Neutrino masses from gravity

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Elusives Webinar

11 February 2020

Outline

- Neutrino masses and mixing
- Consequences of neutrino masses
- Neutrino masses from gravity, what has been done
- Neutrino masses from gravity: the Schwinger Dyson approach
- Two ways of solving the SDEs
- Discussions and conclusions

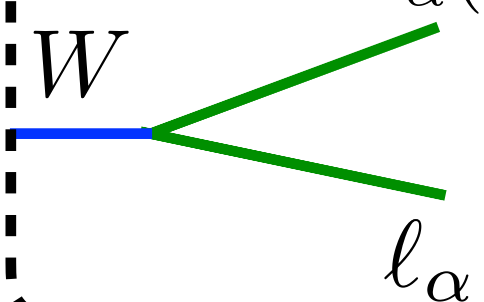
Neutrino Oscillations

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour states — **PMNS matrix** — **mass states**

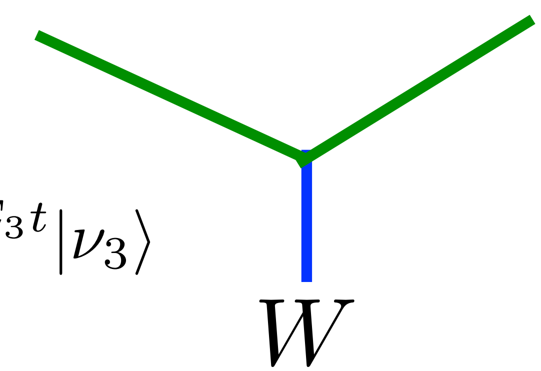
$$U m_\nu U^\dagger = m_{\nu \mathrm{diag}}$$

$$U_{PMNS} = U_e^\dagger U_\nu$$



$\nu_\alpha(t_0) \longrightarrow \nu_\beta(t)$

$$|\nu(t)\rangle = e^{-i\mathcal{H}t}|\nu(0)\rangle = \alpha e^{-iE_1 t}|\nu_1\rangle + \beta e^{-iE_2 t}|\nu_2\rangle + \gamma e^{-iE_3 t}|\nu_3\rangle$$



$$E_i \simeq E + \frac{m_i^2}{E} \implies E_i - E_j \simeq \frac{\Delta m_{ij}^2}{2E}$$

In the simplified two neutrino case:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2\frac{(\Delta m_{ij}^2 L)}{2E}$$

mass splitting

baseline distance between production and detection

mixing angle parametrises misalignment of bases

Energy of neutrino

- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

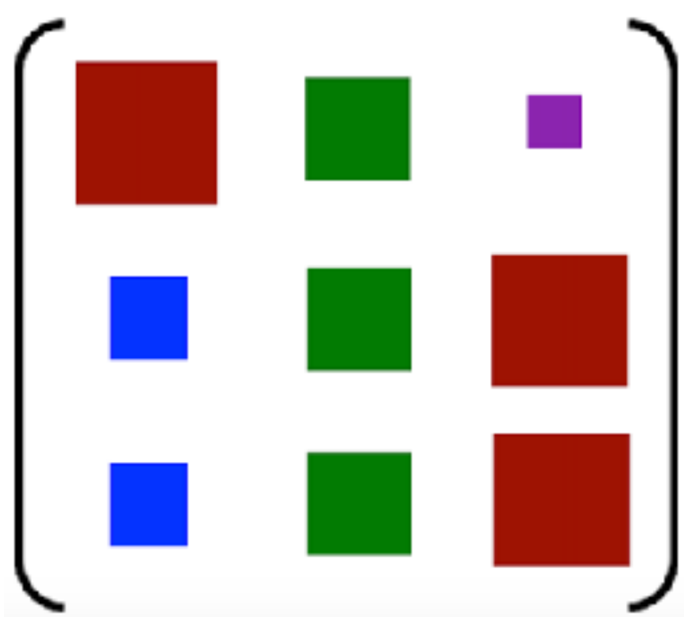
- If neutrinos Dirac fermions, PMNS: 3 mixing angles + 1 phase

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

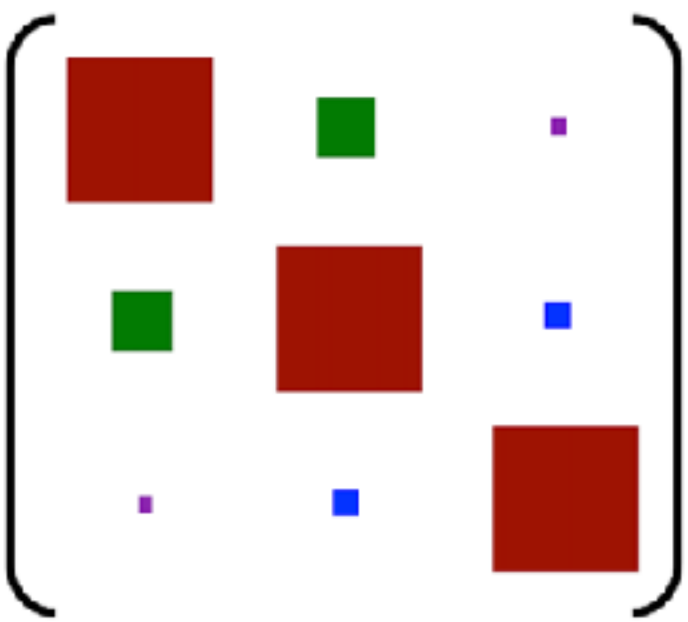
$$41.1 \leq \theta_{23}(^\circ) \leq 51.3 \qquad 8.22 \leq \theta_{13}(^\circ) \leq 8.98 \qquad 31.61 \leq \theta_{12}(^\circ) \leq 36.27$$

$$144 \leq \delta(^\circ) \leq 357$$

nu-fit data 4.1



leptonic mixing



quark mixing

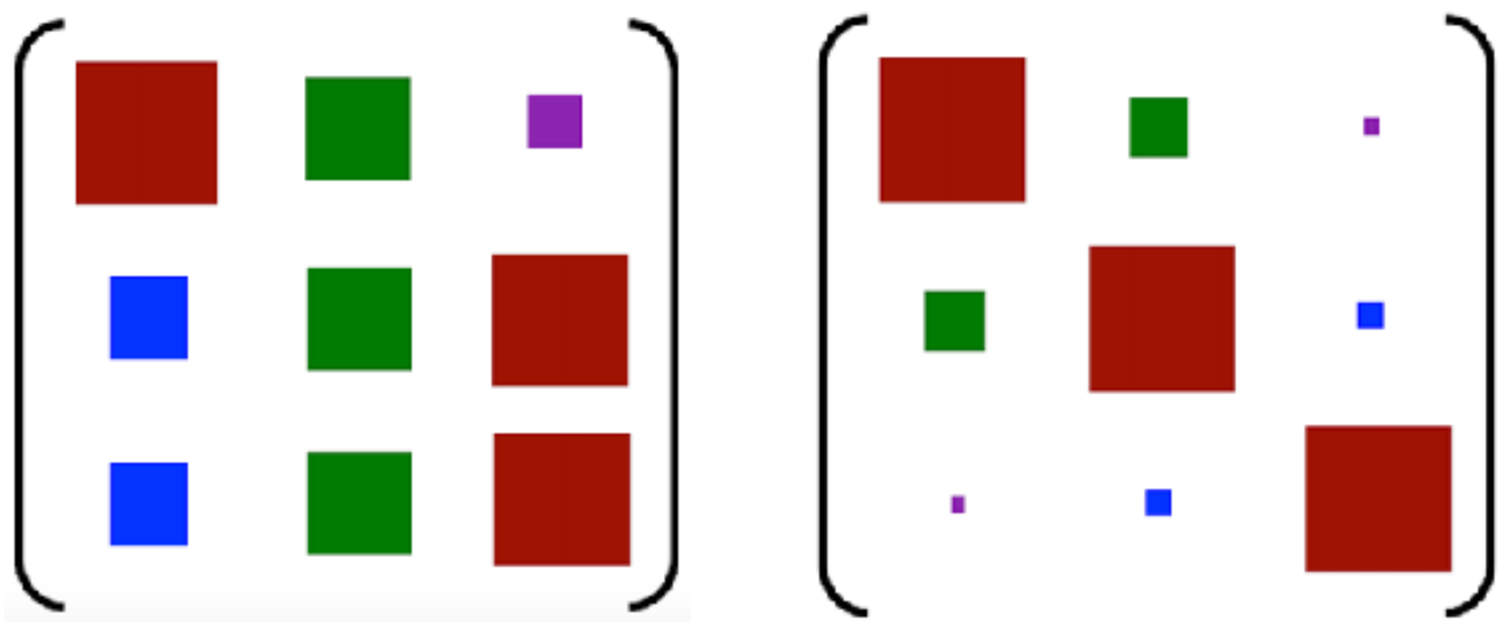
- Neutrinos have (non-degenerate) masses
- Neutrinos mix i.e. PMNS matrix is a non-identity matrix

- If neutrinos Majorana fermions, PMNS: 3 mixing angles + 3 phase

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

- Majorana nature of neutrinos not observable at oscillation experiments

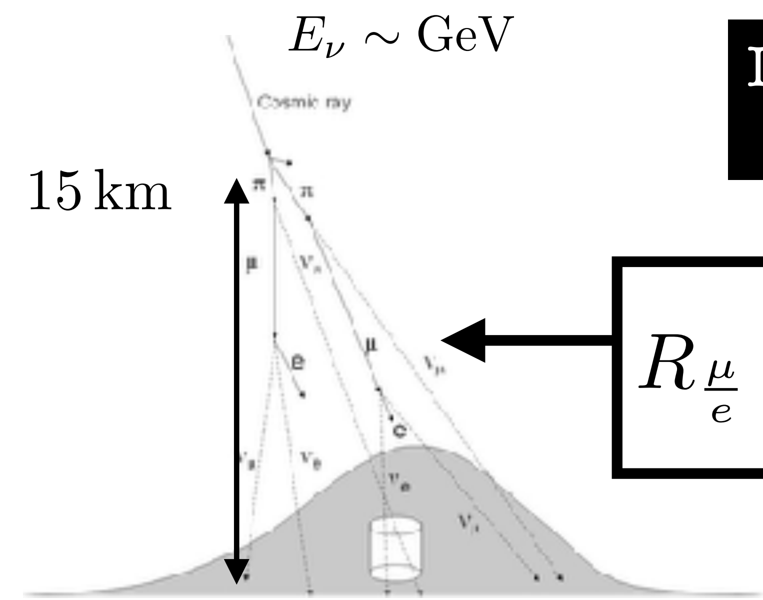
nu-fit data 4.1



leptonic mixing **quark mixing**

Atmospheric

$\nu_{e,\mu}$ produced by interaction of cosmic rays with Earth's atmosphere



Discovered by SK in 1998

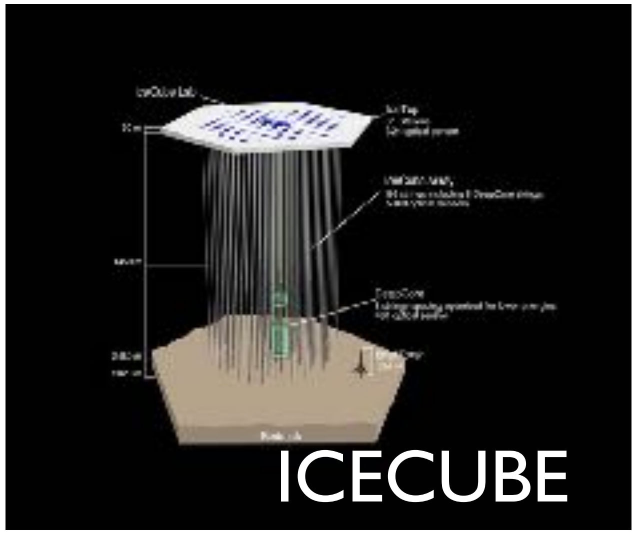
$$R_{\frac{\mu}{e}} = \frac{N_{\nu_\mu} + N_{\overline{\nu_\mu}}}{N_{\nu_e} + N_{\overline{\nu_e}}} \approx 2$$

$$P\left(\nu_\mu \rightarrow \nu_\tau\right)$$

$$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$

$$\theta \sim \frac{\pi}{4}$$

$E_\nu \sim \mathrm{GeV}$



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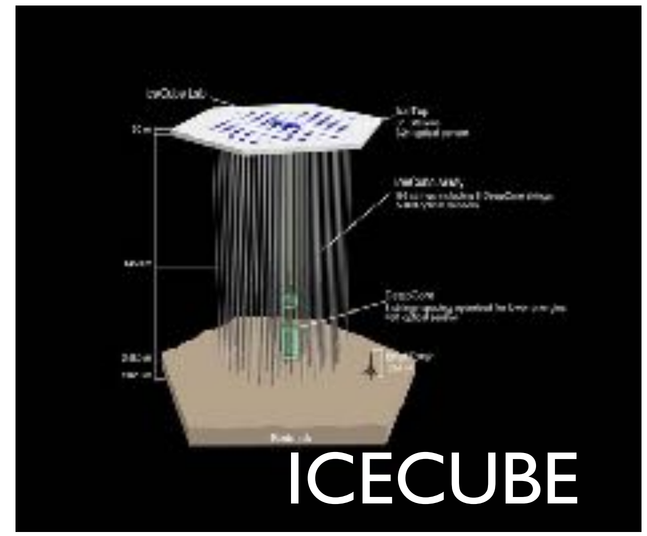

