# Scalar Fields Matter: Democratization, Applications and Type IIB

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# Outline of talk

## Background

- In supergravity theories fields are described by p-forms
- Hodge dualization is a map from p-forms to (d-p)-forms

$$\star(dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}) \equiv \frac{1}{(d-p)!} \varepsilon^{\mu_1 \cdots \mu_p}_{\nu_1 \cdots \nu_{d-p}} (dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_{d-p}})$$

- In the action, the duality between the p-form field potential and their (d-p-2)-duals
- In electromagnetism, we only have the gauge field and in 4 dimensions electric and magnetic charges are duals of each other
- In higher dimensions it is possible to have fields described by higher forms that are not self-duals
- These magnetic charges may play an important role

## Electromagnetic duality

- Maxwell'theory (d=4) is an example of the electric-magnetic dual theory. In a vacuum the equations of motions

$$\partial_\mu F^{\mu\nu} = 0, \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

where the dual

$$\tilde{F}^{\mu\nu} \equiv \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- Left invariant by the change of variables

$$F^{\mu\nu} \to F'^{\mu\nu} = i\tilde{F}^{\mu\nu}$$

exchanging electric and magnetic fields

$$E_i \to E'_i = -B_i, \qquad B_i \to B'_i = E_i$$

- Democratic electromagnetic action

$$S = \int dx^4 \left[\tfrac{1}{8} F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{8}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

# Objectives

- We are then interested in developing an action accounting for electric and magnetic fields
  - Coupling magnetic frames
  - Flux compactification
  - Obtain a thermodynamic description containing electric/magnetic charges
- The duals account for the same number of degrees of freedom that the fields, a further relation must constrain them
- Dualization of higher form fields (E.Bergshoeff, R. Kallosh, T. Ortín,D. Roest, A. Van Proeyen 2001, hep-th/0103233)
- Dualization of scalars non-linearly realized, preserving the symmetries of the $\sigma$-model (Type IIB)

# Democratization of higher form fields

- Proposed pseudo-action containing the potentials for type IIA and IIB
- Pseudo-action: a mnemonic tool to derive the equations of motion but not all equations of motion follow from varying the fields in a pseudo-action. An additional constraint, that does not follow from the pseudo-action, has to be substituted by hand into the set of equations of motions.
- A democratic pseudo-action accounts for all fields of the theory, electric and magnetic
- For these two supergravity theories the extended bosonic field content is

$$IIA: \quad \{e^a, B, \phi, C^{(1)}, C^{(3)}, C^{(5)}, C^{(7)} C^{(9)}\}$$
$$IIB: \quad \{e^a, B, \phi, C^{(0)}, C^{(2)}, C^{(4)}, C^{(6)} C^{(8)}\}$$

# Democratic action for Type IIA, IIB

- Democratic pseudo-action for Type IIA, B in differential form language

$$S = \int - \star (e^a \wedge e^b) \wedge R_{ab} + \tfrac{1}{2} d\phi \wedge \star d\phi + \tfrac{1}{2} H \wedge \star H + \tfrac{1}{2} \sum_n G^{(2n)} \wedge \star G^{(2n)} + \dots$$

  where for Type IIA $n$ is summed over the integers $(n = 0, 1, \dots, 5)$ and over the half-integers for Type IIB $(n = \frac{1}{2}, \dots, \frac{9}{2})$.

- $H$ and $G^{(2n)}$ bosonic field strengths

$$H = dB,$$
$$G = dC - dB \wedge C + G^{(0)} e^B$$

  where these $G$ and $C$ will be different forms based on the considered field strength

- Pseudo-action, we must use a duality relation to relate different potentials and the correct number of degrees of freedom
- Absence of complete scalar democratization and of the Chern-Simons term

# Dualization of a real scalar

- Action
$$S[e^a,\phi] = \int \left\{(-1)^{d-1} \star (e^a \wedge e^b) \wedge R_{ab} + \tfrac{1}{2} d\phi \wedge \star d\phi\right\}$$
where $e^a$ is the Vielbein, $\phi$ the real scalar in $d$ dimensions
- Equations of motion
$$\mathbf{E}_a = \imath_a \star (e^c \wedge e^d) \wedge R_{cd} + \frac{(-1)^d}{2} (\imath_a d\phi \wedge \star d\phi + d\phi \wedge \imath_a \star d\phi)$$
$$\mathbf{E} = -d \star d\phi$$
- Dualization through the introduction of the $(d-2)$-form
$$G \equiv dC = \star d\phi$$
- The eom become
$$\mathbf{E}_a = \imath_a \star (e^c \wedge e^d) \wedge R_{cd} + \tfrac{1}{2} (\imath_a G \wedge \star G + G \wedge \imath_a \star G)$$
$$\mathbf{E} = -d \star G$$
equivalent action in terms of the dual field

