β-symmetry and α' corrections

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IAFE-CONICET-UBA
Buenos Aires, Argentina

GRASS-SYMBHOL meeting, Avila, Nov 2023

Based on 2209.02079 and 2307.02537, Baron, DM, Nuez

and previous work with
Baron, Bedoya, Codina, Hohm, Lescano, F.-Melgarejo and Nuñez

Plan

- A map to perturbative α' corrections in string theory
- The role of T-duality in constraining higher-derivative interactions
- Progress and obstructions in T-duality covariant α' corrections
- The β-symmetry of supergravity

The focus is on

- The first orders of perturbative tree-level α' corrections.
  - Scattering amplitudes
  - Beta functions
  - SUSY
- Neither g_s corrections nor non-perturbative effects.
- NSNS sector: metric, two-form and dilaton.

Universal starting point

Common sector for all strings

$$S = \int dx \sqrt{-g}\, e^{-2\phi} \left[R + 4\,(\partial\phi)^2 - \frac{1}{12}\, H^2 \right]$$

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The α' corrections depend on

- The string: bosonic, heterotic, type II
- The scheme: ambiguous versus unambiguous terms

First order α' corrections

Nepomechie; Gross, Harvey, Martinec and Rhom 1985

Metsaev and Tseytlin, 1987

$$S_{MT} = \int dx \sqrt{-g} e^{-2\phi} \Big[R + 4 (\partial\phi)^2 - \frac{1}{12} H^2 + \frac{a-b}{4} H^{\mu\nu\rho} \Omega_{\mu\nu\rho} - \frac{a+b}{8} \Big(\mathrm{Riem}^2 - \frac{1}{2} H\, H\, \mathrm{Riem} + \frac{1}{24} H^4 - \frac{1}{8} H^2_{\mu\nu} H^{2\mu\nu} \Big) \Big]$$

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	Bosonic	Heterotic	HSZ	Type II
$a+b$	$-2\alpha'$	$-\alpha'$	0	0
$a-b$	0	$-\alpha'$	$-2\alpha'$	0

First order α' corrections

Bergshoeff and de Roo, 1989

$$S_{BdR} = \int dx \sqrt{-g} e^{-2\phi} \left[R + 4\,(\partial\phi)^2 - \frac{1}{12}\,\widehat{H}^2 + \frac{a}{8}\,R_{\rm iem}^{(-)2} + \frac{b}{8}\,R_{\rm iem}^{(+)2} \right]$$

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Hiddenly contains higher orders

$$\omega^{(\pm)} = \omega \pm \tfrac{1}{2} \widehat{H} \, , \qquad \widehat{H} = H - \tfrac{3}{2} a \, \Omega^{(-)} + \tfrac{3}{2} b \, \Omega^{(+)}$$

$$\Omega^{(\pm)} = {\rm tr}[\omega^{(\pm)} d\omega^{(\pm)} + \tfrac{2}{3} \omega^{(\pm)3}]$$

Second order α' corrections

Metsaev and Tseytlin, 1987

$$L^{(2)}_{bos} = R^3_{\text{iem}} + \text{cubic Gauss Bonnet}$$

Naseer and Zwiebach; DM and Lescano 2016

$$L^{(2)}_{HSZ} = -L^{(2)}_{bos} + (\text{Chern Simons})^2$$

Metsaev and Tseytlin, 1987; Bergshoeff and de Roo, 1989

$$L^{(2)}_{het} = (\text{Chern Simons})^2$$

Metsaev and Tseytlin, 1987

$$L^{(2)}_{type\ II} = \text{none}$$

Cubic order α' corrections

Unknown for bosonic and HSZ.

Cai and Nuñez; Gross and Sloan 1986

$$L^{(3)}_{het} = \text{Gauge symmetic } R^4_{\text{iem}} + \zeta(3)(t_8 t_8 - \tfrac{1}{8}\epsilon_{10}\epsilon_{10})R^4_{iem}$$

Gross and Witten; Grisaru and Zanon 1986

$$L^{(3)}_{type\ \ II} = \zeta(3)(t_8 t_8 - \tfrac{1}{8}\epsilon_{10}\epsilon_{10})R^4_{iem}$$

T-duality and α'

Sen 1991: tori reductions yield continuous $O(d, d)$ symmetry to all orders in α'.