Solar

dominates energy production of Sun

CNO cycle

pp chain

ν_e copiously produced by the Sun

$$P\left(\nu_\mu \to \nu_\tau\right)$$

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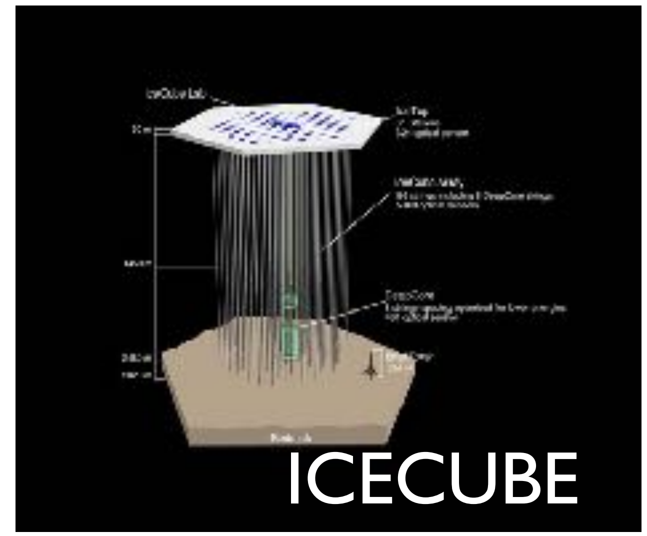

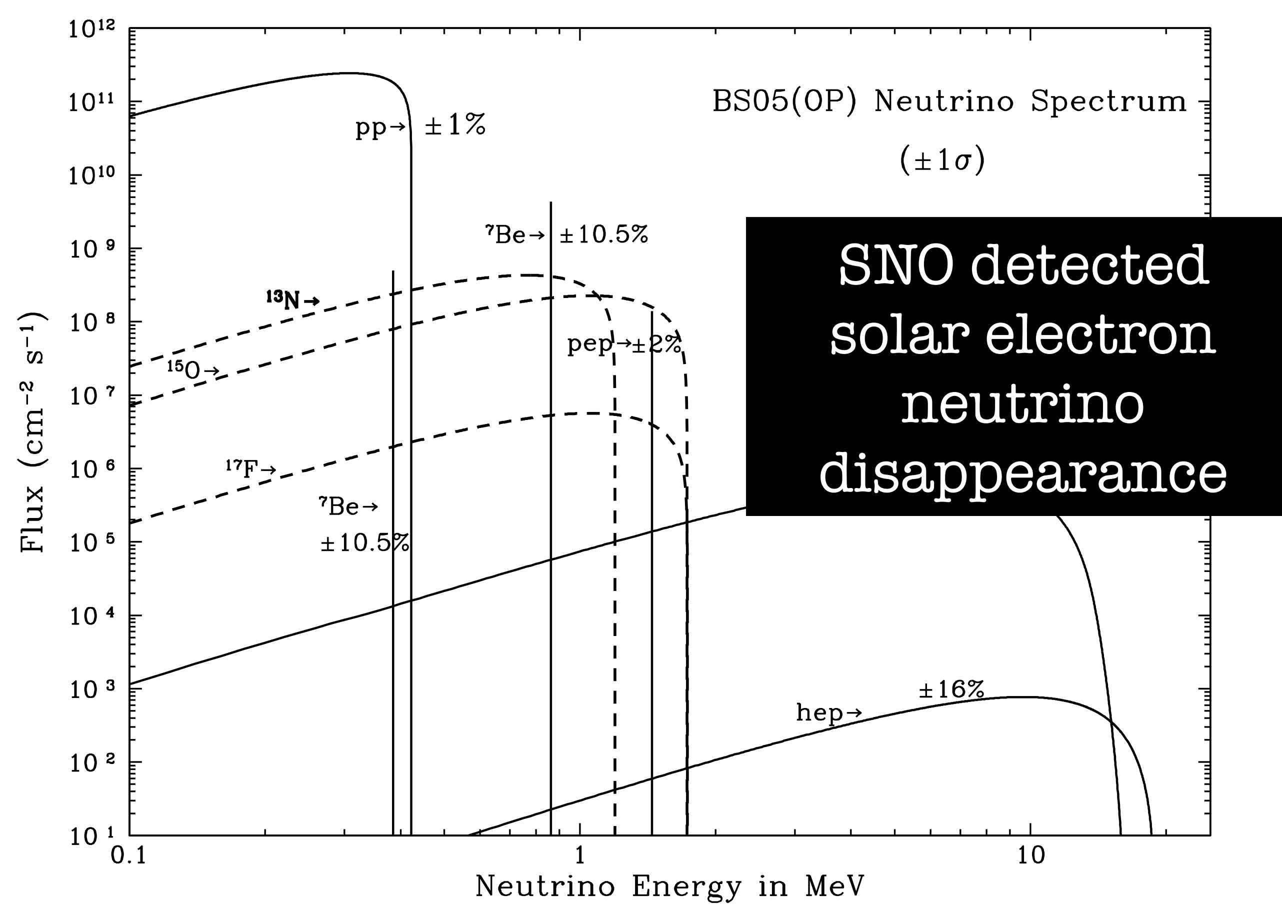

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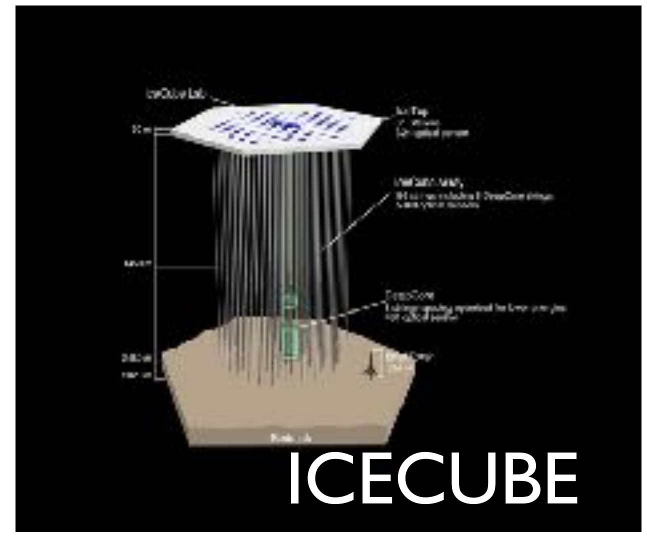

$P\left(\nu_e \to \nu_{\mu/\tau}\right)$

$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$

terrestrial source

$\theta \sim \frac{\pi}{6}$

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

$$P\left(\nu_\mu \to \nu_\tau\right)$$

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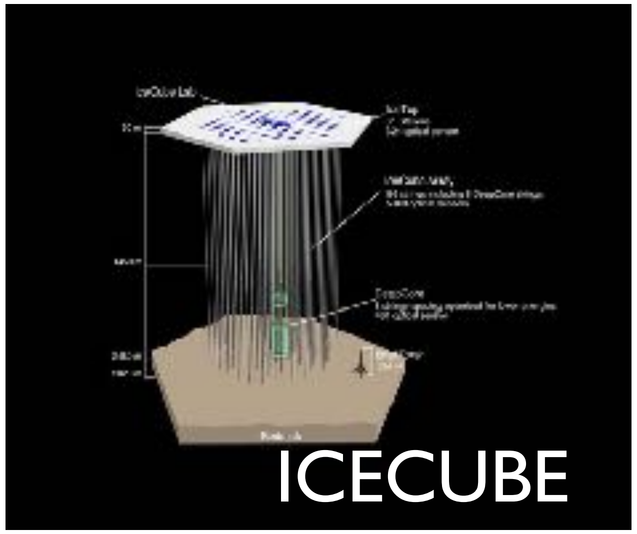
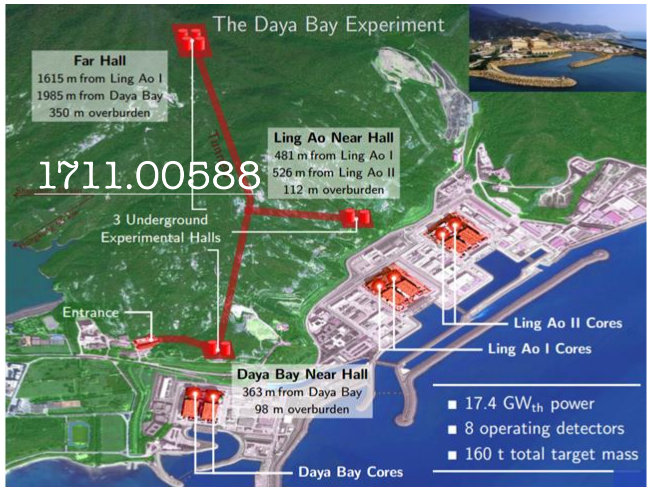


Reactor

DAYA, RENO and Double Chooz all use neutrinos produced at reactors



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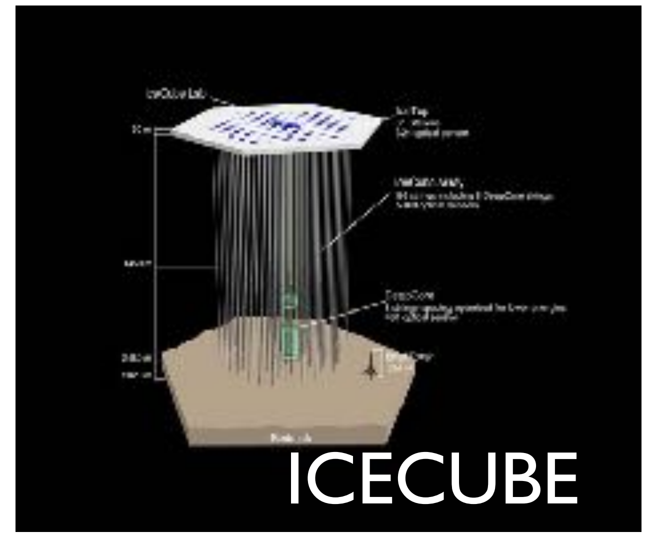
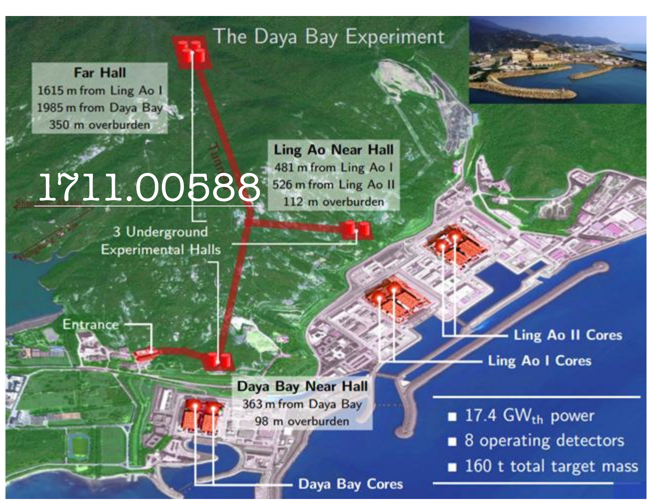


$P\left(\overline{\nu}_e \to \overline{\nu}_e\right)$

$\theta \sim \frac{\pi}{20}$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$



$P\left(\nu_e \to \nu_{\mu/\tau}\right)$

$\Delta m^2 \sim 7\times 10^{-5}\,\mathrm{eV}^2$

$\theta \sim \frac{\pi}{6}$

terrestrial source

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

Accelerator

NOVA $L \sim 810\,\mathrm{km}$

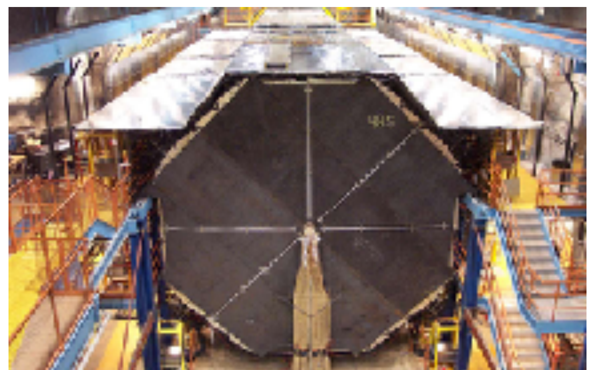
$E_\nu \sim \mathcal{O}(1)\,\mathrm{GeV}$

