

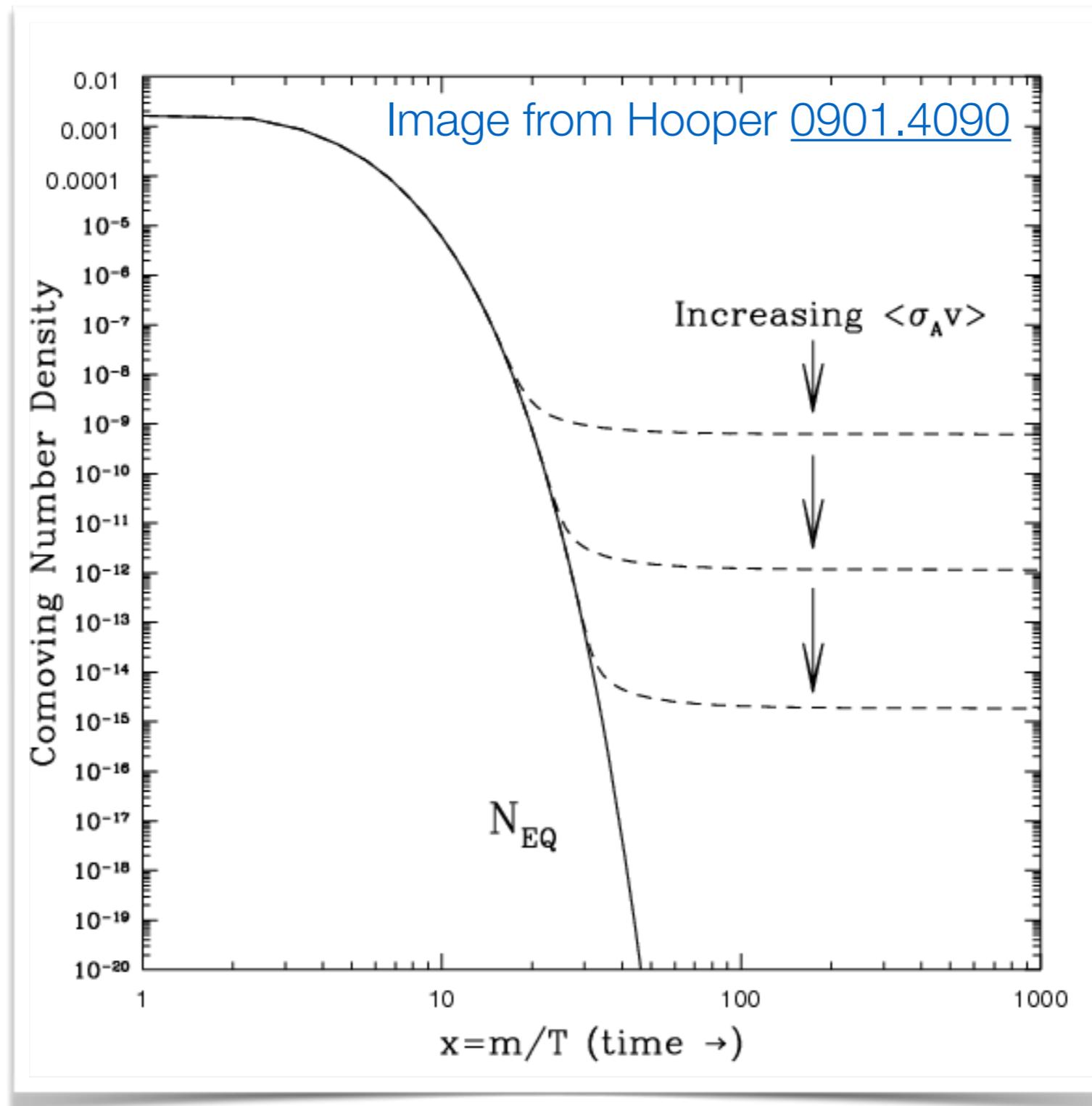
# Filtered Dark Matter

## Setting the DM Abundance Through a First-Order Phase Transition

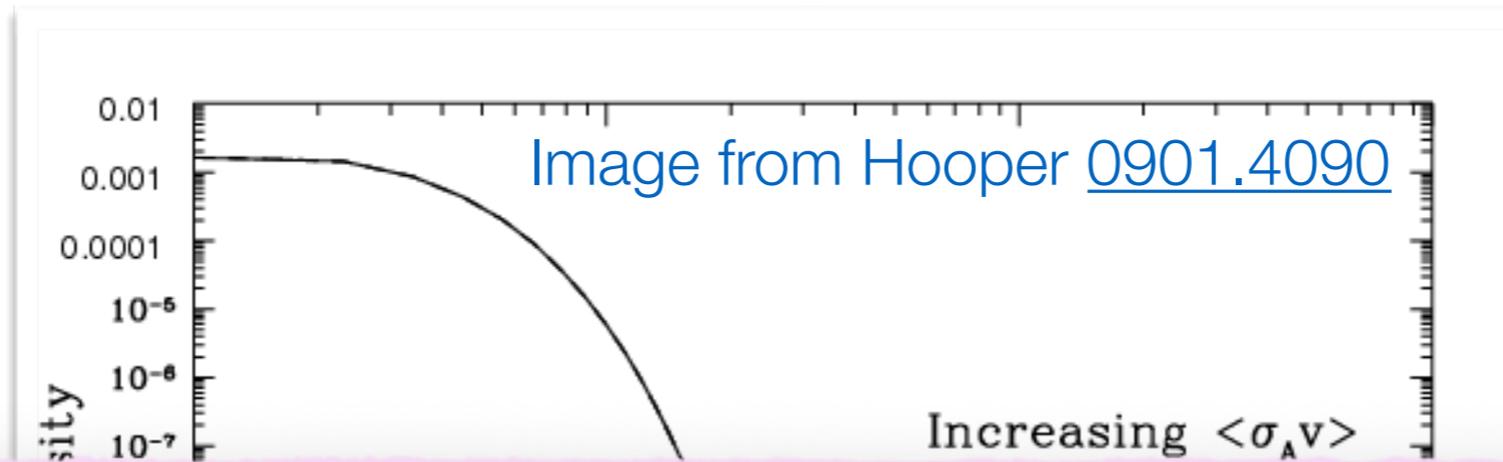
Joachim Kopp (CERN & Uni Mainz)  
Invisibles / Elusives Webinar | 14.01.2020



# DM in the early Universe: Thermal Freeze-Out

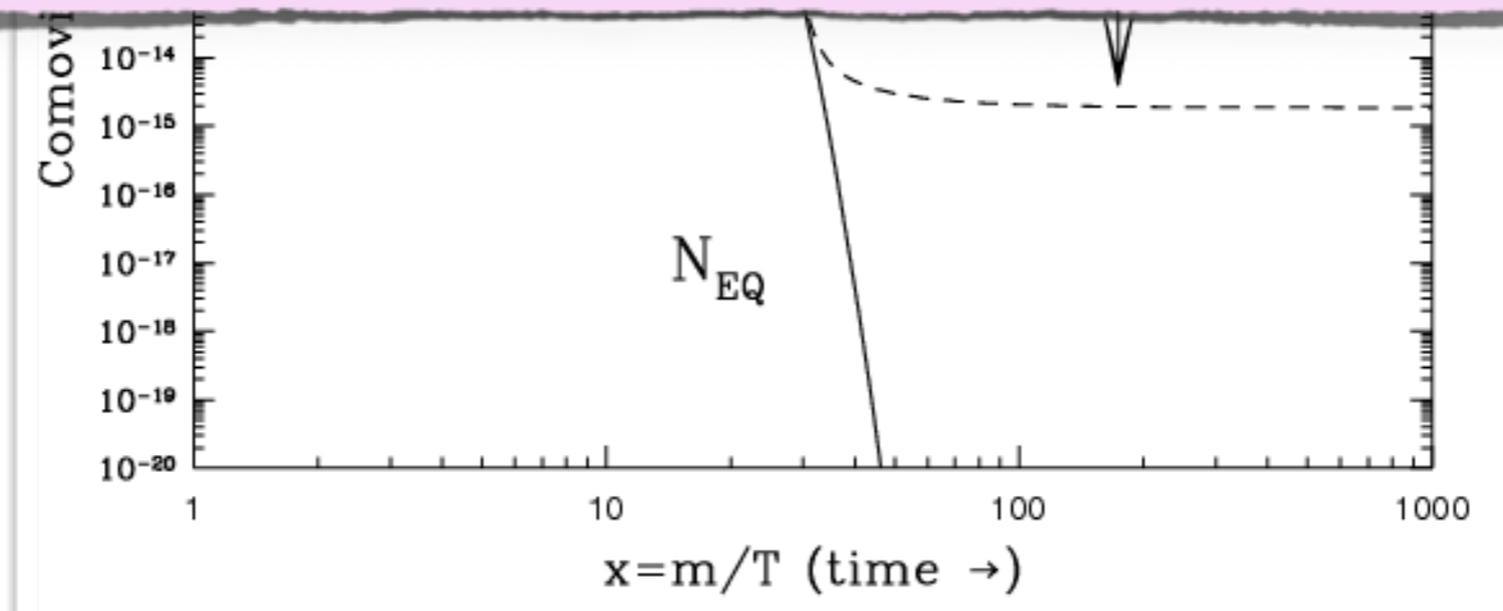


# DM in the early Universe: Thermal Freeze-Out



observed relic abundance obtained for

$$\langle \sigma(\chi\chi \rightarrow \bar{f}f)v_{\text{rel}} \rangle \simeq 2.2 \times 10^{-26} \text{ cm}^3/\text{sec}$$



# Beyond Thermal Freeze-Out



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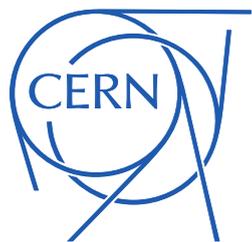


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  - indirect searches (cosmic rays from DM annihilation)
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- ☑ No showstoppers yet, but the community is beginning to worry
- ☑ One alternative: setting the DM abundance in a cosmological phase transition
  - ➡ this talk



# Phase Transitions Primer





# Phase Transitions in Everyday Life

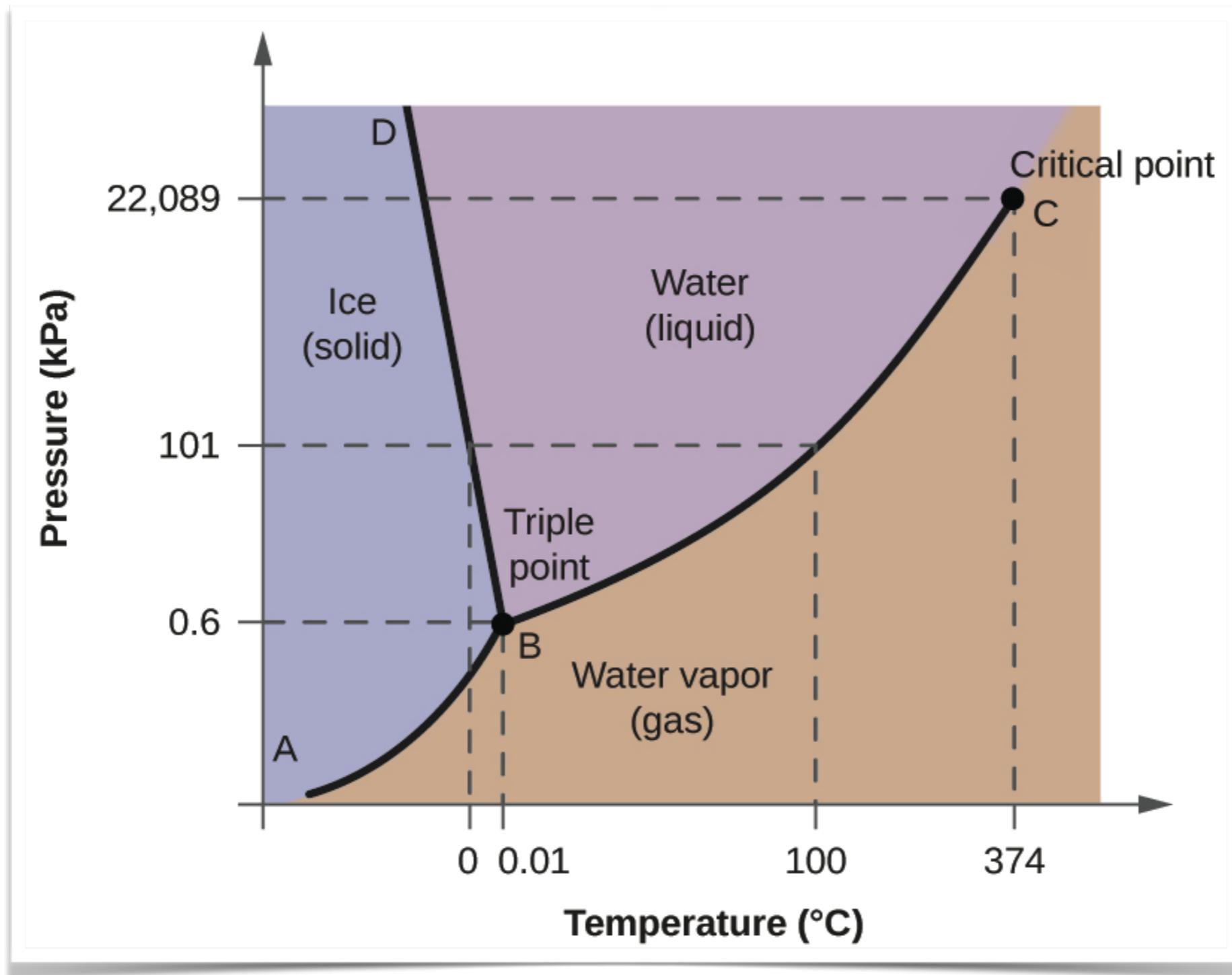


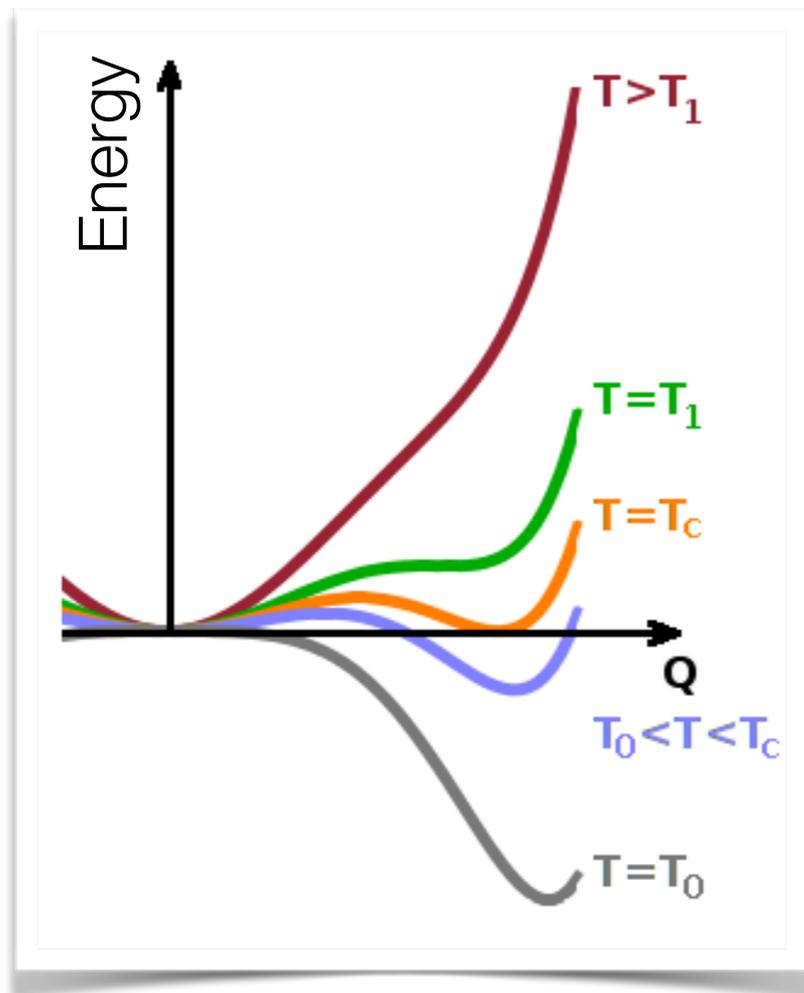
Image Credit: [libretexts.org](http://libretexts.org)

# The Order of a Phase Transition

- ☑ **Order Parameter  $Q$** : a quantity measuring the change in the system across the phase transition
  - for liquid–gas transition: **density  $\rho$**

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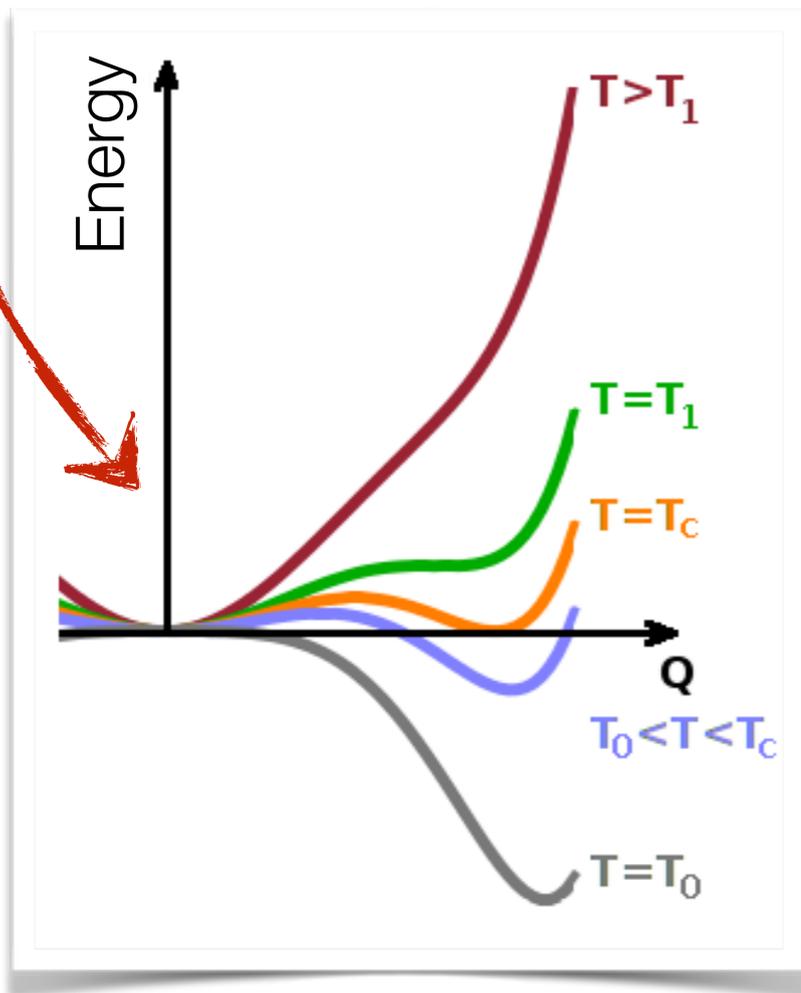
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## 1<sup>st</sup> order transition

order parameter changes discontinuously

for liquid-gas transition: density  $\rho$

quantity measuring the change in the transition



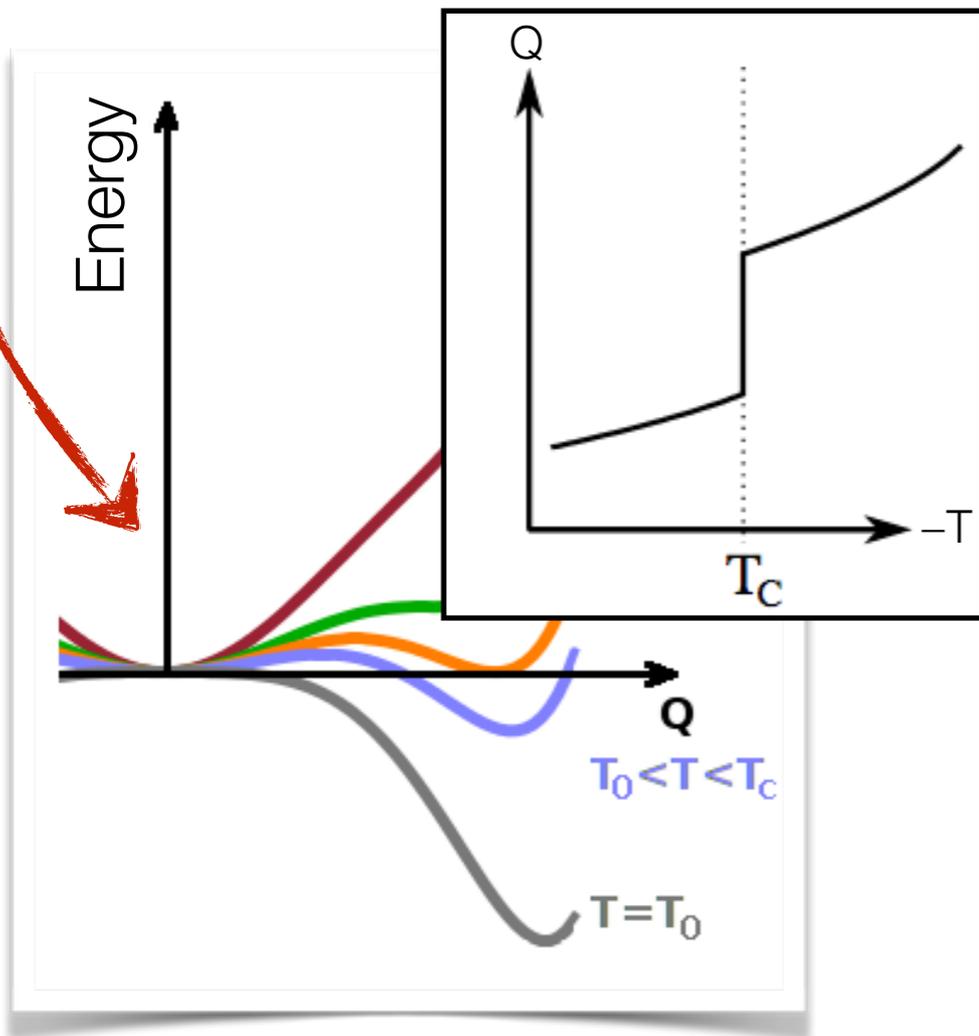
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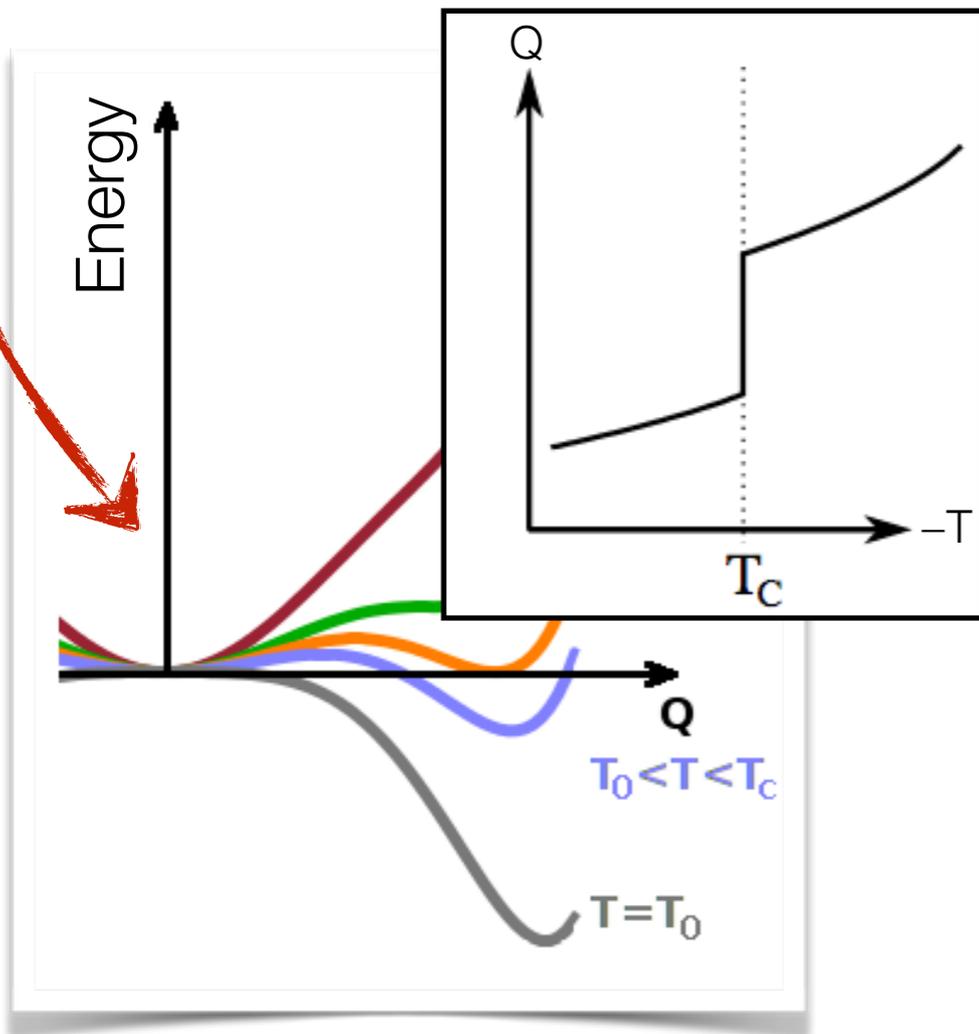


