

# Entanglement Entropy with Non-Invertible Symmetries

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Universidad  
Politécnica  
de Cartagena

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- Anomalies and Generalized Symmetries
- Symmetry Resolved Entanglement Entropy
- Some Examples: Ising Model
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# Generalized Symmetries and Charges

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A **global symmetry** has an associated Noether current that can be written as a p-form:

$$\nabla_{\mu_1} j_p^{\mu_1 \dots \mu_p} = 0 \quad d \star j_p = 0$$

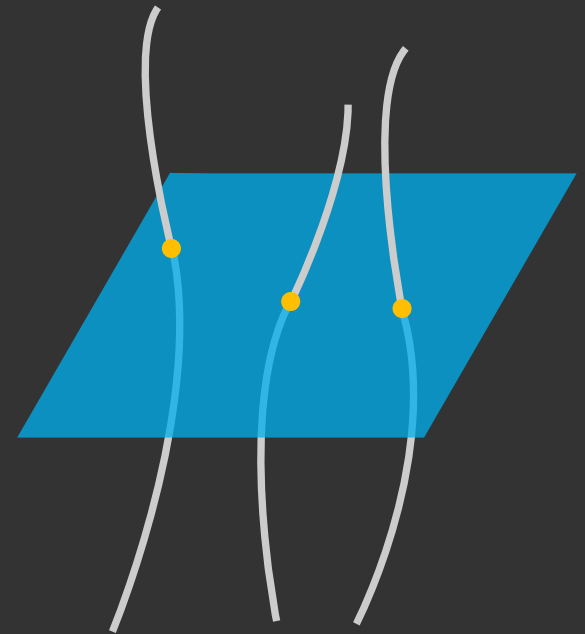
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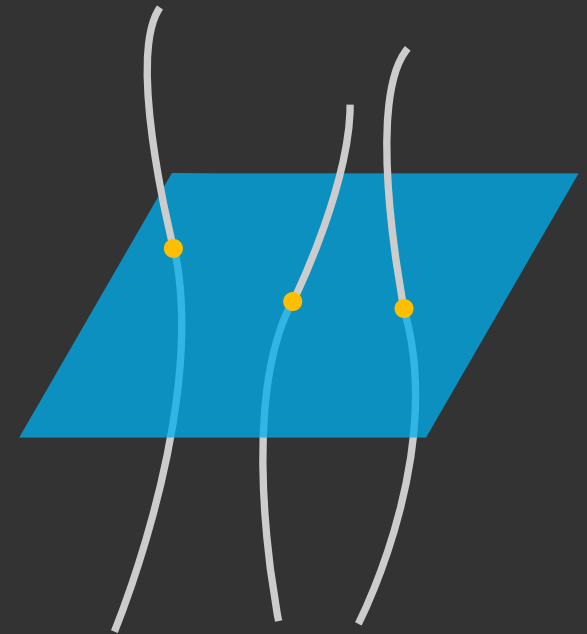
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The **charge** associated to the current can be obtained by integrating in a co-dimension p subspace.

$$Q = \int_{\mathcal{M}_{d-p}} \star j_p$$

This defines a U(1)-valued **topological operator**:

$$U_\alpha(\mathcal{M}_{d-p}) = \exp \left( i\alpha \int_{\mathcal{M}_{d-p}} \star j_p \right)$$



Gaiotto, Kapustin, Seiberg, Willett, 2015

# Chiral Anomaly

---

In 4-dimensional massless fermion theory, one can define 2 conserved currents associated to **global symmetries**:

$$S = \int d^4x \bar{\psi}(\not{\partial} + \not{A})\psi$$

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi$$

$$j_A^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$$

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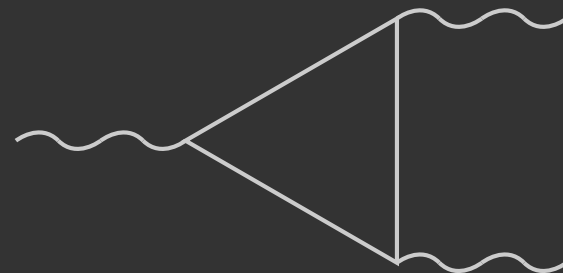
$$j_V^\mu = \bar{\psi}\gamma^\mu\psi$$