T2K

$L \sim 295\,\mathrm{km}$

$E_\nu \sim \mathcal{O}(100)\,\mathrm{MeV}$

MINOS $L \sim 735\,\mathrm{km}$

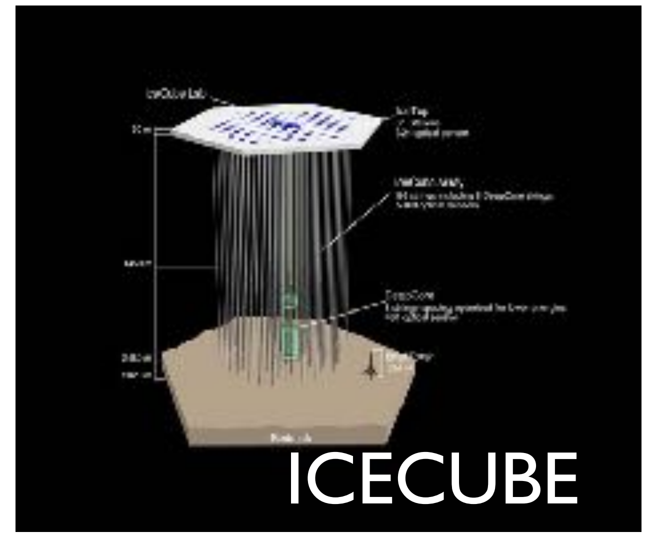


$P\left(\nu_\mu \to \nu_\tau\right)$

$\Delta m^2 \sim 2 \times 10^{-3}\,\mathrm{eV}^2$

$\theta \sim \frac{\pi}{4}$

$E_\nu \sim \mathrm{GeV}$



$P\left(\overline{\nu}_e \to \overline{\nu}_e\right)$

$\theta \sim \frac{\pi}{20}$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$P\left(\nu_e \to \nu_{\mu/\tau}\right)$

$\Delta m^2 \sim 7 \times 10^{-5}\,\mathrm{eV}^2$

$\theta \sim \frac{\pi}{6}$

terrestrial source

$L \sim 10^8\,\mathrm{km}$

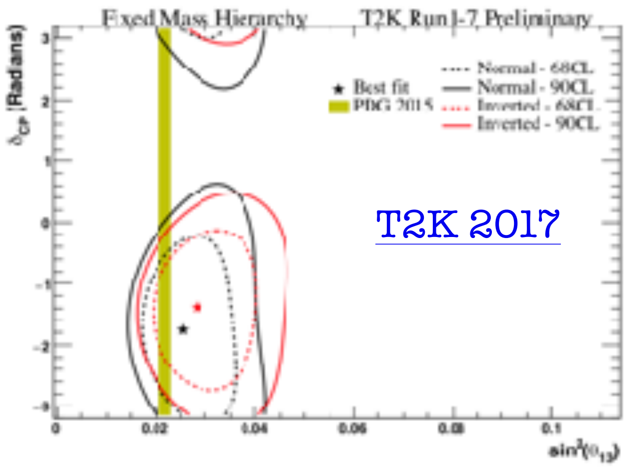
$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

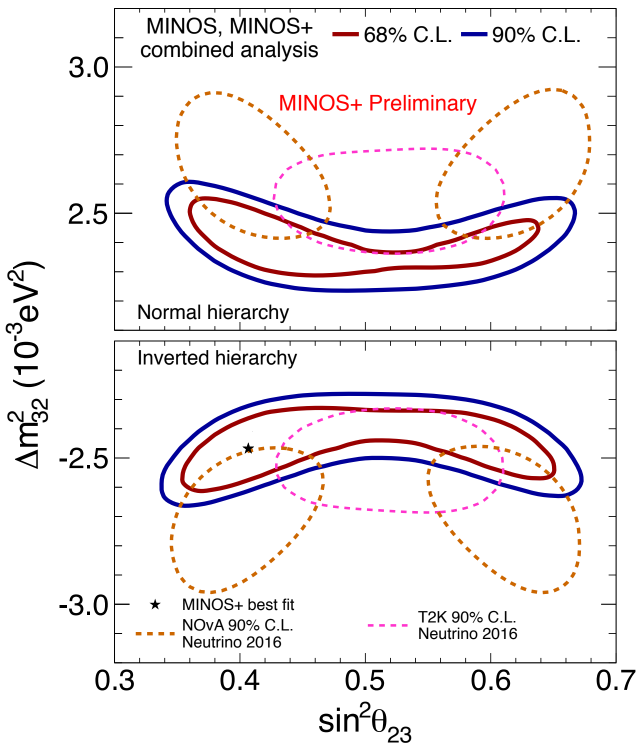
$L \sim 180\,\mathrm{km}$

Accelerator $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$

$P(\nu_\mu \to \nu_{\alpha\neq\mu})$ θ_{23}

$\delta \sim \frac{3\pi}{2}$



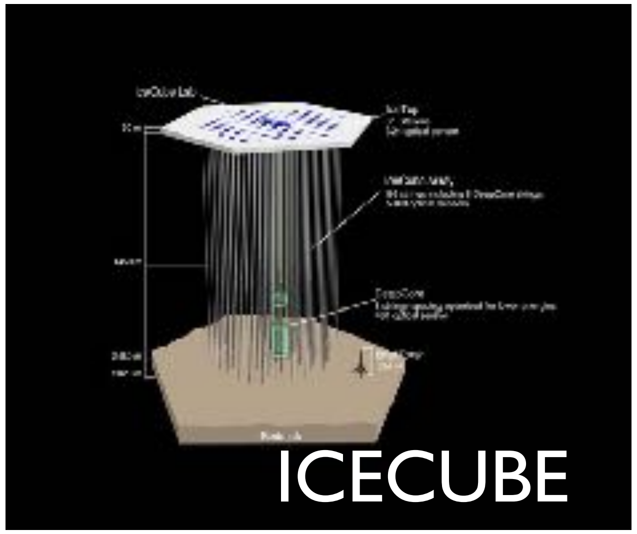


$P(\nu_\mu \to \nu_\tau)$

$\Delta m^2 \sim 2\times 10^{-3}\,\mathrm{eV}^2$

$\theta_{23} \sim \frac{\pi}{4}$

$E_\nu \sim \mathrm{GeV}$

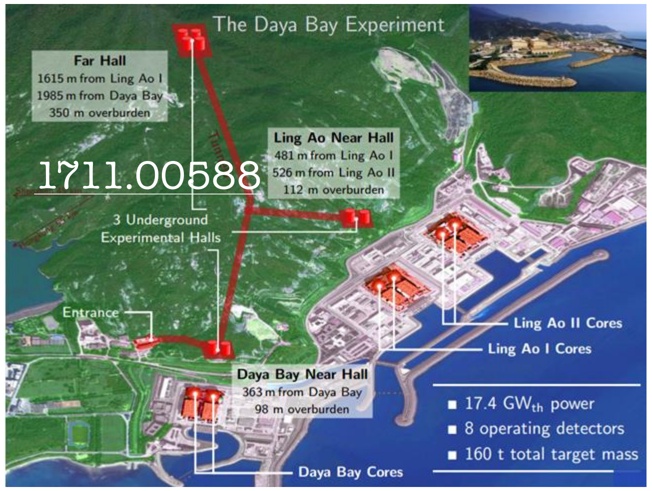


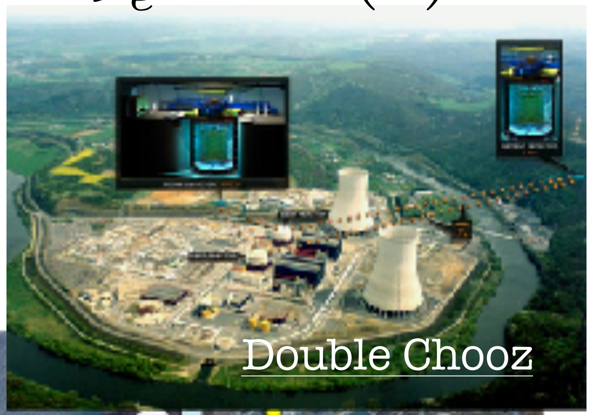
$P(\overline{\nu}_e \to \overline{\nu}_e)$

$L \sim \mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$\theta_{13} \sim \frac{\pi}{20}$





$P(\nu_e \to \nu_{\mu/\tau})$

$\Delta m^2 \sim 7\times 10^{-5}\,\mathrm{eV}^2$

terrestrial source

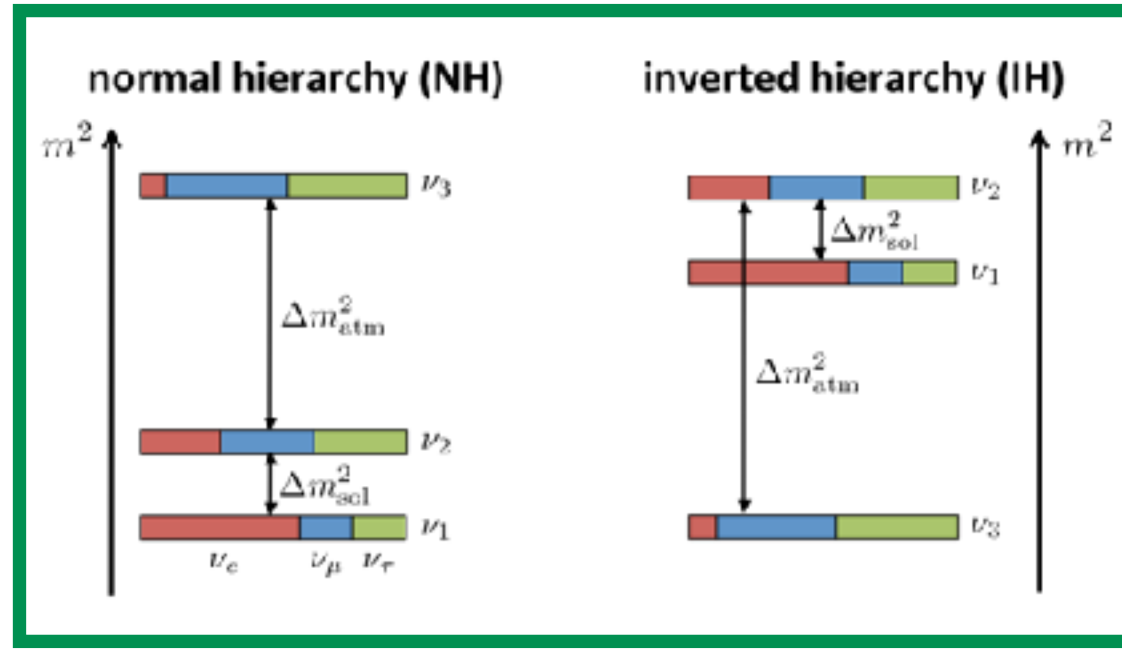
$\theta_{12} \sim \frac{\pi}{6}$

$L \sim 10^8\,\mathrm{km}$

$E_{\overline{\nu}_e} \sim \mathcal{O}(1)\,\mathrm{MeV}$

$L \sim 180\,\mathrm{km}$

- How are the masses ordered?



- What are the precise values of the mixing angles?

- Is leptonic CP maximally violated?

- Are neutrinos Dirac or Majorana fermions?

- What is the mass of the lightest neutrino?

$$\sum_{i=1}^{3} m_i \leq 0.2\,\mathrm{eV}$$

cosmology, many groups see PDG

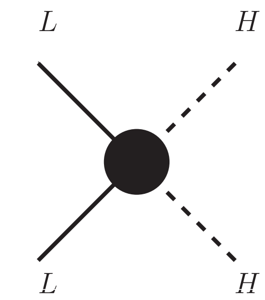
$$m_\nu < 1.1\,\mathrm{eV}\,(90\%)\mathrm{C.L}$$

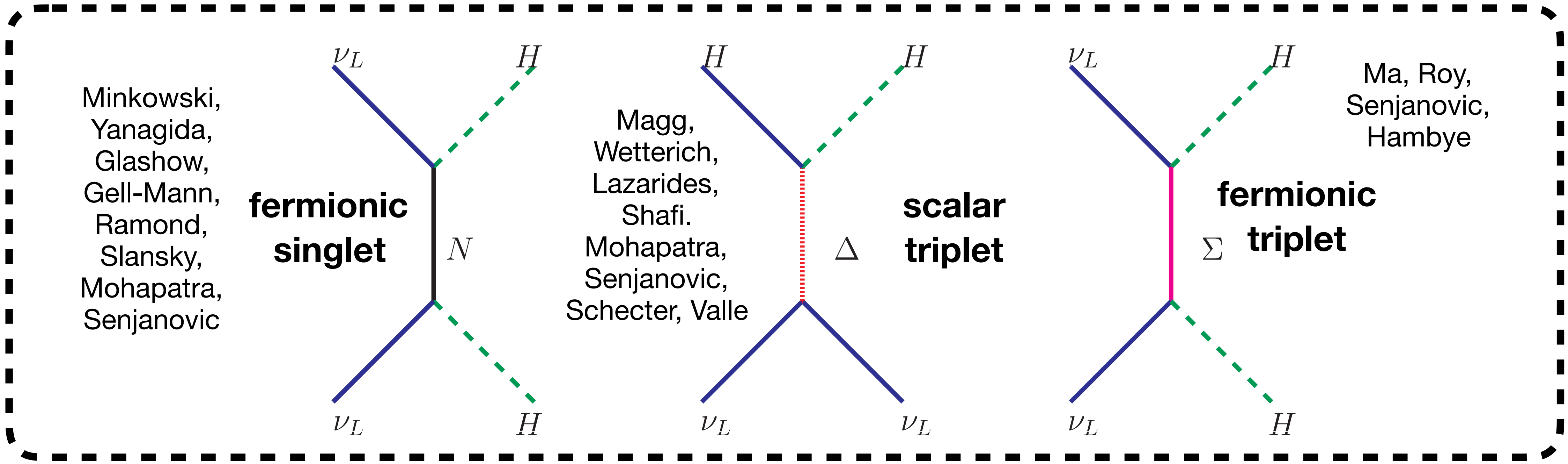
recent measurement by KATRIN 1909.06048

As neutrino are significantly lighter than the other known fermions this suggests they acquire their mass in a different way.