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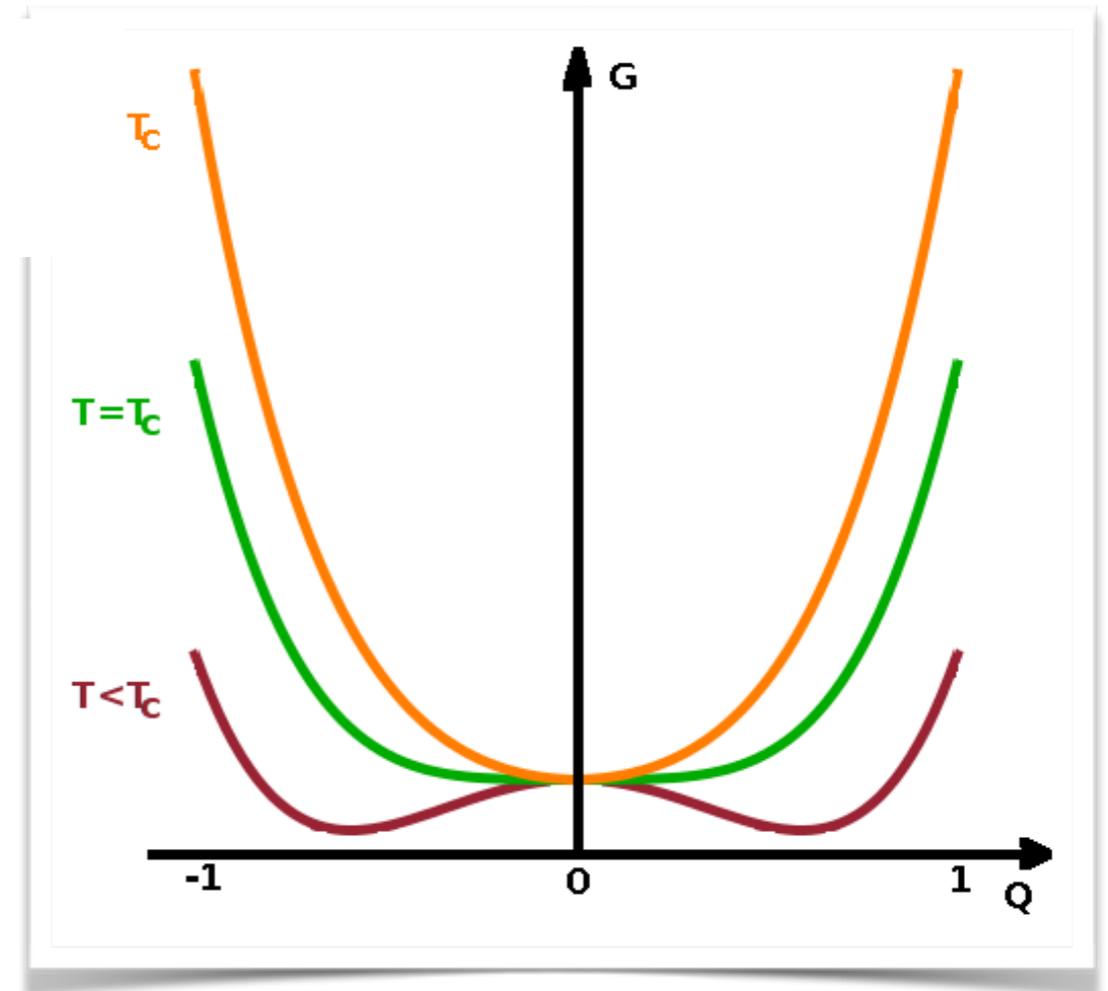
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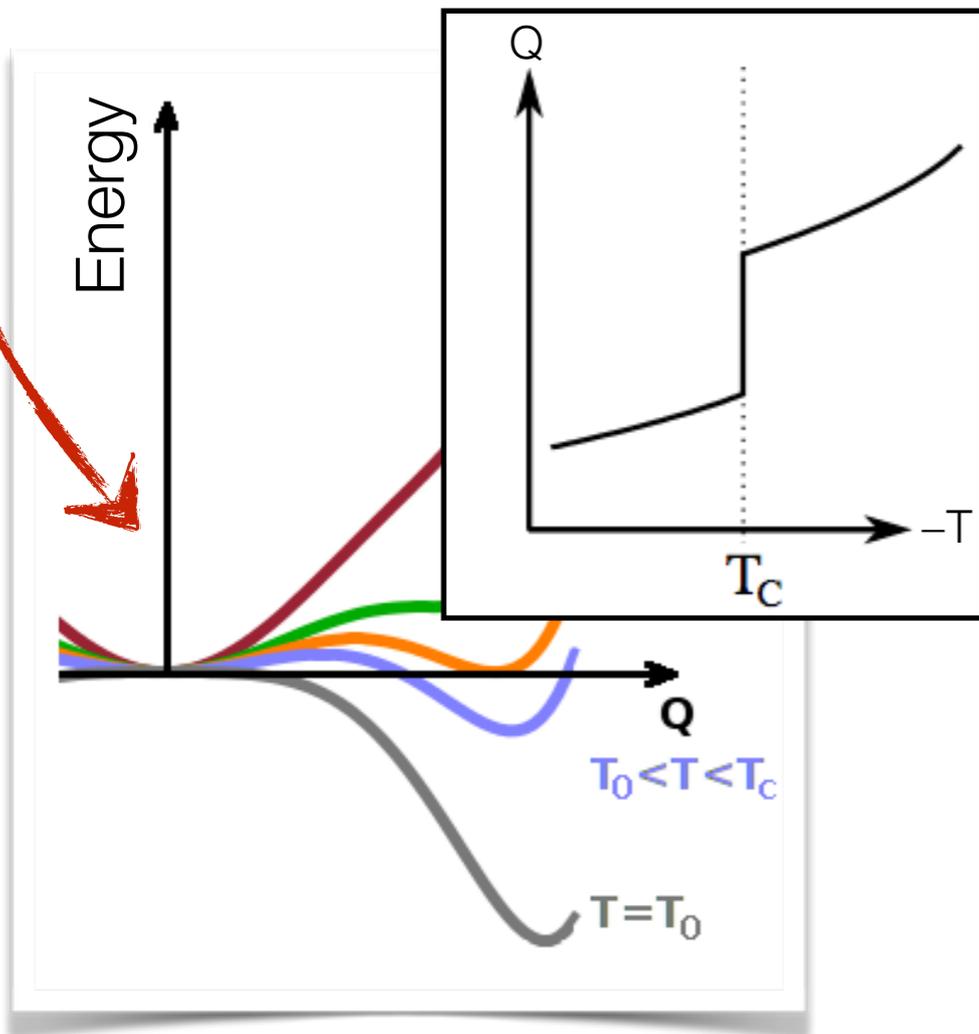
Images: [Rudi Winter](#),  
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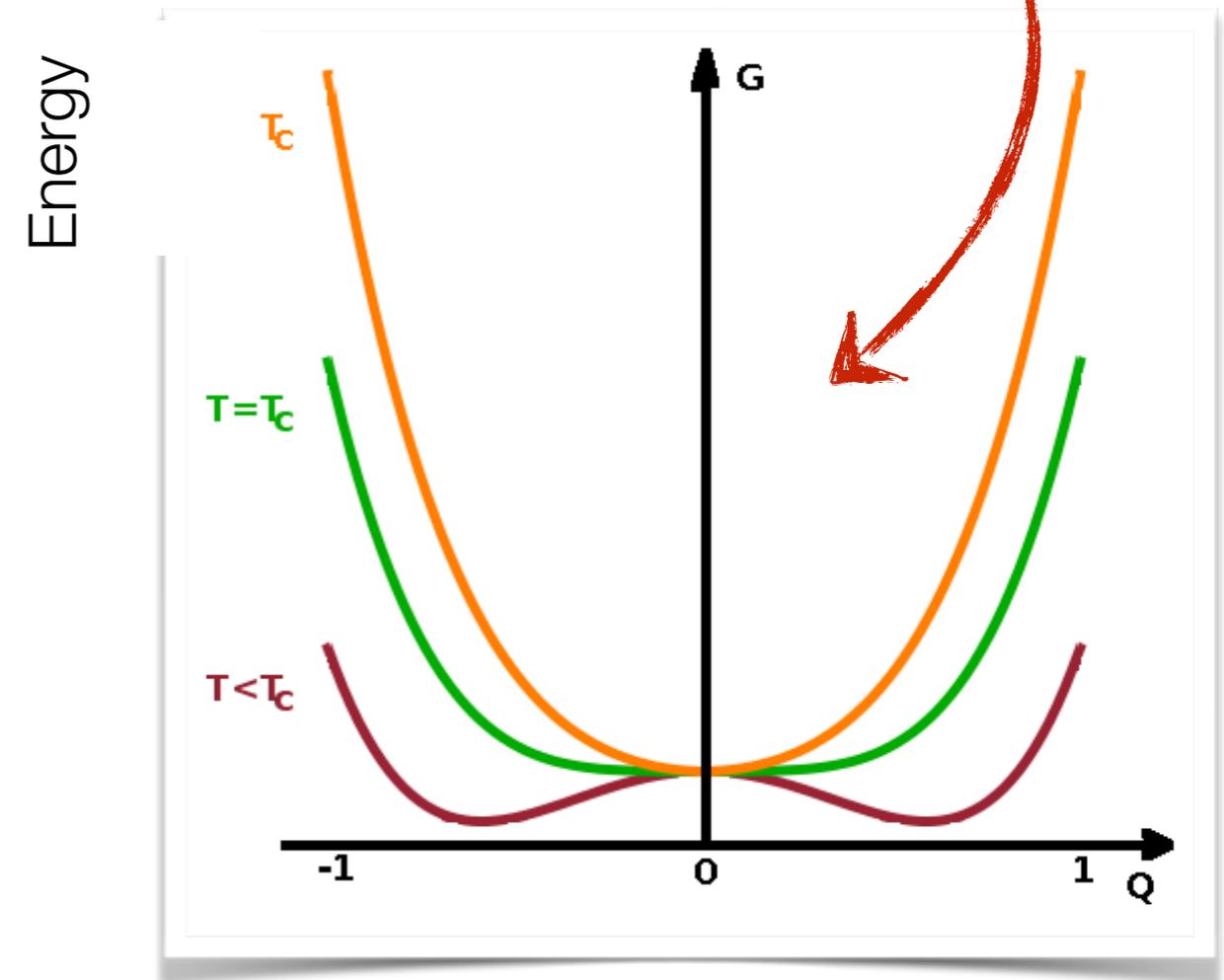
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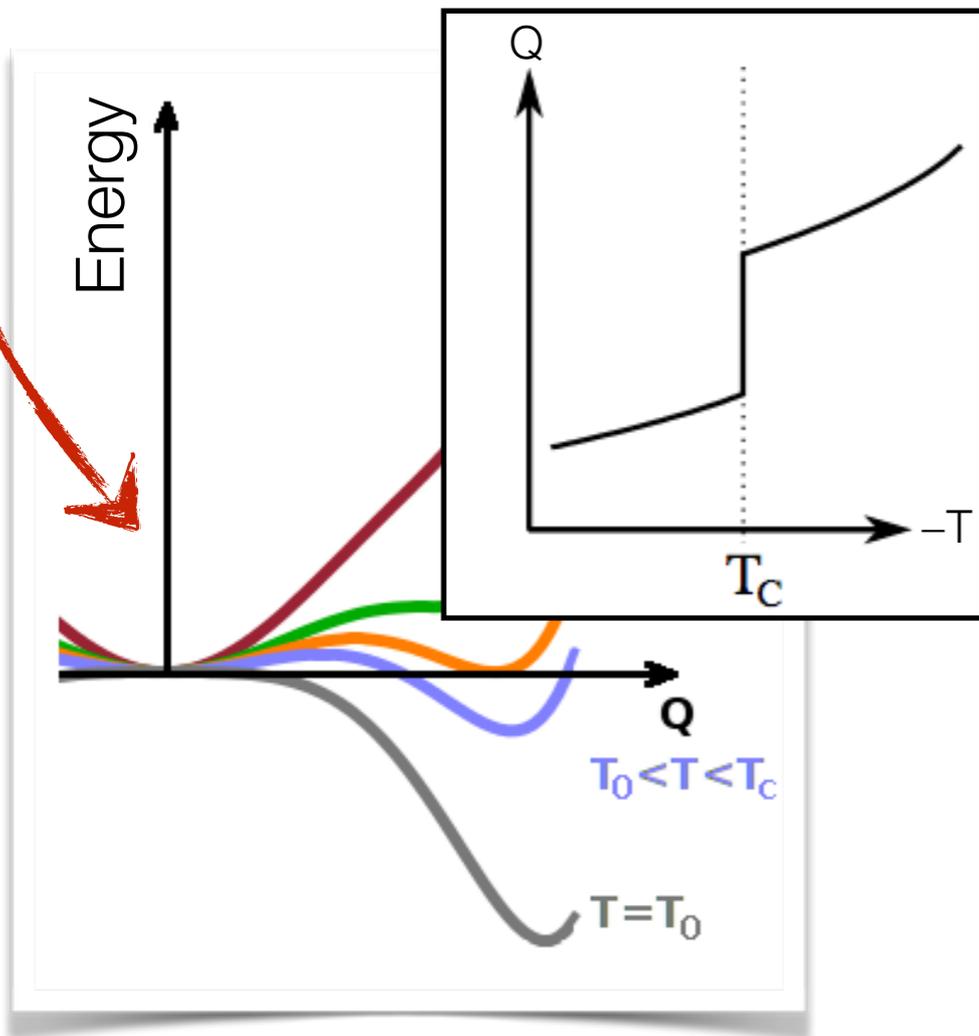
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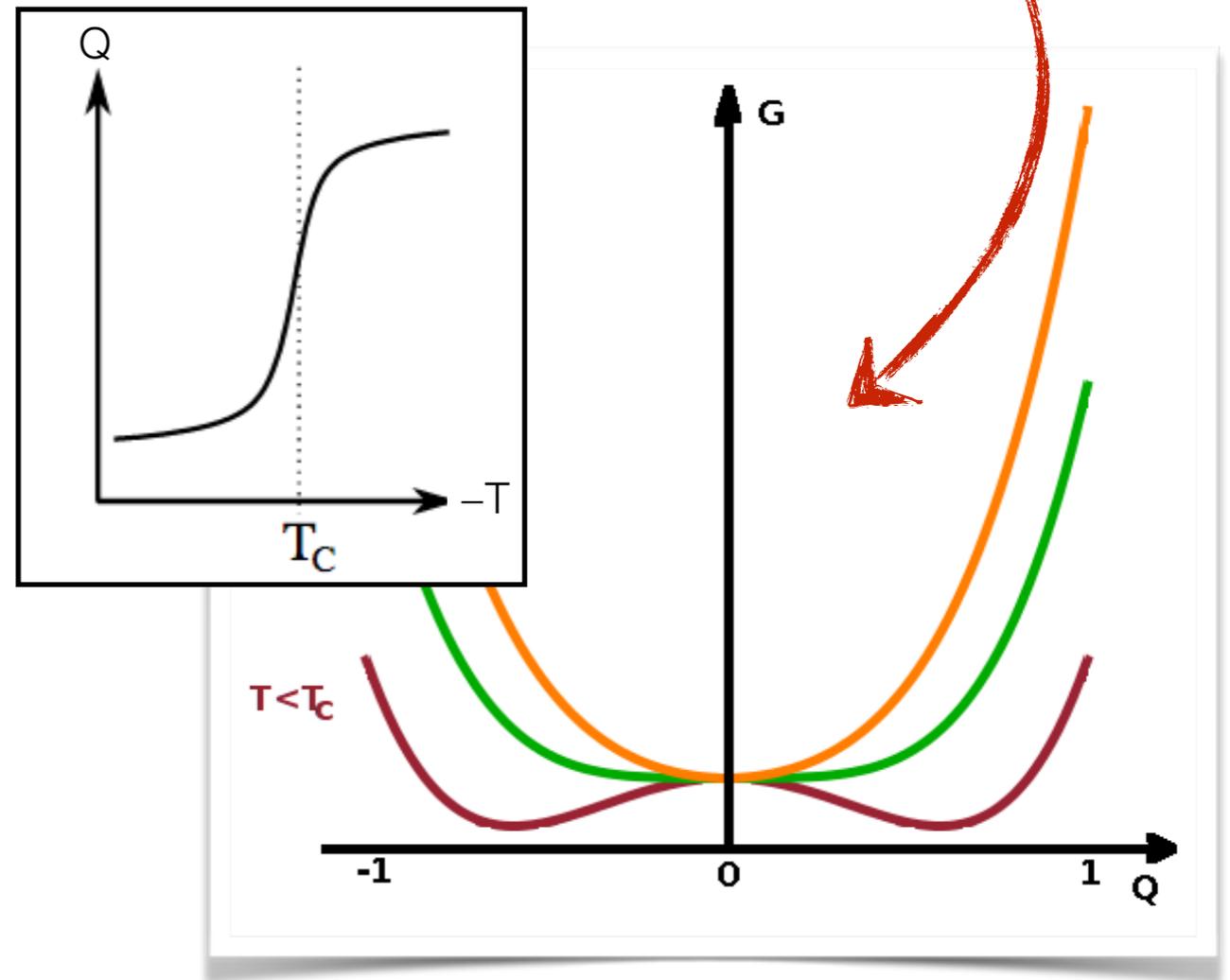
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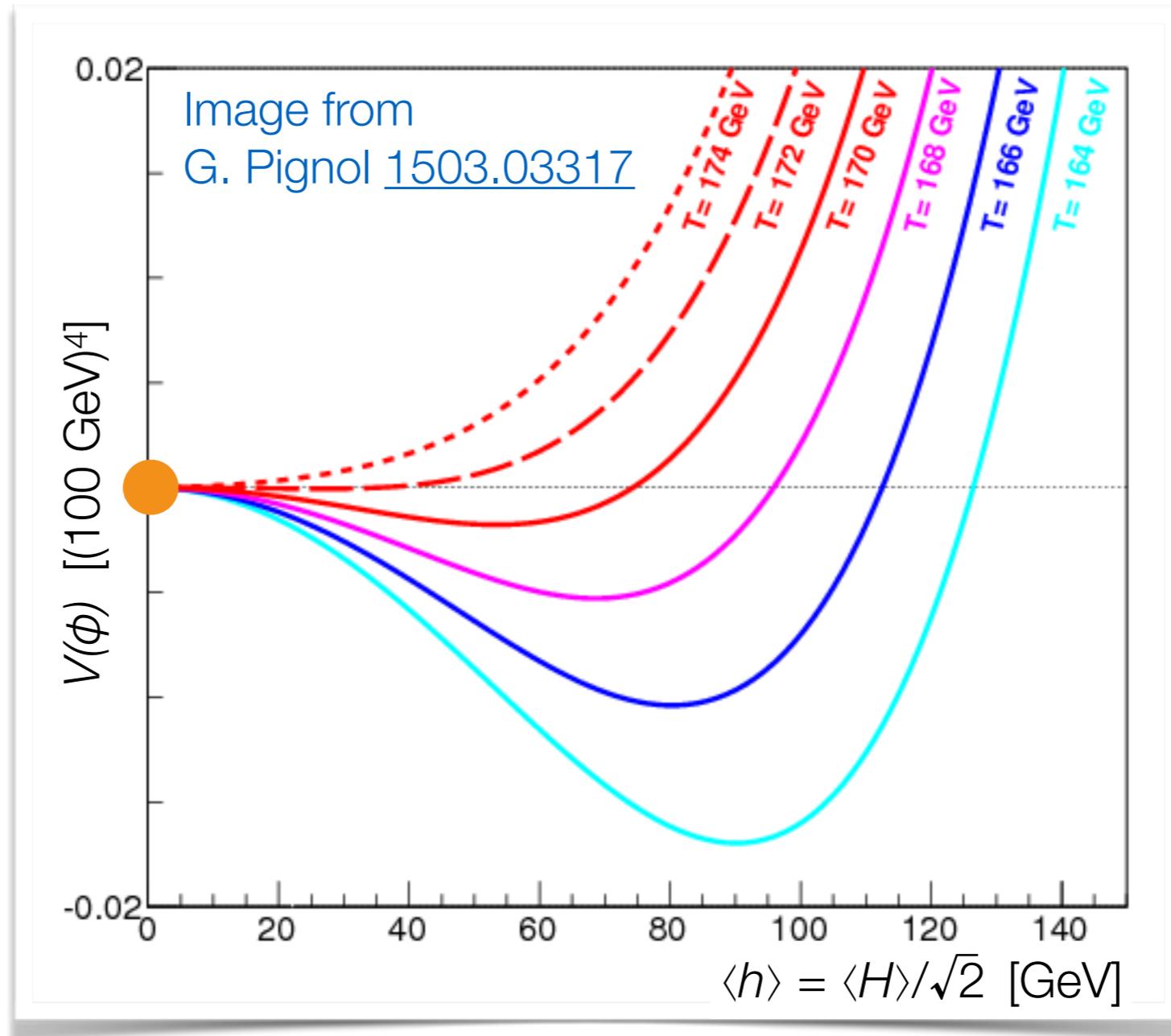
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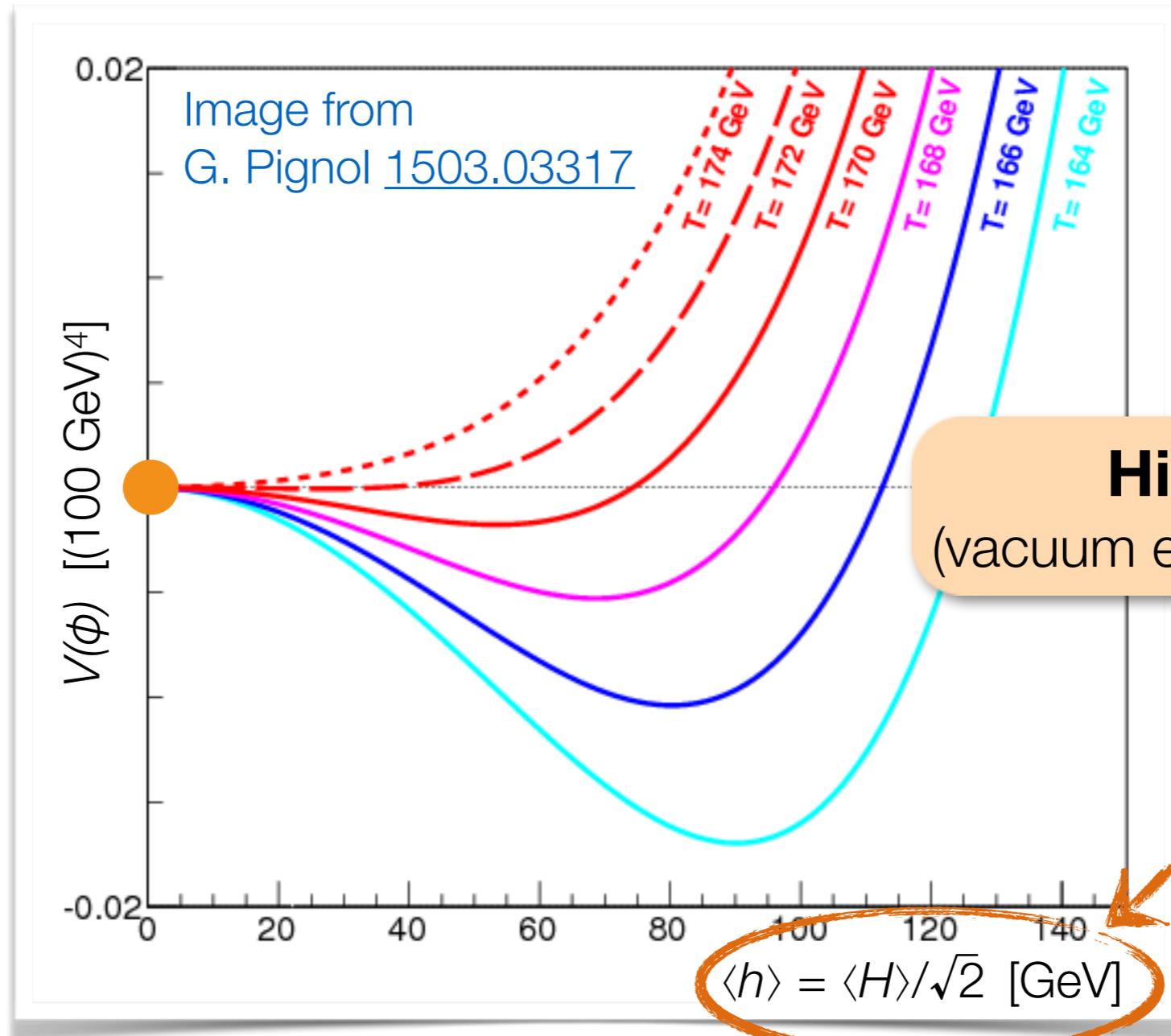


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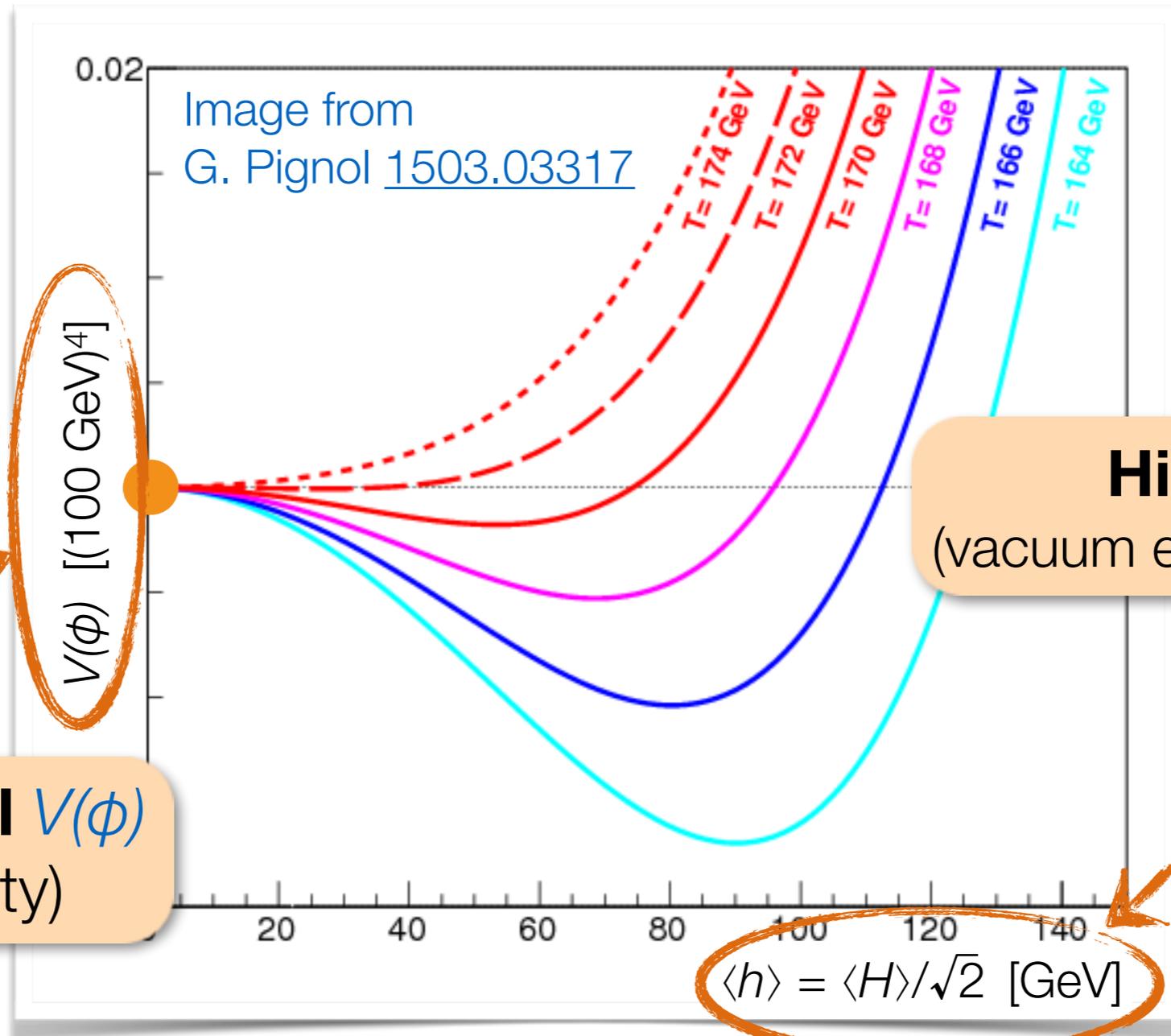
# The Electroweak Phase Transition