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When the theory is quantized an extra term appears in the Ward Identity of the axial current, breaking its conservation. This is the **chiral anomaly**.

$$d \star j_V = 0$$

$$d \star j_A = F \wedge F$$



**Axial charge is not conserved** in the quantum theory!



Can we define a **conserved charge** associated to the axial current?

$$d \star j_A = F \wedge F = d(A \wedge F) \quad \Rightarrow \quad d(\star j_A - A \wedge F) = 0$$

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$$Q = \int_{\mathcal{M}_3} \star J$$

$$U_\alpha(\mathcal{M}_3) = \exp(i\alpha Q)$$

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But we **lose gauge invariance!**

A way to restore it is adding an extra field. We introduce a compact scalar field  $\theta(x)$  on  $\mathcal{M}_3$ : García-Etxebarria and Iqbal, 2023

$$A \rightarrow A + d\Lambda \quad \theta \rightarrow \theta + \Lambda$$

$$\star j_A - A \wedge F + d\theta \wedge F$$

$$U_\alpha(\mathcal{M}_3) = \int \mathcal{D}\theta \exp i\alpha \left( \int_{\mathcal{M}_3} \star j_A - (A - d\theta) \wedge F \right)$$

But now the operator is non-invertible  $\longrightarrow$  **Non-Invertible Symmetry**

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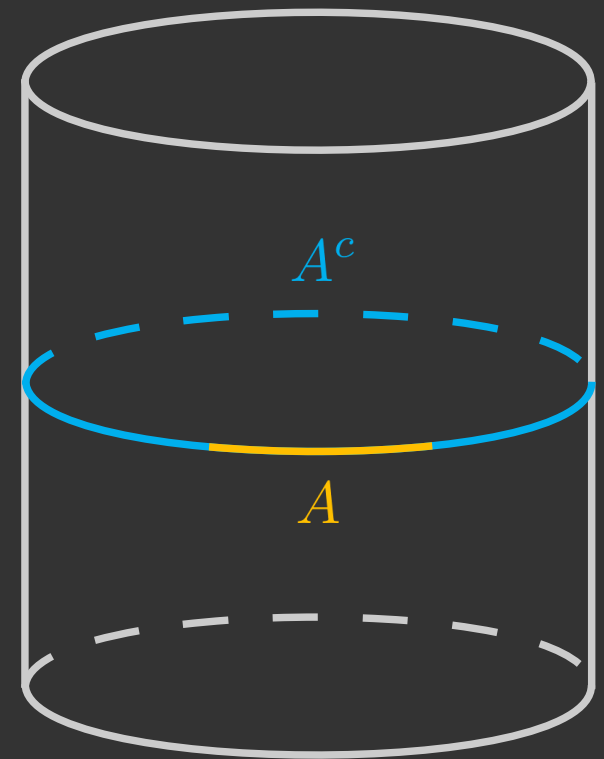
- Anomalies and Generalized Symmetries
- **Symmetry Resolved Entanglement Entropy**
- Some Examples: Ising Model
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# Symmetry Resolved Entanglement Entropy

**Entanglement entropy** measures the entanglement between two subsystems. It is defined as the von Neumann entropy of the density matrix after tracing out the complementary surface.