- Write a Dirac mass term analogous to other SM fermions

$$-\mathcal{L}_{d=5} = \lambda \frac{L.H L.H}{M}$$



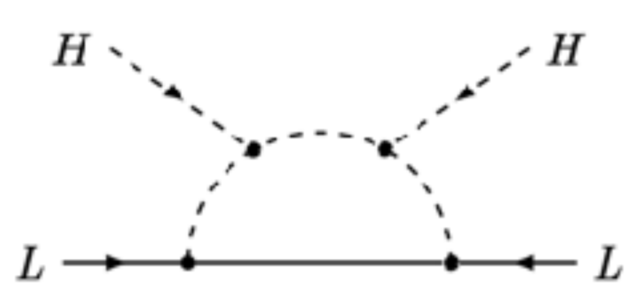


$$\mathcal{L} = -Y_\nu \overline{N} L H - \frac{1}{2}\overline{N^C} M_N N$$

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

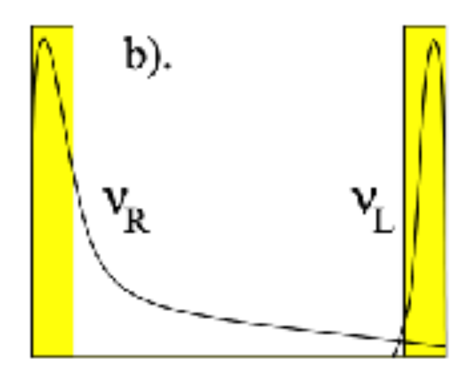
$$m_\nu = \frac{Y_\nu^2 v^2}{M_N} \sim 0.1\,\mathrm{eV}$$

- radiative models, extra dimensions



Babu, Leung, Bhattacharya, Wudka, Farzan, Schmidt, Gouvea, Jenkins, Kobach, Ma

higher dimensional bulk **4D brane**



Arkani-Hamed, Dimopoulos, Dvali, March-Russell

Neutrino Masses from gravity

Dvali & Funcke (1602.03191)

Logic: make an analogue of gravity with QCD

u, d, s are light relative to c, b and t so **approximate** flavour symmetry

$$U(3)_A \times U(3)_V$$

At energies below $\Lambda_{\rm QCD} \sim 300\,\mathrm{MeV}$ quarks confine into hadrons

Once confinement occurs, relevant d.o.f baryons and mesons $\langle \overline{q} q \rangle$

Ground state break symmetry

9 generators

$$U(3)_A \times U(3)_V \to U(3)_V$$

quark confinement spontaneously breaks axial symmetry

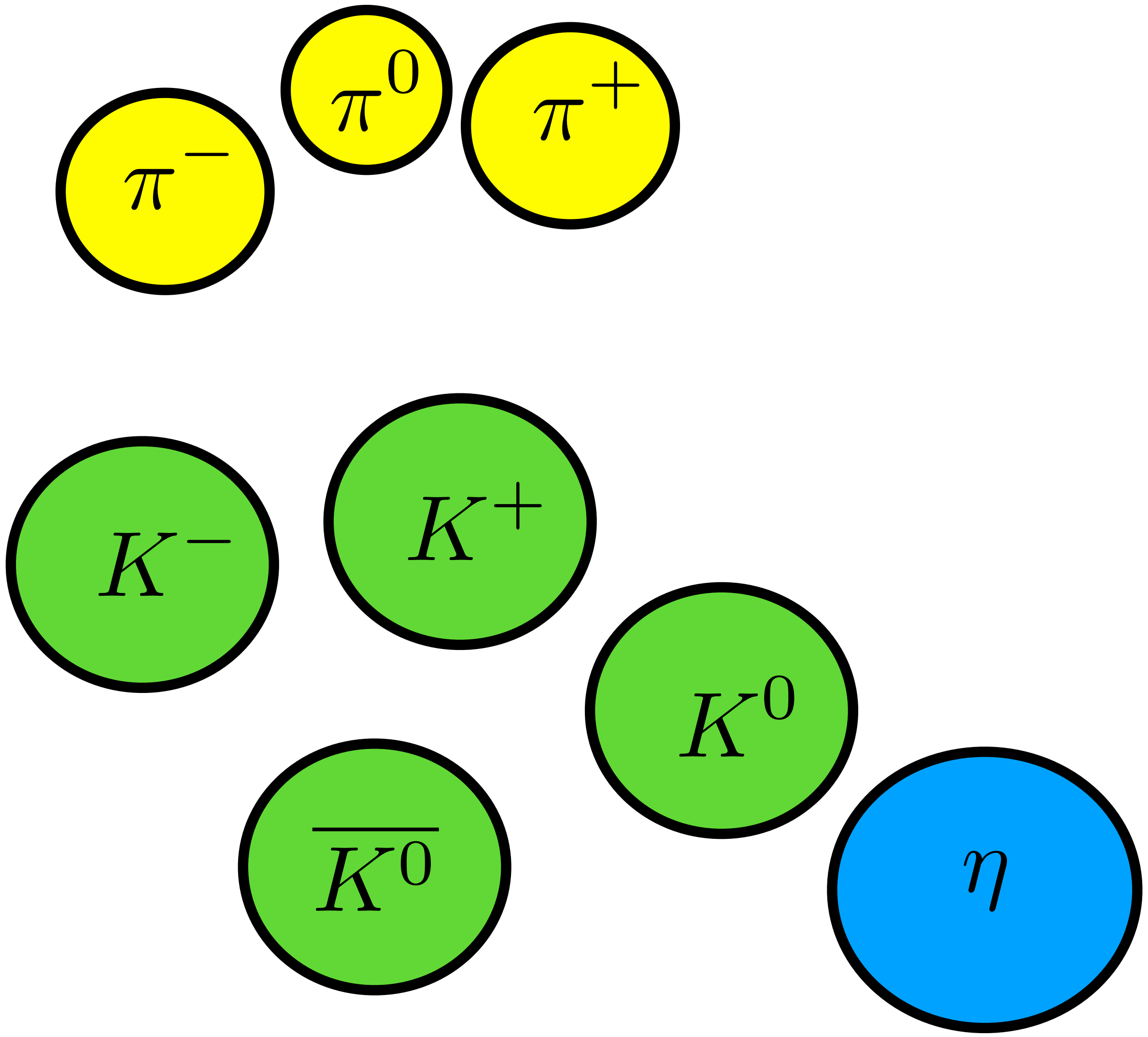
Broken symmetry contains $U(1)_A$ and an SU(3) part which are broken and via Goldstone's theorem give

$$1(\eta') + 8(\pi, \eta, K)$$

Neutrino Masses from gravity (1602.03191)

η' is heavy relative to the other mesons as its mass gets raised due to non-perturbative QCD effects.

not to scale ;)



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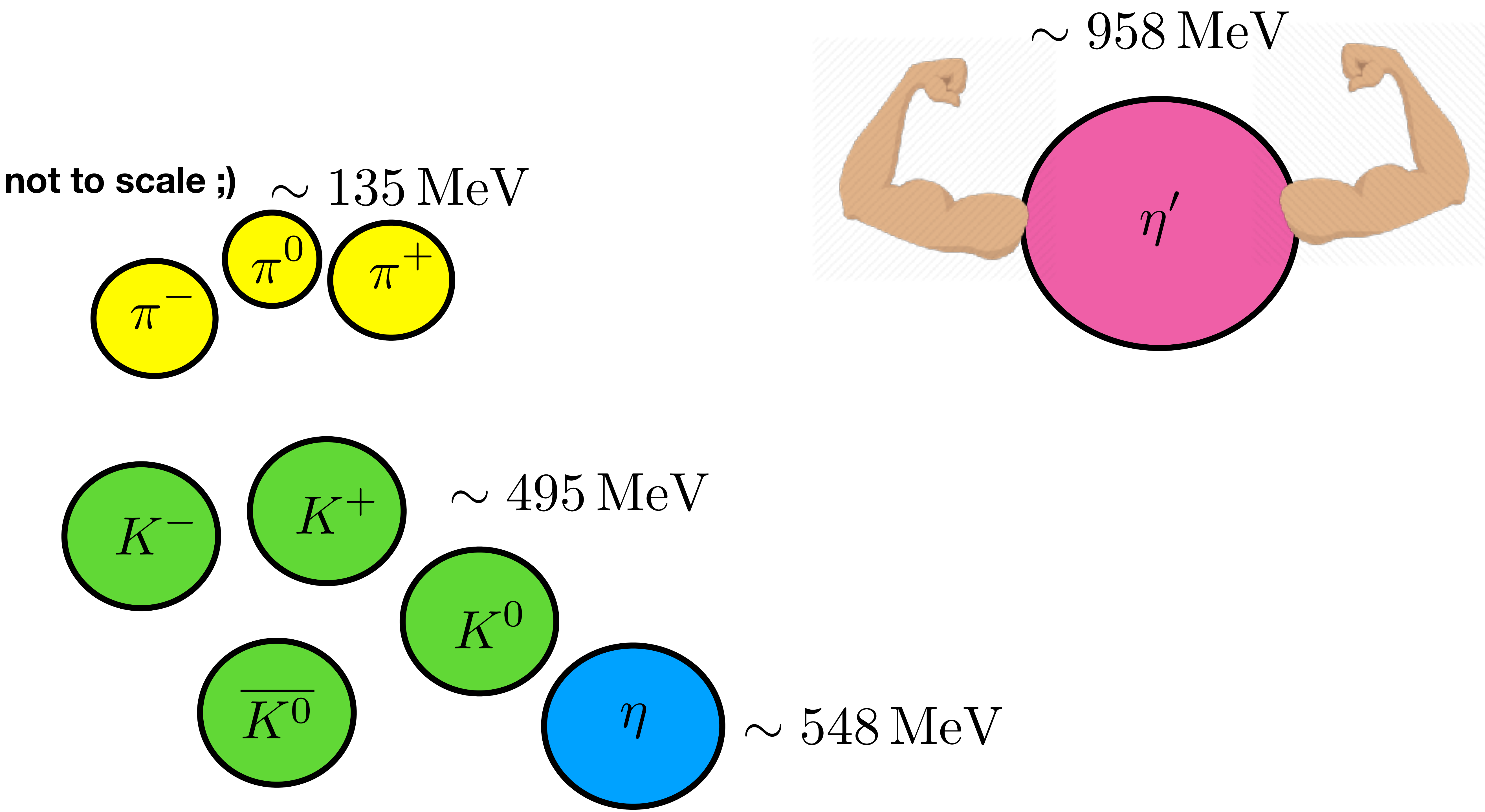

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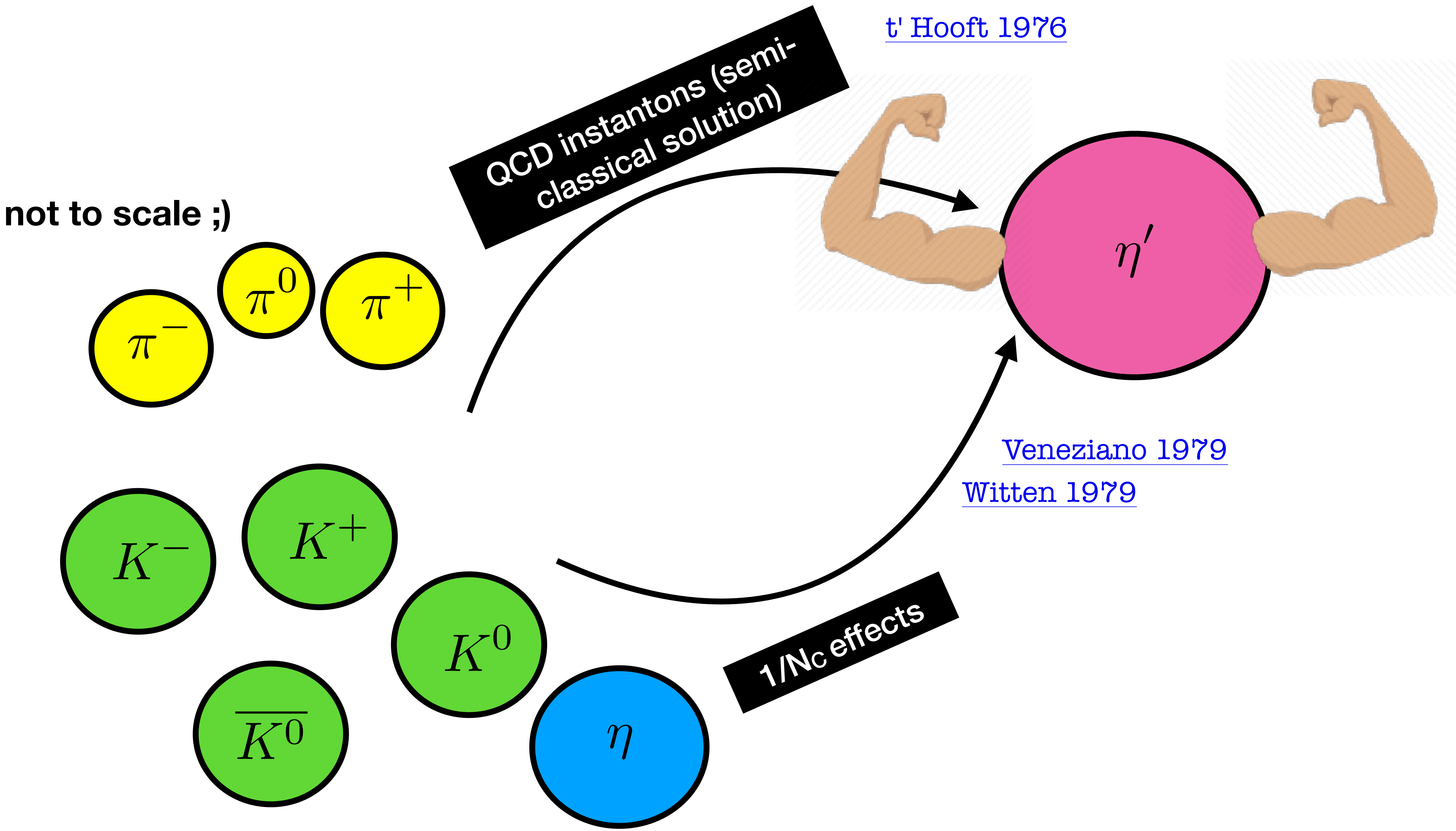