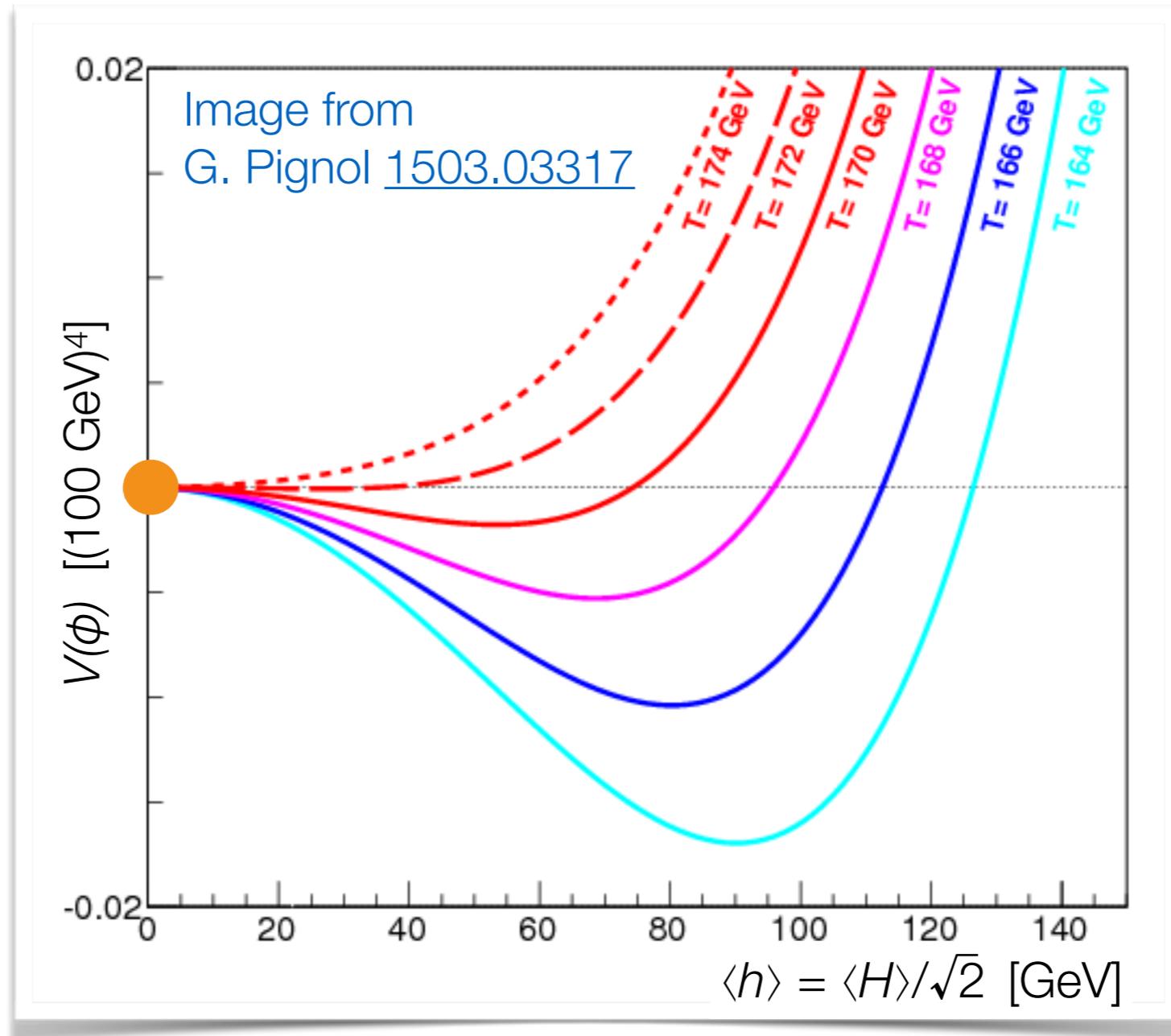
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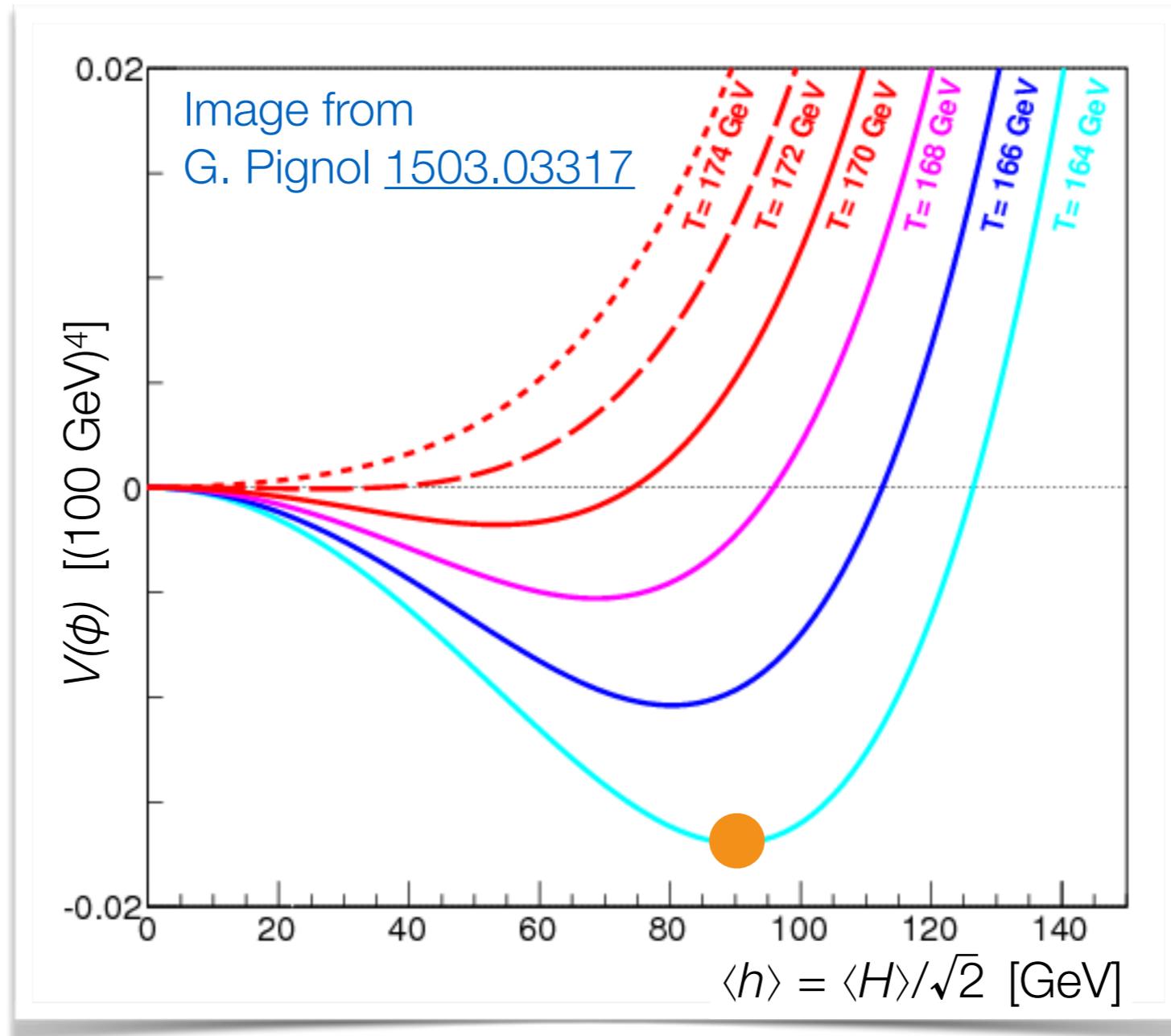
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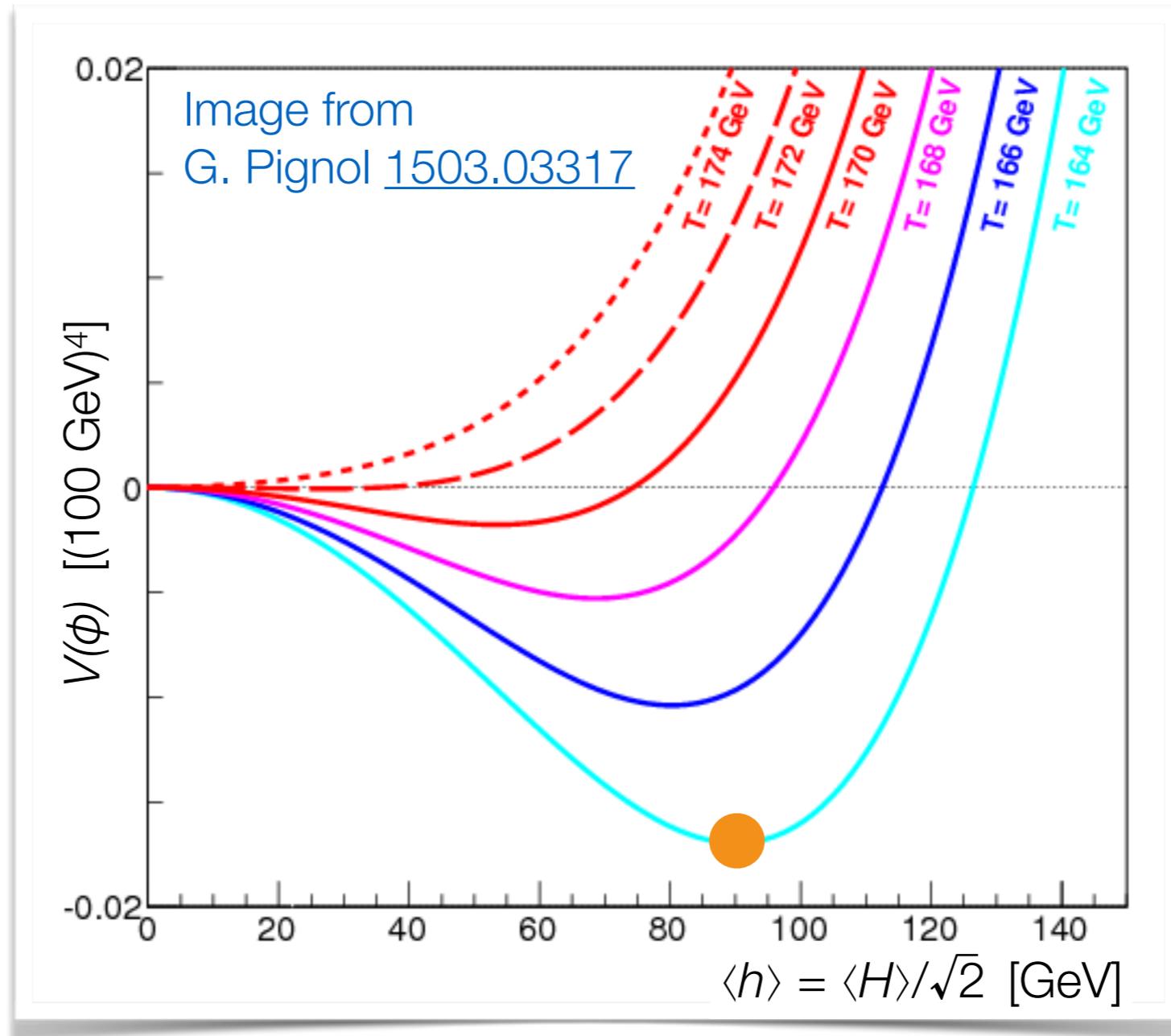
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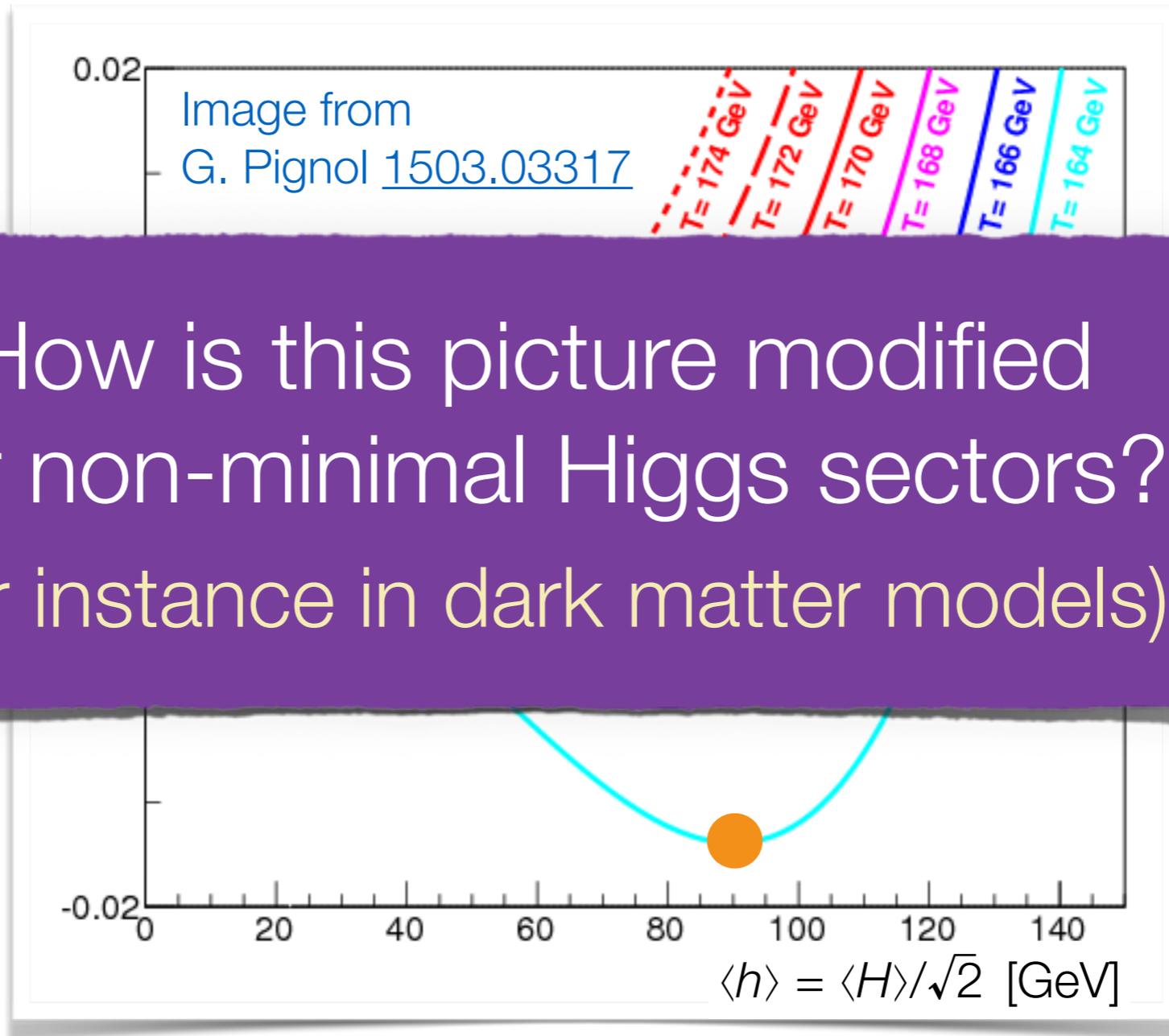


# The Electroweak Phase Transition



computational details ➡ backup slides

# The Electroweak Phase Transition



How is this picture modified  
for non-minimal Higgs sectors?  
(for instance in dark matter models)

computational details  $\Rightarrow$  backup slides

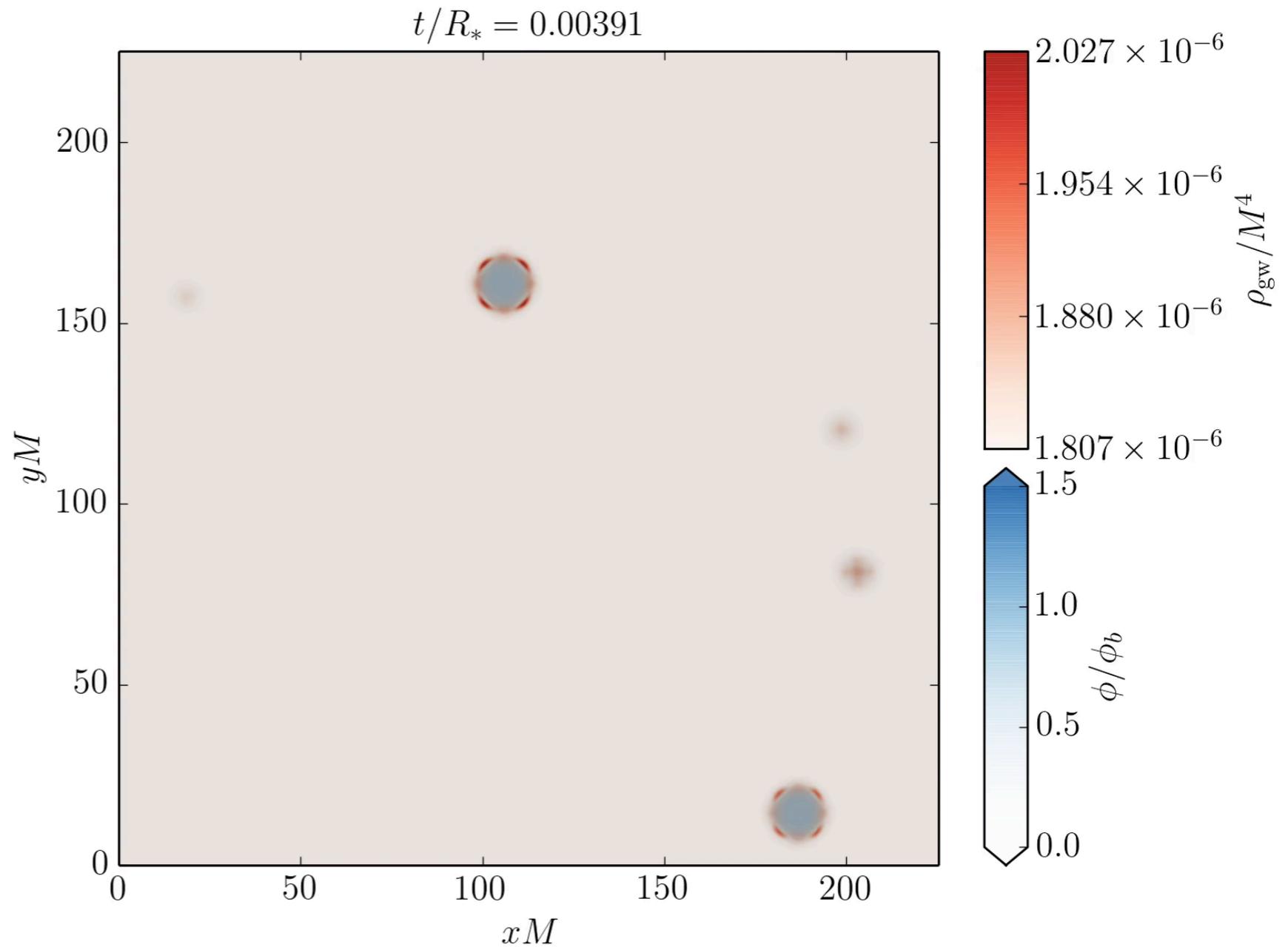
# “Filtered” Dark Matter



# Example 2: DM Filtering at Bubble Walls

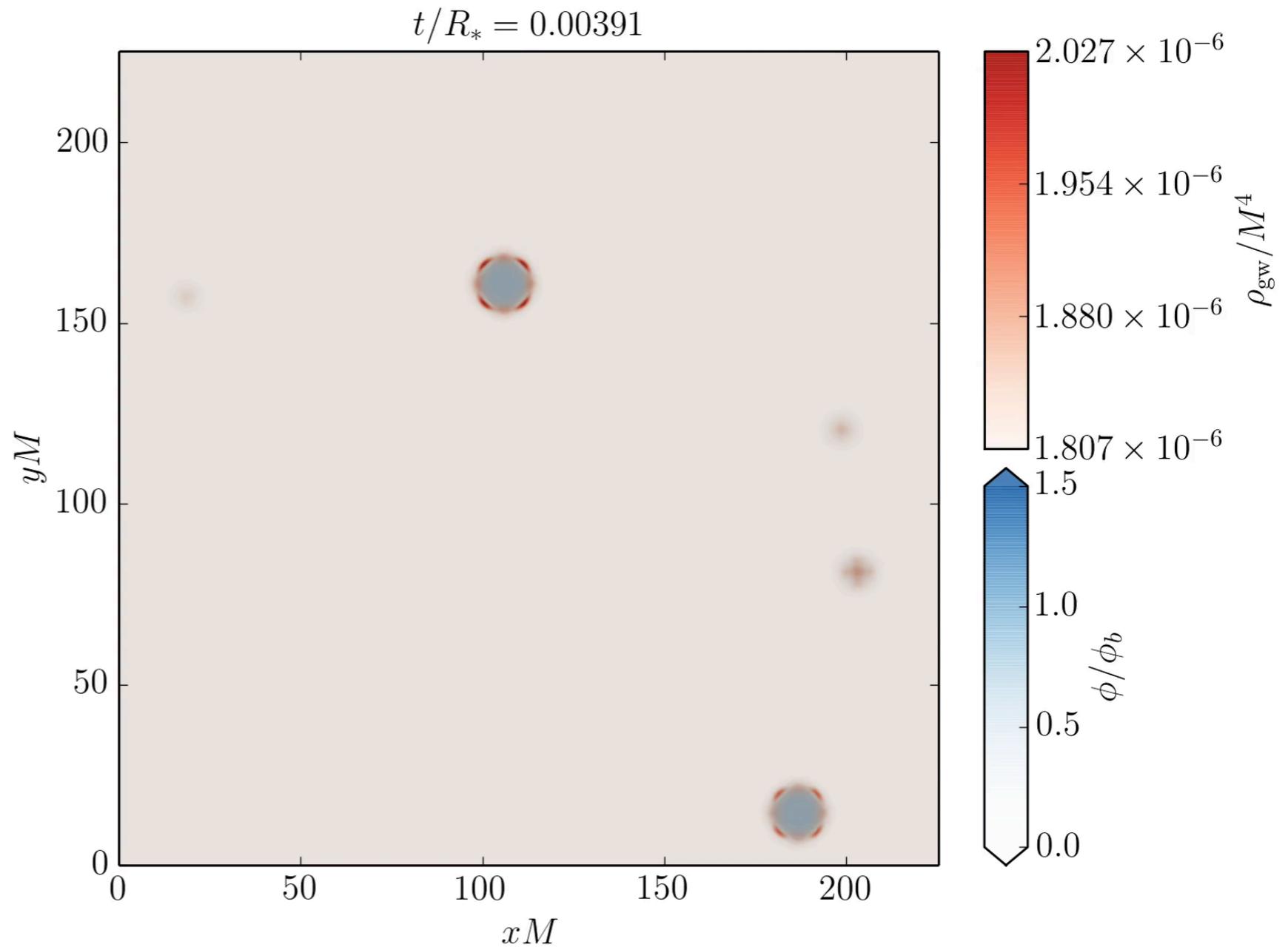


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[Witten 1984](#), [Cutting Hindmarsh Weir 2018](#)

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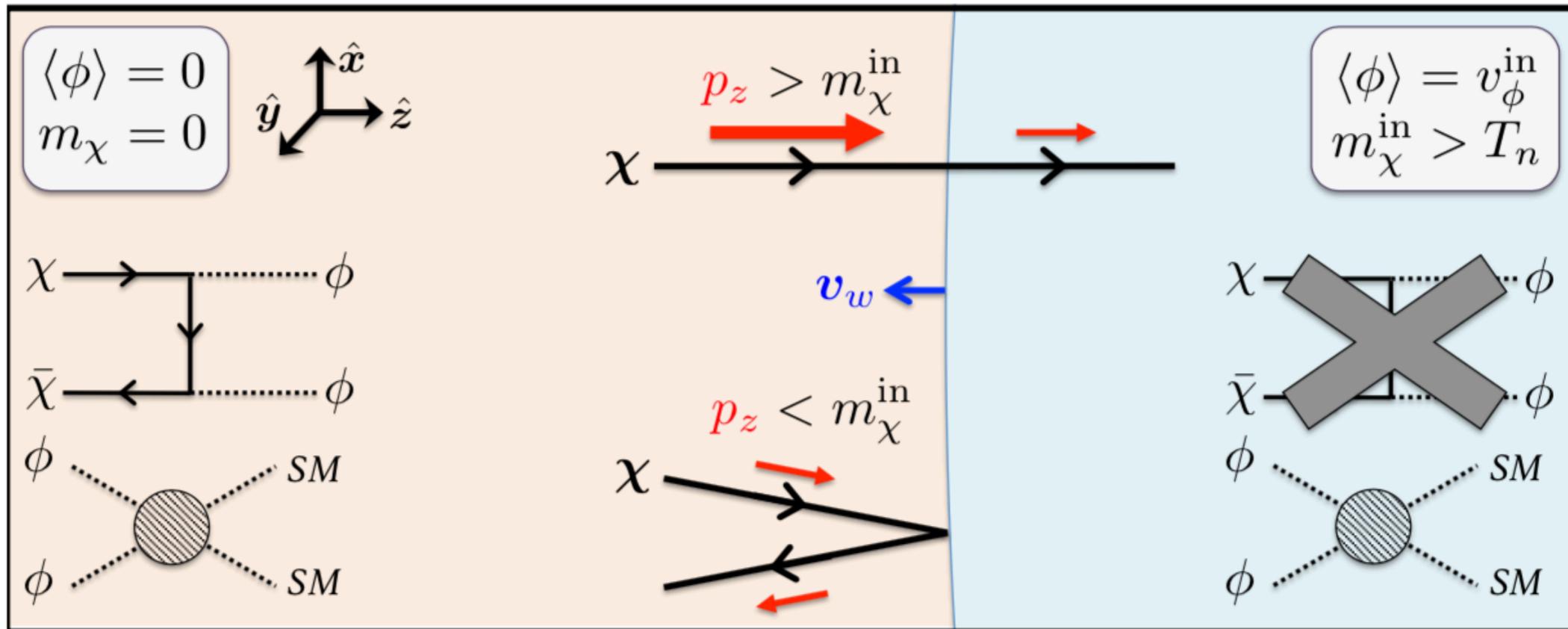
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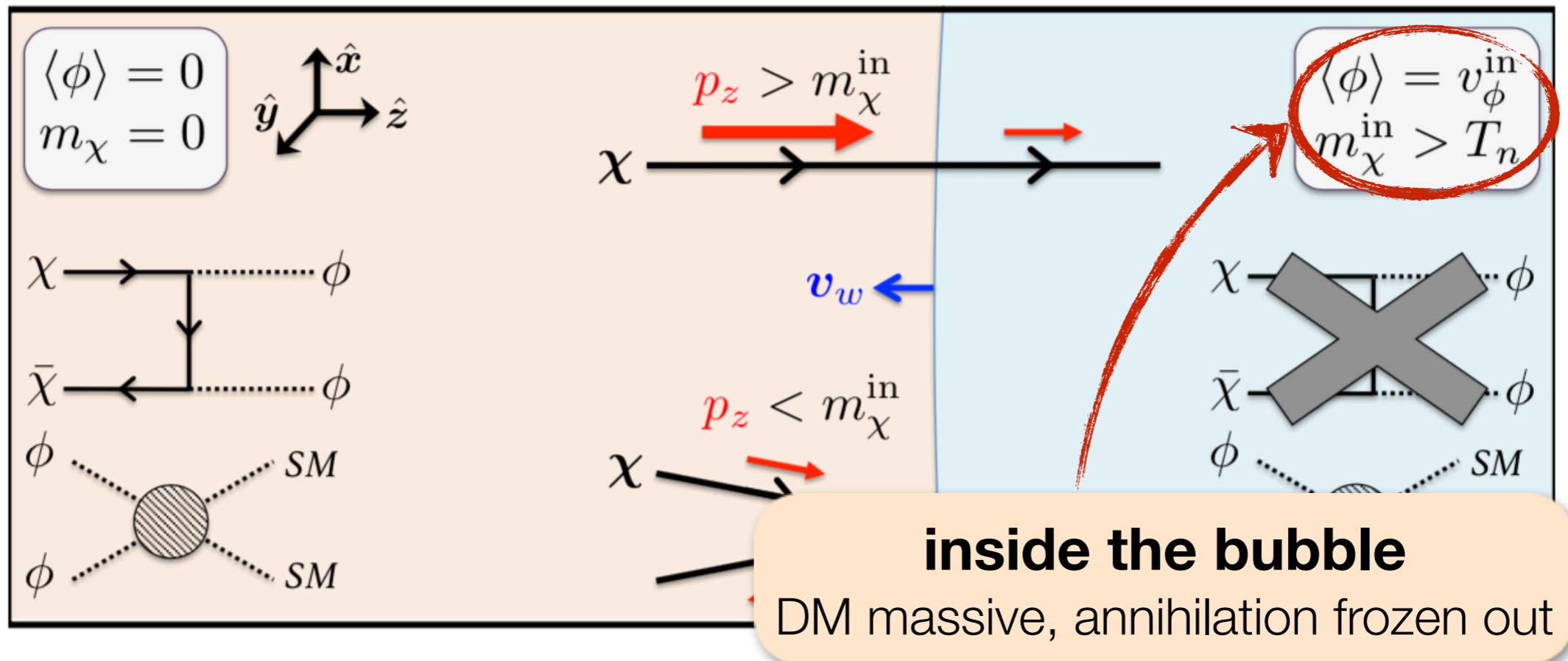
Baker JK Long, arXiv:1912.02830

