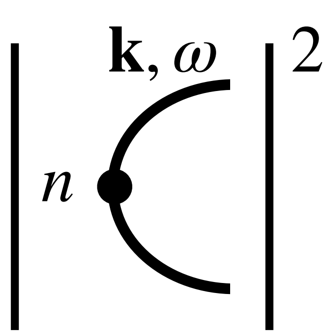
$$\frac{1}{\epsilon(\omega, \mathbf{k})} = 1 + \frac{4\pi e^2}{k^2}\chi(\omega, \mathbf{k})$$

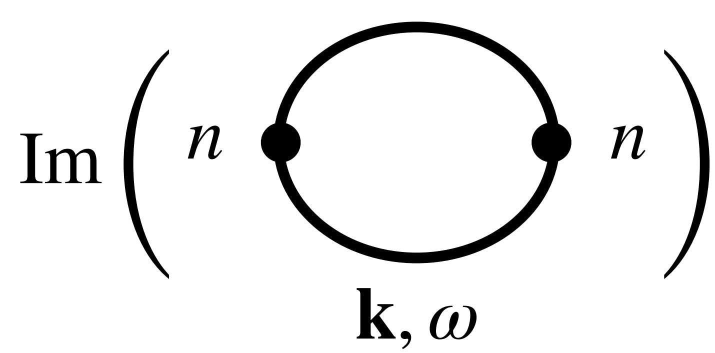
$$H = -\,en(\mathbf{r})\,\Phi_{\text{ext}}(\mathbf{r})$$



Fluctuation-dissipation [Optical] theorem

Spectrum of fluctuations $S(\omega, \mathbf{k}) = \frac{2}{(1 - e^{-\beta\omega})}\,\text{Im}\left(-\chi(\omega, \mathbf{k})\right)$ Dissipation





Induced charge is related to amount of screening:

$$\frac{1}{\epsilon(\omega, \mathbf{k})} = 1 + \frac{4\pi e^2}{k^2}\chi(\omega, \mathbf{k})$$

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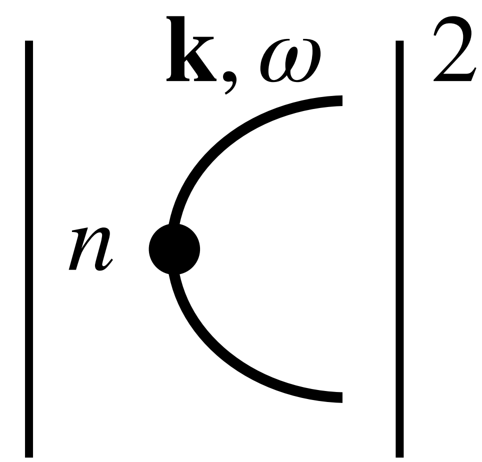


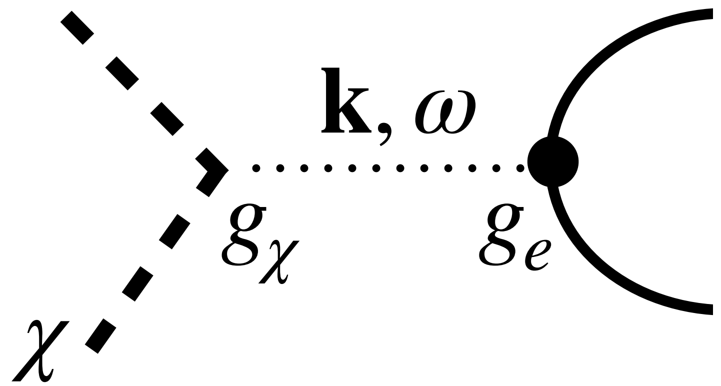
Fluctuation-dissipation [Optical] theorem

$$S(\omega, \mathbf{k}) = \frac{2}{(1 - e^{-\beta\omega})} \,\mathrm{Im}\left(-\chi(\omega, \mathbf{k})\right)$$

$$= \frac{k^2}{2\pi\alpha_{em}(1 - e^{-\beta\omega})} \mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Energy Loss Function (ELF)

DM-electron scattering



$$H = -e \int d^3\mathbf{k}\; n_{\mathbf{k}} \frac{g_\chi g_e e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 + m_V^2}$$

$$\frac{d\sigma}{d^3\mathbf{k}\,d\omega} \propto \bar{\sigma}_e F_{\mathrm{med}}^2(\mathbf{k})\, S(\omega, \mathbf{k}) \propto \bar{\sigma}_e F_{\mathrm{med}}^2(\mathbf{k})\, \mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Charge fluctuations

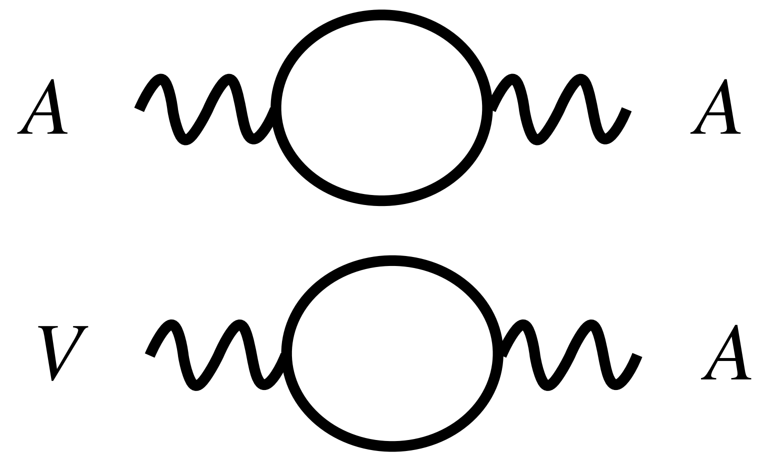
Energy Loss Function (ELF)

Implications
1. Screening effects for vector and scalar mediators
2. Many approaches to calculate or measure ϵ

Vector interactions are screened

Interaction basis: $g_e V_\mu \bar{e} \gamma^\mu e$

In-medium mass and mixing terms



$$\Pi_{AA}$$

$$\Pi_{VA} = \frac{g_e}{e} \Pi_{AA}$$

$$\Pi_{AA}(\omega, \mathbf{k}) = k^2(1 - \epsilon(\omega, \mathbf{k}))$$

In-medium (longitudinal) scattering amplitude: $\sim \dfrac{1}{\epsilon(\omega, \mathbf{k})} \dfrac{g_\chi g_e}{k^2 + m_V^2}$