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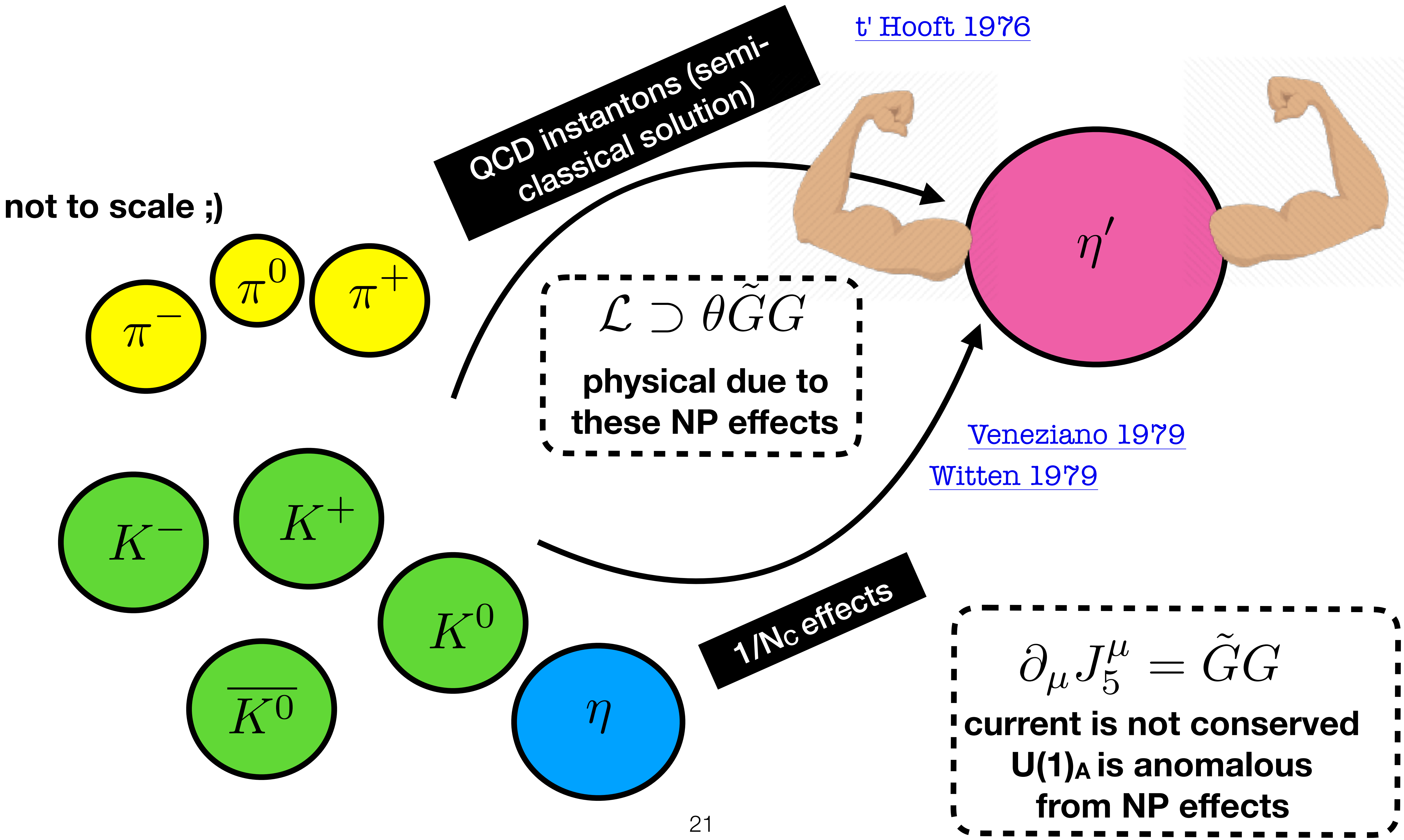


Neutrino Masses from Gravity (1602.03191)

Assume gravity has a theta term: $\mathcal{L}_G \supset \theta_G \tilde{R}R$ ← **Riemann tensor**

Neutrinos have zero bare mass and condense via NP gravitational effects and use dimensional analysis in analogue with QCD:

$$\nu\overline{\nu} = \langle \nu\overline{\nu} \rangle = v e^{i\phi} \implies \Lambda_G \sim v \sim m_\nu \sim \nu\overline{\nu}$$

$$U(3)_V \times U(3)_A \to U(1)^3$$
$$1(\eta_\nu) + 14(\phi)$$

$3^2+3^2-3 = 15$

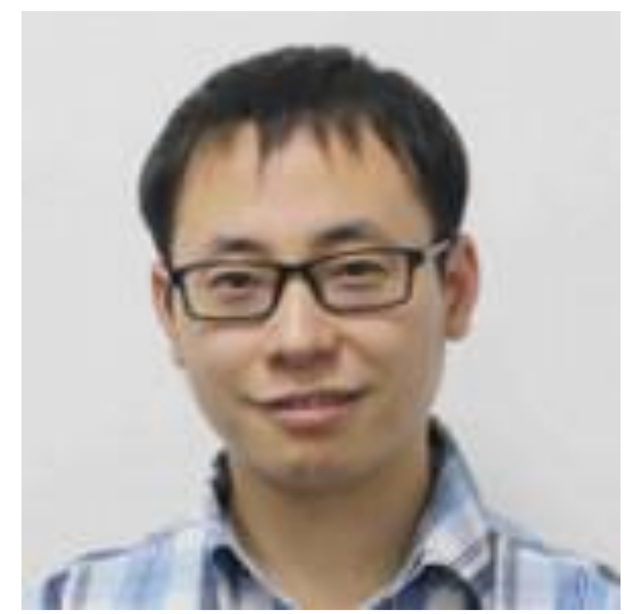
small neutrino masses from this gravitational θ term triggers neutrino condensate and introduces an **infrared gravitational scale**.

Neutrino mass splittings from scalar potential:

$$V(\hat{X}) = \sum_n \frac{1}{n} c_{2n} \mathrm{Tr}[(\hat{X}^+ \hat{X})^n]$$

$$\langle \bar{\nu}_{\alpha_L} \nu_{\alpha_R} \rangle \equiv \hat{X}^{\alpha_R}_{\alpha_L}$$

Work in collaboration with
Gabriela Barenboim and Ye-Ling Zhou



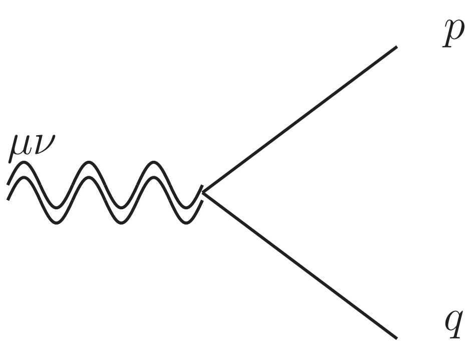
Neutrino Masses from Gravity

1909.04675

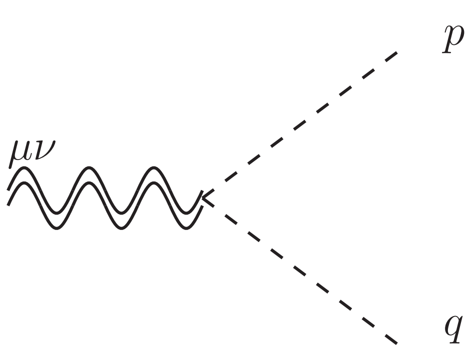
With Gabriela Barenboim (Valencia) and Ye-Ling Zhou (Southampton)

- Treat gravity as an EFT similar to Donoghue see 9405057v1 for a review. Start with flat metric and perturb around it, gravity non-Abelian gauge theory with spin-2 gauge boson.

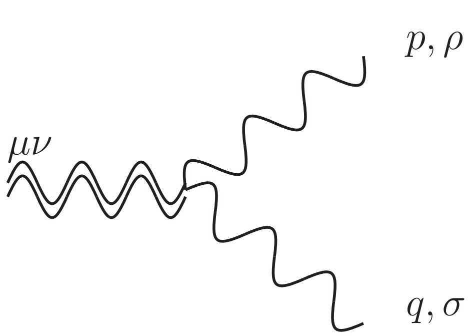
$$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad G = 1/M_{\rm pl}^2 \qquad \kappa = \sqrt{32\pi G} \approx 10^{-18}\,{\rm GeV}^{-1}$$



$$\tau_1^{\mu\nu}(p,q) = \frac{i\kappa}{8}\left[(q-p)^\mu\gamma^\nu + (q-p)^\nu\gamma^\mu - 2\eta^{\mu\nu}(\not{q} - \not{p})\right]$$



$$\tau_2^{\mu\nu}(p,q) = \frac{i\kappa}{2}\left[p^\mu q^\nu + p^\nu q^\mu - \eta^{\mu\nu} p\cdot q\right]$$



$$\tau_3^{\mu\nu\rho\sigma}(p,q) = i\kappa[-\mathcal{P}^{\mu\nu\rho\sigma} - \frac{1}{2}\eta^{\mu\nu}p^\sigma q^\rho + \eta^{\sigma\rho}(p^\mu q^\nu + p^\nu q^\mu) + \frac{1}{2}(\eta^{\mu\sigma}p^\nu q^\rho + \eta^{\nu\sigma}p^\rho q^\mu + \eta^{\nu\rho}p^\sigma q^\mu + \eta^{\mu\rho}p^\nu q^\sigma)]$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$

- “Gravity triggered neutrino condensates” (1009.2504) used SDEs as a means of studying RH neutrino condensation. We revisit this paper and calculation techniques for light neutrinos.

- Here RHN are heavy and light neutrinos get mass from type-I ss. RHN condensate can drive inflation (0811.2998)

- SDE are an infinite tower of integral coupled equations which relate the Green’s functions of a theory to each other.

- We have to choose some truncation skim. For us this is one loop improved.

- This allows us to derive a neutrino gap equation.

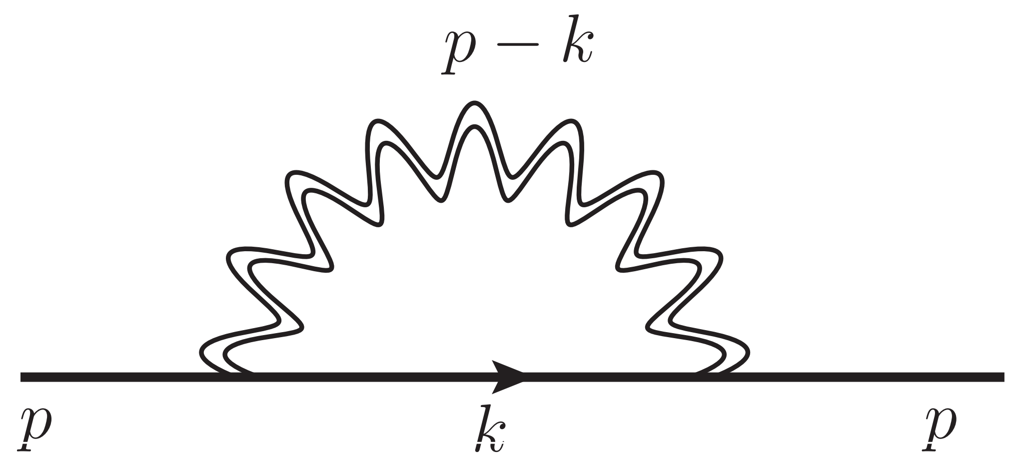
Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

Assume neutrino has zero valued bare mass and Dirac fermion.

graviton-fermion-fermion vertex



undressed graviton propagator

Leading self-energy correction

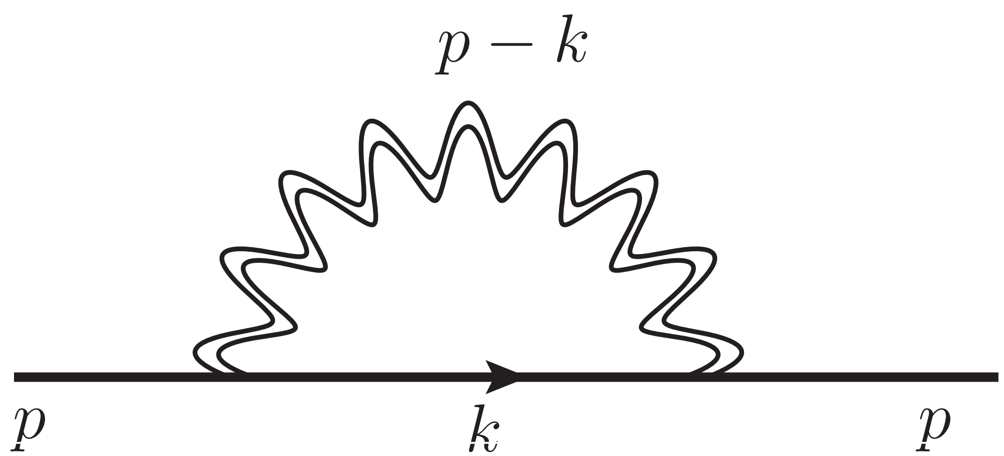
$$-i\Sigma(p) = \int \frac{d^4k}{(2\pi)^4} \tau_1^{\mu\nu}(p,-k) S'_F(k) G_{\mu\nu\rho\sigma}(p-k) \tau_1^{\rho\sigma}(k,-p)$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

Assume neutrino has zero valued bare mass and Dirac fermion.