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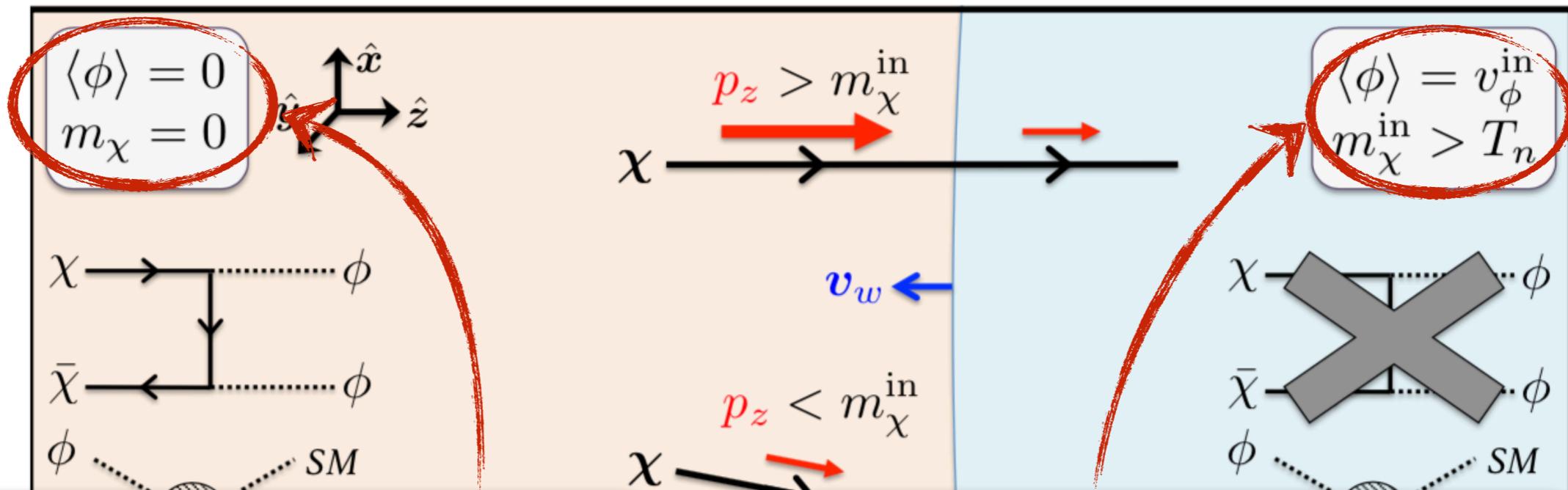
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**outside the bubble**

DM massless, annihilates efficiently

**inside the bubble**

DM massive, annihilation frozen out

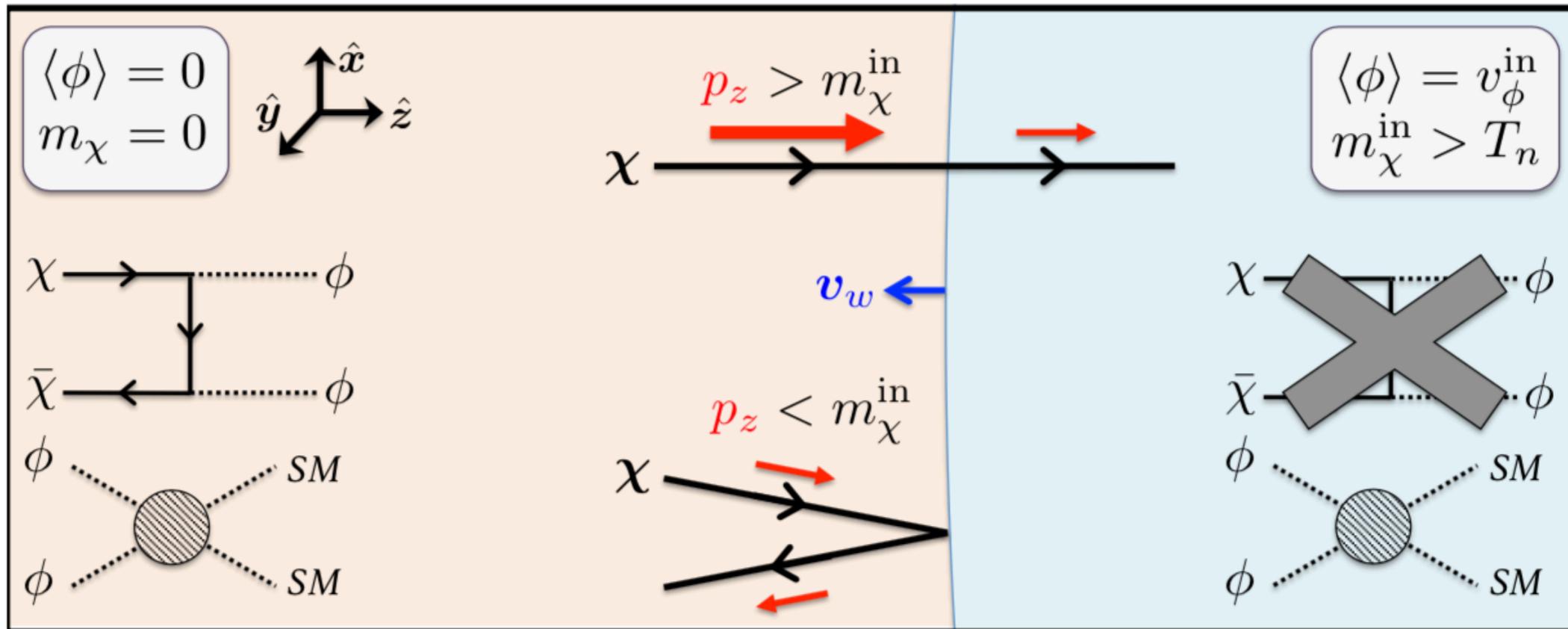
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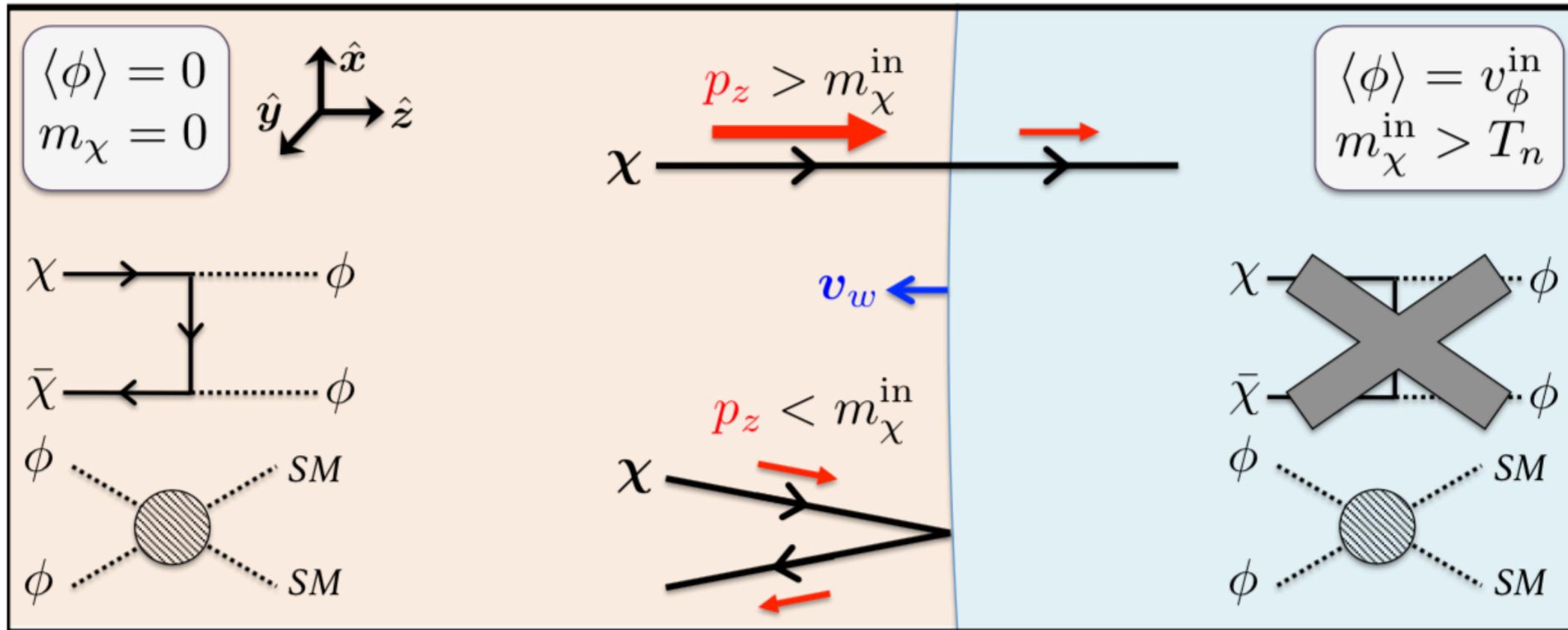
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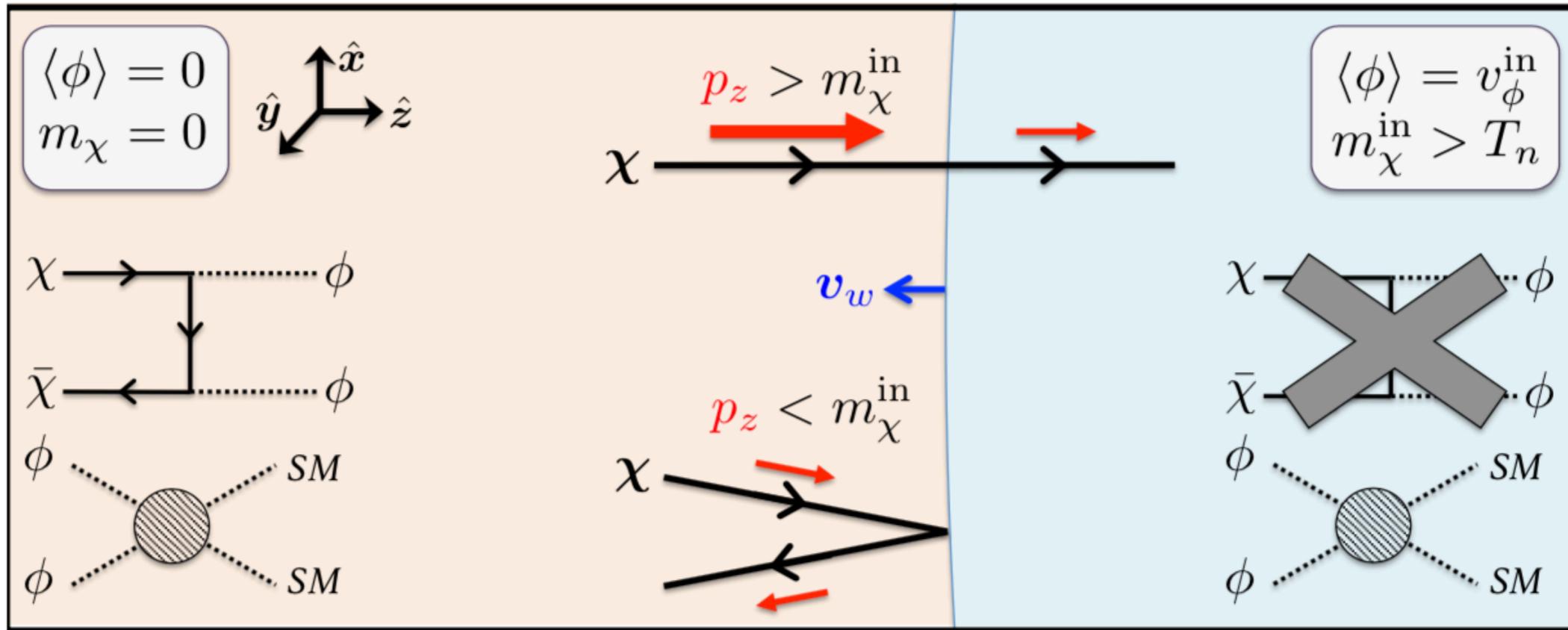


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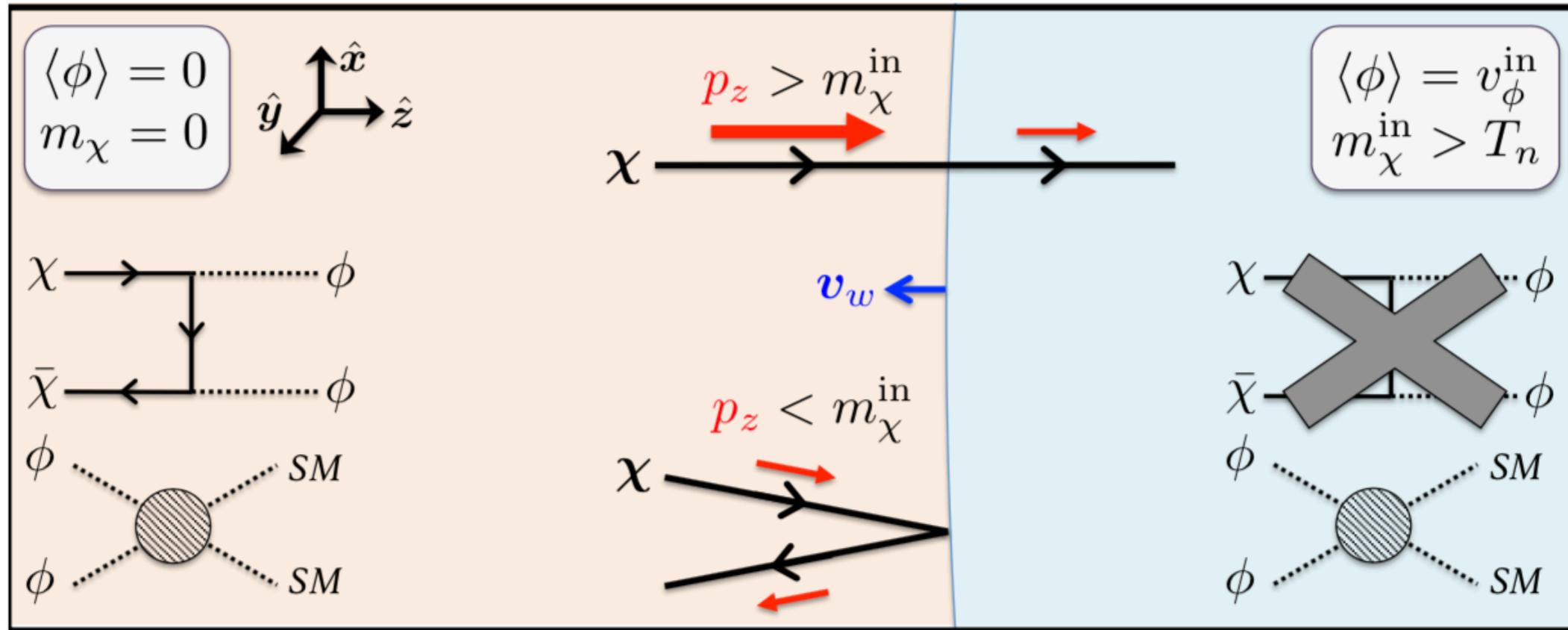


# Example 2: DM Filtering at Bubble Walls



small DM abundance inside the bubble persists

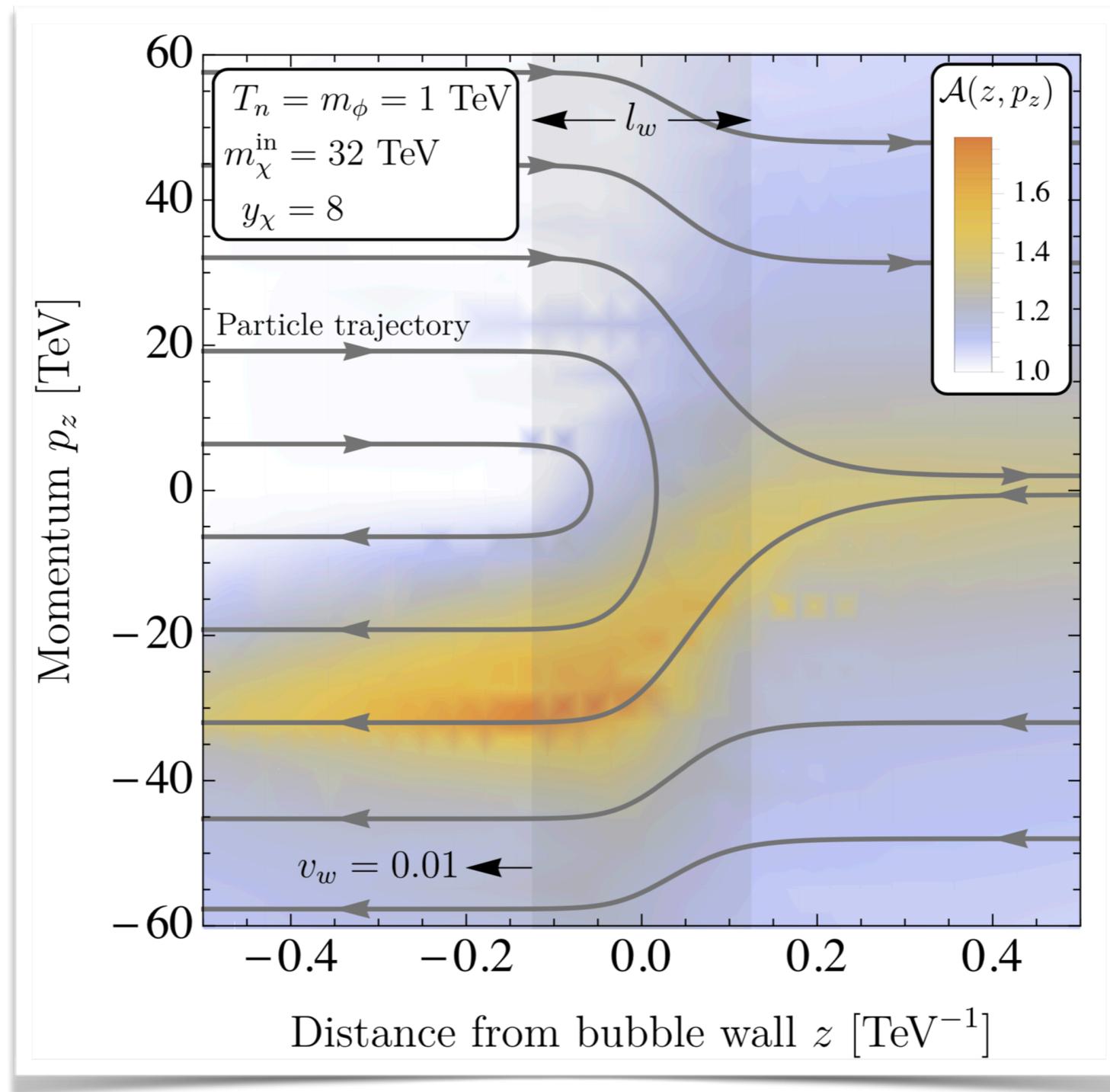
# Example 2: DM Filtering at Bubble Walls



- ☑ small DM abundance inside the bubble persists
- ☑ most DM particles remain outside, annihilate efficiently

Baker JK Long, arXiv:1912.02830

# Dark Matter at Bubble Walls



# Solving the Boltzmann Equations



# Solving the Boltzmann Equations

## General Boltzmann Equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

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## ☑ General Boltzmann Equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

**Liouville operator**  
total time derivative of  
phase space distribution

**collision term**  
change in phase space distribution  
due to collision and annihilation

# Solving the Boltzmann Equations

- ☑ General Boltzmann Equation

$$\mathbf{L}[f_\chi] = \mathbf{C}[f_\chi]$$

- ☑ Liouville operator

$$\mathbf{L}[f_\chi] = \frac{df_\chi}{dt^w} = \frac{\partial f_\chi}{\partial t^w} + \frac{\partial \mathbf{x}^w}{\partial t^w} \frac{\partial f_\chi}{\partial \mathbf{x}^w} + \frac{\partial \mathbf{p}^w}{\partial t^w} \frac{\partial f_\chi}{\partial \mathbf{p}^w}$$

# The Liouville Operator

$$\mathbf{L}[f_\chi] = \frac{df_\chi}{dt^w} = \frac{\partial f_\chi}{\partial t^w} + \frac{\partial \mathbf{x}^w}{\partial t^w} \frac{\partial f_\chi}{\partial \mathbf{x}^w} + \frac{\partial \mathbf{p}^w}{\partial t^w} \frac{\partial f_\chi}{\partial \mathbf{p}^w}$$

## ☑ Simplifications:

- stationarity ( $\partial f_\chi / \partial t^w = 0$ )
- translation invariance in  $x$  and  $y$
- integrate over  $x$  and  $y$  (to reduce number of variables)
- make ansatz  $f_\chi = \mathcal{A}(z^w, p_z^w) \exp\left(-\frac{E^p}{T}\right)$   
(superscript “w”: wall rest frame, “p”: plasma rest frame)

full details in Baker JK Long, arXiv:1912.02830

# The Collision Term

$$g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{C}[f_\chi] = \sum_{\text{spins}} \int \frac{dp_x dp_y}{(2\pi)^2} d\Pi_{q^p} d\Pi_{k^p} d\Pi_{l^p} \frac{(2\pi)^4}{2E_p^p} \delta^{(4)}(p^p + q^p - k^p - l^p) |\mathcal{M}|^2 \\ \cdot \left[ f_{\chi_p} f_{\bar{\chi}_q} (1 \pm f_{\phi_k})(1 \pm f_{\phi_l}) - f_{\phi_k} f_{\phi_l} (1 \pm f_{\chi_p})(1 \pm f_{\bar{\chi}_q}) \right],$$

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phase space integrals

full details in Baker JK Long, arXiv:1912.02830

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matrix element

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phase space integrals

distribution functions,  
Pauli blocking / Bose enhancement

full details in Baker JK Long, arXiv:1912.02830

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## Simplifications:

- same as for the Liouville operator, but also
- neglect Pauli blocking / Bose enhancement

full details in Baker JK Long, arXiv:1912.02830

# Solving the Boltzmann Equation

✓ After simplifications, Boltzmann equation takes the form

$$\left[ \left( \frac{p_z}{m_\chi} \frac{\partial}{\partial z} - \left( \frac{\partial m_\chi}{\partial z} \right) \frac{\partial}{\partial p_z} - \left( \frac{\partial m_\chi}{\partial z} \right) \frac{v_w}{T_n} \right) \mathcal{A}(z, p_z) \right] \frac{g_\chi m_\chi T_n}{2\pi} \exp \left[ \frac{v_w p_z - \sqrt{m_\chi^2 + (p_z)^2}}{T_n} \right] = g_\chi \int \frac{dp_x dp_y}{(2\pi)^2} \mathbf{C}[f_\chi]$$

✓ A PDE of the form

$$a(z^w, p_z^w) \frac{\partial \mathcal{A}}{\partial z^w} + b(z^w, p_z^w) \frac{\partial \mathcal{A}}{\partial p_z^w} = c(\mathcal{A}, z^w, p_z^w)$$

can be solved by the **method of characteristics**

✓ Define parametric curve via

$$\frac{dz^w(\lambda)}{d\lambda} = a(z^w, p_z^w), \quad \frac{dp_z^w(\lambda)}{d\lambda} = b(z^w, p_z^w)$$