An, Pospelov, Pradler 2013, 2014
Hochberg, Pyle, Zhao, Zurek 2015

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F^2_{\rm med}(\mathbf{k})\,\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

$$\propto \bar{\sigma}_e F^2_{\rm med}(\mathbf{k})\,\frac{\mathrm{Im}\,\epsilon(\omega,\mathbf{k})}{|\epsilon(\omega,\mathbf{k})|^2}$$

Proportional to DM-electron scattering form factor in the independent-electron approximation (RPA)

$$\mathrm{Im}\,\epsilon^{\rm RPA}(\omega,\mathbf{k}) = \frac{4\pi^2\alpha_{em}}{Vk^2}\sum_{\ell,\ell'}\sum_{\mathbf{p},\mathbf{p}'}|\langle\mathbf{p}',\ell'|e^{i\mathbf{k}\cdot\mathbf{r}}|\mathbf{p},\ell\rangle|^2$$
$$\times f^0(\omega_{\mathbf{p},\ell})\left(1-f^0(\omega_{\mathbf{p}',\ell'})\right)\,\delta(\omega+\omega_{\mathbf{p},\ell}-\omega_{\mathbf{p}',\ell'})$$

$|\epsilon(\omega,\mathbf{k})|^2$ screening for vector mediators considered in superconductors, Dirac materials.

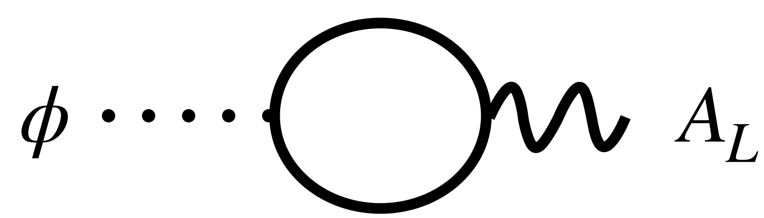
Not included in signal rates for semiconductors. Assumed to be absent for scalar mediators.

Screening of scalars

Interaction basis: $g_e \phi \bar{e} e$

Mixing of scalar mediator with SM photon?

In-medium mixing



$$\Pi_{\phi A} = \frac{g_e}{e} \, \Pi^{0\mu}_{AA} \varepsilon^L_\mu$$

In-medium scattering amplitude
In the non-relativistic limit:

$$\sim \frac{1}{\epsilon(\omega, \mathbf{k})} \frac{g_\chi g_e}{k^2 + m_\phi^2}$$

Hardy and Lasenby 2016
Gelmini, Takhistov, Vitagliano 2020

Vector and scalar mediators

$$-\mathcal{L} \supset g_\chi \phi \bar{\chi}\chi + g_e \phi \bar{e} e \qquad \rightarrow g_\chi \phi n_\chi + g_e \phi n$$

$$-\mathcal{L} \supset g_\chi V_\mu \bar{\chi}\gamma^\mu \chi + g_e V_\mu \bar{e}\gamma^\mu e \quad \rightarrow g_\chi V_0 n_\chi + g_e V_0 n$$

Non-relativistic scattering ($k \gg \omega$) is dominated by scattering through Yukawa potential

$$H = -e \int d^3\mathbf{k}\ n_\mathbf{k} \frac{g_\chi g_e e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 + m_V^2}$$

DM-electron scattering via vector or scalar mediators is identical in the nonrelativistic limit

The energy loss function (ELF)

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Theory	Experiment
Many established approaches to ϵ	Optical measurements
Include local field effects	X-ray scattering
Include electron-electron interactions	Fast electron scattering (EELS)

See Kurinsky, Baxter, Kahn, Krnjaic 2020 and Hochberg, Kahn, Kurinsky, Lehmann, Yu, and Berggren 2021 for complementary work and more emphasis on experimental calibration of dielectric function

Understanding the ELF and implications for DM-electron scattering

The dielectric function

Random-phase approximation (RPA):