Taking the appropriate Dirac trace

$$\alpha(p^2) = 1 - \frac{1}{4p^2}\mathrm{tr}(\not{p}\Sigma(p)) \quad \beta(p^2) = \frac{1}{4}\mathrm{tr}(\Sigma(p))\,.$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

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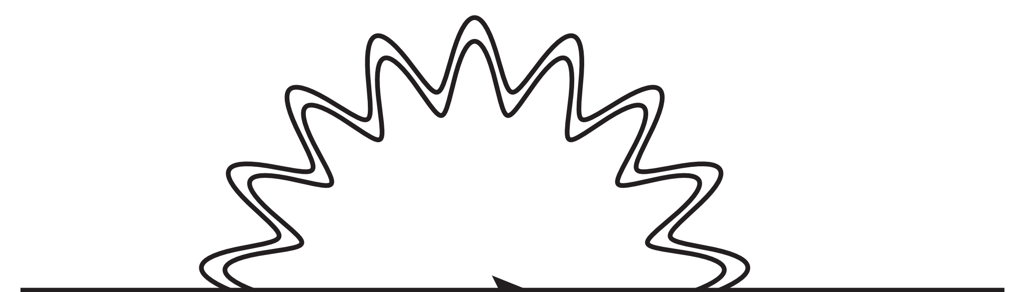


$$\beta(p^2) = 0 \ \forall p$$

$$\alpha(p^2) = 1 - i2\pi G \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(k^2)}{\alpha^2(k^2)k^2 - \beta^2(k^2)} \frac{\left[2(k\cdot p)^2 + 4k^2p^2 + 3k\cdot p(k^2+p^2)\right]}{p^2(p-k)^2}$$

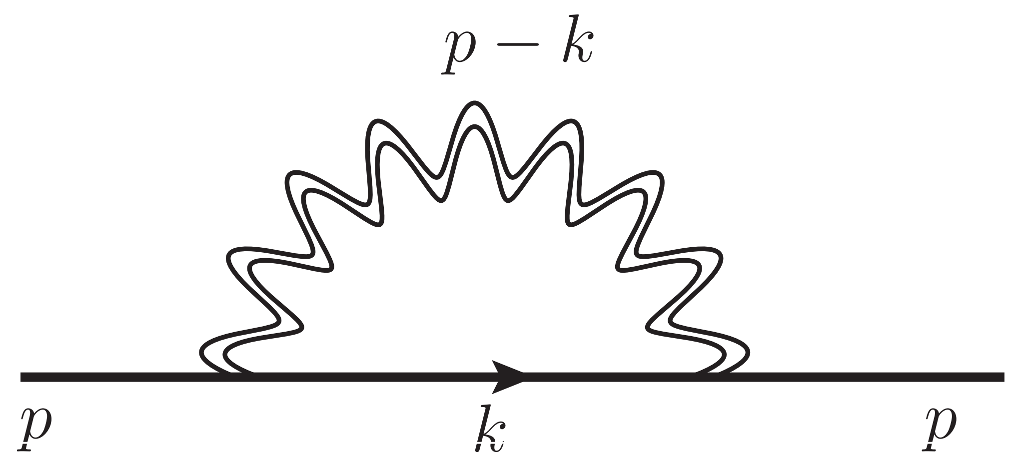
Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

Assume neutrino has zero valued bare mass. Leading contribution from undressed graviton propagator is vanishing. Need to dress graviton.

no mass dynamically induced if graviton is undressed



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Neutrino Masses from Gravity

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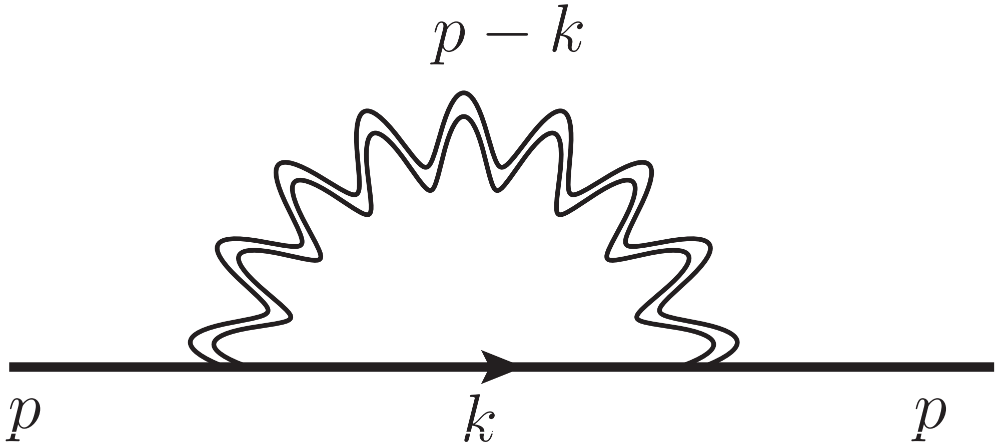

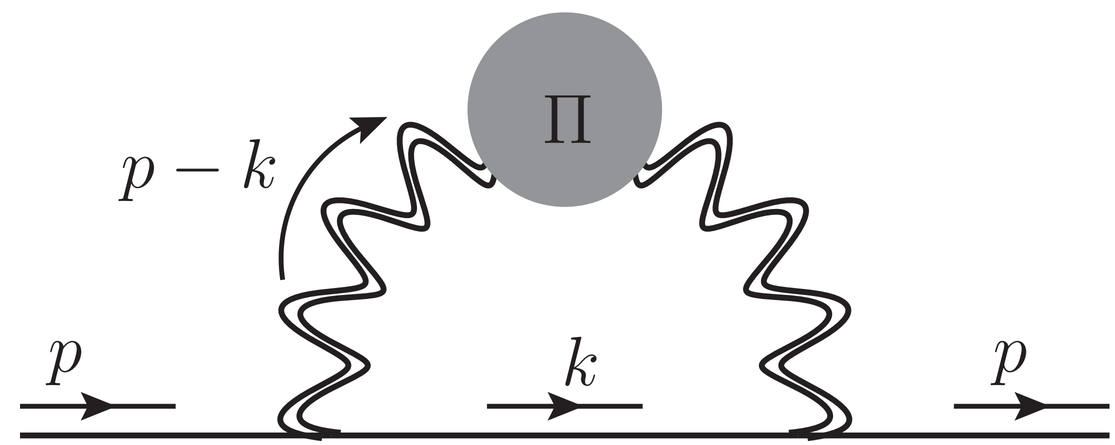

$$G'_{\mu\nu\rho\sigma}(p-k) \to G_{\mu\nu\rho\sigma}(p-k) + G_{\mu\nu\alpha\beta}(p-k)\Pi^{\alpha\beta,\gamma\delta}(p-k)G_{\rho\sigma\gamma\delta}(p-k)$$

Neutrino Masses from Gravity

Apply Schwinger-Dyson equation to find non-trivial vacuum.

$$S'_F(p) = \frac{i}{\not{p} - \Sigma(p)} = \frac{i}{\alpha(p^2)\not{p} - \beta(p^2)} \qquad m_F = \beta(p^2)/\alpha(p^2)$$

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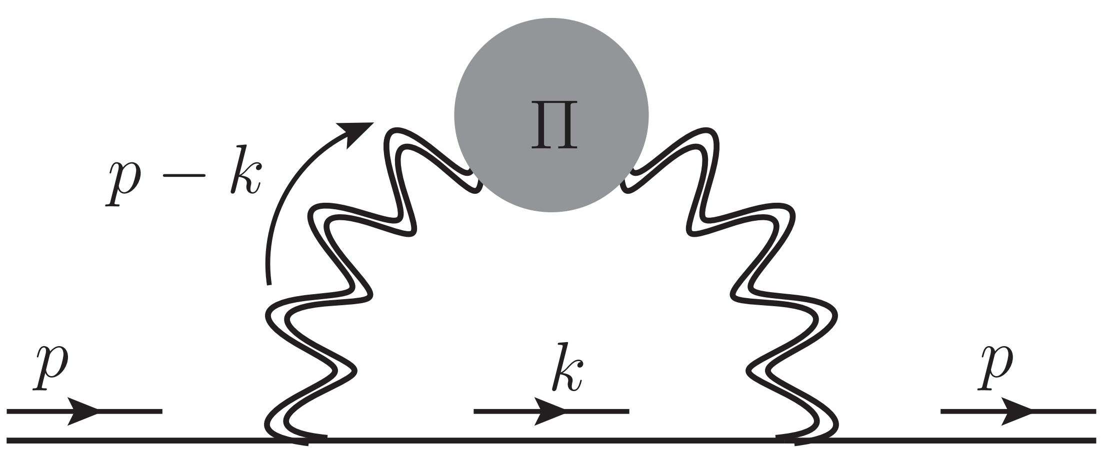
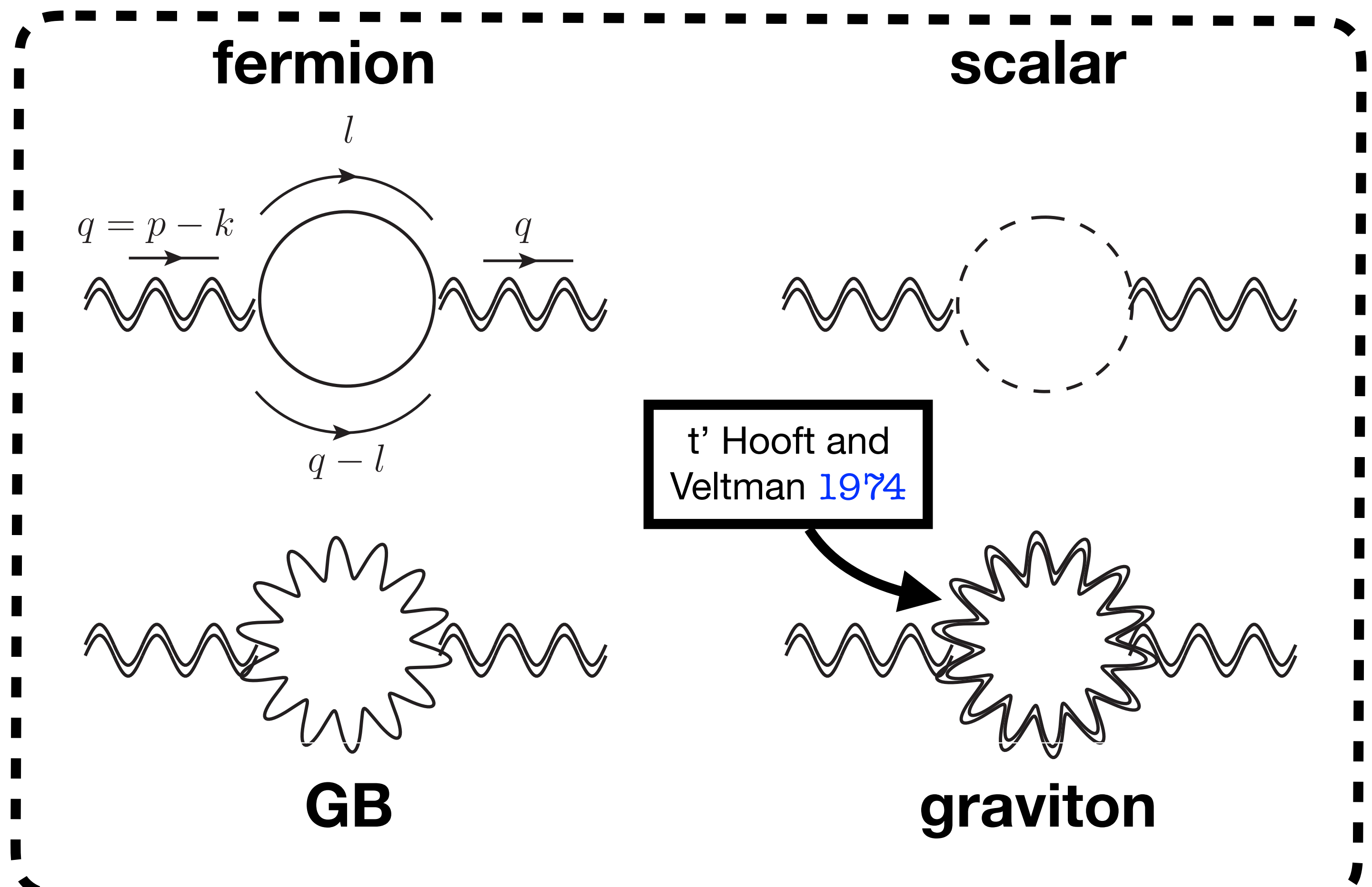