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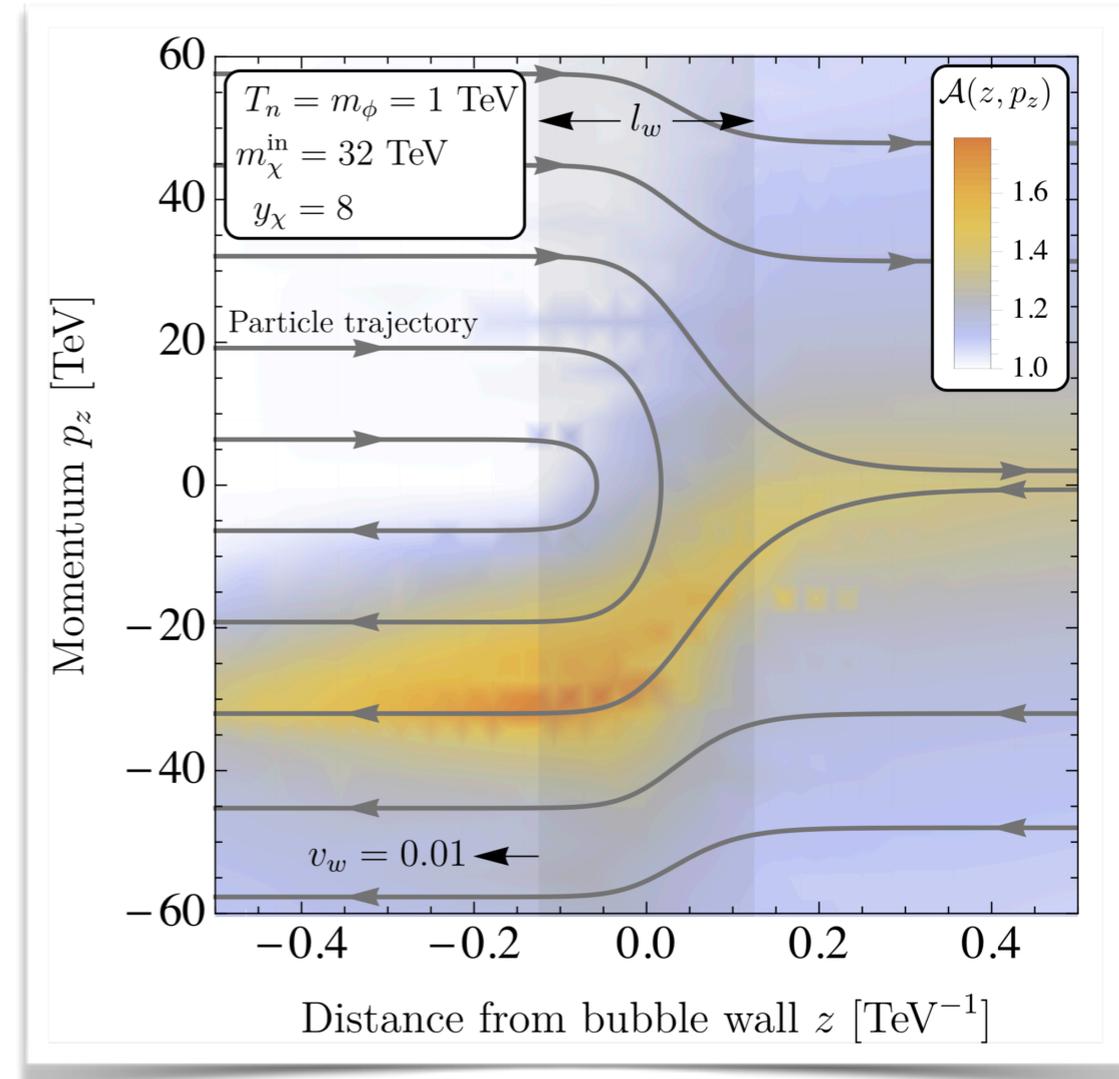
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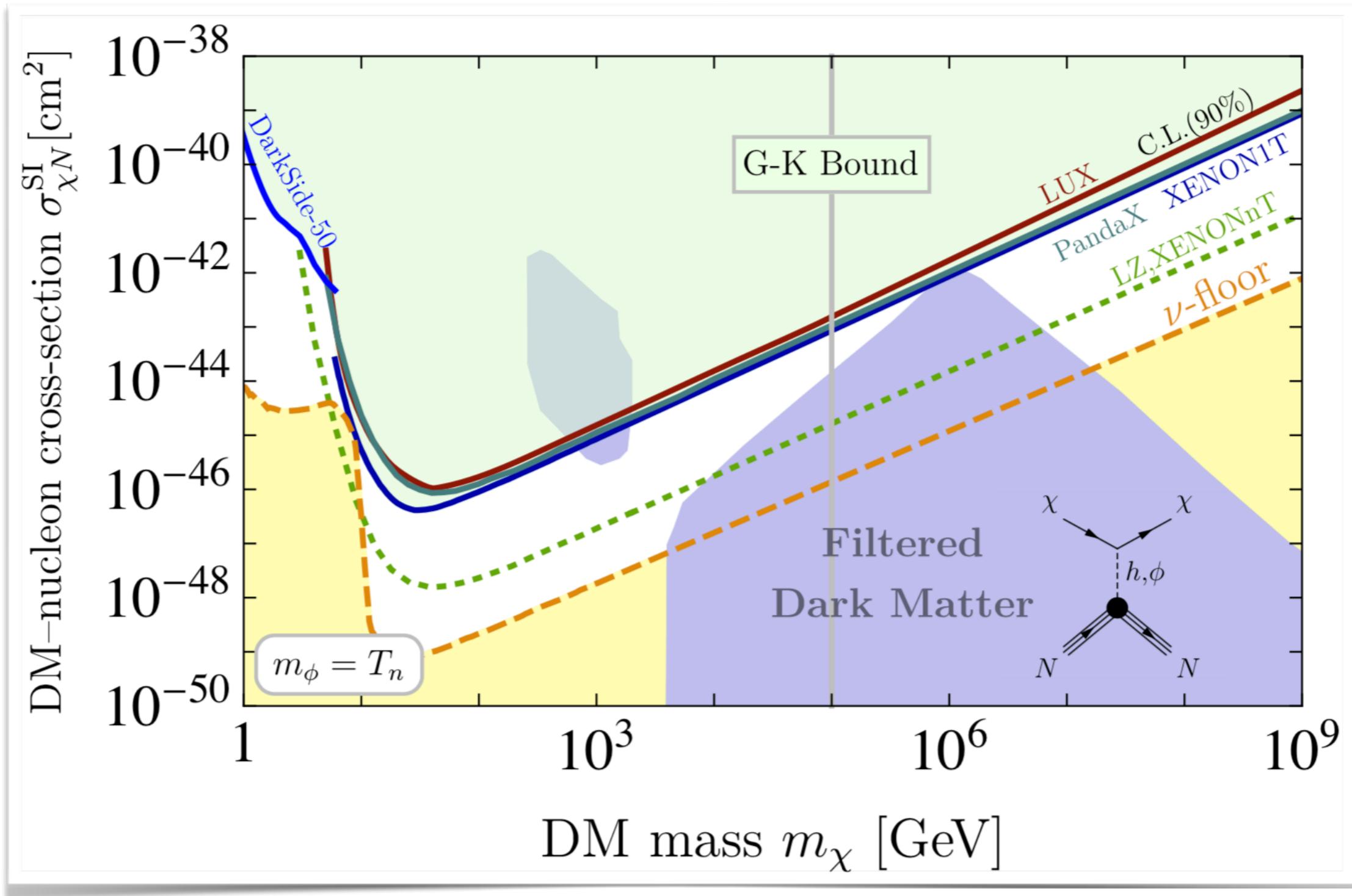
$$\frac{d\mathcal{A}(z^w(\lambda), p_z^w(\lambda))}{d\lambda} = c(\mathcal{A}(\lambda), z^w(\lambda), p_z^w(\lambda))$$

- ☑ Physical interpretation:

- curves = particle trajectories

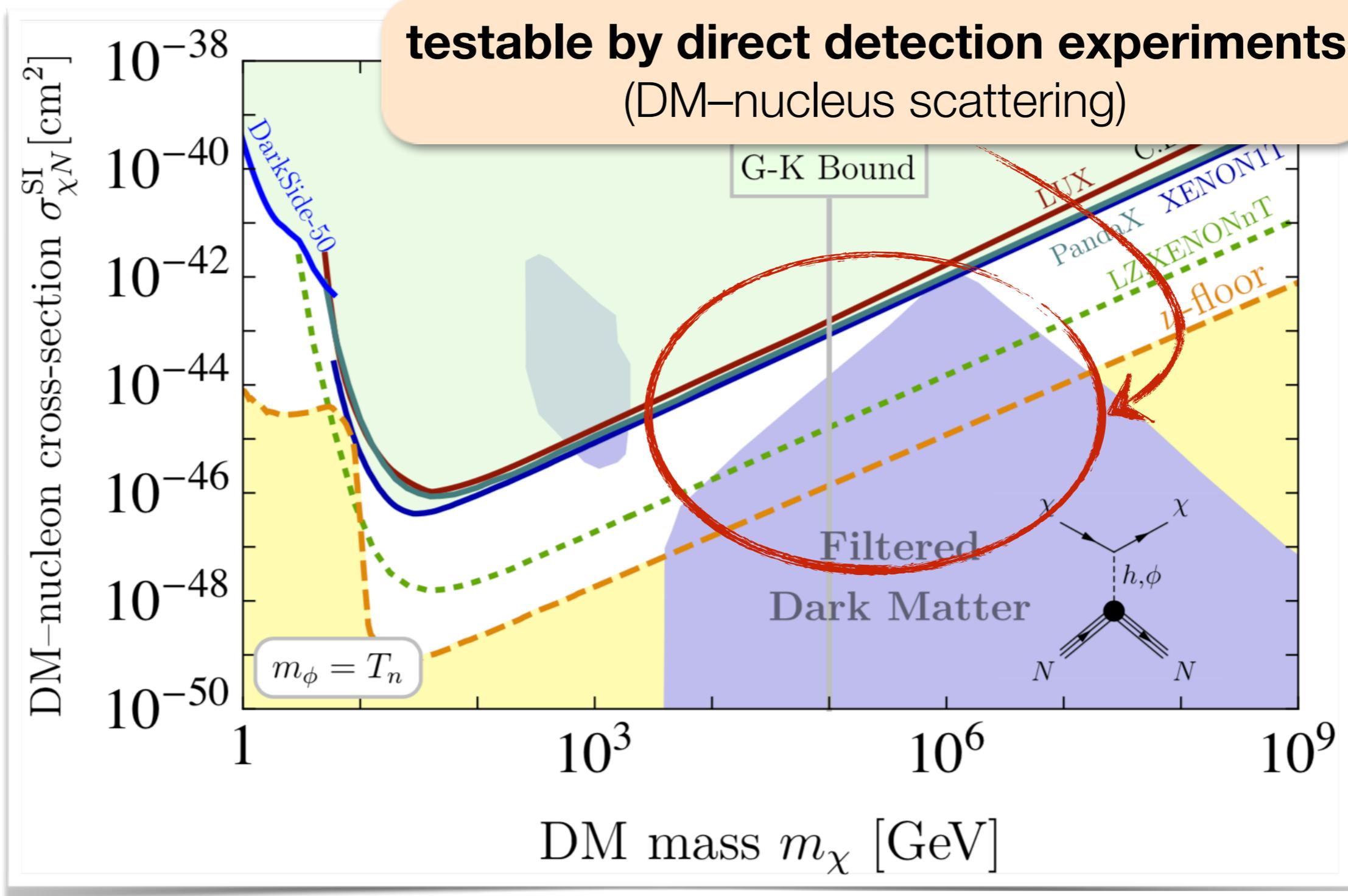


# Parameter Space



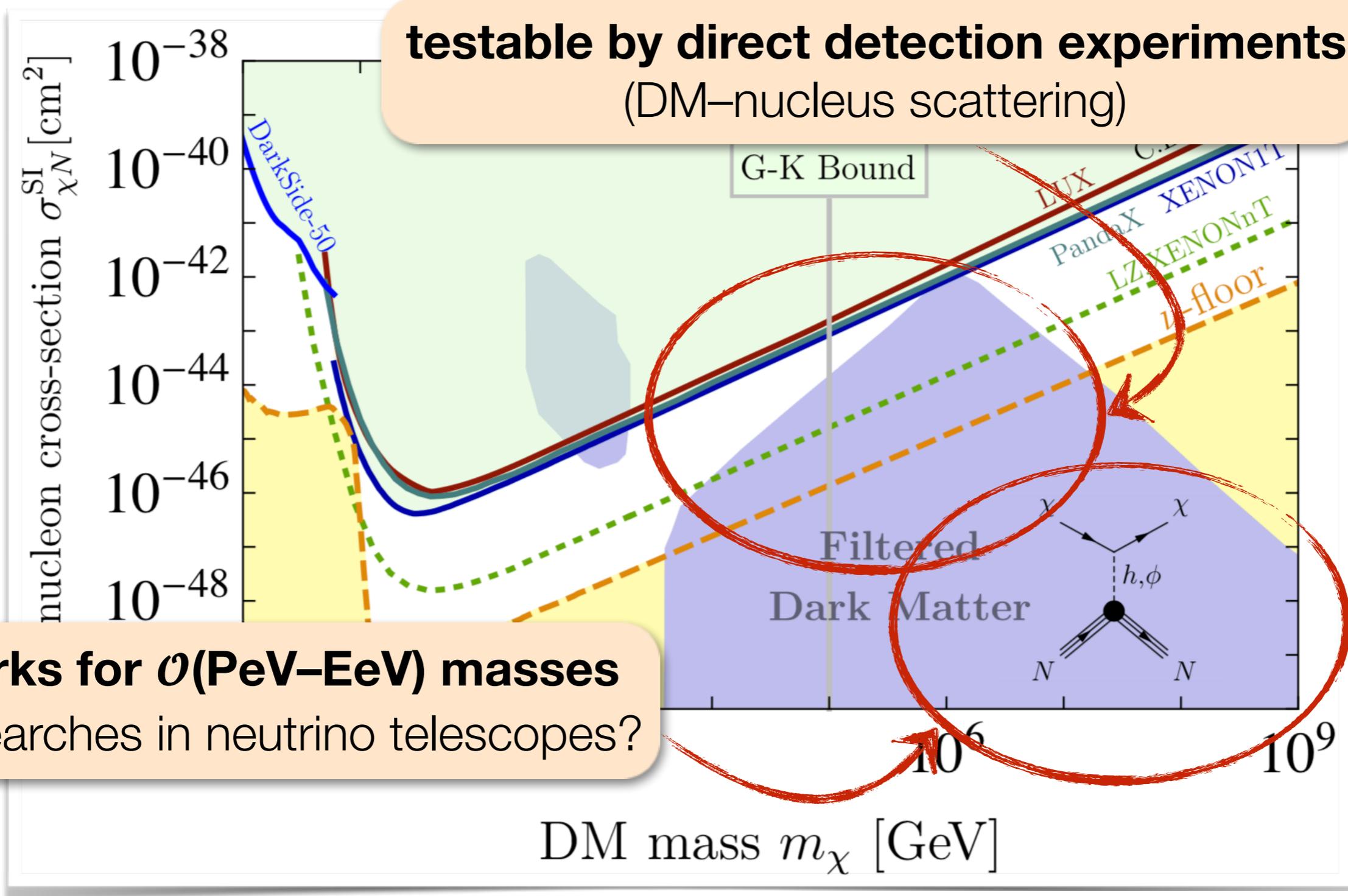
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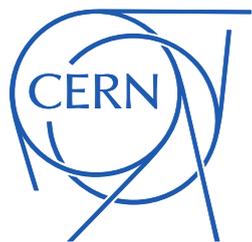
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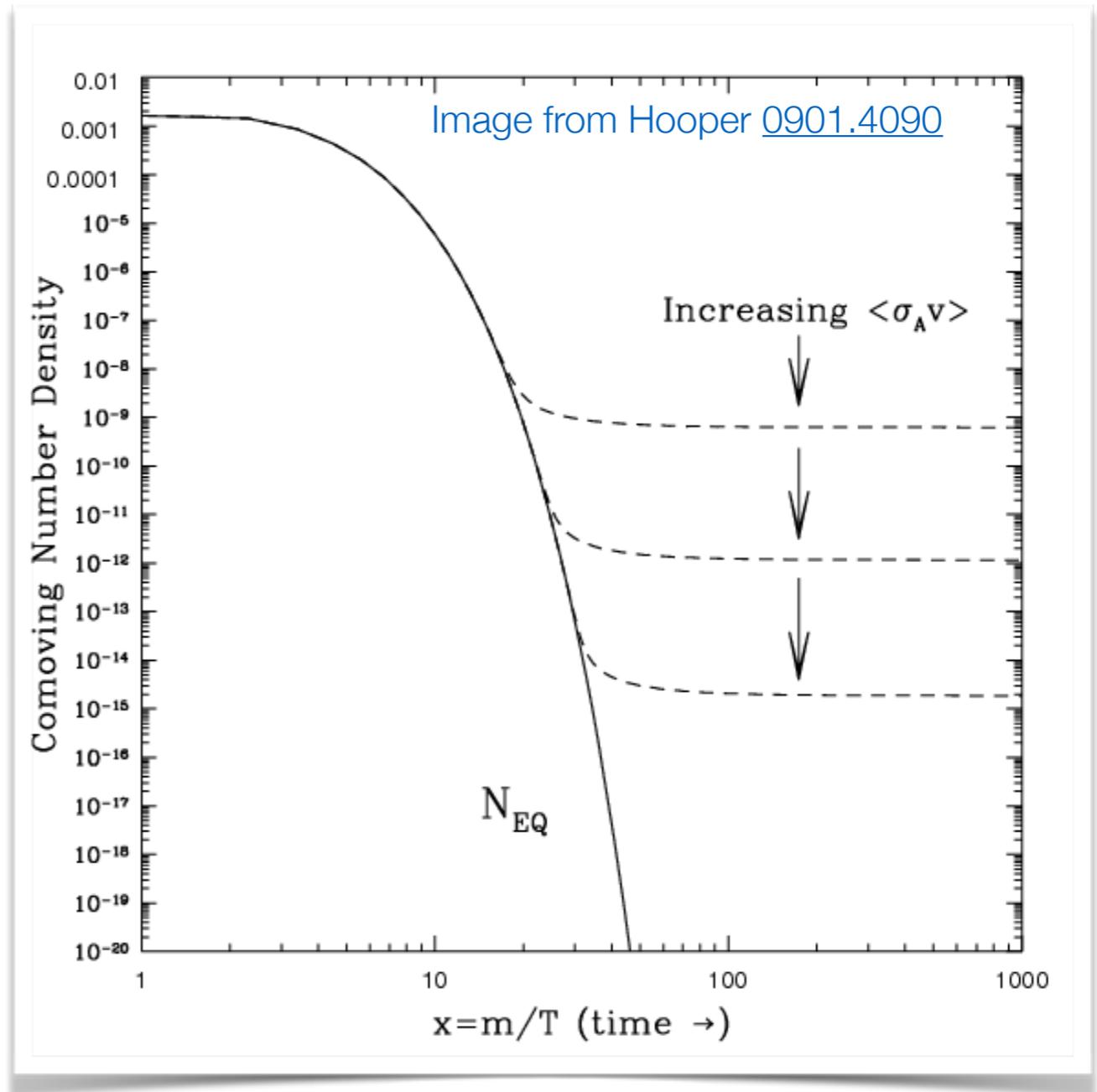


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# Dark Matter Decay Between Phase Transitions

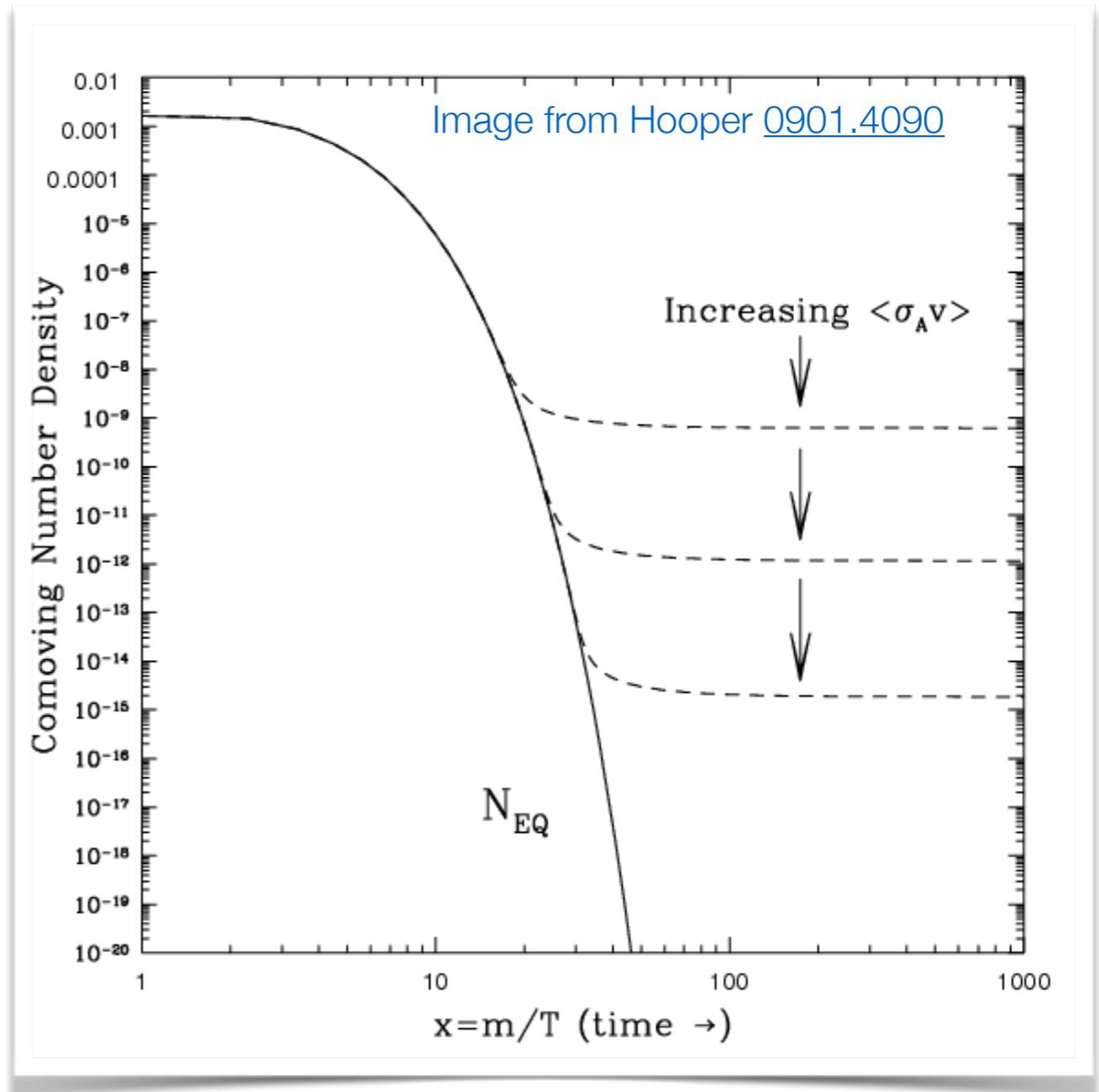


# DM Decay Between Phase Transitions



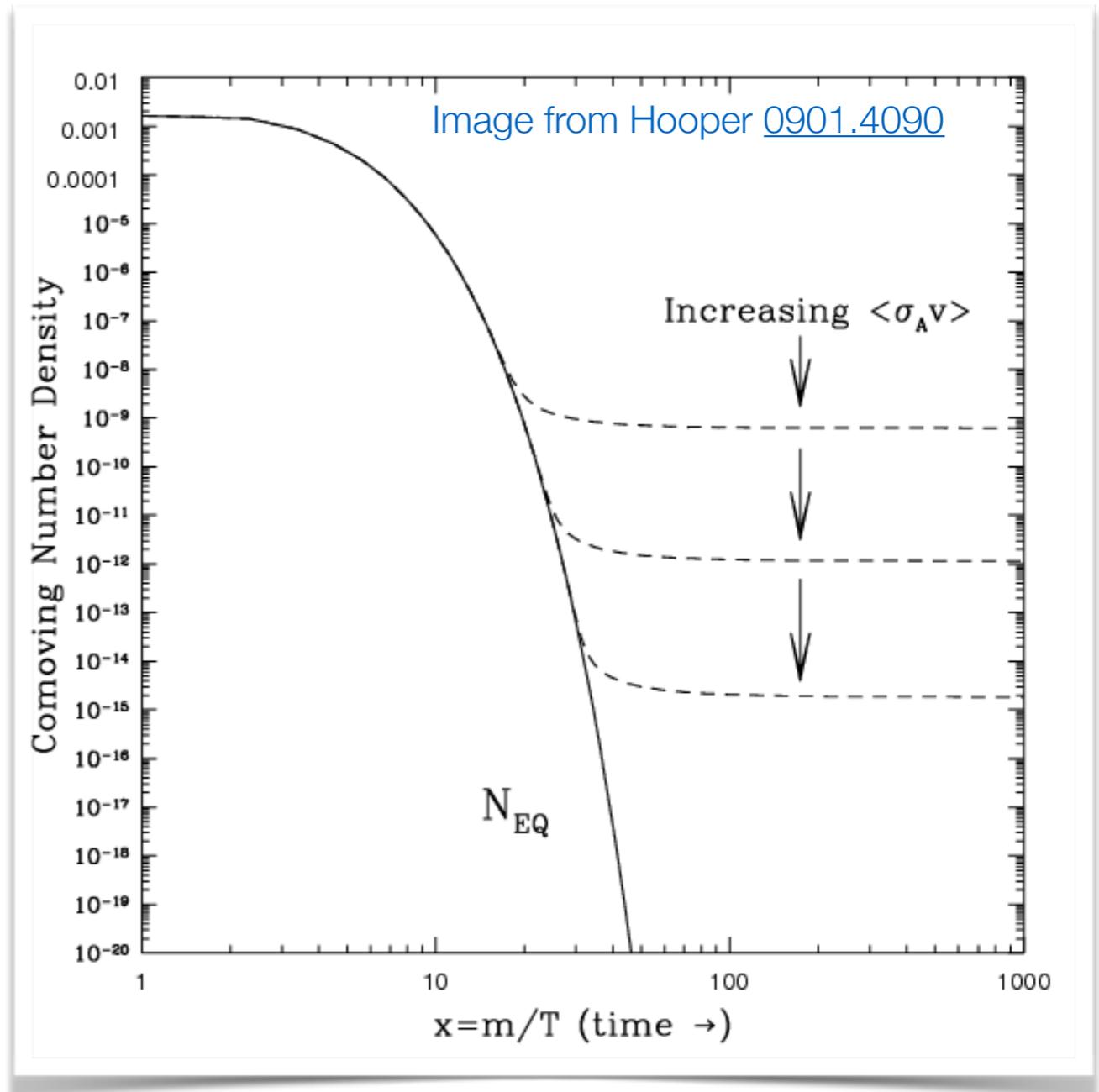
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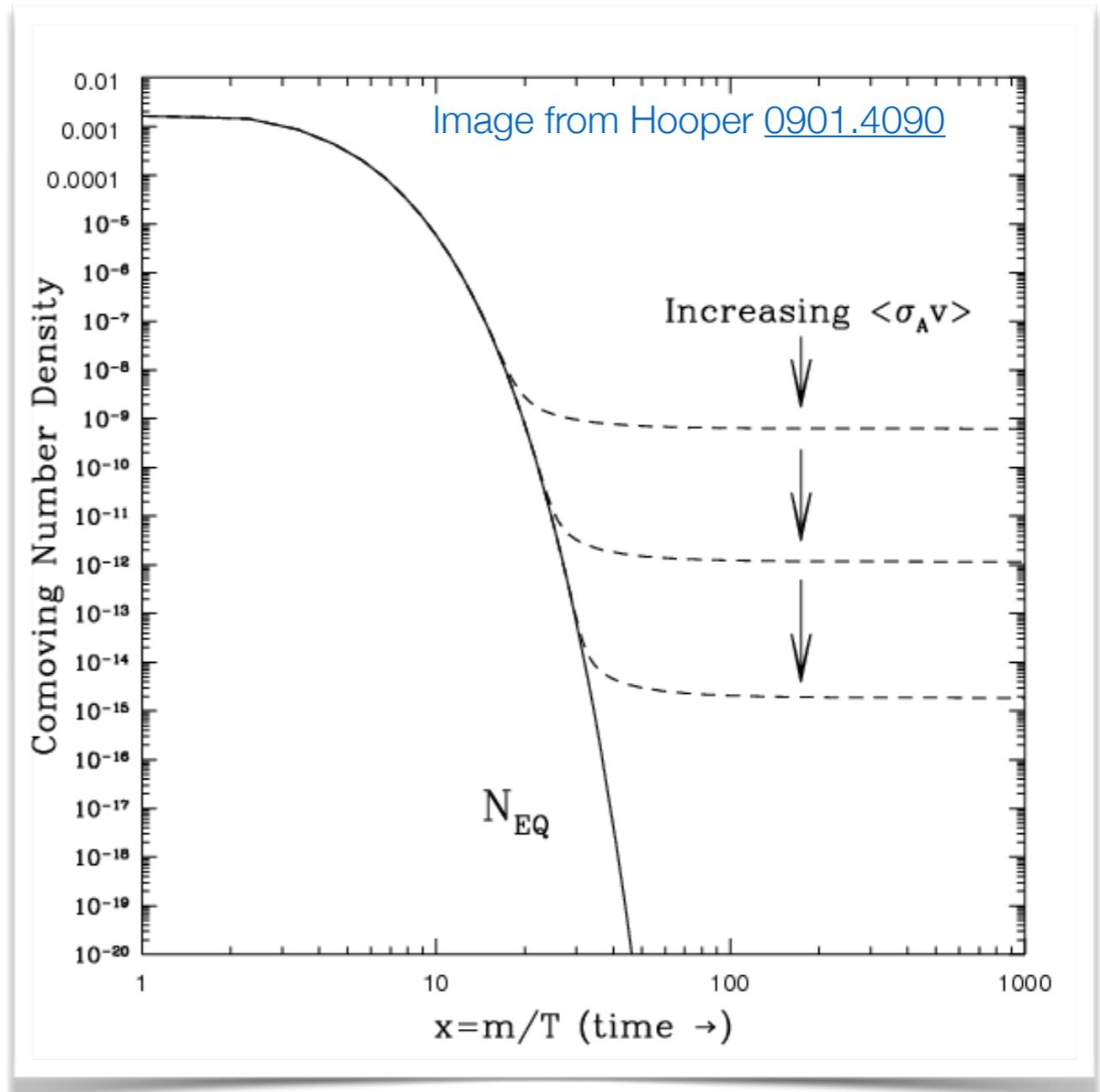
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# DM Decay Between Phase Transitions