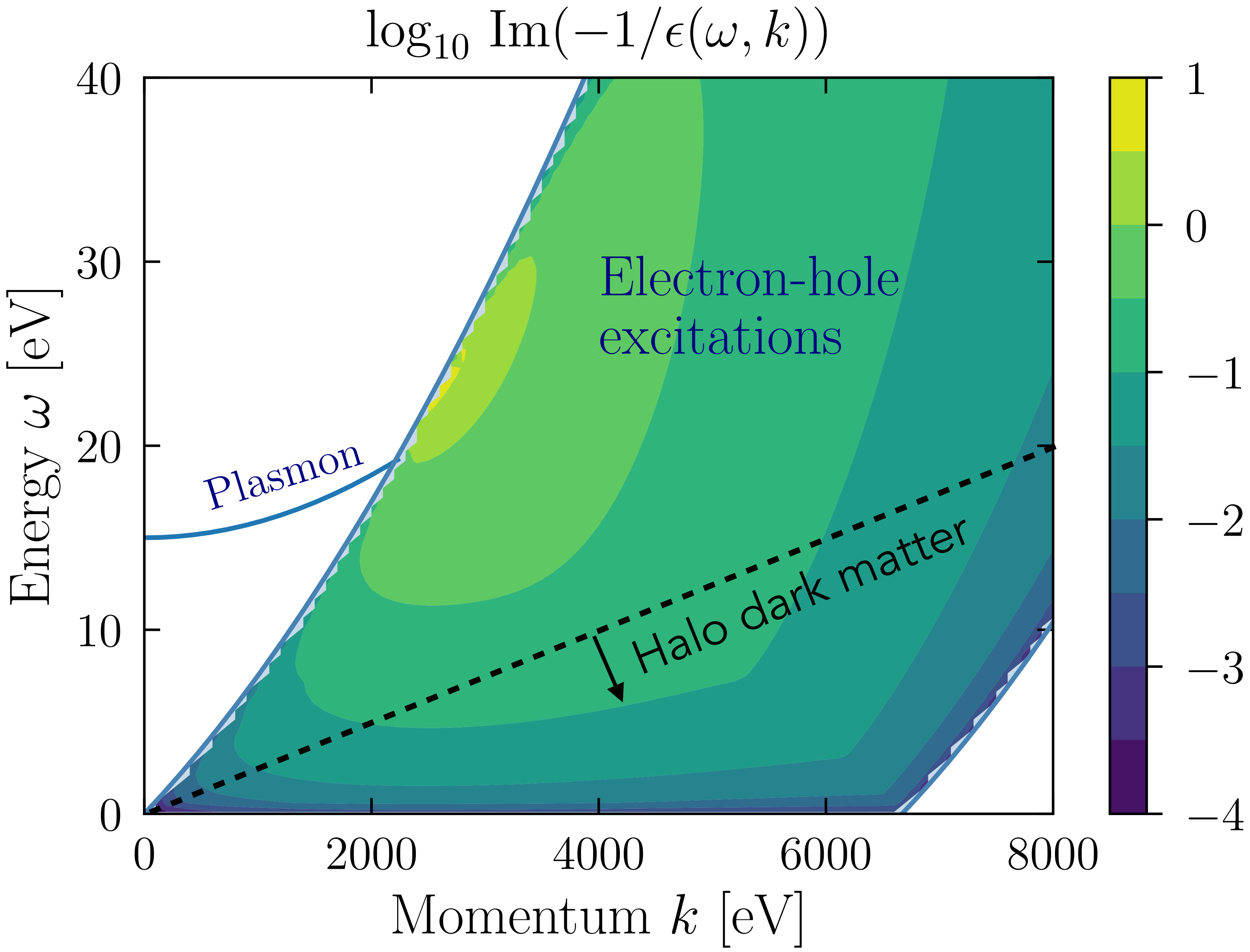
Emission – absorption

$$\epsilon^{\mathrm{RPA}}(\omega, \mathbf{k}) = 1 + \frac{4\pi\alpha_{em}}{Vk^2} \underbrace{\lim_{\eta \to 0} \sum_{\ell,\ell'} \sum_{\mathbf{p},\mathbf{p}'} |\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2 \frac{f^0(\omega_{\mathbf{p}',\ell'}) - f^0(\omega_{\mathbf{p},\ell})}{\omega + \omega_{\mathbf{p},\ell} - \omega_{\mathbf{p}',\ell'} + i\eta}}$$

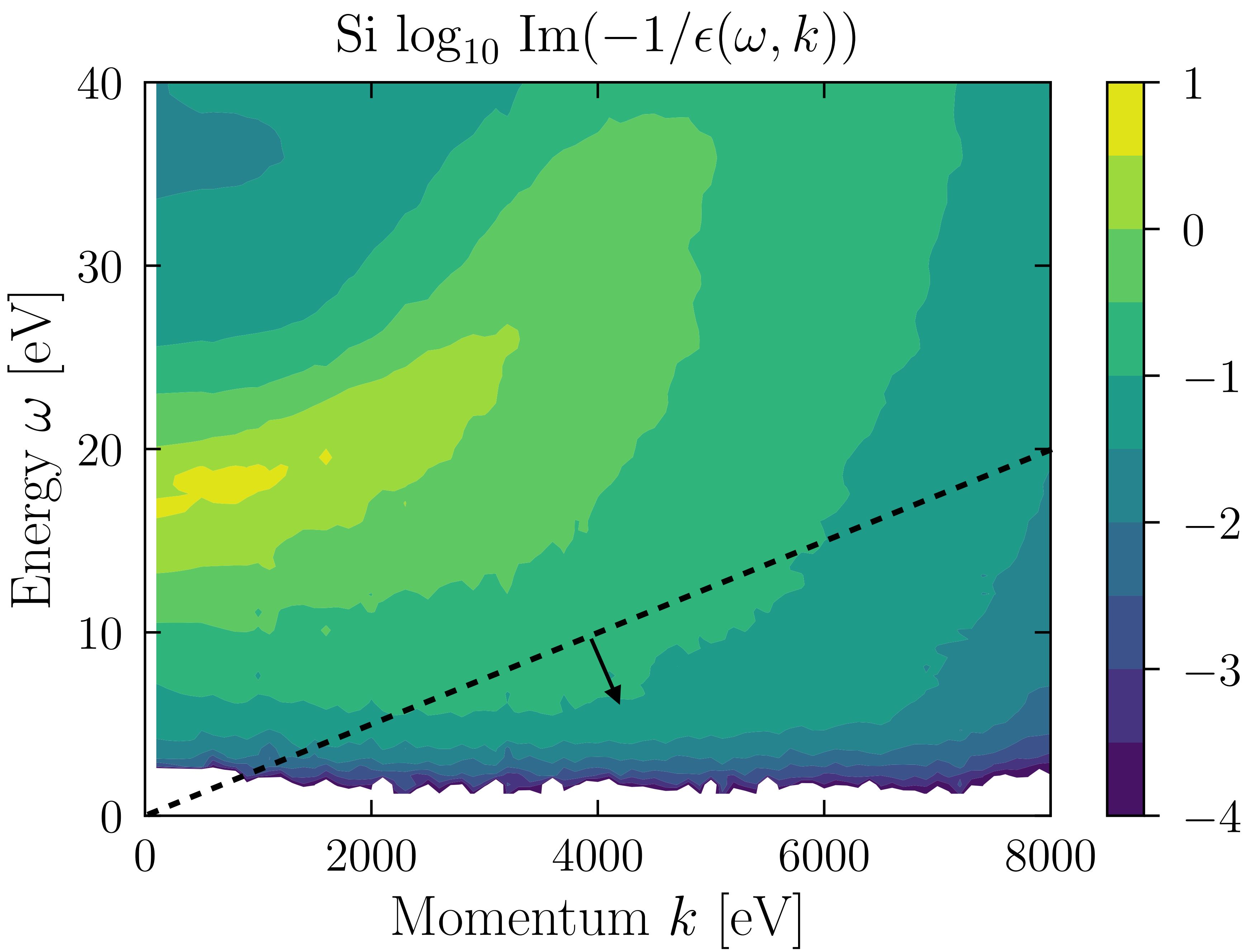
Leading order polarization assuming independent electron response

Can be evaluated analytically in free degenerate electron gas with Fermi momentum p_F and plasma frequency $\omega_p = \sqrt{4\pi\alpha_{em} n_e / m_e}$

