Hamiltonian mechanics and quantization of simplest 3D counterpart of multiple D0-brane system: progress report

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Contents

Outline

Dirichlet p-branes (or Dp-branes) are supersymmetric extended objects

- On which the fundamental $D = 10$ superstrings can have its ends attached.
- In 10D, there exist supersymmetric Dp-branes:
  - $p = 0,\ 2,\ 4,\ 6,\ 8$ in type IIA superspace.
  - $p = 1,\ 3,\ 5,\ 7,\ 9$ in type IIB superspace.
- Its worldvolume action is given by the sum of the nonlinear Dirac-Born-Infeld (DBI) term and Wess-Zumino (WZ) term [Cederwall, von Gussich, Nilsson, Westerberg, 1996; Cederwall, von Gussich, Nilsson, Sundell, Westerberg 1996; Aganagic, Popescu, Schwarz 1996; Bergshoeff, Townseng 1996; Bandos, Sorokin, Tonin 1997].

Systems of multiple branes

- In 1995, E. Witten argued that the system of N nearly coincident Dp-branes
  - carries non-Abelian gauge fields on center of energy worldvolume.
  - Its gauge fixed description at very low energy limit is given by the action of non-Abelian U(N) SUSY Yang-Mills (SYM) theory at low energy.
- In it, the $N = 1$ case gives the action for Abelian U(1) SYM which can be identified as a weak field limit of gauge fixed version of the single Dp-brane.

Problem statement

- Despite a number of very interesting results and certain progress during these years [Tseytlin 1997; Emparan 1998; Myers 1999; Lozano, Janssen et al 2002-2005; Howe, Lindstrom, Wulff 2005,2007] the complete supersymmetric action for mDp-branes had not been known even for the simplest case of $p = 0$. However,
  - it is widely believed that the **bosonic limit** of this system is given by the Myers's "dielectric brane" action [Myers 1999] (but it still resists the supersymmetric generalization).
  - A very interesting supersymmetric **'-1 quantization level'** approach was proposed in [Howe, Lindstrom, Wulff:2005,2007] and its quantization should reproduce the desired mDp action (but the complete consistent realization of this step seems to require the quantization of the complete interacting system of supergravity and super-Dp-brane).
- A complete set of candidates for mD0-brane system was constructed in our [PRD 2022, PRD 2023].

- In this talk we will
  - present our (complete set of) 3D mD0 action(s) [Bandos, Sarraga; JHEP 2022] i.e. the $\mathrm{D} = 3$ counterpart of 10D mD0-brane system which is doubly supersymmetric (spacetime supersymmetry + worldline supersymmetry)
  - construct the Hamiltonian approach of the simplest 3D counterpart of mD0 model and proceed with its quantization.
  - Some problems found in this way are also present in the case (3D counterpart) of single D0-brane.
  - To solve this, we quantized first this single D0-brane system using a new basis.

Outline

Dynamical variables describing the 3D mD0 system

- The set of center of energy variables contains coordinate functions

$$Z^M(\tau) = \big(x^a(\tau), \theta^\alpha(\tau), \bar{\theta}^\alpha(\tau)\big) \ , \qquad a = 0, 1, 2 \ , \quad \alpha = 1, 2 \ ,$$

  given by bosonic 3-vector and two complex conjugate fermionic spinors, describing the embedding of the center of energy worldline $\mathcal{W}^1$ in flat D $= 3$ $\mathcal{N} = 2$ target superspace,

$$\mathcal{W}^1 \subset \Sigma^{(3|4)} : \quad Z^M = Z^M(\tau) \ .$$

- The relative motion of the constituents is described by the matrix fields from the D $= 3$ $\mathcal{N} = 2$ SU(N) SYM model dimensionally reduced to d $= 1$.
- We also use some auxiliary fields: Spinor moving frame variables and momenta for the bosonic matrix fields.