- ☑ Observed DM abundance requires a mechanism that depletes DM by several orders of magnitude, then stops
- ☑ Idea: DM decay!
- ☑ Example:
  - Phase transition shifts particle masses, making DM unstable
  - DM partly decays
  - 2<sup>nd</sup> phase transition restores stability



# The Vev Flip-Flop

☑ Toy Model: SM + singlet scalar  $S$

$$V^{\text{tree}} = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 + \lambda_p (H^\dagger H)(S^\dagger S)$$

☑ Typical behavior: 2-step phase transition

○ High  $T$ :  $\langle S \rangle = 0, \langle H \rangle = 0$

Profumo *et al.* [0705.2425](#)

Cline *et al.* [0905.2559](#)

○ Intermediate  $T$ :  $\langle S \rangle \neq 0, \langle H \rangle = 0$

Espinosa Konstandin Riva [1107.5441](#)

Cui Randall Shuve [1106.4834](#)

○ Low  $T$ :  $\langle S \rangle = 0, \langle H \rangle \neq 0$

Cline Kainulainen [1210.4196](#)

Fairbairn Hogan [1305.3452](#)

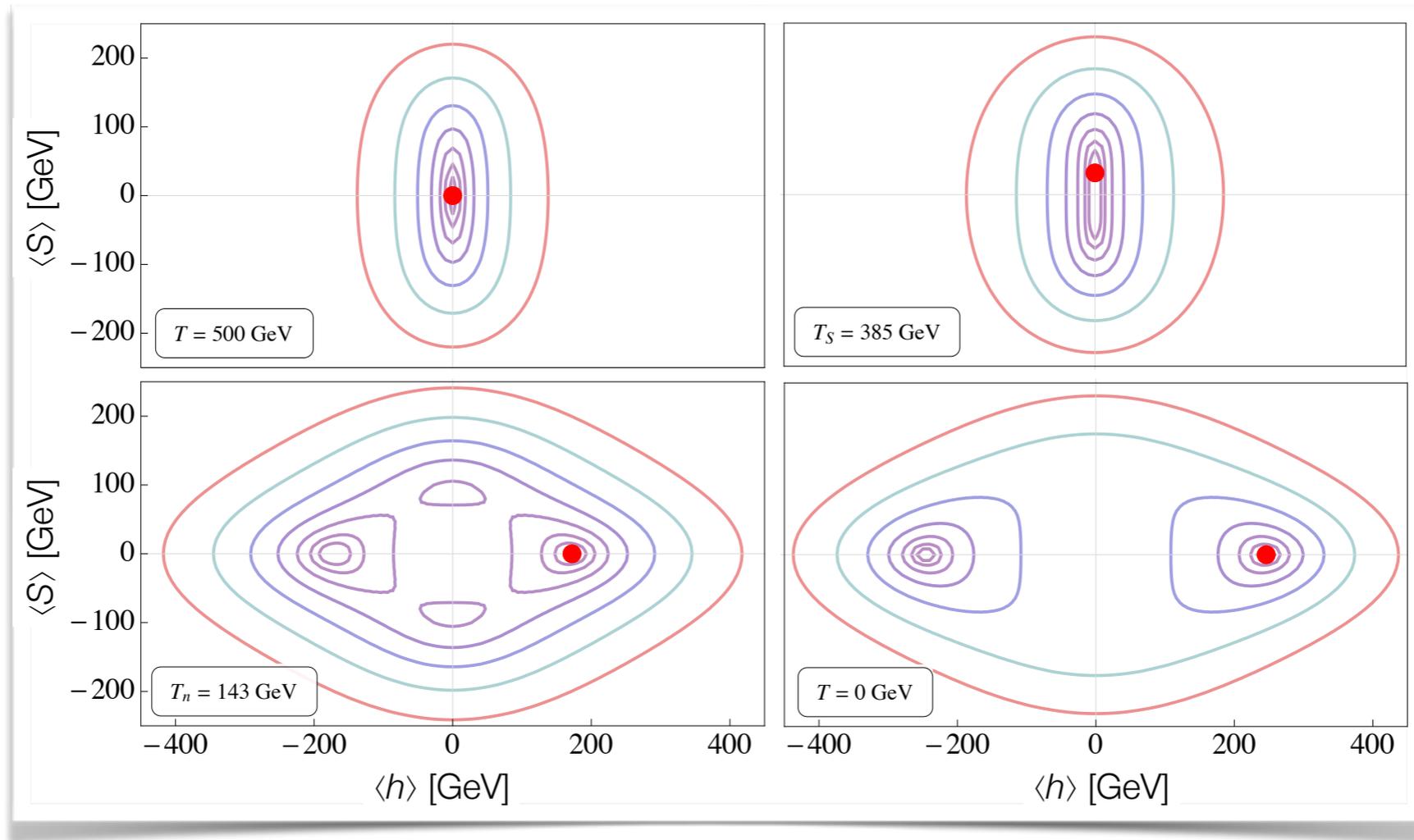
Curtin Meade Yu [1409.0005](#)

Baker JK [1608.07578](#)

Baker Breitbach JK Mittnacht [1712.03962](#)

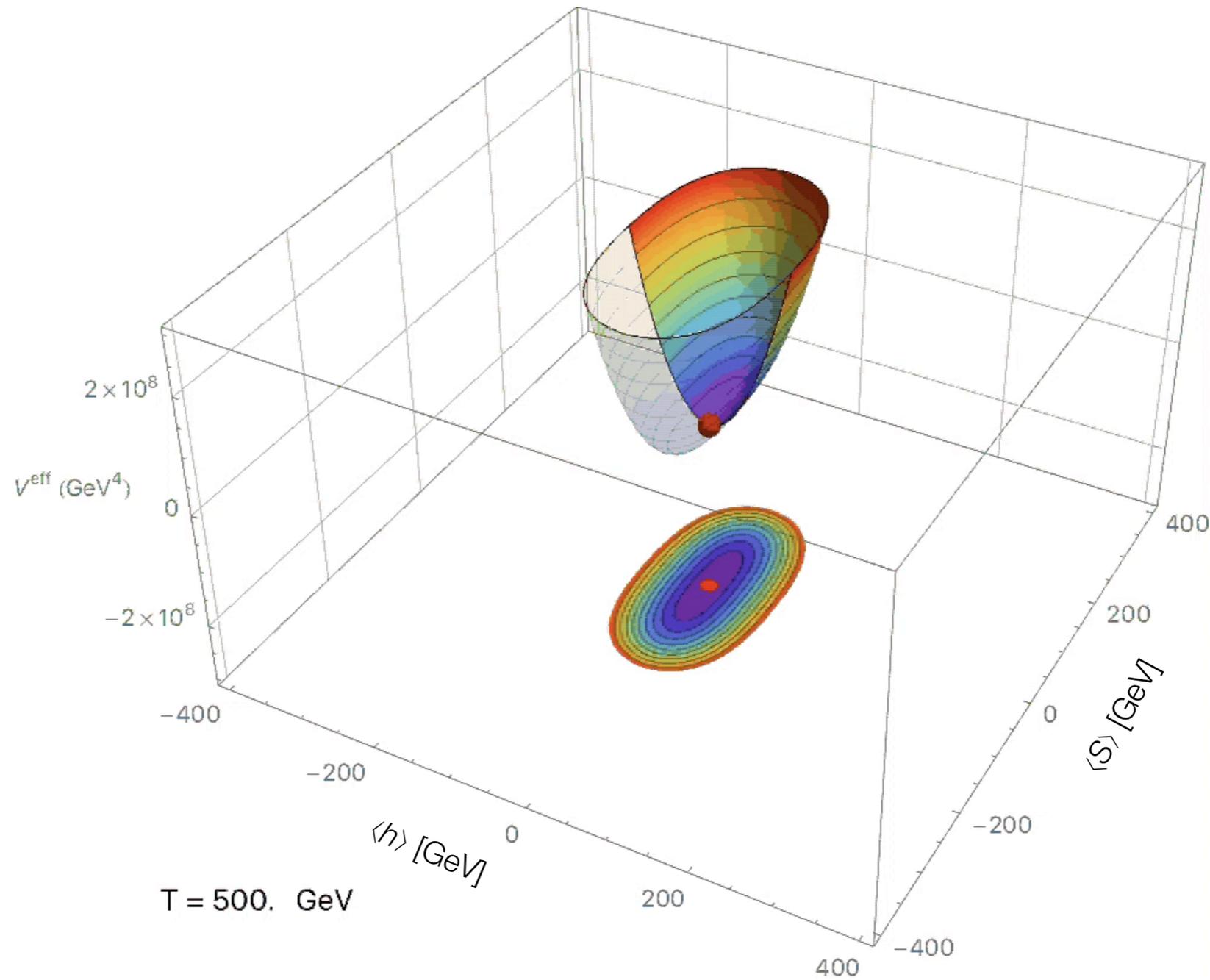
Baker Mittnacht [1811.03101](#)

# The Vev Flip-Flop



- ☑  $T > 400$  GeV:  $\langle S \rangle = 0$ ,  $\langle H \rangle = 0$  (thermal corrections dominate  $V_{\text{eff}}$ )
- ☑  $T \sim 400$  GeV:  $S$  develops vev  $\Rightarrow$  DM unstable
- ☑  $T \sim 150$  GeV:  $H$  develops vev  $\Rightarrow$  Feedback through  $\lambda_p (H^\dagger H)(S^\dagger S)$   
 $\Rightarrow m_{S,\text{eff}}$  changes sign,  $\langle S \rangle \rightarrow 0$ , DM stable

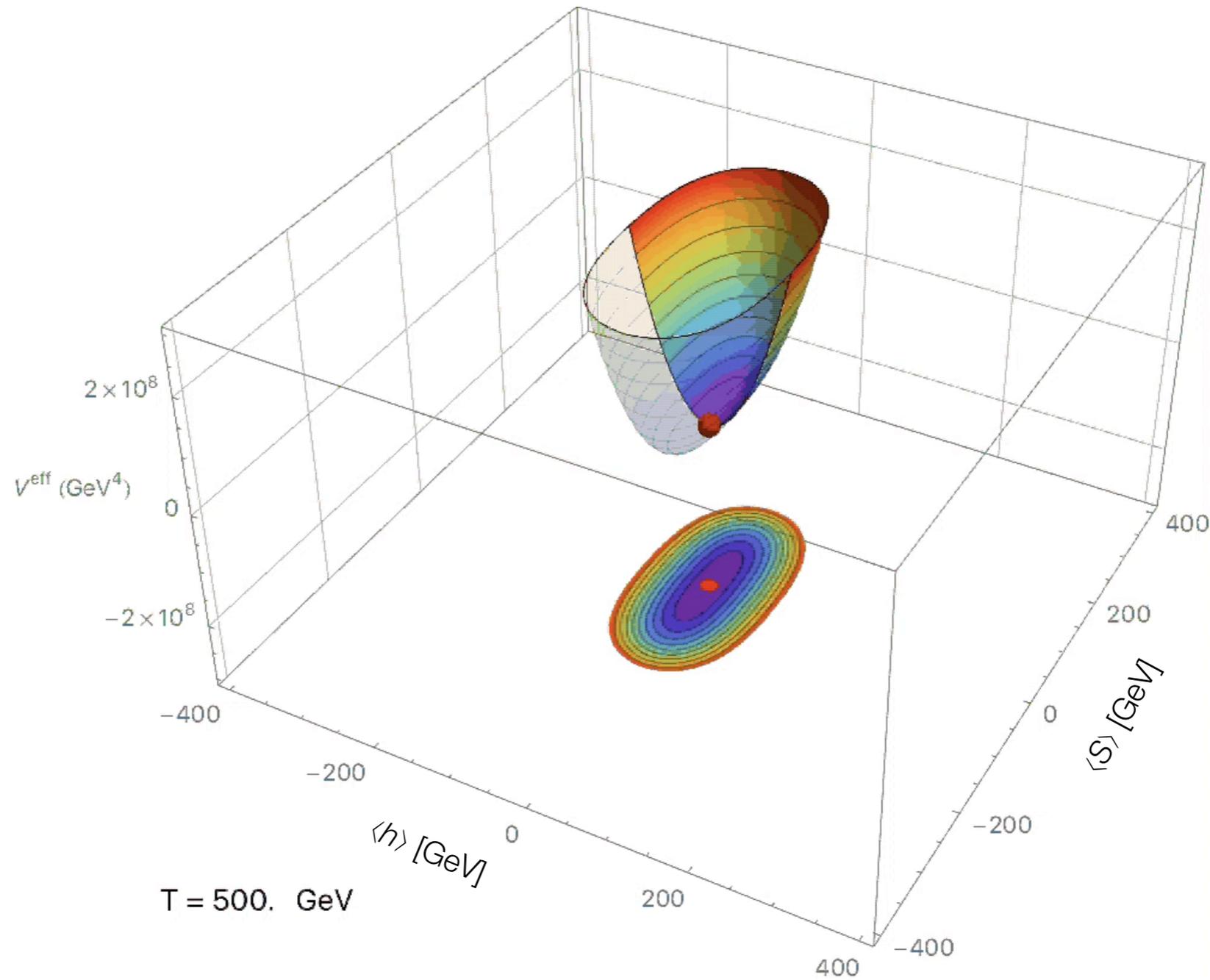
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Computed by Mike Baker using CosmoTransitions

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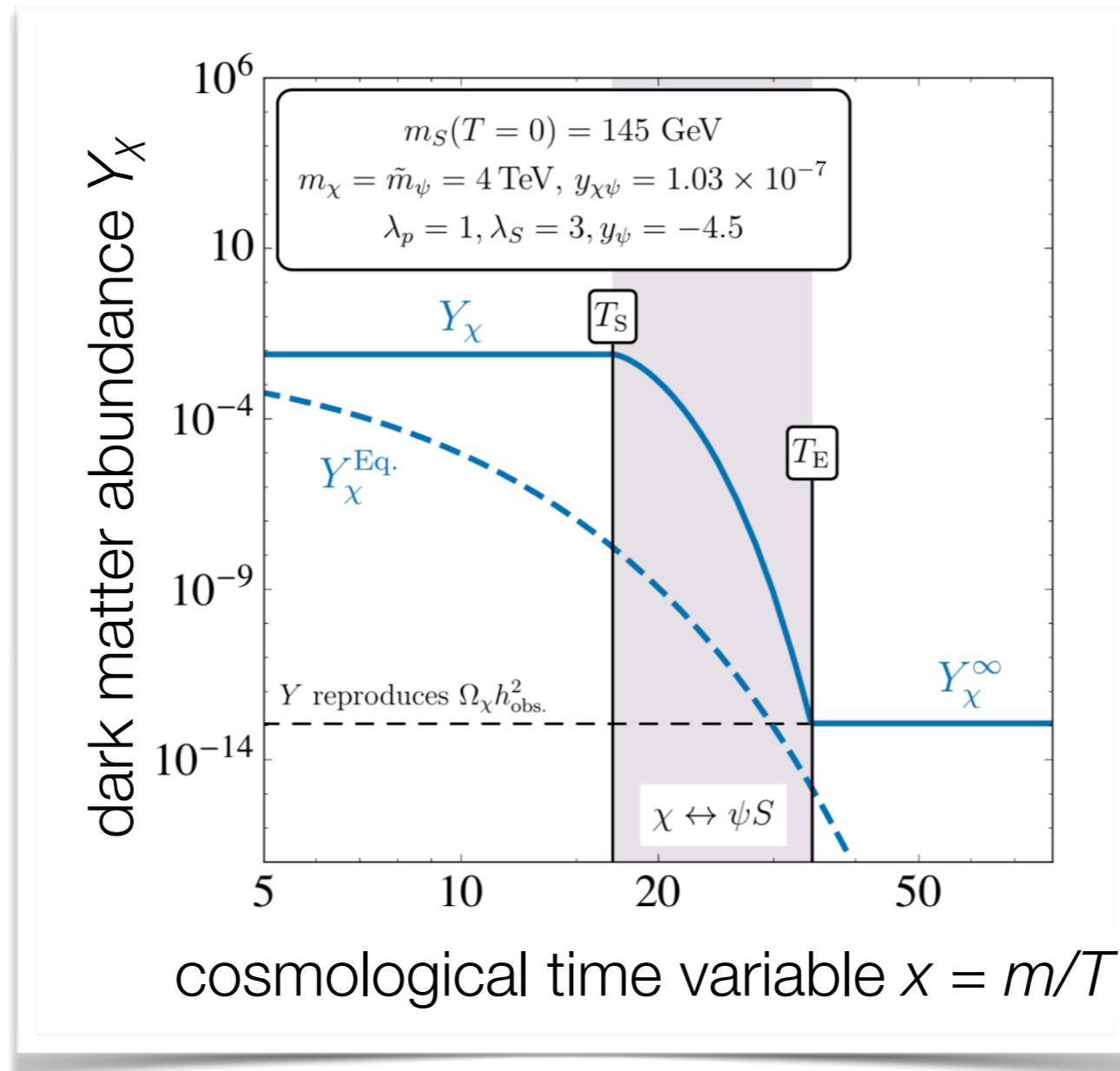


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# Example 1: Decay Between Phase Transitions

## Evolution of DM Abundance



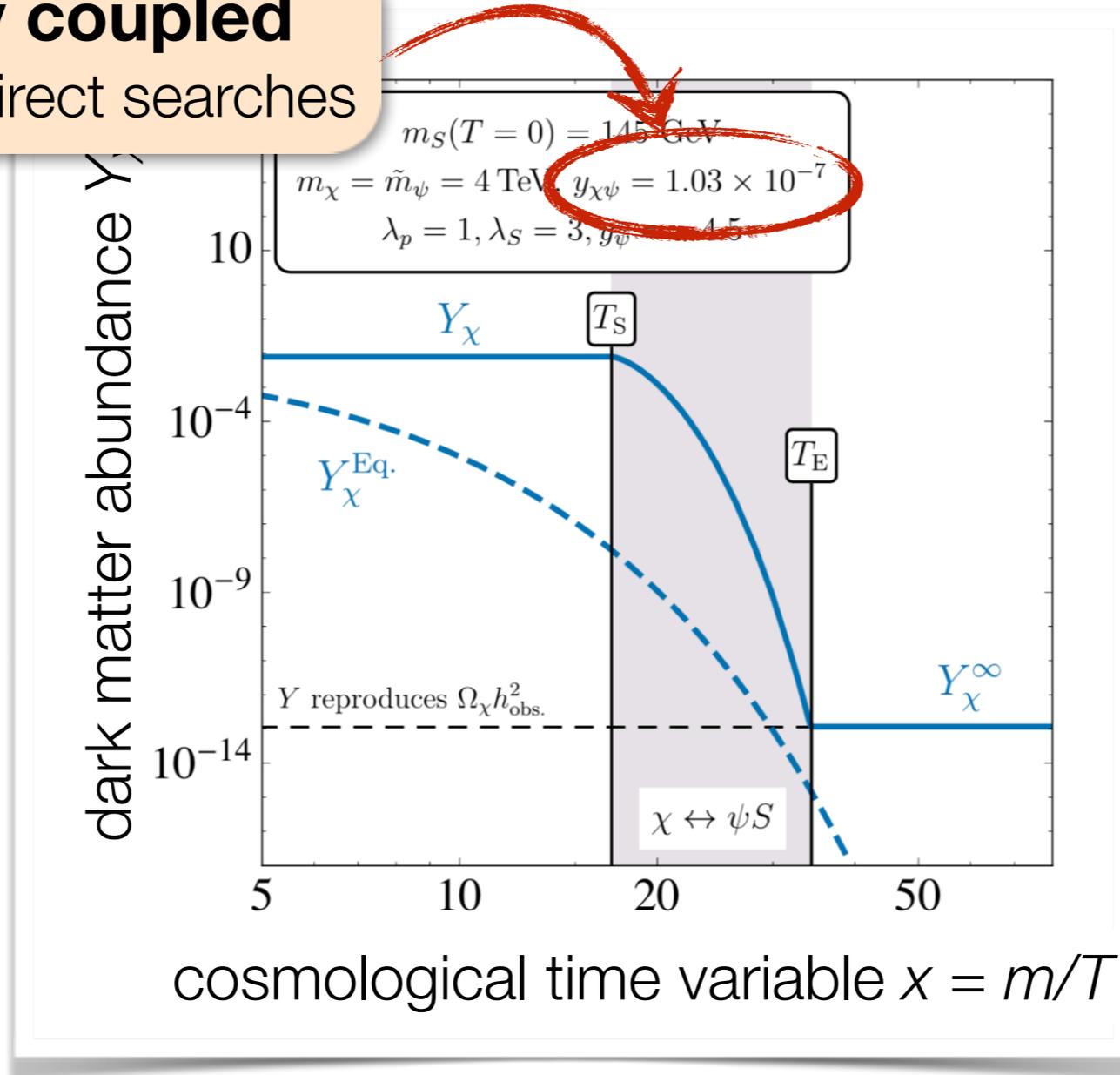
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# Example 1: Decay Between Phase Transitions

## Evolution of DM Abundance

**DM very weakly coupled**

difficult for direct & indirect searches



Baker Mitnacht [arXiv:1811.03101](https://arxiv.org/abs/1811.03101)  
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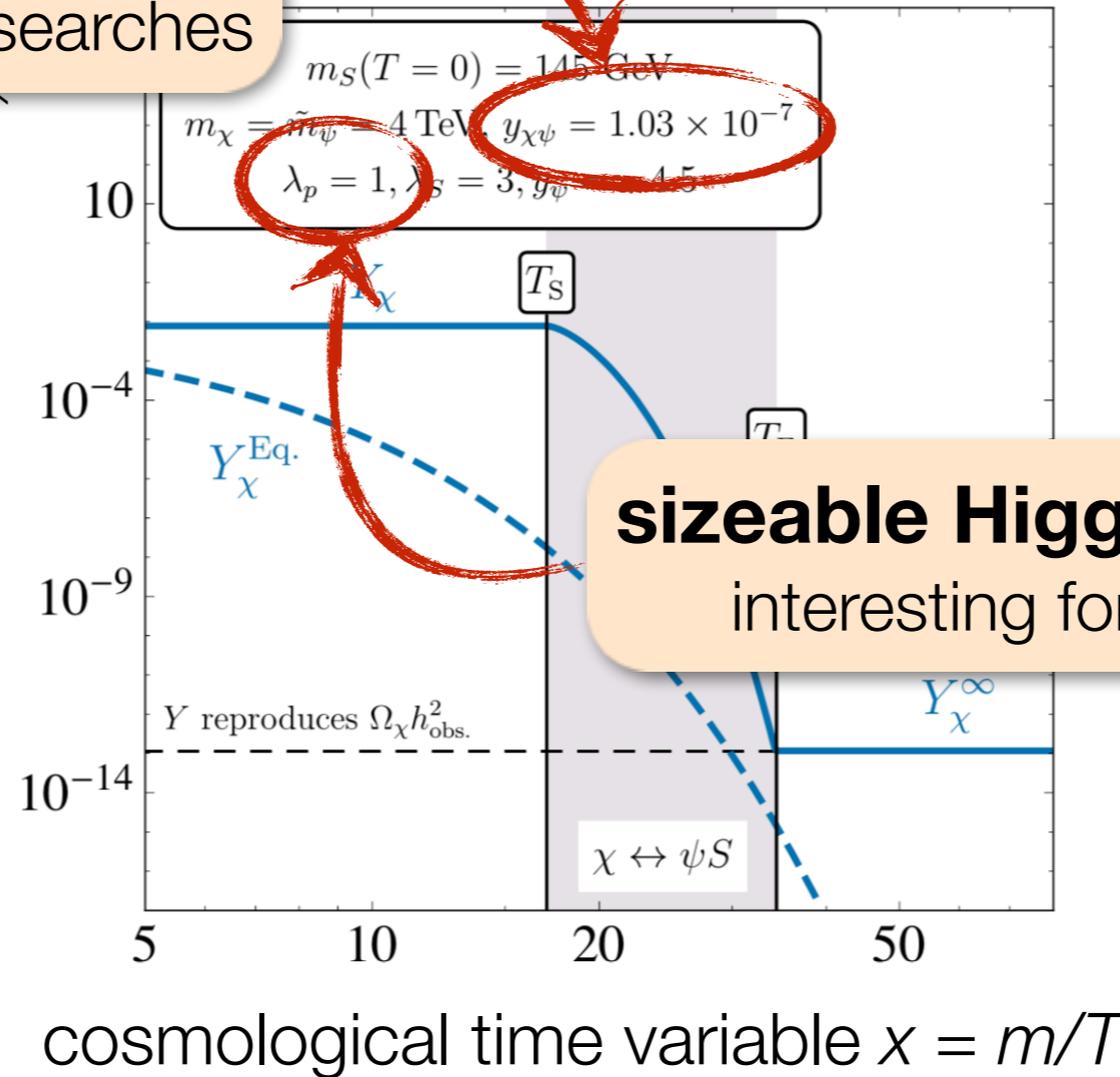
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dark matter abundance  $Y_\chi$



**sizeable Higgs portal coupling**  
interesting for collider searches

Baker Mittnacht [arXiv:1811.03101](https://arxiv.org/abs/1811.03101)  
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# Implications for the LHC



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    - electroweak precision observables ( $S$ ,  $T$ ,  $U$  parameters)
    - modified  $H$  branching ratios
    - direct observation of  $S$   
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  - Precision measurements of Higgs self-coupling  
(e.g. in di-Higgs production)

Barger *et al.*, <https://arxiv.org/abs/0706.4311>  
Robens & Stefaniak, [arXiv:1601.07880](https://arxiv.org/abs/1601.07880)

# Summary



# Summary



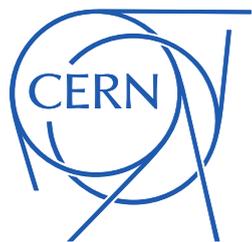
## Phase Transition in the early Universe

- ☑ imply **abrupt change** in the primordial plasma
- ☑ often depend on the dynamics of **scalar particles**
- ☑ can determine the **dark matter** abundance in multiple ways
- ☑ have consequences for
  - Higgs precision measurements
  - gravitational wave observations      ➡ backup slides
  - baryogenesis                                      ➡ backup slides

# Thank you!



# Bonus Slides



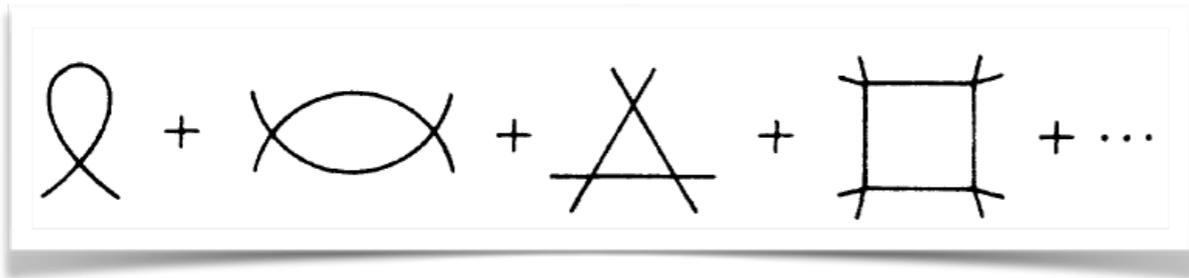
# Scalar Potentials at Finite Temperature

## ☑ Tree level potential

$$V^{\text{tree}} = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

## ☑ Coleman—Weinberg

[Coleman Weinberg 1973](#), [Dolan Jackiw 1974](#)



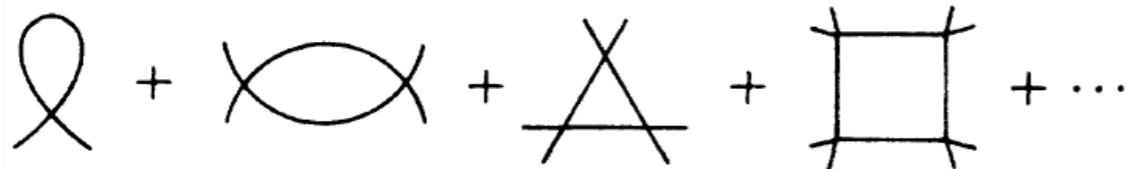
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$$V^{\text{CW}}[\phi] = \sum_{n=1}^{\infty} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2n} \left( \frac{2\lambda\phi}{k^2 - m^2} \right)^n$$

- Sum over  $n$
- Regularize, evaluate integral
- Renormalize by adding counterterms

$$V^{\text{CW}} = \sum_i \frac{n_i}{64\pi^2} m_i^4(h, S) \left[ \log \frac{m_i^2(h, S)}{\Lambda^2} - \frac{3}{2} \right]$$

# Scalar Potentials at Finite Temperature

## 1-loop, finite temperature corrections [Dolan Jackiw 1974](#)

- Evaluate 1-loop diagrams

- Replace vacuum propagators by **thermal propagators**

propagator = correlation function  $\langle \Phi(x) \Phi(y) \rangle$

in vacuum, points  $x$  and  $y$  become correlated if a particle propagates from  $x$  to  $y$ .