Electron gas model

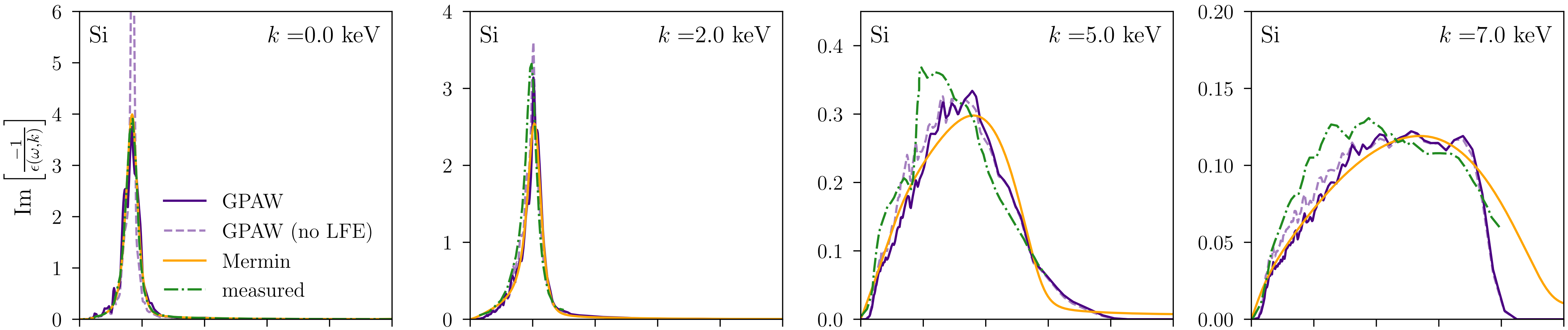


ELF in semiconductors



Density function theory (DFT) calculation using GPAW

ELF in Silicon

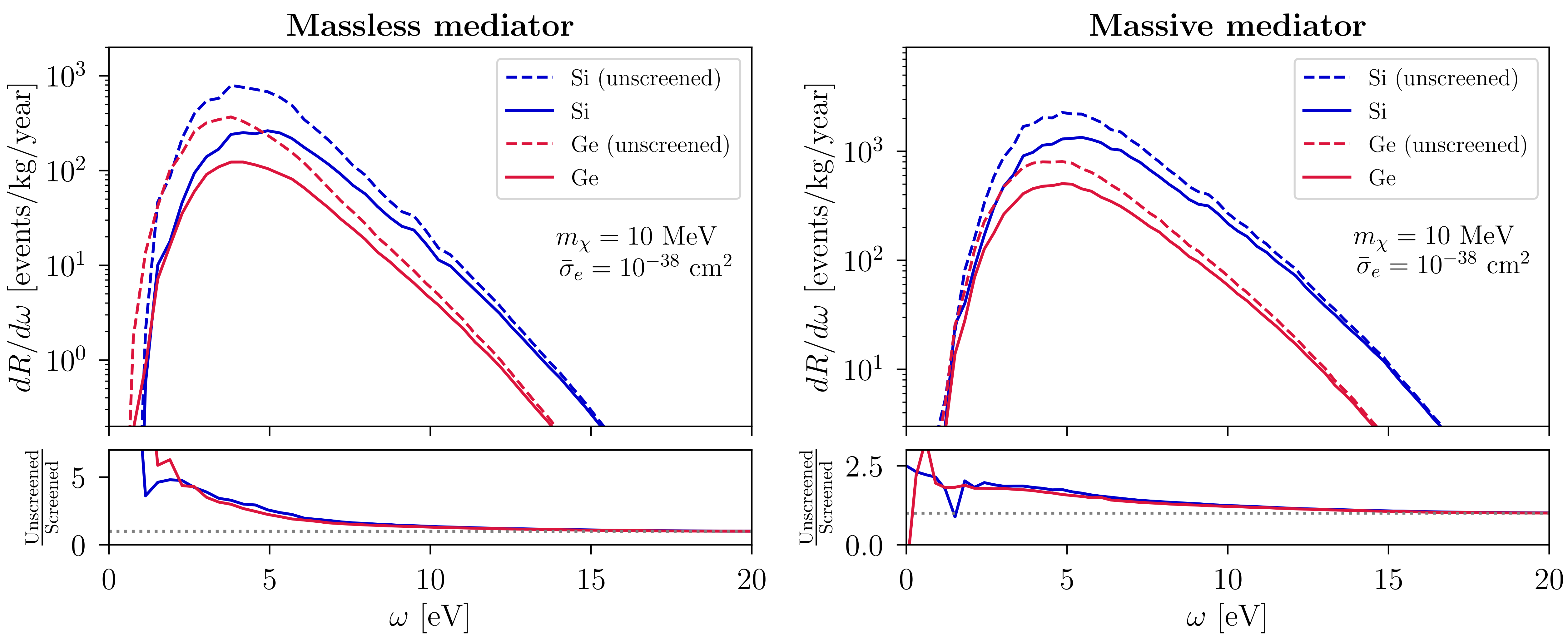


GPAW
GPAW (no LFE)
Mermin
measured

DFT-based calculation
— Phenomenological approach
— X-ray scattering

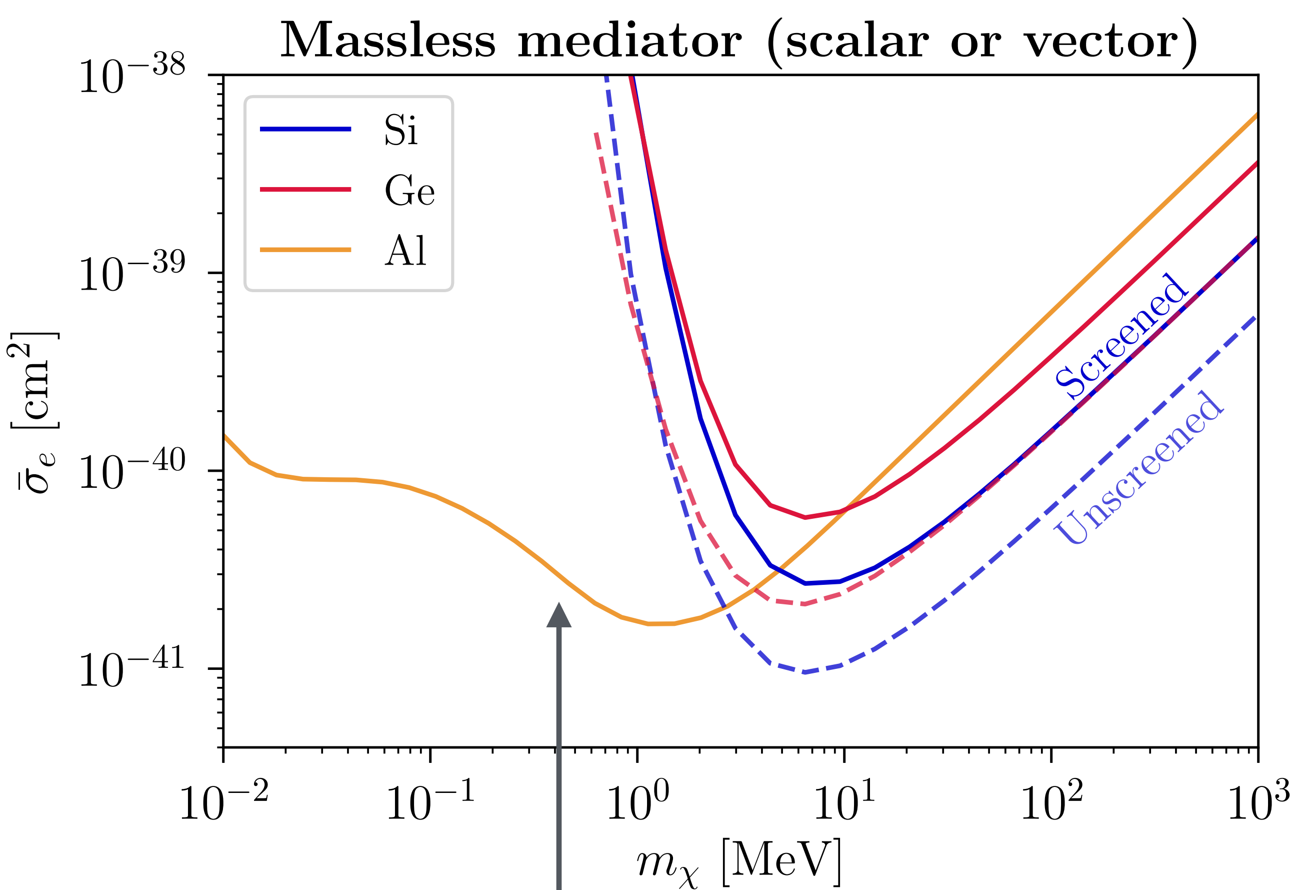
GPAW: Mortensen, Hansen, Jacobsen 2005;
Enkovaara, Rostgaard, Morstensen + 2010;
Mermin approach: Vos and Grande 2021
X-ray: Weissker et al. 2010