Spinor moving frame in 3D

- Spinor moving frame (also called Lorentz harmonics [Bandos 1990]) matrix

$$(v_\alpha^1, v_\alpha^2) \in \mathsf{SL}(2,\mathbb{R}) \iff v^{\alpha 2} v_\alpha^1 = 1 \ ,$$

  it is used as basis to construct our 3D mD0 candidate.
- However, it is more convenient to describe the spinor moving frame by

$$w_\alpha = \frac{1}{\sqrt{2}}(v_\alpha^1 - i v_\alpha^2) \ , \qquad \bar{w}_\alpha = \frac{1}{\sqrt{2}}(v_\alpha^1 + i v_\alpha^2) \ \text{ which obey } \ \bar{w}^\alpha w_\alpha = i \ .$$

- These variables are called spinor (moving) frame variables because it can be considered as a kind of square root of a vector frame in the sense that

Spinor moving frame = $\sqrt{\text{moving frame}}$

- we can construct the moving frame vectors

$$u_a^{(0)} = w\gamma_a\bar{w} \ , \qquad u_a = w\gamma_a w \ , \qquad \bar{u}_a = \bar{w}\gamma_a\bar{w} \ .$$

  These obey

$$u^{(0)a}u_a^{(0)} = 1 \ , \qquad u^a\bar{u}_a = -2 \ , \qquad u^{(0)a}u_a = 0 = u^{(0)a}\bar{u}_a \ .$$

- These moving frame vectors provide a 3D version of the 4D light-like tetrade of the Newman-Penrose formalism [Newman-Penrose 1962] and can be collected in the SO$(1,2)$ valued matrix

$$u_a^{(b)} = \left(u_a^{(0)}, \frac{1}{2}(u_a + \bar{u}_a), \frac{1}{2i}(u_a - \bar{u}_a) \right) \in \mathsf{SO}(1,2) \ .$$

Outline

The complete nonlinear action for the description of 3D multiple D0-brane system

$$S^{3D}_{mD0} = -m\int_{\mathcal{W}^1} \mathrm{E}^0 - m\int_{\mathcal{W}^1}(\mathrm{d}\theta^\alpha\bar{\theta}_\alpha - \theta^\alpha \mathrm{d}\bar{\theta}_\alpha) + \frac{1}{\mu^6}\int_{\mathcal{W}^1}\frac{\mathrm{d}\mathcal{M}}{\mathcal{M}}\mathrm{tr}\left(\bar{\mathbb{P}}\mathbb{Z} + \mathbb{P}\bar{\mathbb{Z}}\right) +$$

$$+\frac{1}{\mu^6}\int_{\mathcal{W}^1}\left[\mathrm{tr}\left(\bar{\mathbb{P}}\mathrm{D}\mathbb{Z} + \mathbb{P}\mathrm{D}\bar{\mathbb{Z}} + \frac{i}{8}\mathrm{D}\boldsymbol{\Psi}\,\bar{\boldsymbol{\Psi}} - \frac{i}{8}\boldsymbol{\Psi}\mathrm{D}\bar{\boldsymbol{\Psi}}\right) - \frac{2}{\mathcal{M}}E^0\mathcal{H}\right] -$$

$$-\frac{1}{\mu^6}\int_{\mathcal{W}^1}\frac{i}{\sqrt{\mathcal{M}}}\left(\mathrm{d}\theta^\alpha w_\alpha\bar{\nu} + \mathrm{d}\bar{\theta}^\alpha\bar{w}_\alpha\nu\right)\ ,$$

where $\quad \nu := \mathrm{tr}(-\boldsymbol{\Psi}\mathbb{P} + \bar{\boldsymbol{\Psi}}[\mathbb{Z},\bar{\mathbb{Z}}])\ , \qquad \bar{\nu} := \mathrm{tr}(-\bar{\boldsymbol{\Psi}}\bar{\mathbb{P}} + \boldsymbol{\Psi}[\mathbb{Z},\bar{\mathbb{Z}}])$

and $\quad \mathcal{H} = \mathrm{tr}\left(\mathbb{P}\bar{\mathbb{P}} + [\mathbb{Z},\bar{\mathbb{Z}}]^2 - \frac{i}{2}\mathbb{Z}\boldsymbol{\Psi}\boldsymbol{\Psi} + \frac{i}{2}\bar{\mathbb{Z}}\bar{\boldsymbol{\Psi}}\bar{\boldsymbol{\Psi}}\right)\ .$

- It is written in terms of the variables used for the single D0-brane (now the center of mass variables) and
- traceless $N \times N$ complex bosonic and fermionic matrix matter fields

$$\mathbb{Z} = (\bar{\mathbb{Z}})^\dagger\ , \qquad \mathbb{P} = (\bar{\mathbb{P}})^\dagger\ , \qquad \boldsymbol{\Psi} = (\bar{\boldsymbol{\Psi}})^\dagger\ ,$$

describing the relative motion of the constituents of the system as well as

- the bosonic anti-Hermitean worldline gauge field $\mathbb{A} = \mathrm{d}\tau \mathbb{A}_\tau$.
- The latter enters in the action from the covariant derivatives of matrix matter fields