$$\rho_A = \text{Tr}_{A^c} |\psi\rangle \langle \psi|$$

$$S_{vN} = -\text{Tr} (\rho_A \log \rho_A)$$



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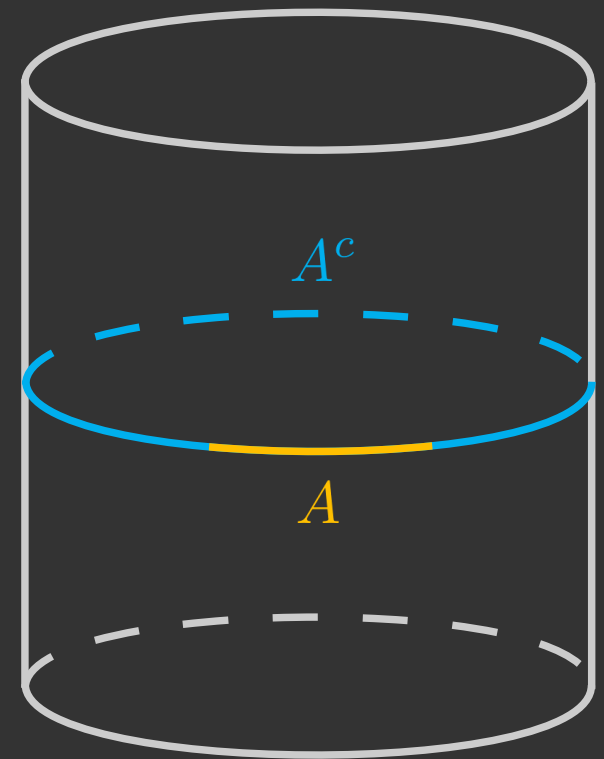
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Symmetry Resolved Entanglement Entropy (**SREE**) is defined as the entanglement entropy of the reduced density matrix projected in **blocks of charge**. This is, how much entropy goes to each charged sector.

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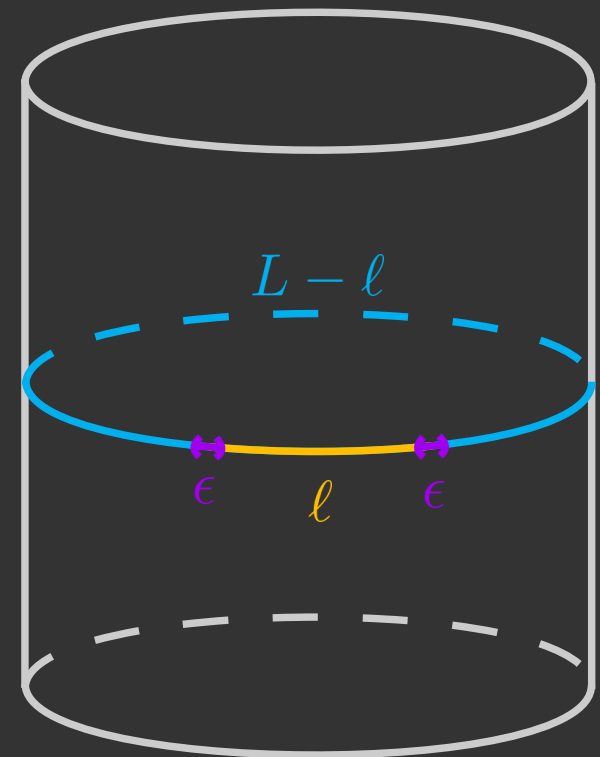
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Entanglement entropy is **UV divergent**, we need a cutoff  $\epsilon$ .