# Democratic action for a real scalar

- Rewriting the action in a democratic form

$$S[e^a,\phi,C]=\int\left\{(-1)^{d-1}\star(e^a\wedge e^b)\wedge R_{ab}+\tfrac{1}{4}d\phi\wedge\star d\phi+\frac{(-1)^d}{4}G\wedge\star G\right\}$$

- Equations of motion

$$\mathbf{E}_a=\imath_a\star(e^c\wedge e^d)\wedge R_{cd}+\frac{(-1)^d}{4}\left(\imath_a d\phi\wedge\star d\phi+d\phi\wedge\imath_a\star d\phi\right)$$

$$+\tfrac{1}{4}\left(\imath_a G\wedge\star G+G\wedge\imath_a\star G\right)$$

$$\mathbf{E}_\phi=-\tfrac{1}{2}d\star d\phi$$

$$\mathbf{E}_C=-\tfrac{1}{2}d\star G$$

- Pseudoaction

## Non-liner $\sigma$ models

- Coset $G/H$
  - $A, B = 1, \ldots, \text{dim}G$ labels the adjoint representation of $G$
  - $i, j = 1, \ldots, \text{dim}H$ of $H$
  - $m, n = 1, \ldots, \text{dim}G - \text{dim}H$ for the scalars
- Maurer-Cartan 1-form

$$v^m = v_x^m d\phi^x$$

  as Vielbeins. The target space metric is

$$g_{xy} = g_{mn} v_x^m v_y^n$$

- $g_{xy}$ admits dim$G$ Killing vectors $k_A^x$
  - dim$G$ $(d-1)$-form associated currents $J_A$
  - dim$G$ $(d-2)$-form $C_A$ fields, with field strength $G_A = dC_A$
  - dim$H$ of which are not-dynamical
- While an equivalent action in terms of the $C_A$ is not obtainable, we can rewrite the action as a democratic pseudo-action

## Democratic pseudo-action for the $\sigma$ model

- The action (1605.05559)

$$S[e^a,\phi^x,C_A]=\int\Big\{(-1)^{d-1}\star(e^a\wedge e^b)\wedge R_{ab}+\tfrac{1}{4}g_{xy}d\phi^x\wedge d\phi^y$$

$$+\frac{(-1)^d}{4}\mathfrak{M}^{AB}G_A\wedge\star G_B-\frac{(-1)^d}{2}g^{AB}G_A\wedge\hat{k}_B\Big\}$$

where the $\mathfrak{M}^{AB}$ is a dim$G\times$dim$G$ metric

$$\mathfrak{M}^{AB}=g^{AC}g^{BD}k_C{}^x k_D{}^y g_{xy}$$

of rank

$$\text{rank}\mathfrak{M}=\text{dim}G-\text{dim}H$$

- Duality relation

$$G_A=J_A=dC_A$$

## Equations of motion of the democratic action

- The Einstein equation

$$\mathbf{E}_a = \imath_a \star (e^c \wedge e^d) \wedge R_{cd} + \frac{(-1)^d}{4} g_{xy} \left( \imath_a d\phi^x \star d\phi^y + d\phi^x \wedge \imath_a \star d\phi^y \right)$$

$$+ \tfrac{1}{4} \mathfrak{M}^{AB} \Big( \imath_a G_A \wedge \star G_B + (-1)^d G_A \imath_a \star G_B \Big)$$

- Eom for the scalars

$$\mathbf{E}_x = -\tfrac{1}{2} g_{xy} \left\{ d \star d\phi^y + \Gamma_{zw}{}^y d\phi^z \wedge \star d\phi^w \right\} + \frac{(-1)^d}{4} \partial_x \mathfrak{M}^{AB} G_A \wedge \star G_B$$

$$- \tfrac{1}{2} g^{AB} k_{A\,x} dG_B + (-1)^{d+1} g^{AB} \nabla_x k_{A\,y} G_B \wedge d\phi^y$$

with duality relation reduces

$$\mathbf{E}_x = -g_{xy} \left\{ d \star d\phi^y + \Gamma_{zw}{}^y d\phi^z \wedge \star d\phi^w \right\}$$

- Eom. for $C_A$

$$\mathbf{E}^A = \tfrac{1}{2} d \left[ \mathfrak{M}^{AB} \star G_B - g^{AB} \hat{k}_B \right]$$

# Dualization of Type IIB

- $N = 2B$, $d = 10$ sugra field content:
  - Vielbein $e^a$
  - 2-form $B^M$ doublet
  - 4-form $D$, a $SL(2,\mathrm{R})$ singlet with self-dual 5-form field strength $F$
  - A complex scalar $\tau$ that parameterizes the coset $SL(2,\mathrm{R})/SO(2)$
- Field strengths

$$\mathcal{H}^M \equiv d\mathcal{B}^M$$

$$\mathcal{F} \equiv d\mathcal{D} - \tfrac{1}{2}\varepsilon_{MN}\mathcal{B}^M \wedge \mathcal{H}^N$$