$$\mathrm{D}\mathbb{Z} = \mathrm{d}\mathbb{Z} + 2ia\mathbb{Z} + [\mathbb{A}, \mathbb{Z}] \, , \qquad \mathrm{D}\boldsymbol{\Psi} = \mathrm{d}\boldsymbol{\Psi} - ia\boldsymbol{\Psi} + [\mathbb{A}, \boldsymbol{\Psi}] \, ,$$

$$\mathrm{D}\bar{\mathbb{Z}} = \mathrm{d}\bar{\mathbb{Z}} - 2ia\bar{\mathbb{Z}} + [\mathbb{A}, \bar{\mathbb{Z}}] \, , \qquad \mathrm{D}\bar{\boldsymbol{\Psi}} = \mathrm{d}\bar{\boldsymbol{\Psi}} + ia\bar{\boldsymbol{\Psi}} + [\mathbb{A}, \bar{\boldsymbol{\Psi}}]$$

which also include the composite $\mathrm{U}(1)$ connection

$$a = -\frac{i}{4}\bar{u}^a \mathrm{d}u_a = \frac{i}{4}u^a \mathrm{d}\bar{u}_a = w^\alpha \mathrm{d}\bar{w}_\alpha = \bar{w}^\alpha \mathrm{d}w_\alpha \ .$$

Below we will also use the $\mathrm{SU}(1,1)/\mathrm{U}(1,1)$ Cartan forms

$$f = w^\alpha \mathrm{d}w_\alpha \ , \quad \bar{f} = \bar{w}^\alpha \mathrm{d}\bar{w}_\alpha \ .$$

- Moving frame vector $u_a^{(0)}$ and complex spinors w_α and $\bar{w}_\alpha$ are used to construct bosonic and fermionic forms on the worldvolume

$$E^0 = \Pi^a u_a^{(0)} \ , \qquad E^w = \mathrm{d}\theta^\alpha w_\alpha \ , \qquad \bar{E}^{\bar{w}} = \mathrm{d}\bar{\theta}^\alpha \bar{w}_\alpha \ ,$$

where

$$\Pi^a = \mathrm{d}x^a - i\mathrm{d}\theta\gamma^a\bar{\theta} + i\theta\gamma^a\mathrm{d}\bar{\theta} = \mathrm{d}\tau\Pi_\tau^a$$

is the 3D Volkov-Akulov 1-form.

- $\mathcal{M} = \mathcal{M}\left(\mathcal{H}/\mu^6\right)$ is an *arbitrary* non-vanishing function.
- The particular case of this action with

$$\mathcal{M} = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{\mathcal{H}}{\mu^6}}$$

can be obtained by dimensional reduction of the 4D counterpart of the 11D multiple M-wave (mM0-brane) action (but this is another story [Bandos, Sarraga; JHEP 2022].

Doubly supersymmetry (spacetime supersymmetry + worldline supersymmetry):

- The target superspace D $= 3$ $\mathcal{N} = 2$ SUSY of this action is manifest.
- The local worldline SUSY
  - acts on the center of mass variables as κ-symmetry of single D0-brane.
  - acts on the matrix matter fields with an important role of the function $\mathcal{M} = \mathcal{M}\left(\mathcal{H}/\mu^6\right)$ [Bandos, Sarraga; JHEP 2022].

The simplest 3D counterpart of mD0-brane system $\Longrightarrow$ $\mathcal{M}(\mathcal{H}/\mu^6) = m$

$$S^{\text{3D}}_{\text{mD0}} = -m \int_{\mathcal{W}^1} \mathrm{E}^0 - m \int_{\mathcal{W}^1} (\mathrm{d}\theta^\alpha \bar{\theta}_\alpha - \theta^\alpha \mathrm{d}\bar{\theta}_\alpha) - \frac{1}{\mu^6} \int_{\mathcal{W}^1} \frac{i}{\sqrt{m}} \left(\mathrm{d}\theta^\alpha w_\alpha \bar{\nu} + \mathrm{d}\bar{\theta}^\alpha \bar{w}_\alpha \nu\right) +$$

$$+ \frac{1}{\mu^6} \int_{\mathcal{W}^1} \left[\mathrm{tr}\left(\bar{\mathbb{P}}\mathrm{D}\mathbb{Z} + \mathbb{P}\mathrm{D}\bar{\mathbb{Z}} + \frac{i}{8}\mathrm{D}\boldsymbol{\Psi}\,\bar{\boldsymbol{\Psi}} - \frac{i}{8}\boldsymbol{\Psi}\mathrm{D}\bar{\boldsymbol{\Psi}}\right) - \frac{2}{m} E^0 \mathcal{H}\right]$$

will be the system we study in this talk.