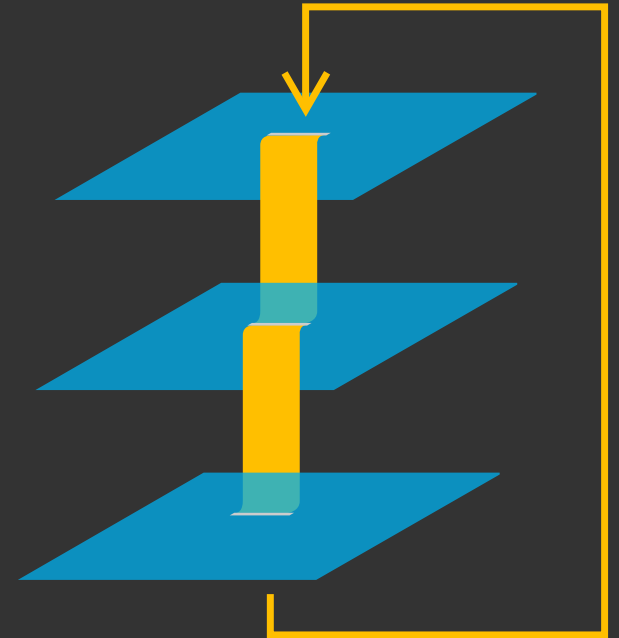


# SREE with Replica Trick

SREE can be computed using the **replica trick**

$$S_{SREE}(q, r) = \lim_{n \rightarrow 1} \frac{1}{1-n} \log \frac{Z(q^n, r)}{(Z(q, r))^n}$$

$$q = e^{-\pi^2 / \log(\ell/\epsilon)}$$





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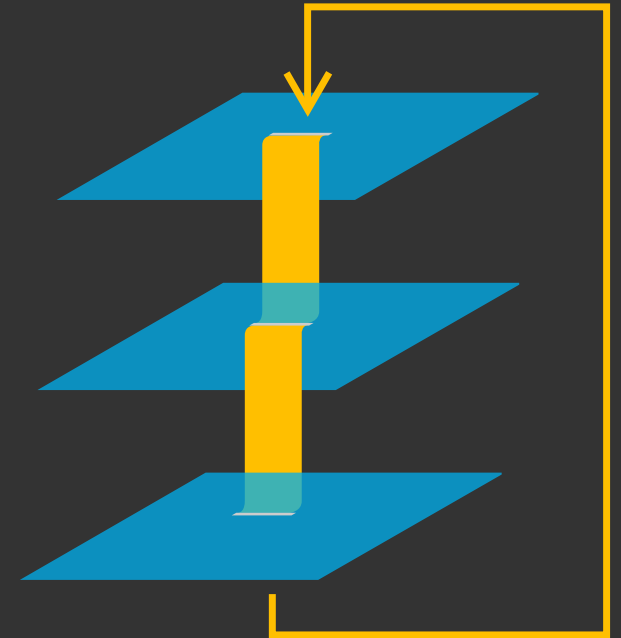
The **partition function** for some irrep is:

$$Z(q^n, r) = \frac{d_r}{|G|} \sum_{g \in G} \chi_r^*(g) \frac{Z(q^n, g)}{(Z(q))^n}$$

With:

$$Z(q) = \text{Tr} q^{L_0 - \frac{c}{24}}$$

$$Z(q^n, g) = \text{Tr} \left( U^A(g) q^{n(L_0 - \frac{c}{24})} \right)$$



Kusuki, Murciano, Ooguri, Pal, 2023

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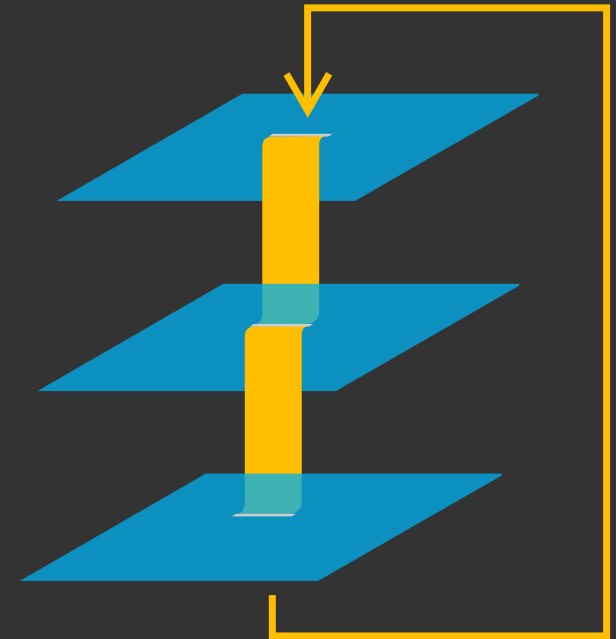
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Kusuki, Murciano, Ooguri, Pal, 2023