Bergshoeff, Janssen and Ortin 1995: Circle reduction of heterotic string.

Meissner, Kaloper and Meissner 1997: Cosmological and circle reductions of bosonic string.

Baron, Melgarejo, DM and Nuñez 2017: Flux compactification of the bi-parametric action.

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In this talk I will discuss how to assess T-duality without compactifying.

Double Field Theory

Siegel 1993; Hohm, Hull and Zwiebach 2009

	Supergravity	DFT
Global Symmetries	GL(D) B-shifts	O(D,D)
Local Symmetries	Lorentz Diffs and gauge	$\underline{\text{Lorentz}} \times \overline{\text{Lorentz}}$ Generalized diffs
Fields	e, B and ϕ	$E(e, \bar{e}, B)$ and $d(\phi, g)$
Space	D-dimensional	$2D$-dimensional Strong constraint

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Gauge fixing and solving the strong constraint

$$\left.\begin{array}{rl} \delta E &= \widehat{\mathcal{L}}_\xi E + E \cdot \Lambda \\ \delta d &= \xi \cdot d - \frac{1}{2}\, \partial \cdot \xi \end{array}\right\} \quad \rightarrow \quad \left\{\begin{array}{l} \delta e = L_\xi e + e \cdot \Lambda \\ \delta B = L_\xi B + d\lambda \\ \delta \phi = L_\xi \phi \end{array}\right.$$

The Green-Schwarz transformation

The three-form in the BdR scheme

$$\widehat{H} = dB - \tfrac{3}{2}\, a\, \Omega^{(-)} + \tfrac{3}{2}\, b\, \Omega^{(+)}$$

is Lorentz invariant due to the Green-Schwarz transformation of B

$$\begin{aligned}
\delta e^a &= L_\xi e + e \cdot \Lambda \\
\delta \Omega^{(\pm)} &= L_\xi \Omega^{(\pm)} + d\ \mathrm{tr}\left(\omega^{(\pm)}\, d\Lambda\right) \\
\delta B &= L_\xi B + d\lambda + \frac{a}{2} \mathrm{tr}\left(\omega^{(-)}\, d\Lambda\right) - \frac{b}{2} \mathrm{tr}\left(\omega^{(+)}\, d\Lambda\right)
\end{aligned}$$

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\end{aligned}$$

A hint that something is missing in DFT.

The Green-Schwarz transformation

The generalized Green-Schwarz transformation
DM and Nunez 2015

$$\delta E_M{}^A = \widehat{\mathcal{L}}_\xi E_M{}^A + E_M{}^B \Lambda_B{}^A + \left(a\, \partial_{[\underline{M}} \Lambda_{\underline{B}}{}^{\underline{C}} F_{\overline{N}]\underline{C}}{}^{\underline{B}} - b\, \partial_{[\overline{M}} \Lambda_{\overline{B}}{}^{\overline{C}} F_{\underline{N}]\overline{C}}{}^{\overline{B}} \right) E^{NA}$$

- Exactly reproduces the Green-Schwarz transformation of B.
- The anomalous transformation of e can be redefined away.
- Finite version: Borsato and Wulff 2020.

The Green-Schwarz transformation

- Preserves the field constraints and closes to first order

$$\left[\delta_{(\xi_1\,,\Lambda_1)},\delta_{(\xi_2\,,\Lambda_2)}\right]=\delta_{(\xi_{21}\,,\Lambda_{21})}$$

w.r.t. the brackets

$$\xi_{12}^M \;=\; [\xi_1\,,\,\xi_2]^M_{(C)}-\frac{a}{2}\,\Lambda_{[1\underline{A}}{}^{\underline{B}}\partial^M\Lambda_{2]\underline{B}}{}^{\underline{A}}+\frac{b}{2}\,\Lambda_{[1\overline{A}}{}^{\overline{B}}\partial^M\Lambda_{2]\overline{B}}{}^{\overline{A}}$$