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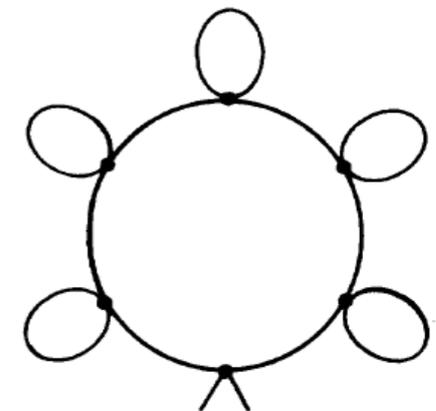
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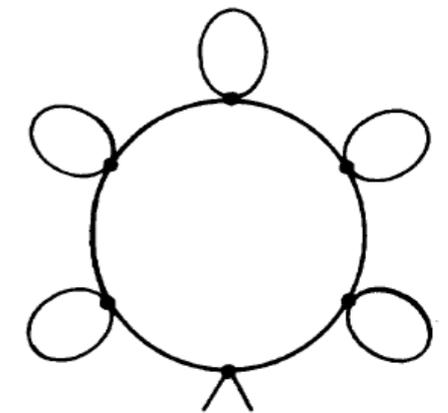
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- $n$  one-vertex bubbles, one  $n$ -vertex bubble:

$$\sum_n \left( \int \frac{d^4 k}{(2\pi)^4} \tilde{D}(k) \right)^n \cdot \int \frac{d^4 k}{(2\pi)^4} (\tilde{D}(k))^n$$



- One-vertex bubbles yield **thermal mass  $\Pi(T)$**

$$V^{\text{daisy}} = -\frac{T}{12\pi} \sum_i n_i \left( [m_i^2(h, S) + \Pi_i(T)]^{\frac{3}{2}} - [m_i^2(h, S)]^{\frac{3}{2}} \right)$$

# A Toy Model

Field	Spin	$\mathbb{Z}_2$	mass Scale
$S$	0	+1	0.1 — 100 GeV
$\chi$	$\frac{1}{2}$	-1	5 GeV — 5 TeV
$\psi$	$\frac{1}{2}$	-1	5 GeV — 5 TeV

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$$\mathcal{L} \supset - [y_{\chi\psi} \bar{\psi} S \chi + h.c.] - y_{\chi} \bar{\chi} S \chi - y_{\psi} \bar{\psi} S \psi$$

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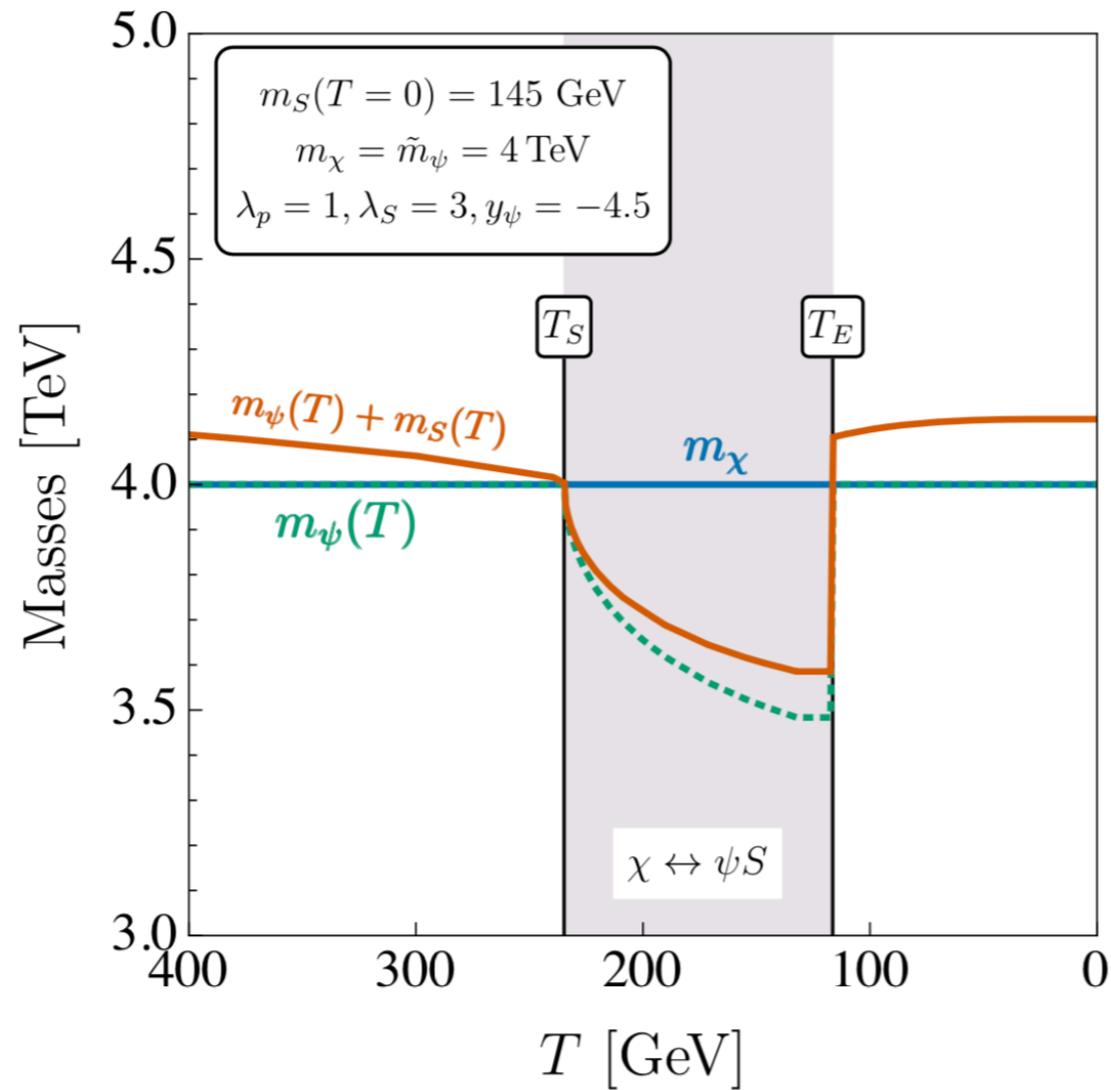
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temporarily allows decay

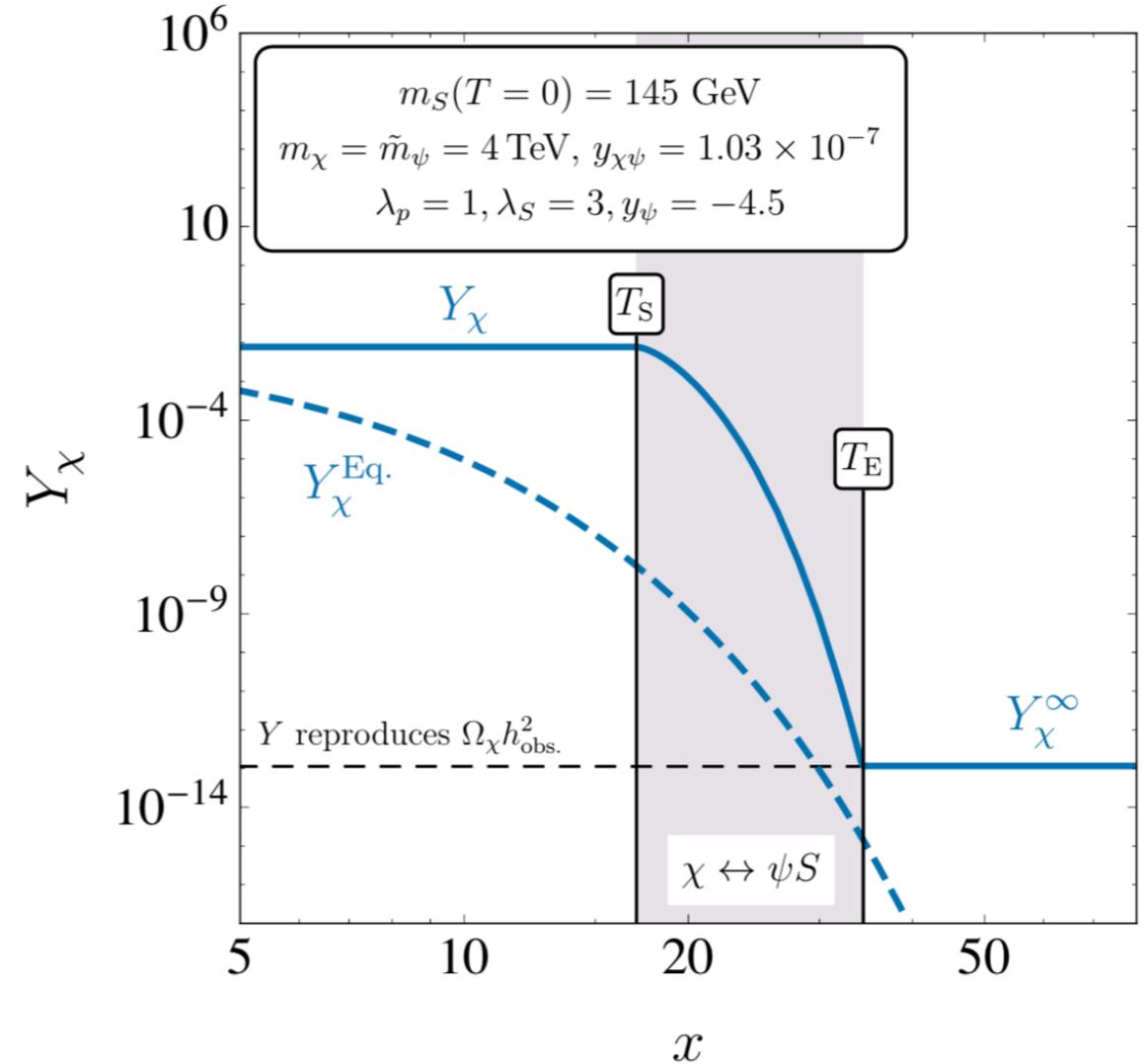


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## Evolution of Particle Masses



## Evolution of DM Abundance



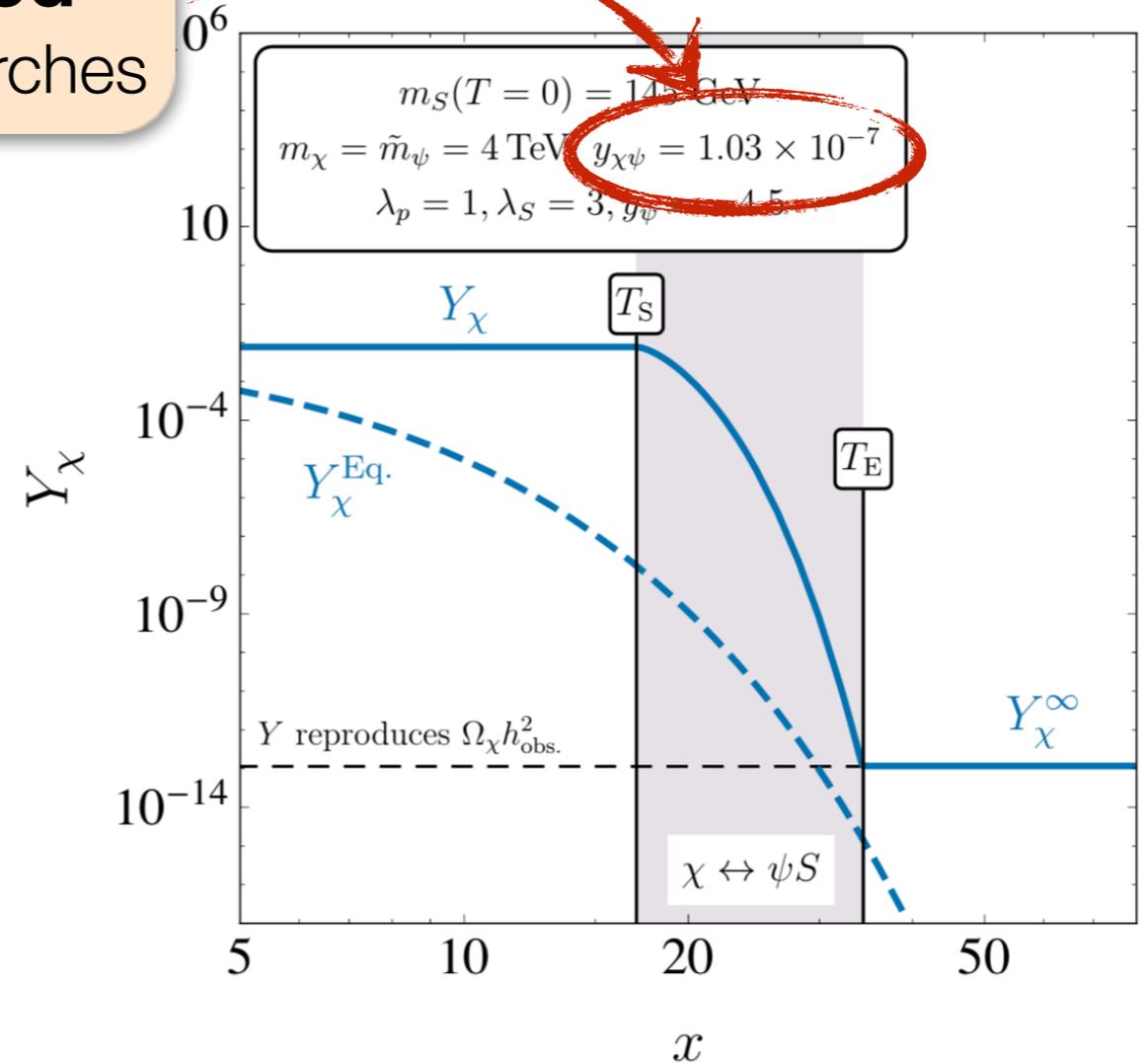
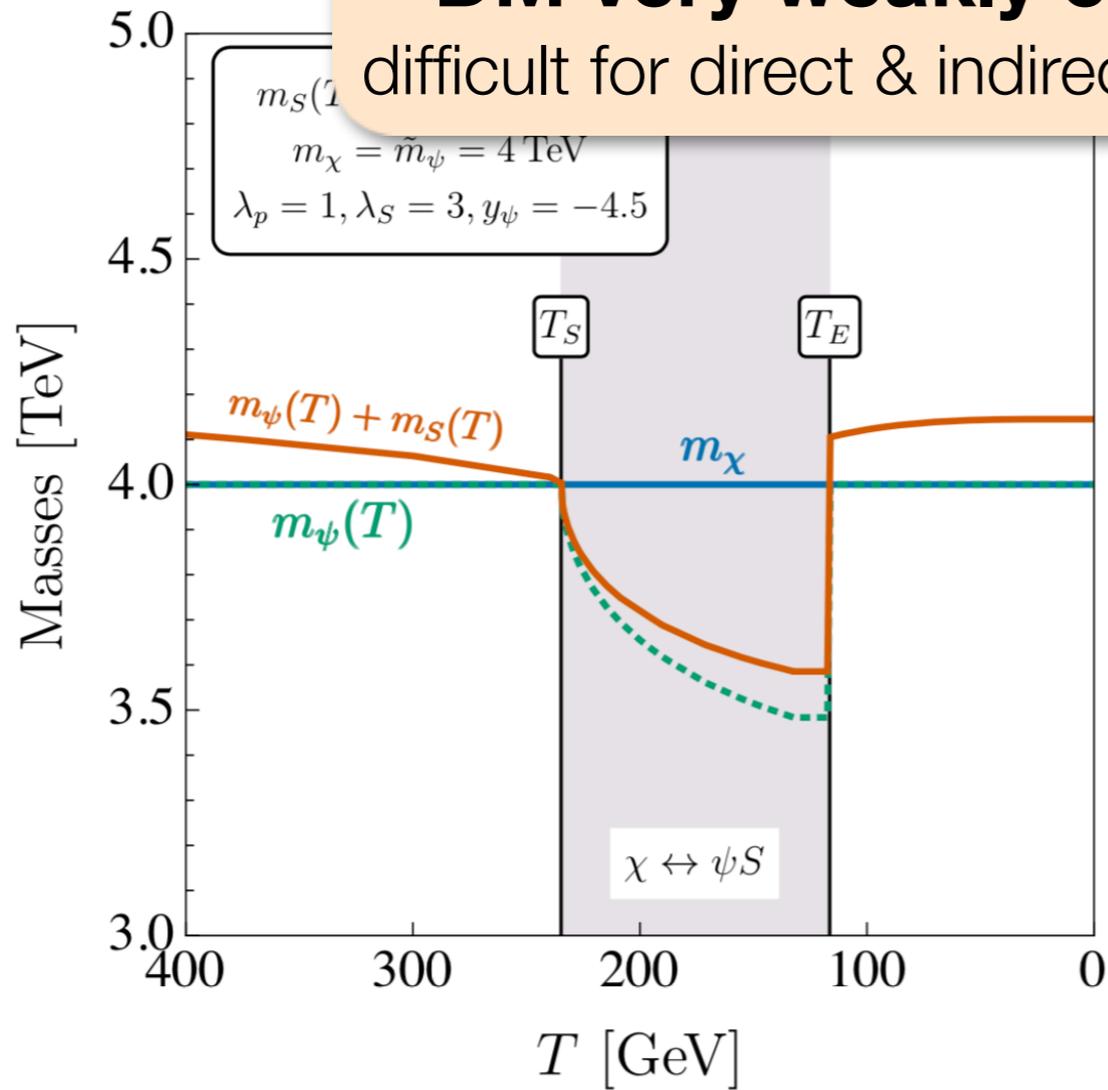
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# Cosmological Evolution