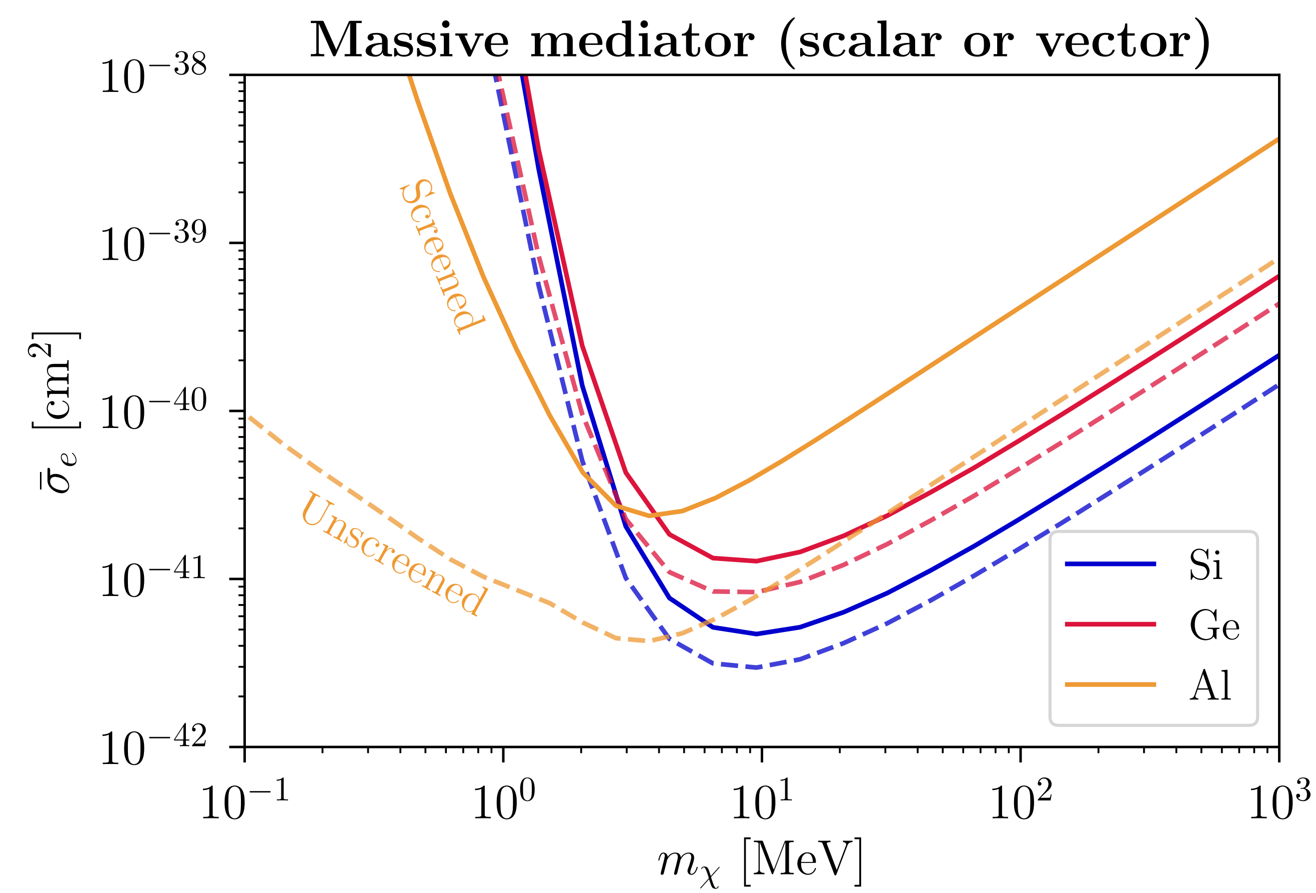
Implications for DM-electron scattering



Unscreened: $\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right) \rightarrow \mathrm{Im}\left(\epsilon(\omega, \mathbf{k})\right)$

Implications for DM-electron scattering





Metal/superconductor: large screening, but also massive gains in rate at low momentum

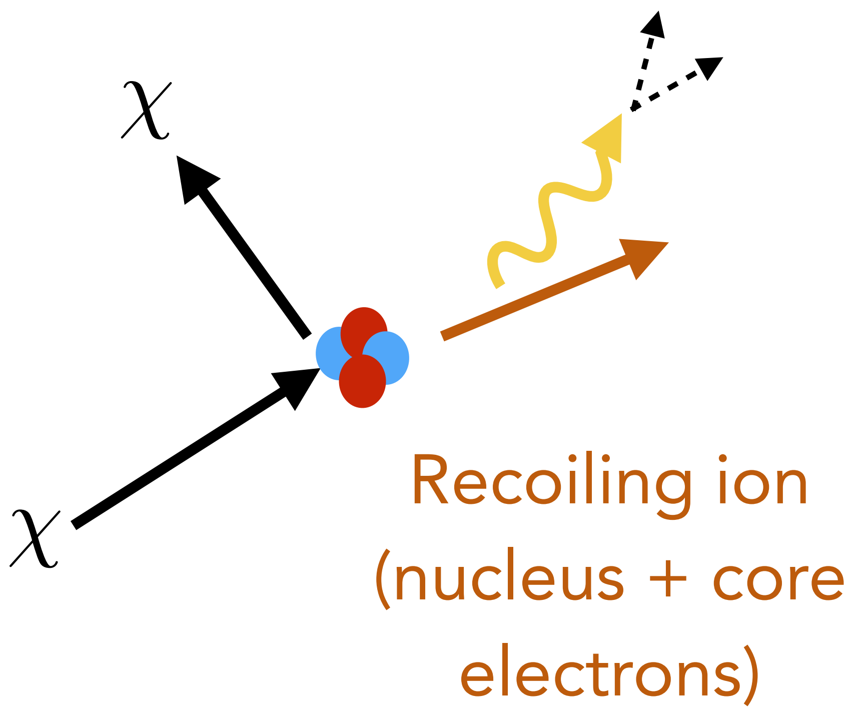
Summary

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F^2_{\text{med}}(\mathbf{k}) \,\text{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