$$\begin{aligned}\Lambda_{12\,AB} \;=\;& 2\xi^P_{[1}\partial_P\Lambda_{2]\,AB}-2\Lambda_{[1\,A}{}^{C}\Lambda_{2]\,CB}\\ &+a\,\partial_{[\overline{A}}\Lambda_1^{\underline{CD}}\partial_{\overline{B}]}\Lambda_{2\underline{DC}}+a\,\partial_{[\underline{A}}\Lambda_1^{\underline{CD}}\partial_{\underline{B}]}\Lambda_{2\underline{DC}}\\ &-b\,\partial_{[\underline{A}}\Lambda_1^{\overline{CD}}\partial_{\underline{B}]}\Lambda_{2\overline{DC}}-b\,\partial_{[\overline{A}}\Lambda_1^{\overline{CD}}\partial_{\overline{B}]}\Lambda_{2\overline{DC}}\end{aligned}$$

The Green-Schwarz transformation

- Induces a first-order correction to the action

$$S = \int dX e^{-2d} \left(\mathcal{R} + a\,\mathcal{R}^{(0,1)} + b\,\mathcal{R}^{(1,0)} \right)$$

The details are not important. We only need to know:

  - $\mathcal{R}^{(0,1)}$ and $\mathcal{R}^{(1,0)}$ depend on the generalized fluxes so are scalars under generalized diffeomorphisms.
  - $\delta_\Lambda^{(1)}\mathcal{R} + \delta_\Lambda^{(0)} \left(a\,\mathcal{R}^{(0,1)} + b\,\mathcal{R}^{(1,0)} \right) = 0$

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- After section, gauge fixing and field redefinitions it reproduces exactly the Bergshoeff-de Roo action

$$\mathcal{R} + a\,\mathcal{R}^{(0,1)} + b\,\mathcal{R}^{(1,0)} = R + 4\,(\partial\phi)^2 - \frac{1}{12}\,\widehat{H}^2 + \frac{a}{8}\,R^{(-)2}_{\rm iem} + \frac{b}{8}\,R^{(+)2}_{\rm iem}$$

The Green-Schwarz transformation

$$\begin{array}{ccccc} & & & & \cdots \\ & & & \mathcal{R}^{(3,0)} & \\ & & \mathcal{R}^{(2,0)} & & \cdots \\ & \mathcal{R}^{(1,0)} & & \mathcal{R}^{(2,1)} & \\ \mathcal{R}^{(0,0)} & & \mathcal{R}^{(1,1)} & & \cdots \\ & \mathcal{R}^{(0,1)} & & \mathcal{R}^{(1,2)} & \\ & & \mathcal{R}^{(0,2)} & & \cdots \\ & & & \mathcal{R}^{(0,3)} & \\ & & & & \cdots \end{array}$$

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 & & & & \cdots
\end{array}
$$

To find higher orders there is another idea in supergravity that can be generalized.

The Bergshoeff-de Roo identification

Start with lowest order heterotic supergravity

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \text{fermions}$$

where

$$\widehat{H} = dB + CS(A) + \text{fermions}$$

The Bergshoeff-de Roo identification

Bergshoeff and de Roo 1988 realized that

$$\begin{array}{rcccl} \text{gauge fields} & A & \leftrightarrow & \omega^{(-)} & \text{spin con. w/torsion} \\ \text{gauginos} & \chi & \leftrightarrow & D\psi & \text{gravitino curvature} \end{array}$$

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based on supersymmetry

$$\begin{array}{lcl} \delta A = \bar{\epsilon}\gamma\chi & \leftrightarrow & \delta\omega^{(-)} = \bar{\epsilon}\gamma D\psi \\ \delta\chi = F_{\mu\nu}\gamma^{\mu\nu}\epsilon & \leftrightarrow & \delta D\psi = R_{-\mu\nu}\gamma^{\mu\nu}\epsilon \end{array}$$

The pair $(\omega^{(-)}, D\psi)$ *effectively* behaves as a gauge multiplet.