Outline

The canonical Hamiltonian H_0

- is defined by the Legendre transformation of the Lagrangian

$$H_0 = \dot{x}^a P_a + \dot{\theta}^\alpha \Pi_\alpha + \dot{\bar{\theta}}^\alpha \bar{\Pi}_\alpha + ia\mathfrak{d}^{(0)} + if\bar{\mathfrak{d}} - i\bar{f}\mathfrak{d} + \frac{1}{\mu^6}\mathrm{tr}(\dot{\mathbb{Z}}\bar{\mathbb{P}}) + \frac{1}{\mu^6}\mathrm{tr}(\dot{\bar{\mathbb{Z}}}\mathbb{P}) + $$
$$+ \frac{i}{8\mu^6}\mathrm{tr}(\dot{\mathbf{\Psi}}\bar{\mathbf{\Psi}}) + \frac{i}{8\mu^6}\mathrm{tr}(\dot{\bar{\mathbf{\Psi}}}\mathbf{\Psi}) + \mathrm{tr}(\dot{\mathbb{A}}\mathbb{P}_{\mathbb{A}}) - \mathcal{L}^{\mathrm{3D}}_{\mathrm{mD0}} \, .$$

- In it, we denote the momenta conjugate to the bosonic and fermionic coordinates functions by

$$P_a = \frac{\partial \mathcal{L}}{\partial \dot{x}^a} \, , \qquad \Pi_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{\theta}^\alpha} \, , \qquad \bar{\Pi}_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{\bar{\theta}}^\alpha}$$

satisfying the non-vanishing Poisson brackets

$$[P_a, x^b]_{\mathrm{PB}} = -\delta_a^b \, , \qquad \{\Pi_\alpha, \theta^\beta\}_{\mathrm{PB}} = -\delta_\alpha^\beta \, , \qquad \{\bar{\Pi}_\alpha, \bar{\theta}^\beta\}_{\mathrm{PB}} = -\delta_\alpha^\beta \, ,$$

- and the covariant momenta of complex spinor variables

$$\mathfrak{d} = w_\alpha P^\alpha \, , \qquad \bar{\mathfrak{d}} = \bar{w}_\alpha \bar{P}^\alpha \, , \qquad \mathfrak{d}^{(0)} = \bar{w}_\alpha P^\alpha - w_\alpha \bar{P}^\alpha$$

which have the following Poisson brackets

$$[\mathfrak{d}, w_\alpha]_{\mathsf{PB}} = 0 \,, \qquad [\bar{\mathfrak{d}}, w_\alpha]_{\mathsf{PB}} = -\bar{w}_\alpha \,, \qquad [\mathfrak{d}^{(0)}, w_\alpha]_{\mathsf{PB}} = w_\alpha \,,$$

$$[\mathfrak{d}, \bar{w}_\alpha]_{\mathsf{PB}} = -\bar{w}_\alpha \,, \qquad [\bar{\mathfrak{d}}, \bar{w}_\alpha]_{\mathsf{PB}} = 0 \,, \qquad \left[\mathfrak{d}^{(0)}, \bar{w}_\alpha\right]_{\mathsf{PB}} = -\bar{w}_\alpha \,.$$

- Calculating the canonical momenta we find the set of *primary constraints*

$$\Phi_a := P_a + \left(m + \frac{2}{\mu^6}\frac{\mathcal{H}}{m}\right) u_a^0 \approx 0 \,,$$

$$d_\alpha := \Pi_\alpha + iP_a(\gamma^a\bar{\theta})_\alpha + m\bar{\theta}_\alpha + \frac{i}{\mu^6\sqrt{m}} w_\alpha\,\bar{\nu} \approx 0 \qquad \text{with } \bar{\nu} := \mathrm{tr}(-\bar{\mathbf{\Psi}}\bar{\mathbb{P}} + \mathbf{\Psi}[\mathbb{Z}, \bar{\mathbb{Z}}]) \,,$$

$$\bar{d}_\alpha := \bar{\Pi}_\alpha + iP_a(\gamma^a\theta)_\alpha - m\theta_\alpha + \frac{i}{\mu^6\sqrt{m}} \bar{w}_\alpha\,\nu \approx 0 \qquad \text{with } \nu := \mathrm{tr}(-\mathbf{\Psi}\mathbb{P} + \bar{\mathbf{\Psi}}[\mathbb{Z}, \bar{\mathbb{Z}}]) \,,$$