DM-electron scattering is determined by the energy loss function (ELF)

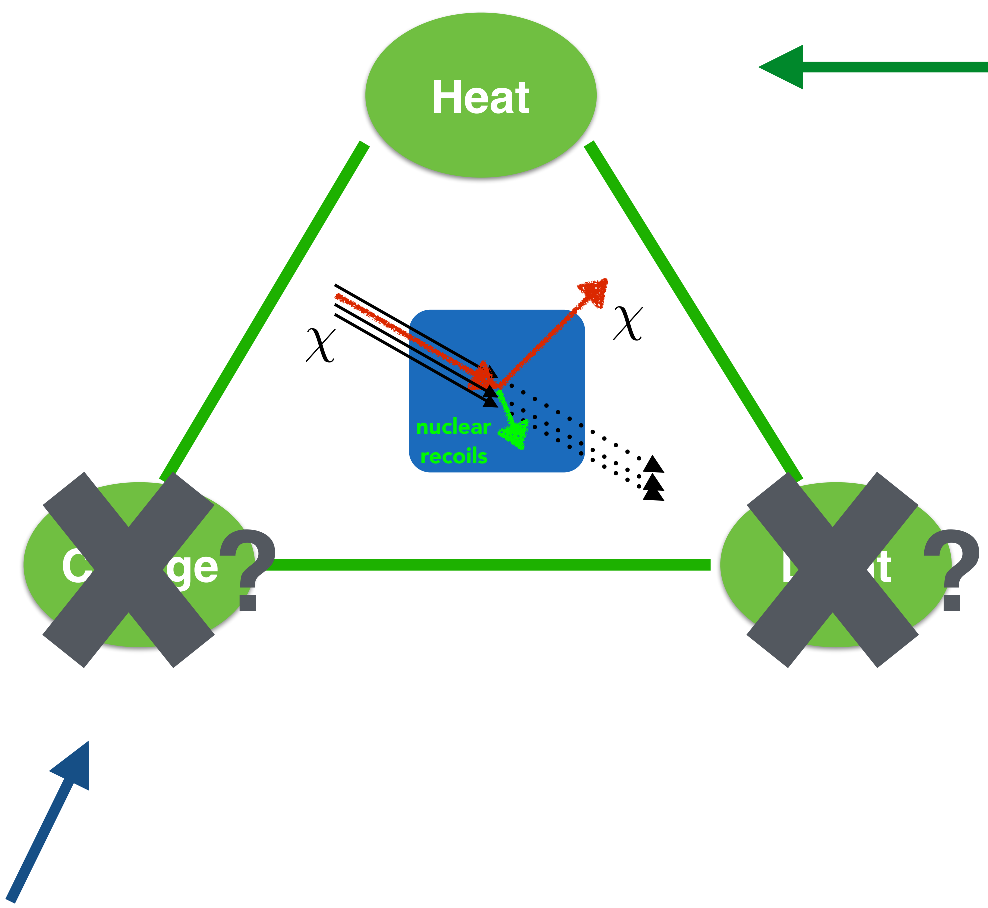
Account for screening effects (scalar and vector mediators) and many-body physics to desired accuracy. Effects in semiconductors impact sensitivity of current/upcoming experiments

The Migdal effect in semiconductors



with Simon Knapen and Jonathan Kozaczuk
2011.09496, 2101.08275

Challenges of low-energy nuclear recoils



Lower the heat threshold

- Detectors in development to reach ~eV scale thresholds and lower
- Search for single phonon excitations with sub-eV thresholds

Search for rare inelastic processes where electron recoil accompanies nuclear recoil

- Bremsstrahlung $\chi + N \rightarrow \chi + N + \gamma$
- Migdal effect $\chi + N \rightarrow \chi + N + e^-$

Graphic from talk by Kaixuan Ni

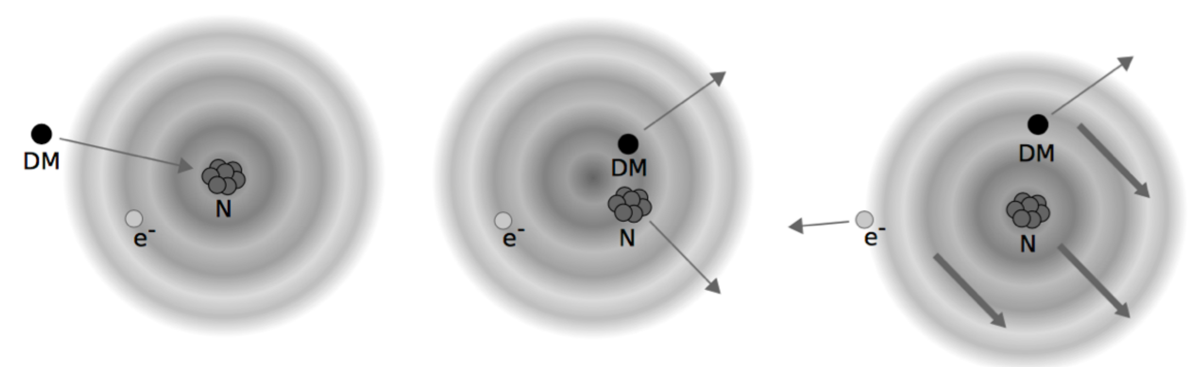
Atomic Migdal effect

Electrons have to 'catch up' to recoiling nucleus

Boost initial state to frame of moving nucleus:

$$|i\rangle \to e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle$$

From 1711.09906 (Dolan, Kahlhoefer, McCabe)



Nucleus recoils with velocity $\mathbf{v}_N$

Transition probability $|\mathscr{M}_{if}|^2$

$$\mathcal{M}_{if} = \langle f| \, e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} \, |i\rangle \approx im_e \, \langle f| \, \mathbf{v}_N \cdot \textstyle\sum_\beta \mathbf{r}_\beta \, |i\rangle$$

Small probability for "shake-off" electron, but allows low-energy nuclear recoil to be above the e- recoil threshold