Symmetry Operator

Charged moment

Under the modular transformation  $q \rightarrow \tilde{q} = e^{-4 \log(\ell/\epsilon)}$  the **charged moment**:

$$Z(q^n, g) = {}_g \langle a_1 | \tilde{q}^{\frac{1}{n}} (L_0 - \frac{c}{24}) | a_2 \rangle_g$$

Kusuki, Murciano, Ooguri, Pal, 2023

We may demand our states to be **symmetric** under the action of every element of the group.

$$U^A(g) |a\rangle = \langle U^A(g) \rangle |a\rangle$$

**Strongly Symmetric**

$$U^A(g) |a\rangle = |a\rangle + \dots$$

**Weakly Symmetric**

Choi, Rayhaun, Sanghavi, Shao, 2023

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Choi, Rayhaun, Sanghavi, Shao, 2023

The operator implementing the symmetry is a topological **co-dimension 1 operator**.

$$U_\alpha^A(\mathcal{M}_{d-1}) = \exp \left( i\alpha \int_{\mathcal{M}_{d-1}} \star j \right)$$

For  $d = 2$  they are lines

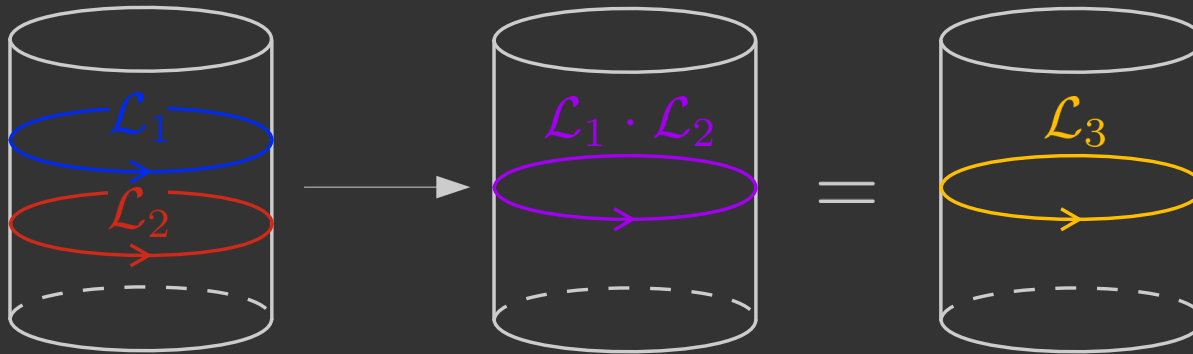


**Verlinde Lines**

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Verlinde lines are topological 1-dimensional operators. They have a defined product over them called **fusion**.

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The product may not be one of a group but something wider: **Fusion Category**.

Some elements of the category may not have an inverse: **non-Invertible lines!**

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Some elements of the category may not have an inverse: **non-Invertible lines!**



We can compute **SREE** for **non-Invertible symmetries** with Verlinde Lines.

# General Result for Finite Groups

---

Verlinde lines are also useful to compute SREE for **groups**.

For finite groups, the **identity sector dominates** over the other elements of the group in the small cutoff limit:

$$\lim_{\tilde{q} \rightarrow 0} \frac{Z(q^n, g)}{Z(q^n, e)} = \delta_{g,e}$$

Furthermore, the **leading contribution** comes from the ground state:

Kusuki, Murciano, Ooguri, Pal, 2023



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# Critical Ising Model

---

The **Critical Ising Model**  $\left(c = \frac{1}{2}\right)$  has three primaries and three lines

