Stringy compactifications with non-trivial torsional homology

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Our Starting Point

- To perform a dimensional reduction we expand the higher dimension fields in a basis of forms

- It might be non-consistent to expand using only the internal space harmonics

- Observed for SU(3) structures with fluxes

An Interesting Example 1/2

[Gurrieri, Louis, Micu, Waldram 03]

$$\begin{array}{ccc} \text{IIB } M_4 \times Y & \xleftrightarrow{\;mirror\;} & \text{IIA } M_4 \times \tilde{Y} \\ \Big\downarrow {\scriptstyle H(Y,\mathbb{R})} & & \Big\downarrow {\scriptstyle H(\tilde{Y},\mathbb{R})} \\ \mathcal{L}_{4d,\text{IIB}} & \underset{=}{\longleftrightarrow} & \mathcal{L}_{4d,\text{IIA}} \end{array}$$

$$\begin{array}{ccc} \text{IIB } M_4 \times Y + H \text{ fluxes} & \xleftrightarrow{\;mirror\;} & \text{IIA } M_4 \times \tilde{Y}' \\ \Big\downarrow {\scriptstyle H(Y,\mathbb{R})} & & \Big\downarrow {\scriptstyle H(\tilde{Y}',\mathbb{R})} \\ \mathcal{L}_{4d,\text{IIB}} & \underset{\neq}{\longleftrightarrow} & \mathcal{L}_{4d,\text{IIA}} \end{array}$$

An Interesting Example 2/2

[Gurrieri, Louis, Micu, Waldram 03]

$$
\begin{array}{ccc}
\text{IIB } M_4 \times Y + H \text{ fluxes} & \xleftrightarrow{\;\;mirror\;\;} & \text{IIA } M_4 \times \tilde{Y}' \\
\Big\downarrow H(Y,\mathbb{R}) & & \Big\downarrow H(\tilde{Y}',\mathbb{R}) + \text{coexact forms} \\
\mathcal{L}_{4d,\text{IIB}} & \xleftrightarrow[=]{} & \mathcal{L}_{4d,\text{IIA}}
\end{array}
$$

Idea

- A basis of coexact forms is often postulated but not described explicitly for SU(3) structures

Representatives of torsion cohomology can be related in a natural way to this basis

In a Nutshell

- Torsion Cohomology classes represented with delta forms

- EFTs may have access to the smeared delta forms

The Linking Numbers of calibrated torsional cycles can be computed via the smeared delta forms

[Gonzalo C., Fernando M., M.Z. 23]

What is torsion (co)homology?

$H_{\text{free}}(X_n, \mathbb{Z})$

- Counts # Cycles
- Not trivial
- Intersection numbers

$H_{\text{tor}}(X_n, \mathbb{Z})$

- Counts # Torsion Cycles
- Trivial with N windings
- Linking numbers

Smeared delta forms

- Bump Delta forms = Poincare Duals of a cycle

$$\int_{\Pi_p} \omega_p = \int_{X_n} \omega_p \wedge \delta(\Pi_p)$$

- Expansion in a basis of smooth forms

$$\delta(\Pi_p) = \sum_i c_i \, b^i_{n-p} \qquad\qquad c_i = \int_{\Pi_p} \star \, b^i_{n-p}$$

- Smearing

$$\delta^{\mathrm{sm}}(\Pi_p) = \sum_{\lambda_i \ll \ell_s m_{kk}} c_i \, b^i_{n-p}$$

Smeared Linking Number

- Working definition [Horowitz, Srednicki 90]

$$L(\Pi_{n-p-1}, \Pi_p) = \int_{X_n} d^{-1}\delta(\Pi_{n-p-1}) \wedge \delta(\Pi_p)$$

- Expansion

$$\delta(\Pi_p) = \sum_i c_i b^i_{n-p} \qquad \delta(\Pi_{n-p-1}) = \sum_i e_i \frac{1}{\lambda_i} d \star b^i_{n-p} \quad \Rightarrow \quad L = \sum_i \frac{c_i e_i}{\lambda_i}$$

- Smearing

$$L^{\rm sm}(\Pi_{n-p-1}, \Pi_p) = \sum_{\lambda_i \ll \ell_s m_{kk}} \frac{c_i e_i}{\lambda_i}$$

The Proposal

The Smeared Linking Numbers of calibrated torsional cycles is equal to the Exact Linking Number

$$L = L^{\mathrm{sm}}$$

Based on:

- Explicit examples (twisted torus)
- Supersymmetric EFTs structures (4d Aharanov-Bohm strings)

EFT Perspective

1. The δ^{sm} enter in the EFTs as massive modes

   - Form a basis for dimensional reduction

2. The $\mathbb{Z}_n$ factors of $H(X_n, \mathbb{Z})$ produce $\mathbb{Z}_n$ gauge symmetries via Stuckelberg like Lagrangians

   - It is possible to extract L^{sm} from the Lagrangian
   - If $L = L^{\mathrm{sm}}$ we can extract topological information!