Vacuum polarisations Π



Neutrino Masses from Gravity

Rotate to Euclidean space: $k^2 = -k_E^2, \quad p^2 = -p_E^2, \quad d^4k = i d^4 k_E$

$$\alpha(p_E^2) = 1 - 2\pi G \int \frac{d^4 k_E}{(2\pi)^4} \frac{\alpha(k_E^2)}{\alpha^2(k_E^2)k_E^2 + \beta^2(k_E^2)} \frac{\left[2(k_E \cdot p_E)^2 + 4k_E^2 p_E^2 + 3k_E \cdot p_E(k_E^2 + p_E^2)\right]}{p_E^2 (p_E - k_E)^2}$$

$$\beta(p_E^2) = -8G^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{\beta(k_E^2)}{\alpha^2(k_E^2)k_E^2 + \beta^2(k_E^2)} \left[A(k_E + p_E)^2 - B\frac{\left(p_E^2 - k_E^2\right)^2}{8(p_E - k_E)^2}\right] \log\left[\frac{\mu^2}{(p_E - k_E)^2}\right]$$

d.o.f running in the loop are given by:

$$A = \frac{27/2 N_{\rm ms} + 6N_{\rm df} + 12N_{\rm gb} + N_{\rm cs} + 267N_{\rm gr}}{288},$$

$$B = \frac{9N_{\rm ms} + 6N_{\rm df} + 12N_{\rm gb} + N_{\rm cs} + 186N_{\rm gr}}{288}$$

$\mathrm{SM}: \; 12\,\mathrm{gb} \quad 48\,\mathrm{df} \quad 4\,\mathrm{ms}, \quad 1\,\mathrm{gr}$

$A_{\rm SM} = 2.61$

$B_{\rm SM} = 2.27$

Neutrino Masses from Gravity

Rotate to Euclidean space and rescale momenta: $x = \frac{p_E^2}{\Lambda^2} \quad y = \frac{k_E^2}{\Lambda^2}$

Use hyper-spherical coordinates for phase space:

$$d^4 k_E \to 2\pi \Lambda^4 y dy \sin^2\theta d\theta \qquad \theta \in [0, 2\pi]$$

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2} \int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x,y)$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3} \int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x,y)$$

$$K(x,y) = \frac{1}{x}\int_0^\pi \sin^2\theta d\theta \frac{2xy\cos^2\theta + 4xy + 3\sqrt{xy}(x+y)\cos\theta}{x + y - 2\sqrt{xy}\cos\theta},$$

$$L(x,y) = \int_0^\pi \sin^2\theta d\theta \left[A\left(x + y + 2\sqrt{xy}\cos\theta\right) - B\,\frac{(x-y)^2}{8(x+y-2\sqrt{xy}\cos\theta)}\right] \times \log\left[x + y - 2\sqrt{xy}\cos\theta\right]$$

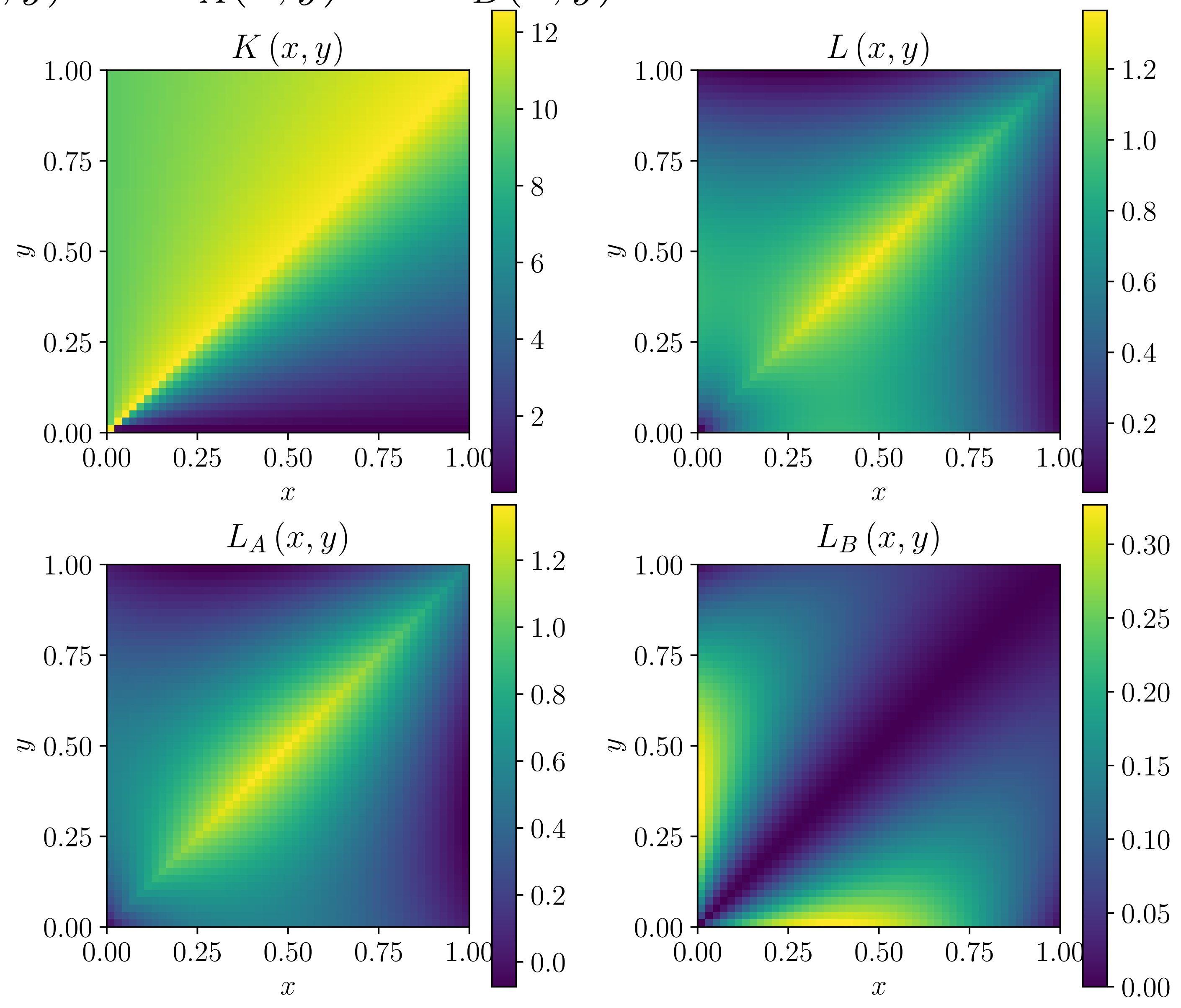
Kernel structure

Rotate to Euclidean space, rescale momentum

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x)+\beta^2(y)} K(x,y)\,,$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3}\int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y)+\beta^2(y)} L(x,y)\,.$$

$L(x,y) = AL_A(x,y) + BL_B(x,y)$



integrated over θ

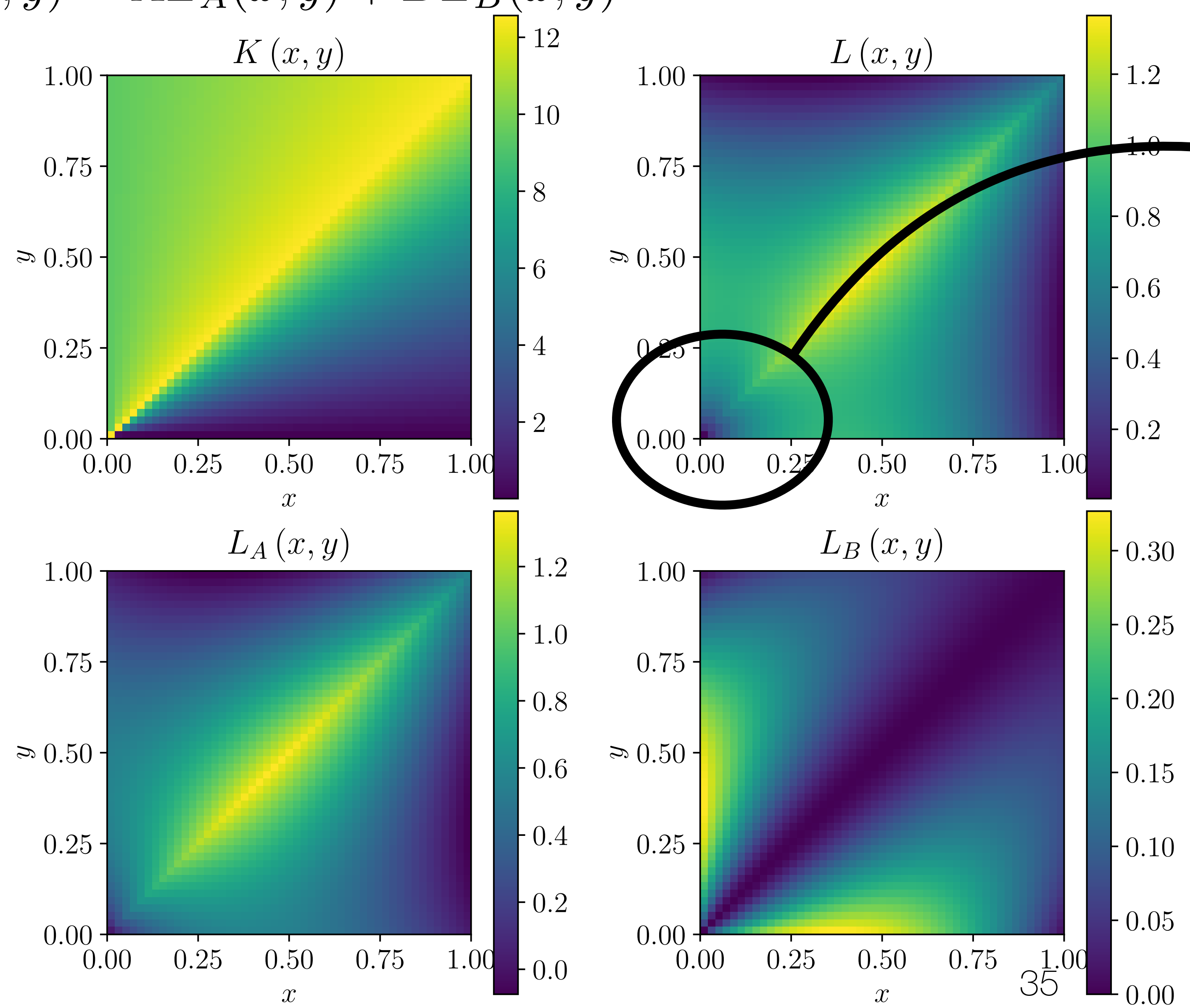
Kernel structure

Rotate to Euclidean space, rescale momentum

$$\alpha(x) = 1 - \frac{G\Lambda^2}{(2\pi)^2}\int_0^1 dy \frac{y\alpha(y)}{y\alpha^2(x) + \beta^2(y)} K(x,y)\,,$$

$$\beta(x) = \frac{8G^2\Lambda^4}{(2\pi)^3}\int_0^1 dy \frac{y\beta(y)}{y\alpha^2(y) + \beta^2(y)} L(x,y)\,.$$

$L(x,y) = AL_A(x,y) + BL_B(x,y)$



gravity $L(0,0) = 0$
$x = y = 0$

QCD $L(0,0) \neq 0$
$x = y = 0$

d-wave interaction due to spin 2 nature of graviton

35
35

Use two methods to solve SDEs using two well known methods

1. **Solve equations iteratively and apply extrapolation**

2. **Make informed Ansatz of the kernel and check for self consistency of non-trivial vacuum.**

Solving SD Equation - Extrapolation

$$
\begin{aligned}
\alpha^{(i+1)}(x) =& 1 - \frac{G\Lambda^2}{(2\pi)^2} \int_0^1 dy \frac{y\alpha^{(i)}(y)}{y{\alpha^{(i)}}^2(x) + {\beta^{(i)}}^2(y)} K(x,y) \\
\beta^{(i+1)}(x) =& \frac{8(G\Lambda^2)^2}{(2\pi)^3} \int_0^1 dy \frac{y\beta^{(i)}(y)}{y{\alpha^{(i)}}^2(y) + {\beta^{(i)}}^2(y)} L(x,y) \,.
\end{aligned}
$$

Start with two trial functions

$$
\alpha^{(0)}(x) = c_1 \,, \quad \beta^{(0)}(x) = c_2
$$

$$
\text{tolerance} \equiv \frac{\beta^{(i+1)}(x)}{\beta^{(i)}(x)} - 1
$$

This method allows us to find the NP non-trivial vacuum.

Solution (true non-trivial vacuum) is not sensitive to trial function value or tolerance value.