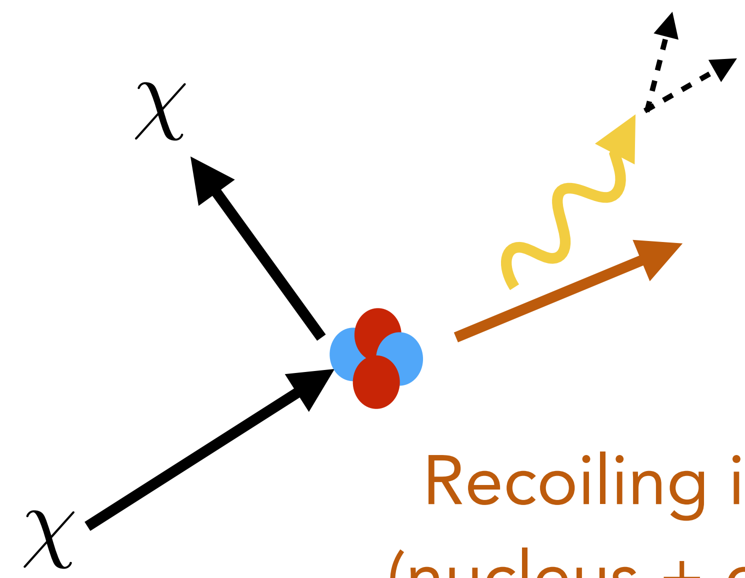
Ibe, Nakano, Shoji, Suzuki 2017

The Migdal effect as bremsstrahlung

Bremsstrahlung calculation

$$\chi + N \to \chi + N + e^-$$

treating N as nucleus with tightly bound core electrons. Valid for $10\ \text{MeV} \lesssim m_\chi \lesssim 1\ \text{GeV}$.



Usual DM-nucleus scattering

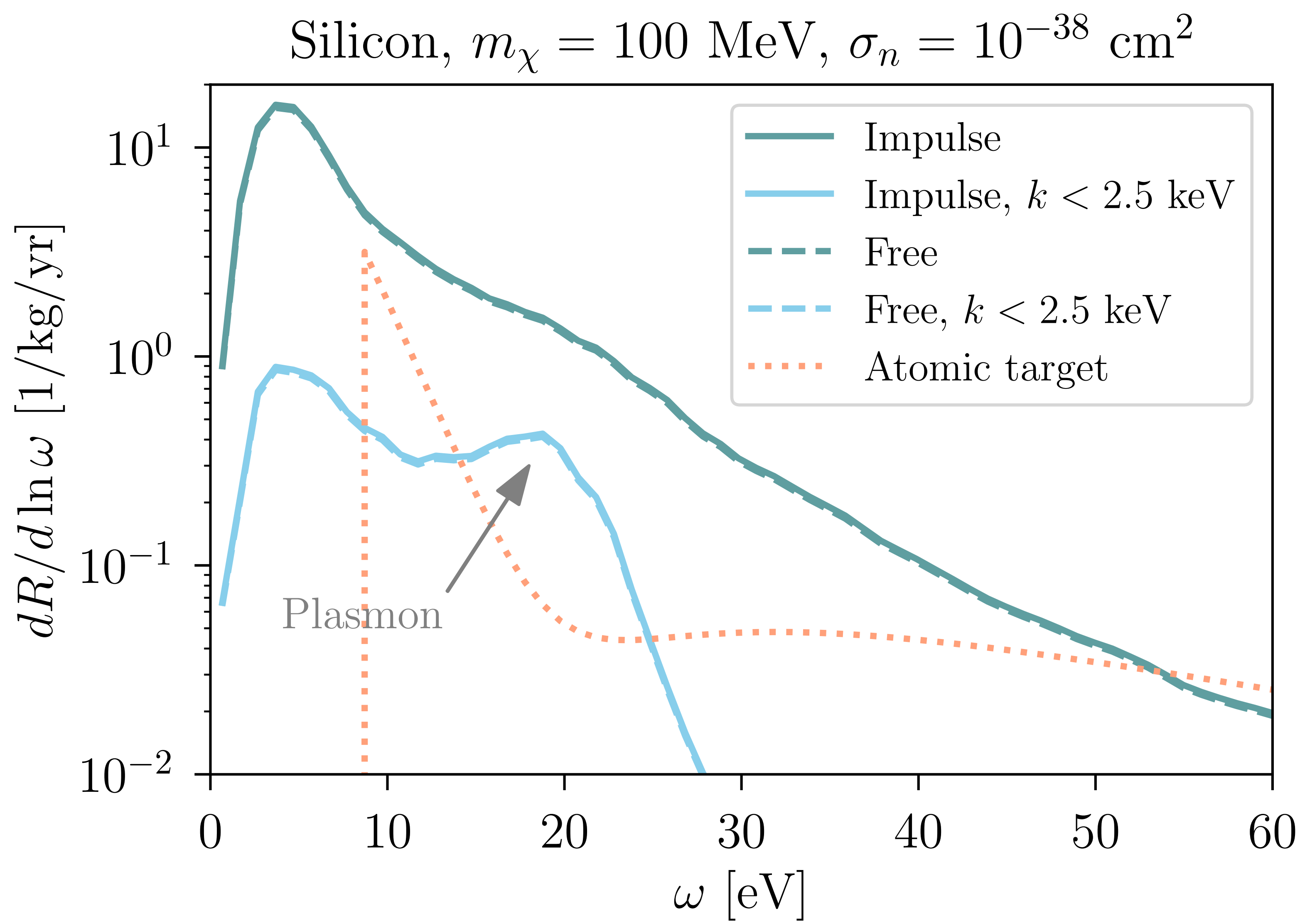
$$\frac{d\sigma}{d\omega} = \frac{2\pi^2 A^2 \sigma_n}{m_\chi^2 v_\chi} \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3} \delta(E_i - E_f - \omega - E_N) \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})^2$$

$$\times 4\alpha_{em} Z_{\text{ion}}^2 \left[\frac{1}{\omega - \mathbf{q}_N \cdot \mathbf{k}/m_N} - \frac{1}{\omega} \right]^2 \text{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