$$\mathfrak{d} \approx 0 \,, \qquad \bar{\mathfrak{d}} \approx 0 \,, \qquad U^{(0)} := \mathfrak{d}^{(0)} - \tfrac{2}{\mu^6}\mathcal{B} \approx 0 \qquad \text{with } \mathcal{B} := \mathrm{tr}\left(\bar{\mathbb{P}}\mathbb{Z} - \mathbb{P}\bar{\mathbb{Z}} - \tfrac{i}{8}\mathbf{\Psi}\bar{\mathbf{\Psi}}\right) \,,$$

$$\mathbb{P}_{\mathbb{A}} := \frac{\partial \mathcal{L}^{\mathsf{3D}}_{\mathsf{mD0}}}{\partial \dot{\mathbb{A}}} \approx 0 \,,$$

- and the *secondary constraint* $\mathbb{G} := \frac{1}{\mu^6}\left([\bar{\mathbb{Z}}, \mathbb{P}] + [\mathbb{Z}, \bar{\mathbb{P}}] + \frac{i}{4}\{\mathbf{\Psi}, \bar{\mathbf{\Psi}}\}\right) \approx 0$.

The canonical Hamiltonian vanishes in the weak sense $H_0 \approx 0$

The total Hamiltonian H of this system

- can then be defined [Dirac 1967] as a sum of constraints with Lagrange multipliers

$$H = b^a \Phi_a + \kappa^\alpha d_\alpha + \bar{\kappa}^\alpha \bar{d}_\alpha + ik\bar{\mathfrak{d}} - i\bar{k}\mathfrak{d} + ik^{(0)} \left(\mathfrak{d}^{(0)} - \frac{2}{\mu^6} \mathcal{B} \right) + \mathrm{tr}(\mathbb{Y}\mathbb{G}) \ .$$

- These coefficients are restricted by $\frac{\mathrm{d}}{\mathrm{d}\tau}(\text{constraints}) = [\text{constraints}, H]_{\mathrm{PB}} \approx 0$.
- This procedure results in some conditions for Lagrange multipliers, which are solved in terms of a few independent functions corresponding to gauge symmetries:

$$H = b\, u^{a(0)} \Phi_a + \kappa\, \bar{w}^\alpha d_\alpha + \bar{\kappa}\, w^\alpha \bar{d}_\alpha + ik^{(0)} \left(\mathfrak{d}^{(0)} - \frac{2}{\mu^6} \mathcal{B} \right) + \mathrm{tr}(\mathbb{Y}\mathbb{G}) \ ,$$

where the arbitrary b, κ, $\bar{\kappa}$, $k^{(0)}$ and $\mathbb{Y}$ reflect the reparametrization, κ-symmetry, $\mathrm{U}(1)$ and $\mathrm{SU}(N)$ symmetries of our model.

Outline

- Some difficulties appear when we proceed to quantize this system using BRST or Gupta-Bleuler methods **even if we only consider single D0-brane**.
- To overcame these problems $\Longrightarrow$ we introduce the so-called analytical basis as follows:
  1. We note that our center of mass superspace is an extended (Lorentz harmonic) superspace: $\Sigma^{(3+3|4)} = \{x^a, \theta^\alpha, \bar{\theta}^\alpha; w_\alpha, \bar{w}_\alpha)\}$.
  2. We define a new basis: $\Sigma^{(3+3|4)} = \{(\mathrm{x}^{(0)}, \mathrm{x}_A, \bar{\mathrm{x}}_A\ , \theta^w, \theta^{\bar{w}}, \bar{\theta}^w, \bar{\theta}^{\bar{w}}; w_\alpha, \bar{w}_\alpha)\}$

$$\mathrm{x}^{(0)} = x^a u_a^{(0)}\ , \qquad \mathrm{x}_A = \mathrm{x} - 2i\theta^w\bar{\theta}^w\ , \qquad \bar{\mathrm{x}}_A = \bar{\mathrm{x}} + 2i\theta^{\bar{w}}\bar{\theta}^{\bar{w}}\ ,$$

with $\mathrm{x} = x^a u_a$, $\bar{\mathrm{x}} = x^a \bar{u}_a$ and

$$\theta^w := \theta^\alpha w_\alpha\ , \qquad \theta^{\bar{w}} := \theta^\alpha \bar{w}_\alpha\ ,$$

$$\bar{\theta}^w := \bar{\theta}^\alpha w_\alpha\ , \qquad \bar{\theta}^{\bar{w}} := \bar{\theta}^\alpha \bar{w}_\alpha\ .$$

  3. We rewrite our Lagrangian in this analytical coordinate basis and we proceed with its quantization but it could be a good warm-up exercise to start with the single D0-brane case.