The Bergshoeff-de Roo identification

First order corrections are obtained by including extra Lorentz multiplets and *identifying* them with $(\omega^{(-)}, D\psi)$

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \frac{1}{4}R^{(-)2} + \text{fermions}$$

where

$$\widehat{H} = dB + CS(A) - CS(\omega^{(-)}) + \text{fermions}$$

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where

$$\widehat{H} = dB + CS(A) - CS(\omega^{(-)}) + \text{fermions}$$

$CS(\omega^{(-)})$ deforms the transformation of $\omega^{(-)}$ itself, rendering the identification ill-defined to second order. Higher orders require a Noether procedure Bergshoeff and de Roo 1989.

The Bergshoeff-de Roo identification

Gauge multiplets can be included in DFT through extensions of the duality group and local symmetries Hohm and Kwak 2011

$$\mathcal{G} = O(D, D+k) \ , \qquad \mathcal{H} = \underline{O(D)} \times \overline{O(D+k)}$$

$$\mathcal{E} \to e \oplus B \oplus A \ , \qquad \Psi \to \psi \oplus \chi$$

Generalized diffeomorphisms $\to$ $GL(D)$ diffs $\oplus$ B-shifts $\oplus$ $\mathcal{K}$

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Generalized diffeomorphisms $\to$ $GL(D)$ diffs $\oplus$ B-shifts $\oplus$ $\mathcal{K}$

One can then implement the identification

Bedoya, DM and Nuñez; Coimbra, Minasian, Triendl and Waldram; Lee 2014

$$\begin{array}{ccc} \mathcal{K} & \leftrightarrow & O(D) \in \overline{O(D+k)} \\ A & \leftrightarrow & \omega^{(-)} \\ \chi & \leftrightarrow & D\psi \end{array}$$

The Bergshoeff-de Roo identification

Instead of decomposing $O(D, D+k)$ w.r.t. $GL(D)$, we preserve $O(D, D)$ covariance, Hohm, Sen and Zwiebach 2014

$$\mathcal{E} \to E \oplus \mathcal{A} \;, \quad E \in O(D,D) \;, \quad E^{M\bar{A}}\, \mathcal{A}_M{}^{\alpha} = 0$$

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Only then one should look for a generalized Bergshoeff-de Roo identification

$$A_\mu{}^\alpha \quad \leftrightarrow \quad \omega^{(-)}_{\mu ab} \qquad | \qquad \mathcal{A}_M{}^\alpha \quad \leftrightarrow \quad ???$$

The Bergshoeff-de Roo identification

There are generalizations of everything in DFT:

$$A_\mu{}^\alpha \quad \leftrightarrow \quad \mathcal{A}_{\underline{A}}{}^\alpha$$

$$\omega^{(-)}_{\mu bc} \quad \leftrightarrow \quad \text{Generalized spin connection}$$

$$O(D) \quad \leftrightarrow \quad \underline{O(D)} \times \overline{O(D+k)}$$

The Bergshoeff-de Roo identification

The correct answer turns out to be... Baron, Lescano and DM 2018

$$A_\mu{}^\alpha \quad \leftrightarrow \quad \mathcal{A}_{\underline{A}}{}^\alpha$$

$$\omega^{(-)}_{\mu bc} \quad \leftrightarrow \quad \mathcal{F}_{\underline{A}\overline{\mathcal{B}\mathcal{C}}}$$

$$O(D) \quad \leftrightarrow \quad \overline{O(D+k)}$$

The Bergshoeff-de Roo identification

From the $O(D, D+k)$ gen diffs we get

$$\delta \mathcal{A}_{\underline{A}\alpha} = \widehat{\mathcal{L}}_\xi \mathcal{A}_{\underline{A}\alpha} - \mathcal{D}_{\underline{A}}\lambda_\alpha + \left[\lambda\,,\, \mathcal{A}_{\underline{A}}\right]_\alpha + \mathcal{A}_{\underline{D}\alpha}\Lambda^{\underline{D}}{}_{\underline{A}}$$