	$\mathbb{1}$	$\varepsilon$	$\sigma$	
$\hat{\mathbb{1}} :$	1	1	1	$\varepsilon : \textit{energy operator}$
$\hat{\eta} :$	1	1	-1	$\sigma : \textit{spin operator}$
$\hat{N} :$	$\sqrt{2}$	$-\sqrt{2}$	0	

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The **fusion rules**:

$$\eta^2 = \mathbb{1} \quad N^2 = \mathbb{1} + \eta \quad \eta N = N\eta = N$$

The lines  $\{\mathbb{1}, \eta\}$  implement the group-like  $\mathbb{Z}_2$  symmetry of the model. The  $N$  line is **non-invertible** and it implements the Krammers-Wannier duality.

One can find **three simple boundary states** in the model:

$$|\uparrow\rangle = \frac{1}{\sqrt{2}} |\mathbf{1}\rangle\rangle + \frac{1}{\sqrt{2}} |\varepsilon\rangle\rangle + \frac{1}{2^{1/4}} |\sigma\rangle\rangle$$

$$|\downarrow\rangle = \frac{1}{\sqrt{2}} |\mathbf{1}\rangle\rangle + \frac{1}{\sqrt{2}} |\varepsilon\rangle\rangle - \frac{1}{2^{1/4}} |\sigma\rangle\rangle$$

$$|f\rangle = |\mathbf{1}\rangle\rangle - |\varepsilon\rangle\rangle$$

Kusuki, Murciano, Ooguri, Pal, 2023



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Only the last one is invariant under the action of the  $\mathbb{Z}_2$  group:

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However, under the **non-invertible** line  $N |f\rangle = |\uparrow\rangle + |\downarrow\rangle$

It is **not possible to compute SREE** for the category of Critical Ising model.

# Tricritical Ising Model

The **Tricritical Ising Model**  $\left(c = \frac{7}{10}\right)$  has six primary operators:

	$\mathbb{1}$	$\varepsilon$	$\varepsilon'$	$\varepsilon''$	$\sigma$	$\sigma'$	
$\widehat{\eta} :$	1	1	1	1	-1	-1	$\varphi = \frac{1 + \sqrt{5}}{2}$
$\widehat{N} :$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0	
$\widehat{W} :$	$\varphi$	$-\varphi^{-1}$	$-\varphi^{-1}$	$\varphi$	$-\varphi^{-1}$	$\varphi$	

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We recognize  $\eta$  and  $N$  lines like in the Critical Ising Model.

We can also notice a new **non-Invertible** line:

$$W^2 = \mathbb{1} + W$$

The lines  $\{\mathbb{1}, W\}$  form a **subcategory** within the whole category: Fibonacci Category. We can compute SREE using this subcategory.

For the Fibonacci subcategory one can find three (weakly) symmetric boundary states.

$$W |W\rangle = |W\rangle + |\mathbb{1}\rangle \quad W |\eta W\rangle = |\eta W\rangle + |\eta\rangle$$

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Choi, Rayhaun, Sanghavi, Shao, 2023

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At leading order, with boundary conditions  $\langle NW|$  and  $|NW\rangle$ :

$$S(q, r) = \log \frac{d_r}{|G|} + \frac{7}{30} \log \frac{\ell}{\epsilon} + \log \left( \sqrt{10 - 4\sqrt{5}} \right)$$

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$\frac{c}{3}$

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- For the **ones we can** compute SREE the result is the **same as the ordinary symmetry** one. So to this quantity they behave exactly as ordinary symmetries.
  - What are the **irreps of a fusion category**? What are their dimensions? **(WIP)**
- There were some lines for which SREE cannot be computed:
  - Does this mean that **SREE is ill-defined in some cases**?
  - On the other hand, this quantity is intimately related to the notion of conserved charge. Could the impossibility of computing SREE for some lines mean there is **no good definition of charge in those cases**? **(WIP)**
- Though we could make a good analysis based on SREE, there are some **other quantities worth computing**. Don't miss **Javi's talk** tomorrow.

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**THANK YOU!**