3D single D0-brane

- Lagrangian of a single 3D D0-brane in spinor moving frame formalism

$$\mathcal{L}_{\mathrm{D0}} = -m\mathrm{E}^{(0)} - m(\mathrm{d}\theta^{\alpha}\bar{\theta}_{\alpha} - \mathrm{d}\bar{\theta}^{\alpha}\theta_{\alpha})$$

can be rewrite in this analytical basis as

$$\mathcal{L}_{\mathrm{D0}} = -m\mathrm{dx}^{(0)} + imf\,\bar{\mathrm{x}}_A - im\bar{f}\,\mathrm{x}_A + 4ma\theta^{\bar{w}}\bar{\theta}^{w} + 2im(\mathrm{d}\theta^{\bar{w}}\bar{\theta}^{w} + \mathrm{d}\bar{\theta}^{w}\theta^{\bar{w}}) \ .$$

- The momenta of the bosonic coordinate functions are related to the momenta in central basis by

$$p^{(0)} := u^{a(0)}p_a \ , \qquad p := -\tfrac{1}{2}u^a p_a \ , \qquad \bar{p} := -\tfrac{1}{2}\bar{u}^a p_a$$

and satisfying the set of Poisson bracket relations with non-vanishing

$$[p^{(0)}, x^{(0)}]_{\mathrm{PB}} = -1 \ , \qquad [p, \bar{\mathrm{x}}_A]_{\mathrm{PB}} = -1 \ , \qquad [\bar{p}, \mathrm{x}_A]_{\mathrm{PB}} = -1 \ .$$

- Similarly, the momenta conjugate to the fermionic coordinates functions $(\theta^w, \theta^{\bar{w}}, \bar{\theta}^w, \bar{\theta}^{\bar{w}})$ are related to the central basis by

$$\Pi^{\theta}_{w} := -i\bar{w}^{\alpha}\Pi_{\alpha} + 2i\bar{\theta}^{w}\bar{p} \,, \qquad \Pi^{\theta}_{\bar{w}} := iw^{\alpha}\Pi_{\alpha} - 2i\bar{\theta}^{\bar{w}}p \,,$$

$$\bar{\Pi}^{\bar{\theta}}_{w} := -i\bar{w}^{\alpha}\bar{\Pi}_{\alpha} - 2i\theta^{w}\bar{p} \,, \qquad \bar{\Pi}^{\bar{\theta}}_{\bar{w}} := iw^{\alpha}\bar{\Pi}_{\alpha} + 2i\theta^{\bar{w}}p \,,$$

  and have the non-vanishing Poisson brackets

$$\{\Pi^{\theta}_{w}, \theta^{w}\}_{\mathsf{PB}} = -1 \,, \quad \{\Pi^{\theta}_{\bar{w}}, \theta^{\bar{w}}\}_{\mathsf{PB}} = -1 \,, \quad \{\bar{\Pi}^{\bar{\theta}}_{w}, \bar{\theta}^{w}\}_{\mathsf{PB}} = -1 \,, \quad \{\bar{\Pi}^{\bar{\theta}}_{\bar{w}}, \bar{\theta}^{\bar{w}}\}_{\mathsf{PB}} = -1 \,.$$

- Now, we proceed to quantize this system:
  1. We compute the constraints and check which are first class and which are second class.
  2. We apply Gupta-Bleuler procedure $\Longrightarrow$ we reduce the number of constraints, leaving only one from a pair of conjugate second class constraints.
  3. We represent our "effective" first class constraints as differential operators.