Form factor accounting for multiphonon response in a harmonic crystal

Differential probability of ion to excite an electron

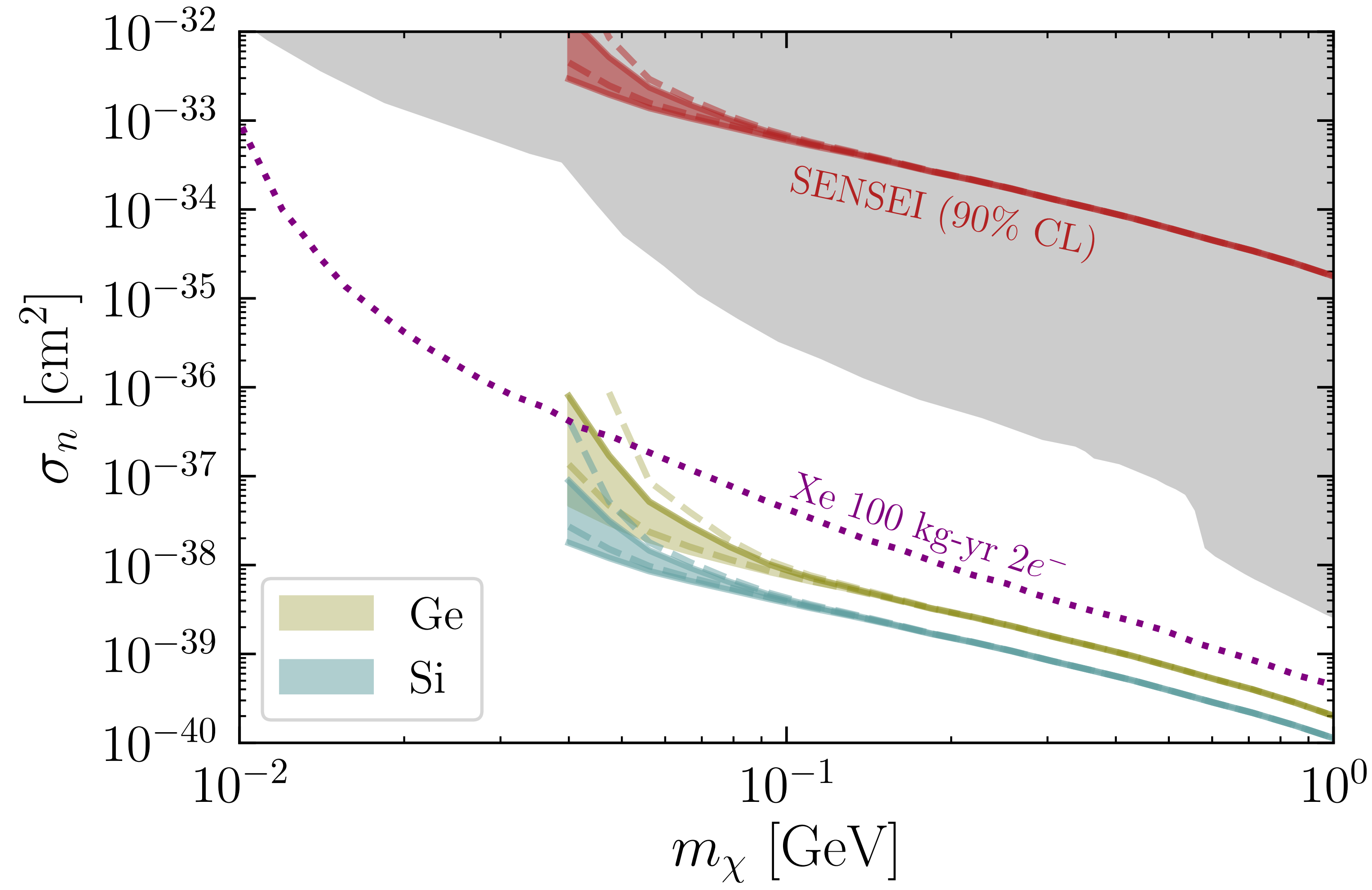
Full rate in semiconductors



Rate in semiconductors is much larger due to lower gap for excitations.

Sensitivity in semiconductors

1 kg-year exposure, with Q > 2 (similar to proposed experiments)



The Migdal effect in semiconductors can enhance sensitivity to nuclear recoils from sub-GeV dark matter

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

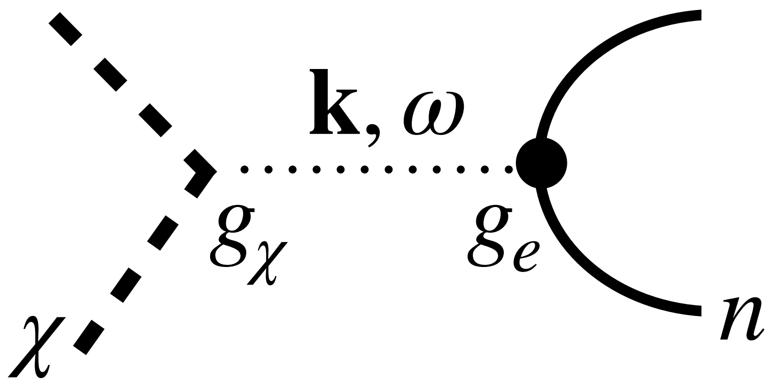
The energy loss function (ELF) in dielectric materials describes response to any electromagnetic probe (Standard Model or DM)

Unifies approach to multiple DM mediators and target materials

First principles calculations accounting for many-body effects

Data-driven and experimental calibration of ELF

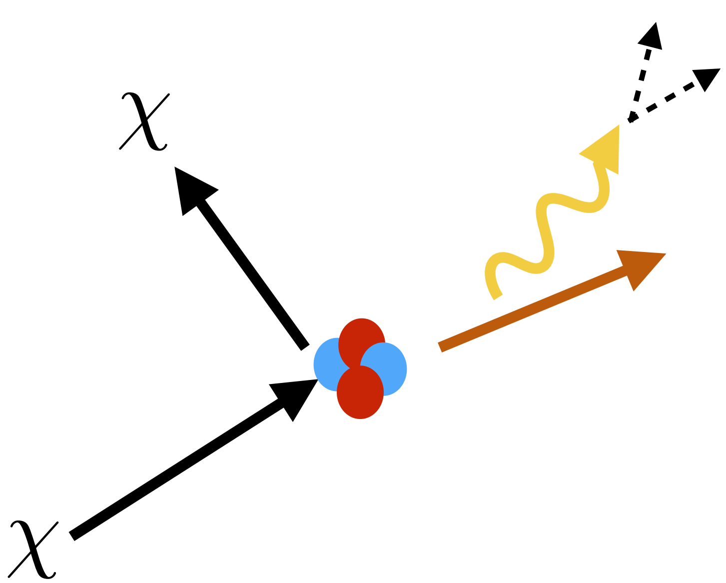
DM-electron scattering



$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \,\text{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

Screening effects in semiconductors and superconductors

DM-nucleus scattering: the Migdal effect



$$\frac{dP}{d^3\mathbf{k}d\omega} \propto \frac{4\pi\alpha_{em}Z_{\text{ion}}^2}{\omega^4}\frac{|\mathbf{v_N}\cdot\mathbf{k}|^2}{k^2}\text{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

First derivation & calculation of Migdal effect in semiconductors