Which transforms as a generalized spin connection

$$\delta \mathcal{F}_{\underline{A}\overline{BC}} = \widehat{\mathcal{L}}_\xi \mathcal{F}_{\underline{A}\overline{BC}} - \mathcal{D}_{\underline{A}}\Lambda_{\overline{BC}} + \left[\Lambda\,,\, \mathcal{F}_{\underline{A}}\right]_{\overline{BC}} + \mathcal{F}_{\underline{D}\overline{BC}}\Lambda^{\underline{D}}{}_{\underline{A}}$$

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$$\delta \mathcal{F}_{\underline{A}\overline{\mathcal{BC}}} = \widehat{\mathcal{L}}_\xi \mathcal{F}_{\underline{A}\overline{\mathcal{BC}}} - \mathcal{D}_{\underline{A}} \Lambda_{\overline{\mathcal{BC}}} + \left[\Lambda\, ,\, \mathcal{F}_{\underline{A}}\right]_{\overline{\mathcal{BC}}} + \mathcal{F}_{\underline{D}\overline{\mathcal{BC}}} \Lambda^{\underline{D}}{}_{\underline{A}}$$

The generalized Bergshoeff-de Roo identification is then Baron, Lescano and DM 2018

$$\begin{aligned} \mathcal{K} &= \overline{O(D+k)} \\ -g\, \lambda_\alpha\, (t^\alpha)_{\overline{\mathcal{AB}}} &= \Lambda_{\overline{\mathcal{AB}}} \\ -g\, \mathcal{A}_{\underline{A}\alpha}\, (t^\alpha)_{\overline{\mathcal{BC}}} &= \mathcal{F}_{\underline{A}\overline{\mathcal{BC}}}[\mathcal{E}[E, \mathcal{A}]] \end{aligned}$$

The Bergshoeff-de Roo identification

- It preserves $O(D,D)$ covariance.
- It is *necessarily* generalized.
- It is exact, no need for a Noether procedure.
- It gives an iterative procedure to compute an infinite tower of higher-derivatives.
- It doesn't need supersymmetry, but is consistent with it

$$
\begin{aligned}
-g\,\mathcal{A}_{\underline{A}\alpha}\,(t^{\alpha})_{\overline{\mathcal{B}\mathcal{C}}} &= \mathcal{F}_{\underline{A}\overline{\mathcal{B}\mathcal{C}}} - \frac{1}{2}\bar{\Psi}_{\overline{\mathcal{B}}}\gamma_{\underline{A}}\Psi_{\overline{\mathcal{C}}} \\
g\Psi_{\overline{\mathcal{D}}}\mathcal{E}_{\alpha}{}^{\overline{\mathcal{D}}}(t^{\alpha})_{\overline{\mathcal{A}\mathcal{B}}} &= 2\left[\nabla_{[\overline{\mathcal{A}}}\Psi_{\overline{\mathcal{B}}]}\right]_{\text{det.}}
\end{aligned}
$$

The Bergshoeff-de Roo identification

It naturally induces an all-order generalized Green-Schwarz transformation for the $O(D, D)$ generalized frame

$$\delta E_{M\underline{A}} = \partial_{\overline{M}}\lambda \cdot \mathcal{A}_{\underline{A}} \rightarrow \partial_{\overline{M}}\Lambda \cdot \mathcal{F}_{\underline{A}}$$

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It naturally induces an all-order generalized Green-Schwarz transformation for the $O(D,D)$ generalized frame

$$\delta E_{M\underline{A}} \; = \; \partial_{\overline{M}}\lambda \cdot \mathcal{A}_{\underline{A}} \; \rightarrow \; \partial_{\overline{M}}\Lambda \cdot \mathcal{F}_{\underline{A}}$$

Perturbatively we get...