- Quantum first class constraints imposed on the state vector Ξ:

$$\hat{\Phi}^{(0)}\Xi = (-i\partial_{\mathrm{x}^{(0)}} + m)\Xi = 0 \; , \qquad \hat{\bar{\Phi}}\Xi = -i\partial_{\mathrm{x}_A}\Xi = 0 \; , \qquad \hat{\Phi}\Xi = -i\bar{\partial}_{\bar{\mathrm{x}}_A}\Xi = 0 \; ,$$

$$\hat{\bar{d}}_w\Xi = -i\left(\bar{\partial}_{\bar{\theta}^w} + 2m\theta^{\bar{w}}\right)\Xi = 0 \; , \qquad \hat{d}_w\Xi = -i\partial_{\theta^w}\Xi = 0 \; , \qquad \hat{\bar{d}}_{\bar{w}}\Xi = -i\bar{\partial}_{\bar{\theta}^{\bar{w}}}\Xi = 0$$

$$\hat{\tilde{U}}^{(0)}\Xi = -i\left(\mathbb{D}^{(0)} - 2x_A\partial_{x_A} + 2\bar{x}_A\partial_{\bar{x}_A} - \bar{\theta}^w\partial_{\bar{\theta}^w} + \theta^{\bar{w}}\partial_{\theta^{\bar{w}}} - q\right)\Xi = 0 \; ,$$

where derivatives and covariant derivatives are defined as follows

$$\partial_{x^{(0)}} = \frac{\partial}{\partial x^{(0)}} \; , \qquad \partial_{x_A} = \frac{\partial}{\partial x_A} \; , \qquad \bar{\partial}_{\bar{x}_A} = \frac{\partial}{\partial \bar{x}_A} \; ,$$

$$\mathbb{D}^{(0)} = \bar{w}_\alpha\frac{\partial}{\partial \bar{w}_\alpha} - w_\alpha\frac{\partial}{\partial w_\alpha} \; , \qquad \mathbb{D} = w_\alpha\frac{\partial}{\partial \bar{w}_\alpha} \; , \qquad \bar{\mathbb{D}} = \bar{w}_\alpha\frac{\partial}{\partial w_\alpha} \; .$$

- And imposing these conditions, the state vector superfield has the form

$$\boxed{\Xi = \Xi^q = e^{-im\mathrm{x}^{(0)}}\left(\phi^q(\bar{w}_\alpha, w_\alpha) + i\theta^{\bar{w}}\xi^{q-1}(\bar{w}_\alpha, w_\alpha) + 2\theta^{\bar{w}}\bar{\theta}^w\phi^q(\bar{w}_\alpha, w_\alpha)\right)}$$

- But let's take it one step further: For the case of $q=0$ (and all $\theta=0$), the state vector can be written as

$$\Xi^{(0)}|_{\theta=0} = e^{-im\mathrm{x}^{(0)}}\phi^{(0)}(\bar{w}_\alpha, w_\alpha) \text{ which is a realization of } e^{-ip_a x^a}\phi(p)|_{p^2=m^2} \; .$$

with $p_a = m u_a^{(0)} = m w\gamma\bar{w}$.

- So, to have a usual state vector in the coordinate representation, we should integrate over on-shell momentum. In our variables $p_a = m u_a^{(0)} = m w\gamma\bar{w}$ this is realized as

$$\phi(x) = \int \bar{f}\wedge f\, e^{-imx^a u_a^{(0)}}\phi^{(0)}(\bar{w}, w) \; , \qquad q=0 \; .$$

This can be easy generalised for the case of non-vanishing q

$$\phi_{\alpha_1\ldots\alpha_q}(x) = \int \bar{f}\wedge f\, w_{\alpha_1}\ldots w_{\alpha_q} e^{-imx^a u_a^{(0)}}\phi^q(\bar{w}, w) \; , \qquad q\geq 0 \; ,$$

and

$$\phi_{\alpha_1\ldots\alpha_{-q}}(x) = \int \bar{f}\wedge f\, \bar{w}_{\alpha_1}\ldots \bar{w}_{\alpha_{-q}} e^{-imx^a u_a^{(0)}}\phi^q(\bar{w}, w) \; , \qquad q< 0 \; .$$

- Restoring θ, we find the state vector superfield reads

$$\Xi_{\alpha_1\ldots\alpha_{|q|}}(x,\theta,\bar\theta) = \begin{cases} \int \bar f \wedge f w_{\alpha_1}\ldots w_{\alpha_q} e^{-imx^a u_a^{(0)} + 2m\theta\bar w \bar\theta w} \chi^q(\bar w, w, \theta\bar w) \ , \ q \geq 0 \\ \int \bar f \wedge f \bar w_{\alpha_1}\ldots \bar w_{\alpha_{-q}} e^{-imx^a u_a^{(0)} + 2m\theta\bar w \, \bar\theta w} \chi^q(\bar w, w, \theta\bar w) \ , \ q < 0 \end{cases}$$

with $\chi(\theta^{\bar w}, \bar w_\alpha, w_\alpha) = \phi(\bar w_\alpha, w_\alpha) + i\theta^{\bar w}\,\xi(\bar w_\alpha, w_\alpha)$. This superfield obeys

$$(\bar D_\alpha - im\theta_\alpha)\,\Xi_{\alpha_1\ldots\alpha_{|q|}}(x,\theta,\bar\theta) = 0 \ .$$