Solving SD Equation - Extrapolation

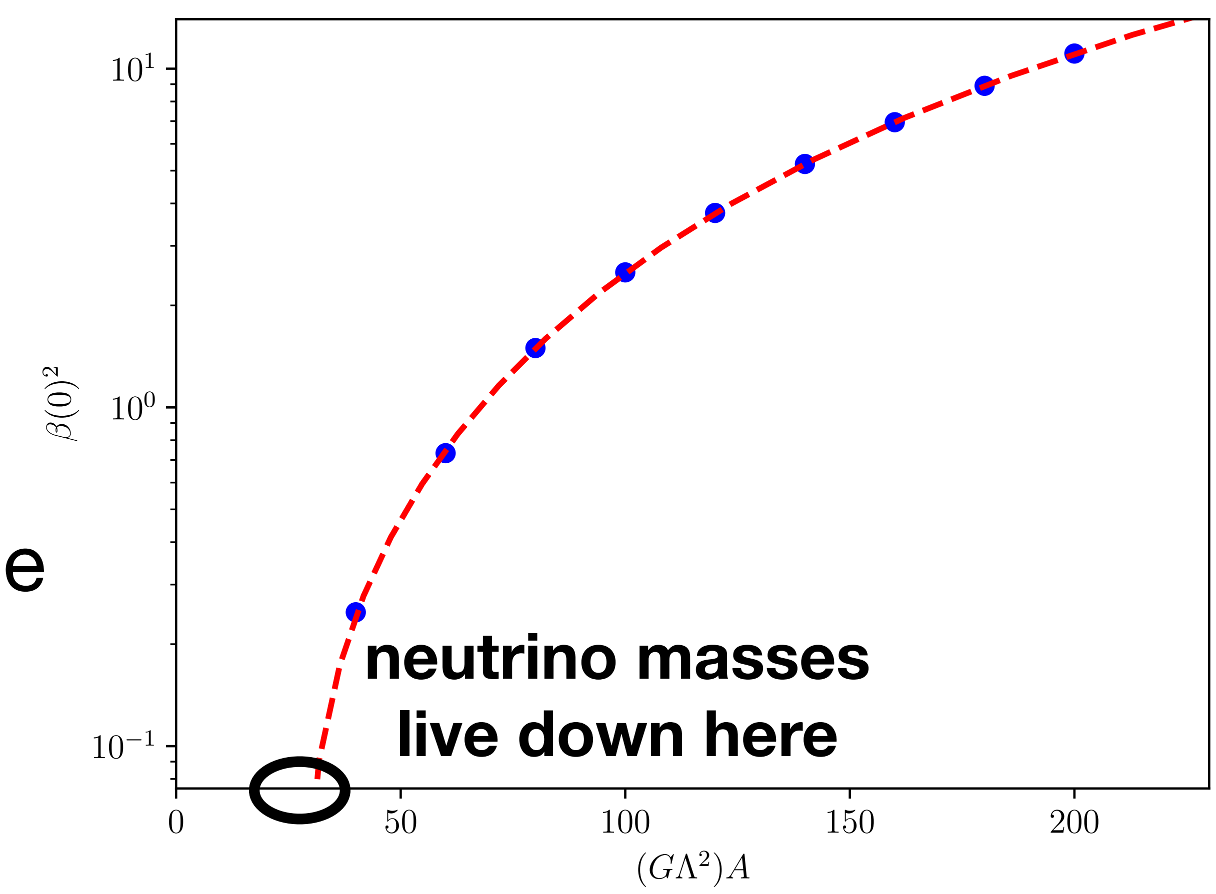
1. Choose a value of $G\Lambda^2$, A, B, tolerance and trial function values.
2. Subdivide the x-interval $[x_{\rm IR}, 1]$ into n bins, where $x_{\rm IR}$ is infrared boundary of the theory.
3. Iteratively solve SDE for each bin.
4. For each bin calculate the tolerance and summate this measure over all bins.
5. Require the tolerance to be close to 0.0. For example, for $G\Lambda^2 = 1.0$, $A = 50.0$ and $B = 43.5$ we choose a tolerance of 10^{-6}.

$$m_\nu = \frac{\beta(0)}{\alpha(0)}\Lambda$$

$$\beta(0) \approx 10^{-29} \text{ for } \Lambda \approx M_{\rm pl}$$

$$G\Lambda^2 \implies A \gtrsim 23$$

Requires beyond SM particle content to support the condensate even if scale is high, also we are tuning around chiral preserving point.



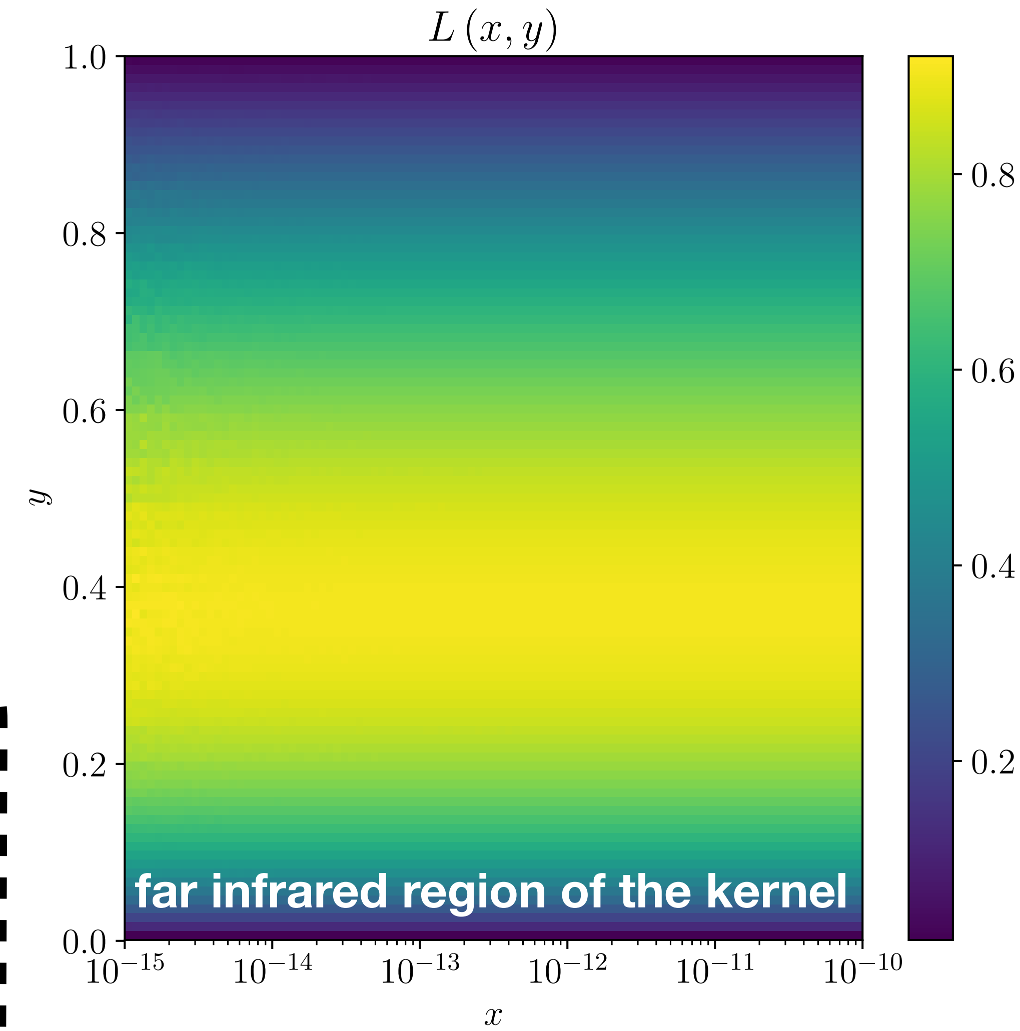
Solving SD Equation - Ansatz

Take quenched limit for simplicity i.e $\alpha \approx 1$

β depends on L(x,y). This kernel is flat in the x-direction even for tiny momentum. Make Ansatz that β is a step function of magnitude ‘a’.

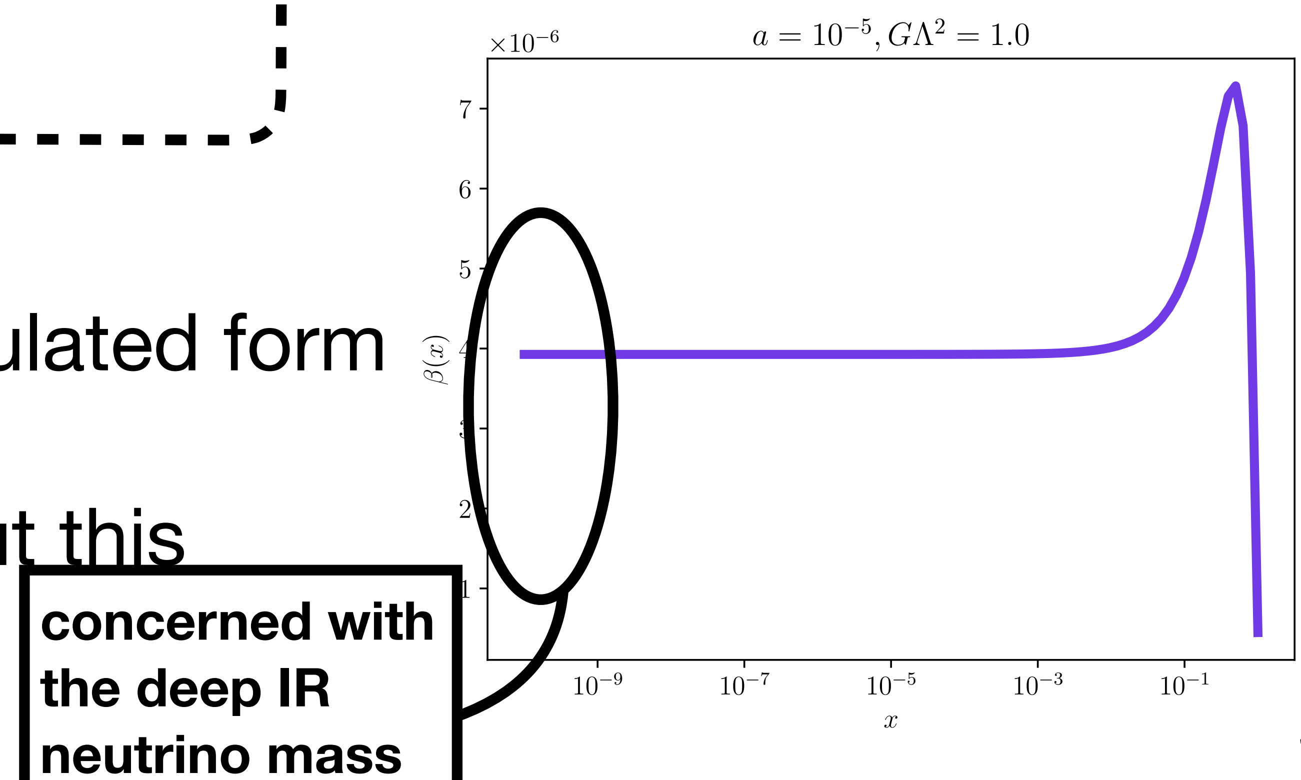
$$\beta(x) = \frac{8(G\Lambda^2)^2}{(2\pi)^3} \int_0^1 \frac{a y dy}{a^2 + y} L_A(x,y)$$

$$A L_A(x,y) \gg B L_B(x,y)$$



Solve SDE for this Ansatz

Checks for self-consistency of postulated form of β with kernel structure.
As before the mass can be tuned but this this approach it is tuned via ‘a’.



Features and bugs

Gravity is democratic and cannot split the neutrinos masses.

To achieve the correct neutrino mass splitting, we need some additional mechanism. We do not assume a mechanism but it must be present.

There are composite d.o.f.s resulting from symmetry breaking. These are pions composed of the constituent neutrinos and are pseudo-Goldstone bosons.

$$m_{\pi_\nu} \sim \frac{\sqrt{\langle \overline{\nu}\nu \rangle m_\nu}}{f_\nu} > \sqrt{\Lambda m_\nu}$$

For high scale condensation the pions heavy > 10^4 GeV and should not affect cosmology i.e. N_{eff}/neutrino free streaming.

Summary

- Neutrinos are unique amongst the Standard Model (SM) fermions in the tininess of their mass, the weakness of their interactions and their capacity to be their own anti-particles. Such features suggest neutrinos acquire their mass in a different way from the quarks and charged leptons.

- Neutrino masses from gravity is an intriguing idea and we have made a first calculational attempt at exploring this possibility.

- An interesting feature is new d.o.fs are necessary to provide finite support to the condensate even if it occurs at a very high scale. SM + gravity is not sufficient unless there are large ED which lowers Planck scale.

- As gravity does not discriminate between the neutrinos, they are mass degenerate, one needs some additional mechanism to induce a mass splitting.

- However a high level of fine-tuning is required if the Planck scale is at $\sim 10^{19}$ GeV.

Thank you for your attention

Back up slides - the action

$$S_g = \int d^4x \sqrt{-g}(\frac{1}{4\pi G} R + \mathcal{L}_m)$$

$$\mathcal{L}_m = D_\mu \phi^* g^{\mu\nu} D_\nu \phi + \frac{i}{2}\left[\bar{\psi}\gamma^a e^\mu_a D_\mu \psi + (D_\mu \bar{\psi})\gamma^a e^\mu_a \psi\right] - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma}$$

where $F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$ and D_μ denotes the covariant derivative with respect to the gravitational field and gauge fields, and e^μ_a is the vierbein to shift frame to the local Minkowski flat frame.

$$g_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

Perturb the metric, the classical gravitational field is fixed at zero.

$$\text{Graviton propagator} : G_{\mu\nu\rho\sigma}(p) = \frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{p^2}$$

$$\mathcal{P}^{\mu\nu\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$$