$$\begin{aligned}\delta E_M{}^{\underline{A}} = {} & \widehat{\mathcal{L}}_\xi E_M{}^{\underline{A}} + E_M{}^{\underline{B}}\,\Lambda_{\underline{B}}{}^{\underline{A}} + \frac{b}{2}\partial_{\overline{M}}\Lambda^{\overline{BC}}\, F^{\underline{A}}{}_{\overline{BC}} \\ & + \frac{b^2}{2}\, E_M{}^{\overline{B}} \Big[\partial_{\overline{B}}\partial^{\overline{C}}\Lambda^{\overline{EF}}\left(F_{\overline{C}\underline{D}}{}^{\underline{A}} F^{\underline{D}}{}_{\overline{EF}} + \partial_{\overline{C}} F^{\underline{A}}{}_{\overline{EF}}\right) - F^{\underline{A}}{}_{\overline{EF}} F_{\underline{C}\overline{D}}{}^{\overline{F}} \left(F^{\underline{C}\overline{HD}}\partial_{\overline{B}}\Lambda_{\overline{H}}{}^{\overline{E}} - F^{\underline{C}\overline{HE}}\partial_{\overline{B}}\Lambda_{\overline{H}}{}^{\overline{D}}\right) \\ & + F^{\underline{C}}{}_{\overline{EF}}\, \partial_{\overline{B}}\Lambda^{\overline{E}}{}_{\overline{G}} \left(F^{\underline{A}}{}_{\underline{CD}} F^{\underline{D}\overline{GF}} - \partial^{\underline{A}} F_{\underline{C}}{}^{\overline{GF}} + 2\, \partial_{\underline{C}} F^{\underline{A}\overline{GF}}\right) - F^{\underline{A}}{}_{\overline{EF}}\partial_{\overline{B}} \left(\partial^{\underline{C}}\Lambda^{\overline{ED}} F_{\underline{C}\overline{D}}{}^{\overline{F}}\right)\Big] + \dots\end{aligned}$$

The Bergshoeff-de Roo identification

The idea can be extended to account for the two parameter (a, b) family of theories Baron and DM 2020

$$
\begin{array}{ccccc}
 & & & & \cdots \\
 & & & \mathcal{R}^{(3,0)} & \\
 & & \mathcal{R}^{(2,0)} & & \cdots \\
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\end{array}
$$

This offers the geometrization of an infinite tower of higher-derivatives that includes (a sector of) the heterotic, the bosonic and HSZ.

The third order α' corrections

The following step naturally would be moving to the third order

$$
\begin{array}{ccccc}
 & & & & \dots \\
 & & & \boxed{\begin{array}{c} \mathcal{R}^{(3,0)} \\ \mathcal{R}^{(2,1)} \\ \mathcal{R}^{(1,2)} \\ \mathcal{R}^{(0,3)} \end{array}} & \dots \\
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 & & \mathcal{R}^{(2,0)} & & \\
 & \mathcal{R}^{(1,0)} & & & \\
\mathcal{R}^{(0,0)} & & \mathcal{R}^{(1,1)} & & \\
 & \mathcal{R}^{(0,1)} & & & \\
 & & \mathcal{R}^{(0,2)} & &
\end{array}
$$

These should account for the gauge symmetric R^4_{iem} couplings of the heterotic string

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 & \mathcal{R}^{(1,0)} & & & \\
\mathcal{R}^{(0,0)} & & \mathcal{R}^{(1,1)} & & \\
 & \mathcal{R}^{(0,1)} & & & \\
 & & \mathcal{R}^{(0,2)} & &
\end{array}
$$

These should account for the gauge symmetric R^4_{iem} couplings of the heterotic string, but not the universal interactions

$$\alpha'^3 \, \zeta(3) (t_8 t_8 - \tfrac{1}{8}\epsilon_{10}\epsilon_{10}) R^4_{iem}$$

The third order α' corrections

Since $\zeta(3)$ is irrational, these interactions require a new $\mathcal{O}(\alpha'^3)$ parameter

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In other words, we need a new Lorentz invariant starting at third order

$$\delta L = (\delta^{(0)} + c\delta^{(3)})[L^{(0)} + cL^{(3)}] = c[\underbrace{\delta^{(3)} L^{(0)}}_{EOM} + \delta^{(0)} L^{(3)}] + \mathcal{O}(c^2) = 0$$

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No Go: under certain assumptions, there is no such invariant in the background independent frame formulation of DFT: Hronek and Wulff 2021.