- This superfield obeys the Klein-Gordon

$$(\partial_a\partial^a + m^2)\,\Xi_{\alpha_1\ldots\alpha_{|q|}}(x,\theta,\bar\theta) = 0$$

and the Dirac equation

$$\partial^{\alpha\alpha_1}\Xi_{\alpha_1\alpha_2\ldots\alpha_{|q|}} = 2m\,\frac{q}{|q|}\,\Xi^{\alpha}{}_{\alpha_2\ldots\alpha_{|q|}} \qquad q \neq 0 \ .$$

- **The quantization of mD0 on this line is on the way** (hence progress report in the title of this talk).

Outline

Conclusions

- The Hamiltonian description of the simplest 3D mD0-brane system has been obtained. The gauge symmetries of the model (reparametrization, κ-symmetry, U(1) and SU(N) symmetries) are generated by the first class constraints in it.
- The quantization of the system in the natural "central basis" of configuration space meets some technical difficulties.
- To overcame these, we reformulate our model in the so-called analytical basis of the configuration superspace of the system.
- As a preparation stage, we have performed the quantization of the 3D single D0-brane system in the analytical basis that
  - provides us with the superfield representation of the state vector as a superfield in the standard D $= 3$ $\mathcal{N} = 2$ superspace obeying

$$(\bar{D}_\alpha - im\theta_\alpha)\, \Xi_{\alpha_1 \ldots \alpha_{|q|}}(x, \theta, \bar{\theta}) = 0 \ .$$

- The quantization of the (simplest) 3D mD0 is on the way.
- This, in its turn, will be a preliminary step to approach the quantum description of 10D mD0 system which in its turn might shed a new light on the properties of String theory.

Outlook

- To complete the quantization of 3D multiple D0-system **[work in progress]** and to obtain and to study the system of equations of a superfield theory in an extended superspace enlarged by bosonic and fermionic matrix coordinates.
- Quantization of 10D mD0-brane system and studying the superfield equations in enlarged superspace which results from these.
- Quantization of 11D mM0-brane system and study its relation with mD0-brane equations.
- Searching for new insights in String/M-theory studying the mD0 and mM0 field theories thus obtained.

The end!

Thank you for your attention!

Appendix I: The canonical Hamiltonian H_0 vanishes in the weak sense

- The canonical Hamiltonian of mD0-brane is defined by the Legendre transformation of the Lagrangian

$$H_0 = \dot{x}^a P_a + \dot{\theta}^\alpha \Pi_\alpha + \dot{\bar{\theta}}^\alpha \bar{\Pi}_\alpha + ia\mathfrak{d}^{(0)} + if\bar{\mathfrak{d}} - i\bar{f}\mathfrak{d} + \frac{1}{\mu^6}\mathrm{tr}(\dot{\mathbb{Z}}\bar{\mathbb{P}}) + \frac{1}{\mu^6}\mathrm{tr}(\dot{\bar{\mathbb{Z}}}\mathbb{P}) + \\ + \frac{i}{8\mu^6}\mathrm{tr}(\dot{\mathbf{\Psi}}\bar{\mathbf{\Psi}}) + \frac{i}{8\mu^6}\mathrm{tr}(\dot{\bar{\mathbf{\Psi}}}\mathbf{\Psi}) + \mathrm{tr}(\dot{\mathbb{A}}\mathbb{P}_{\mathbb{A}}) - \mathcal{L}^{\mathrm{3D}}_{\mathrm{mD0}} \; .$$

- Substituting the $\mathcal{L}^{\mathrm{3D}}_{\mathrm{mD0}}$ expression and taking into account the definitions of constraints, the canonical Hamiltonian reads

$$H_0 = E^a_\tau \Phi_a + \dot{\theta}^\alpha d_\alpha + \dot{\bar{\theta}}^\alpha \bar{d}_\alpha + iaU^{(0)} + if\bar{\mathfrak{d}} - i\bar{f}\mathfrak{d} + \mathrm{tr}(\dot{\mathbb{A}}\mathbb{P}_{\mathbb{A}}) - \mathrm{tr}(\mathbb{A}_\tau \mathbb{G}) \approx 0 \; ,$$

which vanishes if we consider all the constraints.