A Simple Example: $\tilde{T}^6 = \tilde{T}^3 \times T^3$

- Realized in type 2A as half-flat manifold mirror dual to T^6 in type 2B with electric NS flux [Gurrieri, Louis, Micu, Waldram 03]

$$ds^2_{\tilde{T}^3} = \delta_{ab} e^a \otimes e^b$$

$$e^1 = r_1 dx^1 \qquad e^2 = r_2 dx^2 \qquad e^3 = r_3(dx^3 + N x^2 dx^1)$$

$$\Pi_2^{\mathrm{tor}}\,,\ \Pi_3^{\mathrm{tor}} \quad \Rightarrow \quad \Pi_1^{\mathrm{tor}} \times \Pi_1\,,\ \Pi_1^{\mathrm{tor}} \times \Pi_2$$

$$L(\Pi_1^{\mathrm{tor}}, \Pi_1^{\mathrm{tor}}) = L^{\mathrm{sm}}(\Pi_1^{\mathrm{tor}}, \Pi_1^{\mathrm{tor}})$$

Proof:

Step 0: we consider a 3-dim manifold such that

$$\xi \text{ killing}\,, \quad \star d\xi = f\xi\,, \qquad \xi^2 = 1\,, \qquad f \in \mathbb{R}$$

Step 1: we consider a basis of scalars such that

$$\{\phi_i\} \text{ scalars}\,, \qquad \Delta\phi_i = \sigma_i^2\phi_i\,, \qquad \mathcal{L}_\xi\phi_i = i\mu_i\phi_i\,, \qquad \sigma_i^2, \mu_i \in \mathbb{R}$$

$$\Rightarrow \quad \phi_i = e^{i\mu_i\theta}\tilde{\phi}_i\,, \qquad \mu_i \in \mathbb{Z}, \quad \theta \sim \theta + 2\pi$$

Step 2: we build a basis of 1-forms

- We define

$$A_i = d\phi_i\,, \qquad B_i = \star d(\phi_i \xi)\,, \quad C_i = \star dB_i$$

- Closed under $* d$

$$\star dB_i = C_i\,, \qquad \star dC_i = \sigma_i^2 B_i + f C_i$$

- Diagonalize $\Delta D_i^{\pm} = (\lambda_i^{\pm})^2 D_i^{\pm}$

$$D_i^{\pm} = \left(\frac{1}{2} \pm \frac{f}{2\sqrt{f^2+4\sigma_i^2}}\right) C_i \pm \frac{\sigma_i^2}{\sqrt{f^2+4\sigma_i^2}} B_i\,, \qquad (\lambda_i^{\pm})^2 = \sigma_i^2 + \frac{f^2}{2} \pm \frac{f}{2}\sqrt{f^2+4\sigma_i^2}$$

Step 3: Replacing in the Linking number

$$L(\Pi_1, \tilde{\Pi}_1) = \frac{1}{\lambda_0} K_0 \tilde{K}_0 + \sum_{i,\mu=0} \left[\frac{K_i^+ \tilde{K}_i^+}{\lambda_i^+} + \frac{K_i^- \tilde{K}_i^-}{\lambda_i^-} \right] + \sum_{i,\mu\neq 0} \left[\frac{K_i^+ \tilde{K}_i^+}{\lambda_i^+} + \frac{K_i^- \tilde{K}_i^-}{\lambda_i^-} \right]$$

With

$$K_i^{\pm} = \int_{\Pi_1} D_i^{\pm}, \qquad K_0 = \int_{\Pi_1} D^0$$

blue and red term cancels

$$L^{\mathrm{sm}} \equiv L = \frac{1}{N}$$

Conclusions

- Non-Renormalization theorem, verified in examples, conjectured to be valid in calibrated SUSY EFT. Evidences based on how torsion leaks into the EFTs.

- Prescription to build a basis for dimensional reduction in presence of massive modes

Our findings suggest that torsion in Cohomology may result in measurable physics in the EFT