- Self-duality condition

$$\mathcal{F} = \star\mathcal{F}$$

# Action for Type IIB

- The action

$$S[e^a, \tau, \mathcal{B}_M, D] = \int \Big\{ - \star (e^a \wedge e^b) \wedge R_{ab} + \frac{d\tau \wedge \star d\bar{\tau}}{2(\Im\mathrm{m}\,\tau)^2} + \tfrac{1}{2}\mathcal{M}_{MN}\mathcal{H}^M \wedge \star\mathcal{H}^N$$

$$+\tfrac{1}{4}\mathcal{F} \wedge \star\mathcal{F} - \tfrac{1}{4}\varepsilon_{MN}\mathcal{D} \wedge \mathcal{H}^M \wedge \mathcal{H}^N \Big\}$$

and the equations of motion are

$$E_M = -d\left(\mathcal{M}_{MN} \star \mathcal{H}^N\right) - \varepsilon_{MN}\mathcal{H}^N \wedge \mathcal{F}$$

$$E_4 = -\frac{1}{2}dF - \frac{1}{4}\varepsilon_{MN}\mathcal{H}^{M(3)} \wedge \mathcal{H}^{N(3)}$$

# Dualization of the 2-forms

- Dualization condition

$$\tilde{\mathcal{H}}_M = \begin{pmatrix} \tilde{H}_1 \\ \tilde{H}_2 \end{pmatrix} = \mathcal{M}_{MN} \star \mathcal{H}^N$$

- A democratic action would be

$$\tilde{S} = \int \frac{1}{4}\mathcal{M}_{MN}\mathcal{H}^M \wedge \star\mathcal{H}^N + \frac{1}{4}\mathcal{M}^{MN}\tilde{\mathcal{H}}_M \wedge \star\tilde{\mathcal{H}}_N + \frac{1}{4}F \wedge \star F - \frac{1}{4}D \wedge \eta_{MN}\mathcal{H}^M \wedge \mathcal{H}^N$$

where

$$\begin{aligned} \tilde{E}_M &= E_M + dD \wedge \eta_{MN}\mathcal{H}^N \\ \tilde{E}^M &= -\frac{1}{2}d(\mathcal{M}^{MN} \star \tilde{\mathcal{H}}_N) \\ \tilde{E}_4 &= E_4 - \frac{1}{2}\eta_{MN}\mathcal{H}^M \wedge \mathcal{H}^N \end{aligned}$$

# Dualization of the scalars

- We require the action to be invariant under the transformation

$$\delta_\alpha \phi = \alpha^x k^x_A(\phi)$$
$$\delta_\alpha \mathcal{B}^M = \alpha^A T_A{}^M{}_N \mathcal{B}^N$$

Noether current associated with the Killing vector

$$J_A = \star \hat{k}_A - \frac{1}{2} T_A{}^P{}_M \mathcal{M}_{PN} \mathcal{B}^M \wedge \star \mathcal{H}^N - \frac{1}{2} T_{AP}{}^M \mathcal{M}^{PN} \tilde{\mathcal{B}}_M \wedge \star \tilde{\mathcal{H}}_N$$
$$+ \frac{1}{2} T_A{}^P{}_M \mathcal{B}^M \wedge \eta_{PN} \mathcal{H}^{N(3)} \wedge D$$
$$+ \frac{1}{8} T_A{}^N{}_M \mathcal{B}^P \wedge \eta_{PN} \mathcal{B}^M \wedge \eta_{RS} \mathcal{B}^R \wedge \mathcal{H}^S$$

- Considering the dualization condition

$$J_A = dC_A$$

# Type IIB democratic pseudo-action

$$S = \int -\star(e^a \wedge e^b) \wedge R_{ab} + \frac{1}{4} g_{xy} d\phi^x \wedge \star d\phi^y + \frac{1}{4}\mathcal{M}_{MN}\mathcal{H}^M \wedge \star\mathcal{H}^N + \frac{1}{4}\mathcal{M}^{MN}\tilde{\mathcal{H}}_M \wedge \star\tilde{\mathcal{H}}_N + \frac{1}{4} F \wedge \star F + \frac{1}{4} M^{AB} G_A \wedge \star G_B - \frac{1}{2} g^{AB} G_A \wedge \hat{k}_B + \frac{1}{4}\varepsilon_{MN} D \wedge \mathcal{H}^M \wedge \mathcal{H}^N$$

# Conclusions and Outlook

- Democratic pseudo-action for $N = 2B$, $d = 10$ maintaining the symmetries of the $\sigma$ model
- General method for maximal and half-maximal supergravities (given an expression for the generators and the Killing vectors of the coset)
- Studying possible applications in which a democratic action is useful
- Construction of a Komar charge accounting for all electric and magnetic charges