β-symmetry

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gaining a symmetry enhancement

$$O(d,d) : GL(d) \otimes \mathrm{b-shifts} \otimes \beta\mathrm{-transformations}$$

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The last two items are just field redefinitions. If not implemented, the local and global symmetries would still be there, though hidden.

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Good news: the emergence of $O(d,d)$ under toroidal compactifications can be assessed in D dimensions without going through the KK procedure.

β-symmetry

The lowest order β-transformations are ($E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$)

$$\delta E_{\mu\nu} = -E_{\mu\rho}\beta^{\rho\sigma}E_{\sigma\nu} \ , \quad \delta\Phi = \frac{1}{2}\beta^{\mu\nu}E_{\mu\nu}$$

And the assumption of isometries is encoded in the constraint

$$\beta^{\mu\nu}\partial_\nu \cdots = 0$$

enforcing the orthogonality between external derivatives and internal β.

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In the frame formulation, β-symmetry mixes all curvatures

$$\delta\omega_{cab} = \beta_{[a}{}^{d} H_{b]cd} - \frac{1}{2}\beta_{c}{}^{d} H_{abd} \ , \quad \delta H_{abc} = -6\omega^{d}{}_{[ab}\beta_{c]d}$$

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Demanding β-invariance

$$\begin{aligned} 0 &= \delta\left(R + m\,(\nabla\phi)^2 + n\,\Box\phi + p\,H^2\right) \\ &= \beta^{cd}\nabla^b H_{bcd}\left(-2 + \frac{n}{2}\right) + \beta^{cd}\omega_{cab}H_d{}^{ab}\,(5 - n + 12p) \\ &\quad + \beta^{cd} H_{bcd}\nabla^b\phi\,(m+n) \end{aligned}$$

fixes the two-derivative action

$$m = -4 \ , \quad n = 4 \ , \quad p = -\frac{1}{12}$$

β-symmetry

To first order in α' the β-symmetry receives higher derivative corrections (BdR scheme)

$$\begin{aligned}
\delta^{(1)} e_{ab} &= \frac{a+b}{8}\beta_{(a}{}^{e}\left(\omega_{b)cd}H_{e}{}^{cd} + H_{b)cd}\omega_{e}{}^{cd}\right) \\
&\quad + \frac{b-a}{4}\beta_{(a}{}^{e}\left(\omega_{b)cd}\omega_{e}{}^{cd} + \frac{1}{4}H_{b)cd}H_{e}{}^{cd}\right) \\
\delta^{(1)} b_{ab} &= (a+b)\Big[\beta^{ec}\omega_{e[a}{}^{d}\omega_{b]cd} - \beta^{ec}\omega_{[\underline{a}e}{}^{d}\omega_{\underline{b}]cd} \\
&\qquad - \frac{1}{2}\beta_{[a}{}^{c}\omega_{b]de}\omega_{c}{}^{de} - \frac{1}{8}\beta_{[a}{}^{c}H_{b]de}H_{c}{}^{de}\Big] \\
&\quad + \frac{b-a}{2}\Big[\beta^{ec}\omega_{e[a}{}^{d}H_{b]cd} - \beta^{ec}\omega_{[\underline{a}e}{}^{d}H_{\underline{b}]cd} \\
&\qquad - \frac{1}{2}\beta_{[a}{}^{c}\omega_{b]de}H_{c}{}^{de} - \frac{1}{2}\beta_{[a}{}^{c}H_{b]de}\omega_{c}{}^{de}\Big]
\end{aligned}$$

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- The bi-parametric transformations can be derived from the generalized Green-Schwarz transformation in DFT, after solving the strong constraint and gauge-fixing the double Lorentz symmetry.
- β-symmetry is unobstructed, as it is simply a convenient realization of $O(d,d)$ in lower dimensions. How it relates to the $\alpha'^3 \zeta(3) R^4_{iem}$ interactions is work in progress...

Conclusions

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Muchas gracias por su atención!