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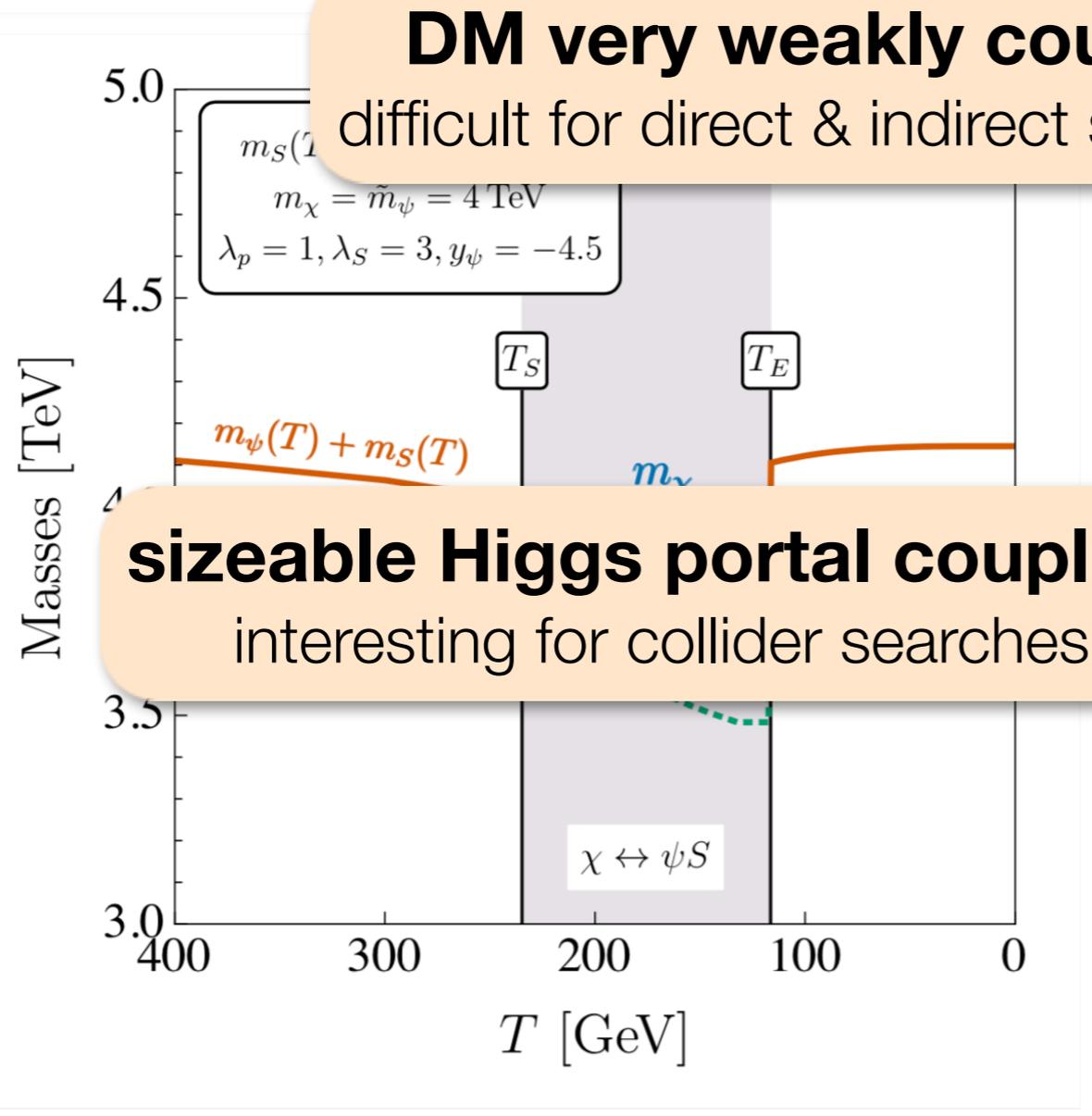
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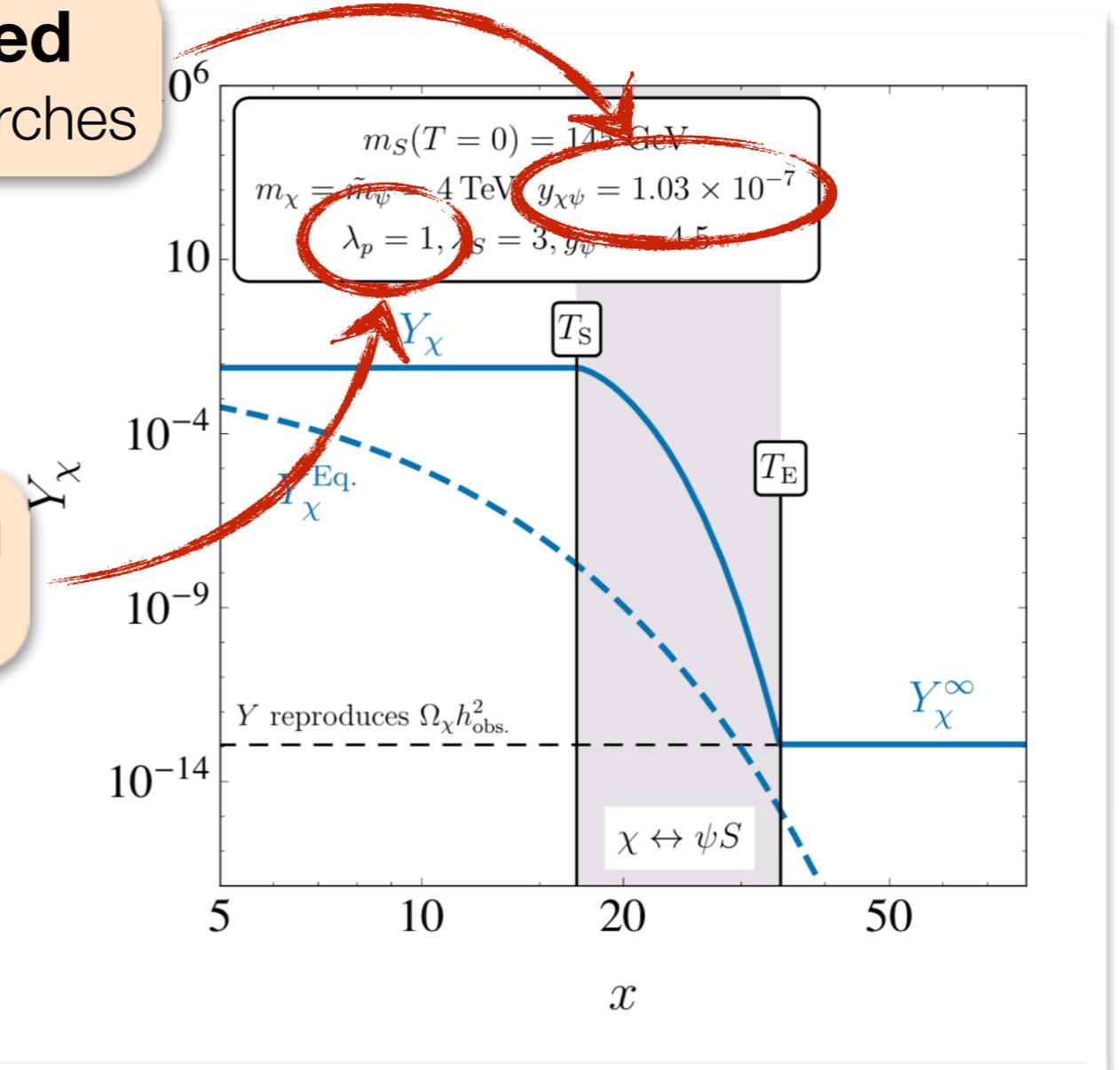
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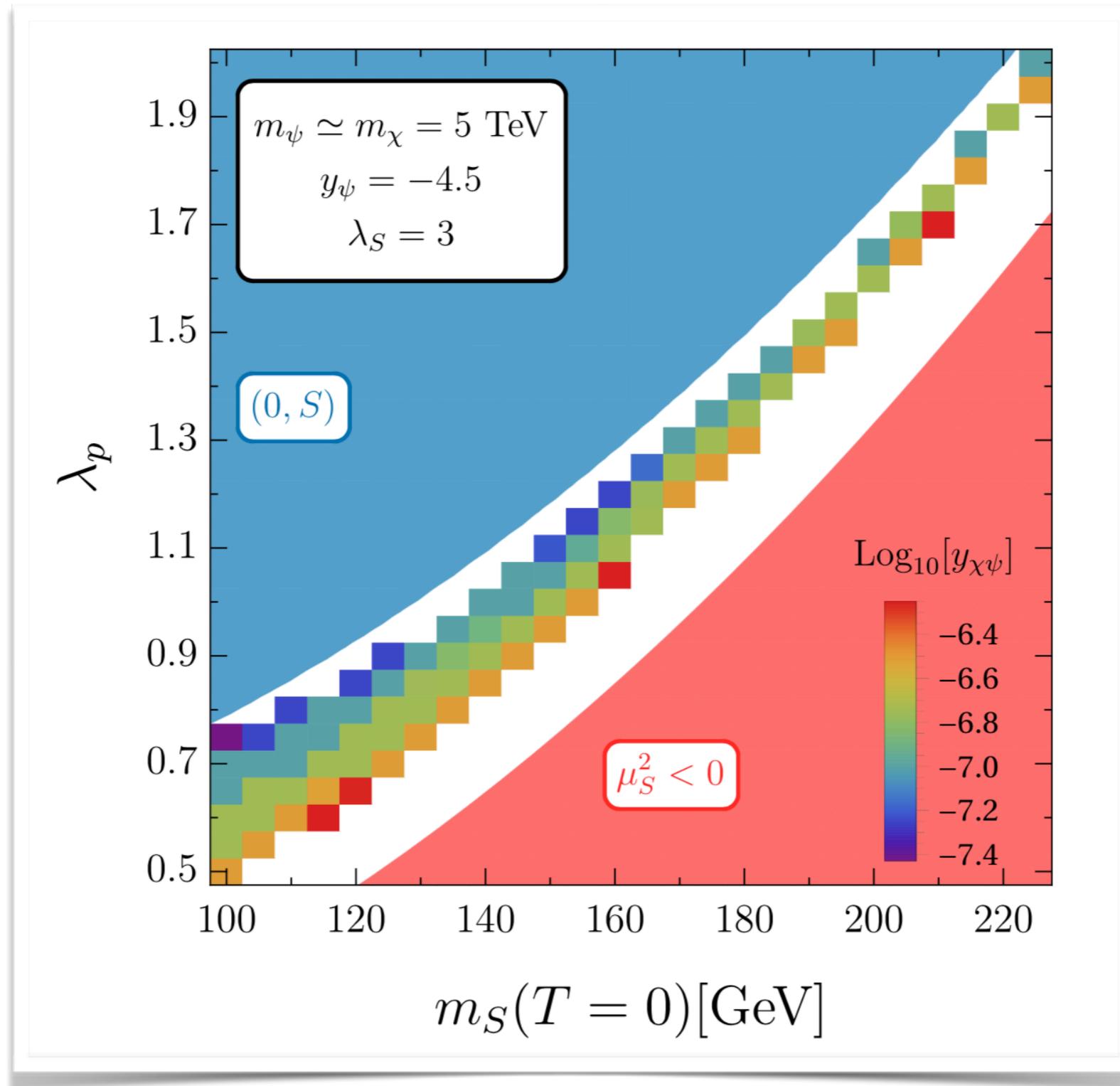


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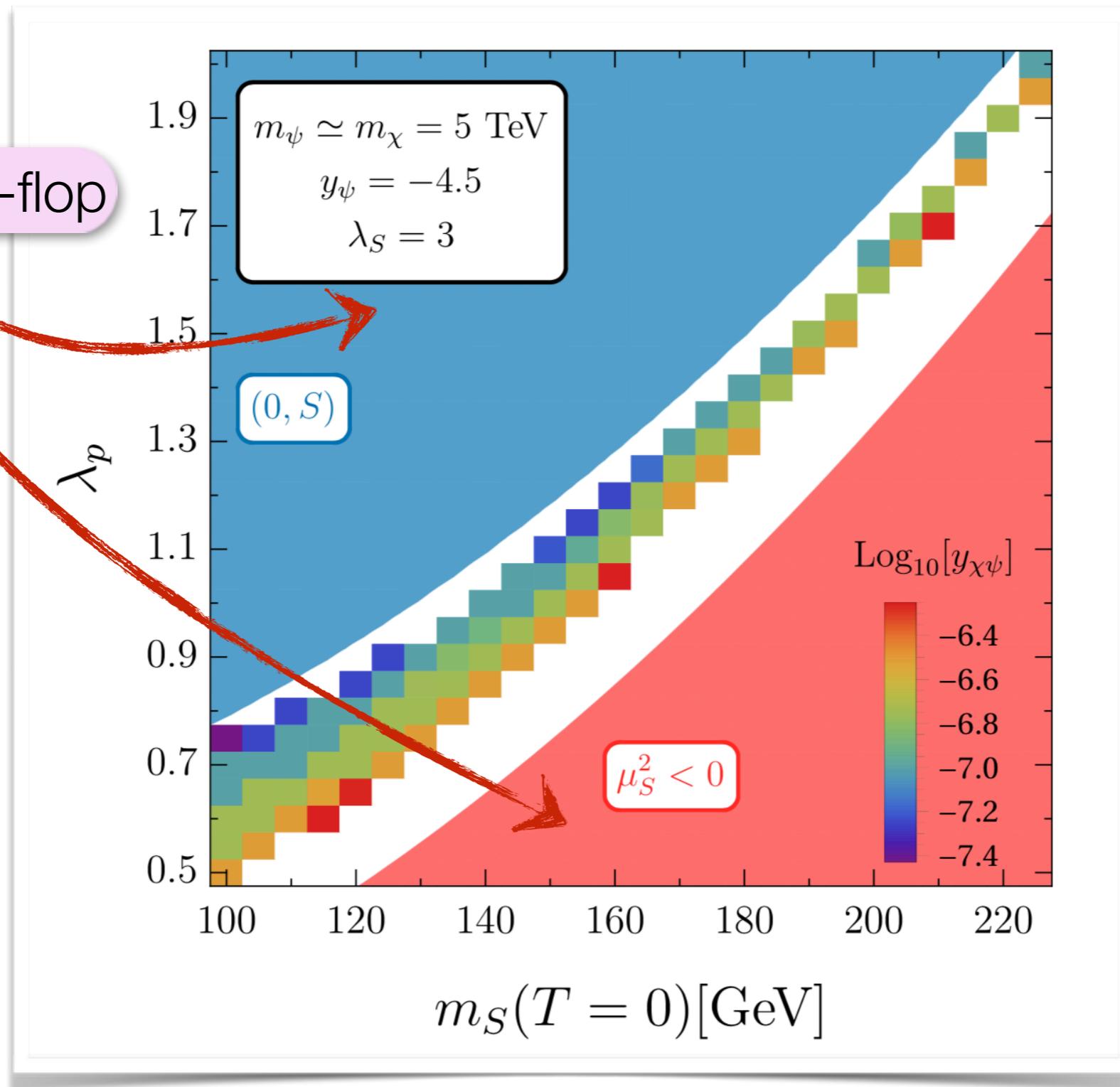
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# Parameter Space



# Parameter Space

no vev flip-flop



# Further Implications

## Gravitational waves

- 1<sup>st</sup> order phase transitions contribute to **stochastic GW background**
- relevant processes: **bubble collisions**, **sound waves**, **turbulence**
- potentially detectable by **LISA** (TeV scale)  
or by **pulsar timing arrays** (GeV scale)

e.g. Breitbach JK Madge Opferkuch Schwaller arXiv:1811.11175

## Electroweak Baryogenesis

- relate **particle–antiparticle asymmetry of the Universe** to different **permeability of bubble walls** for fermions and anti-fermions

# Implications B1

## Baryogenesis



# Electroweak Baryogenesis

- ☑ Consider 1<sup>st</sup> order electroweak phase transition  
e.g. SM + real singlet scalar
- ☑ Penetrating bubble walls is difficult for top quarks  
massless on the outside, massive on the inside  $\Rightarrow$  potential wall
- ☑ Permeability can be larger for  $t_L$  and  $t_R$   
requires new CP-violating interaction
- ☑ Deficit of  $t_L$  outside the bubbles

# Electroweak Baryogenesis

- ☑  $B+L$  (baryon number + lepton number) violated by **sphaleron transitions**
  - effect of the weak interaction  $\Rightarrow$  affect only **LH particles**
  - **active only outside the bubble** (electroweak symmetry broken inside)
  - $B-L$  remains conserved
- ☑ Entropy maximization implies that baryons are regenerated from leptons
- ☑ **Net gain in baryon number**
- ☑ Excess baryons are eventually swept up by advancing bubble walls

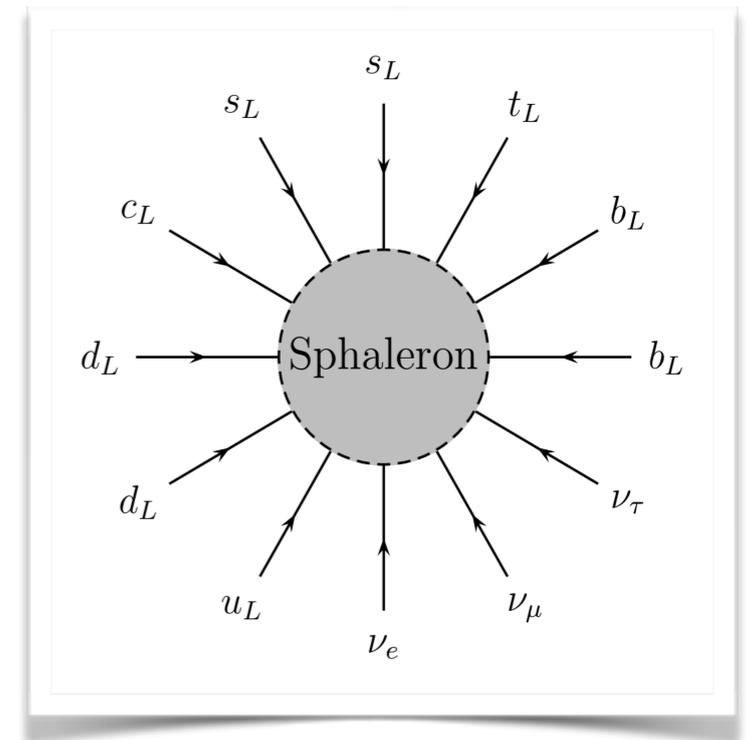


Image: Wilfried Buchmüller, [hep-ph/9812447](https://arxiv.org/abs/hep-ph/9812447)

# Implications B2

## Gravitational Waves

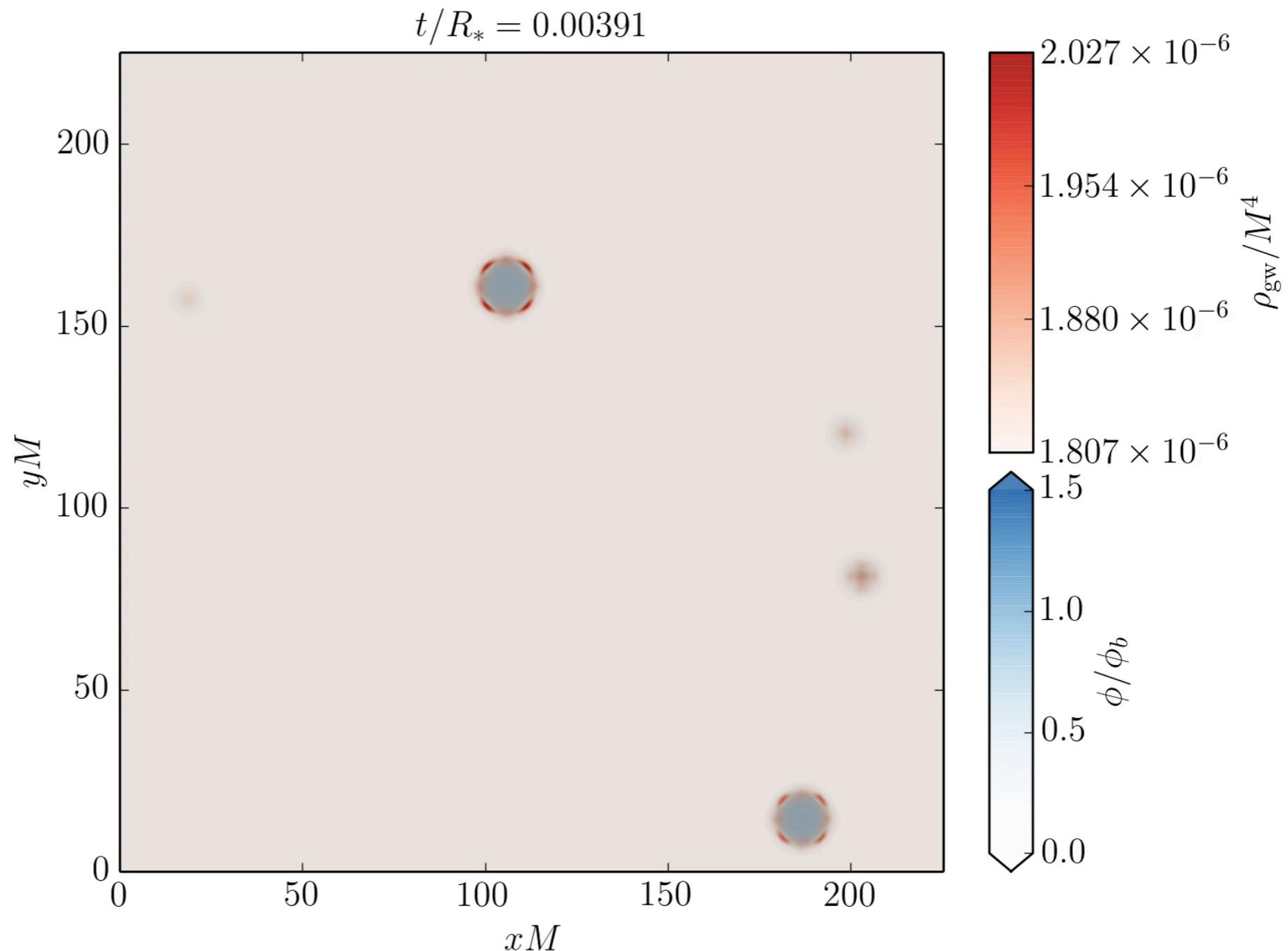


# Gravitational Waves from Phase Transitions

- ☑ Phase transitions in extended scalar sectors often 1<sup>st</sup> order
  - ➡ gravitational wave signals?

[Witten 1984](#)

[Cutting Hindmarsh Weir 2018](#)

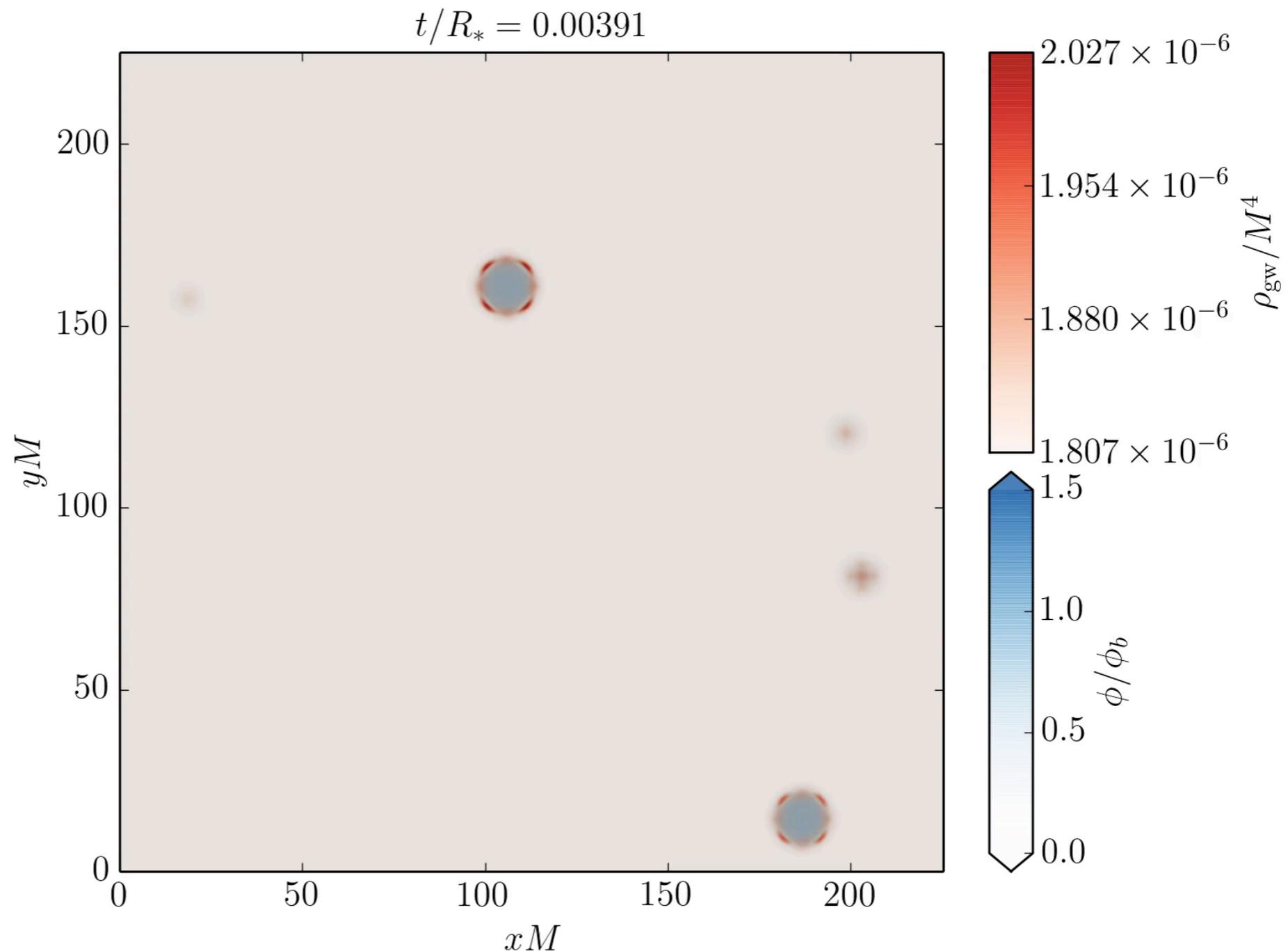


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# Gravitational Waves from Phase Transitions

## ☑ Three contributions

- Bubble collisions
- Collisions of **sound waves** generated during bubble expansion
- **Turbulence** in the plasma

## ☑ How to compute the GW signal from these contributions:

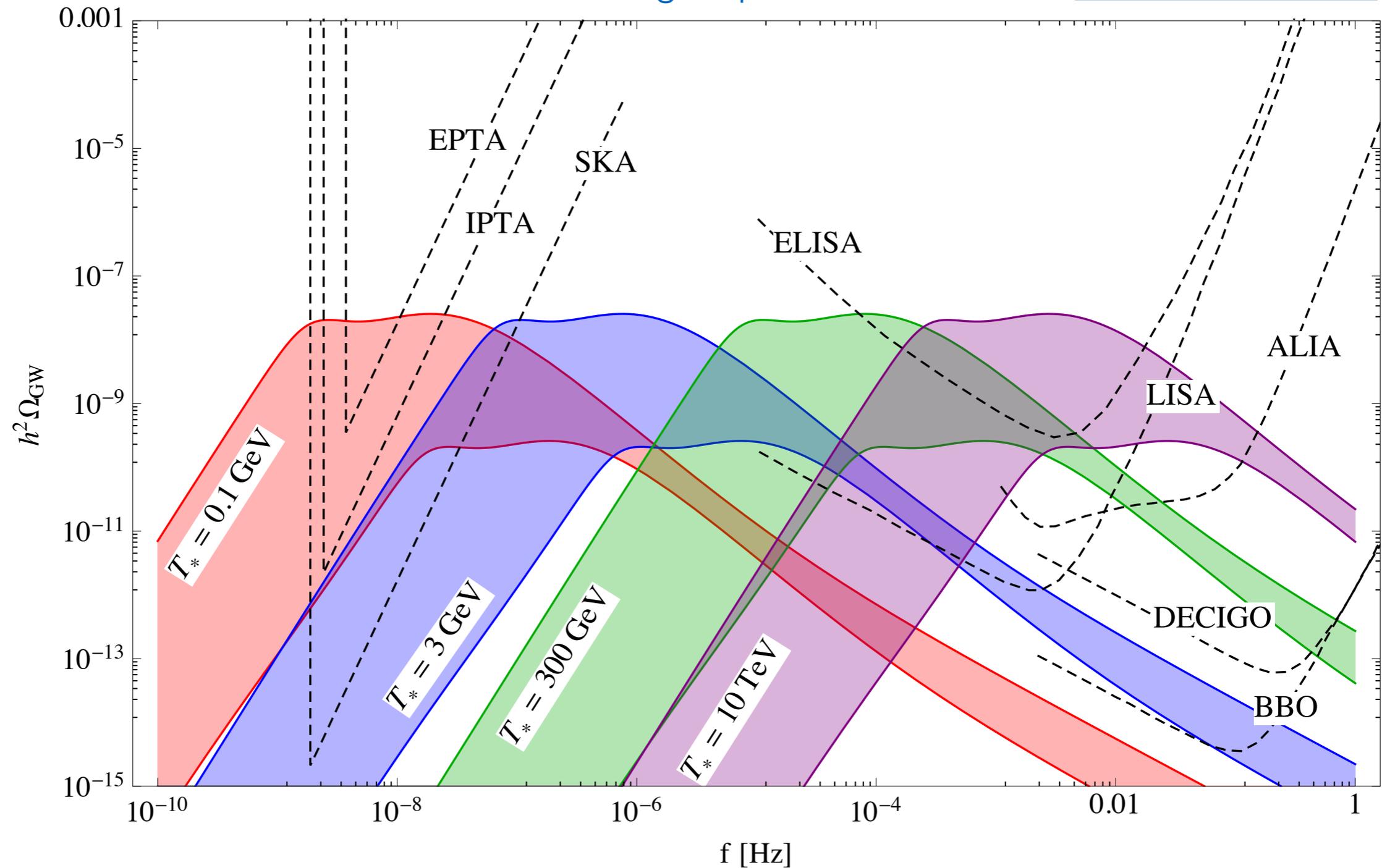
- requires numerical simulations (**large uncertainties!**)
- Parameterize results, e.g. as

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f)}{d \log f} \simeq \mathcal{N} \Delta \left( \frac{\kappa \alpha}{1 + \alpha} \right)^p \left( \frac{H}{\beta} \right)^q s(f)$$

# Gravitational Wave Spectra

plot from Schwaller [arXiv:1504.07263](https://arxiv.org/abs/1504.07263)

see also Breitbach JK Madge Opferkuch Schwaller [arXiv:1811.11175](https://arxiv.org/abs/1811.11175)



# Phase Transition Parameter for GWs

## Four relevant parameters

- Bubble nucleation temperature  $T^{\text{nuc}}$
- Strength of the phase transition

$$\alpha \equiv \frac{\epsilon}{\rho_R} = \frac{1}{\rho_R} \left( -\Delta V + T^{\text{nuc}} \frac{\partial \Delta V}{\partial T} \Big|_{T^{\text{nuc}}} \right)$$

- Inverse duration of phase transition

$$\frac{\beta}{H} = T_h^{\text{nuc}} \frac{dS_E(T)}{dT} \Big|_{T_h^{\text{nuc}}}$$

- Bubble wall velocity  $v_w$

# Phase Transition Parameter for GWs

## ☑ Four relevant parameters

- Bubble nucleation temperature latent heat release
- Strength of the phase transition

$$\alpha \equiv \frac{\epsilon}{\rho_R} = \frac{1}{\rho_R} \left( -\Delta V + T^{\text{nuc}} \frac{\partial \Delta V}{\partial T} \Big|_{T^{\text{nuc}}} \right)$$

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# Phase Transition Parameter for GWs

## ☑ Four relevant parameters

- Bubble nucleation temperature  $T^{\text{nuc}}$  latent heat release
- Strength of the phase transition

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- Inverse duration of phase transition  $\beta/H$  total radiation density

$$\frac{\beta}{H} = T_h^{\text{nuc}} \frac{dS_E(T)}{dT} \Big|_{T_h^{\text{nuc}}}$$

- Bubble wall velocity  $v_w$

# Phase Transition Parameter for GWs

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Euclidean action  
corresponding to the  
transition path in field space

- Bubble wall velocity  $v_w$

# Phase Transition Parameter for GWs

## Four relevant parameters

- Bubble nucleation temperature  $T^{nuc}$
- Strength of the phase transition

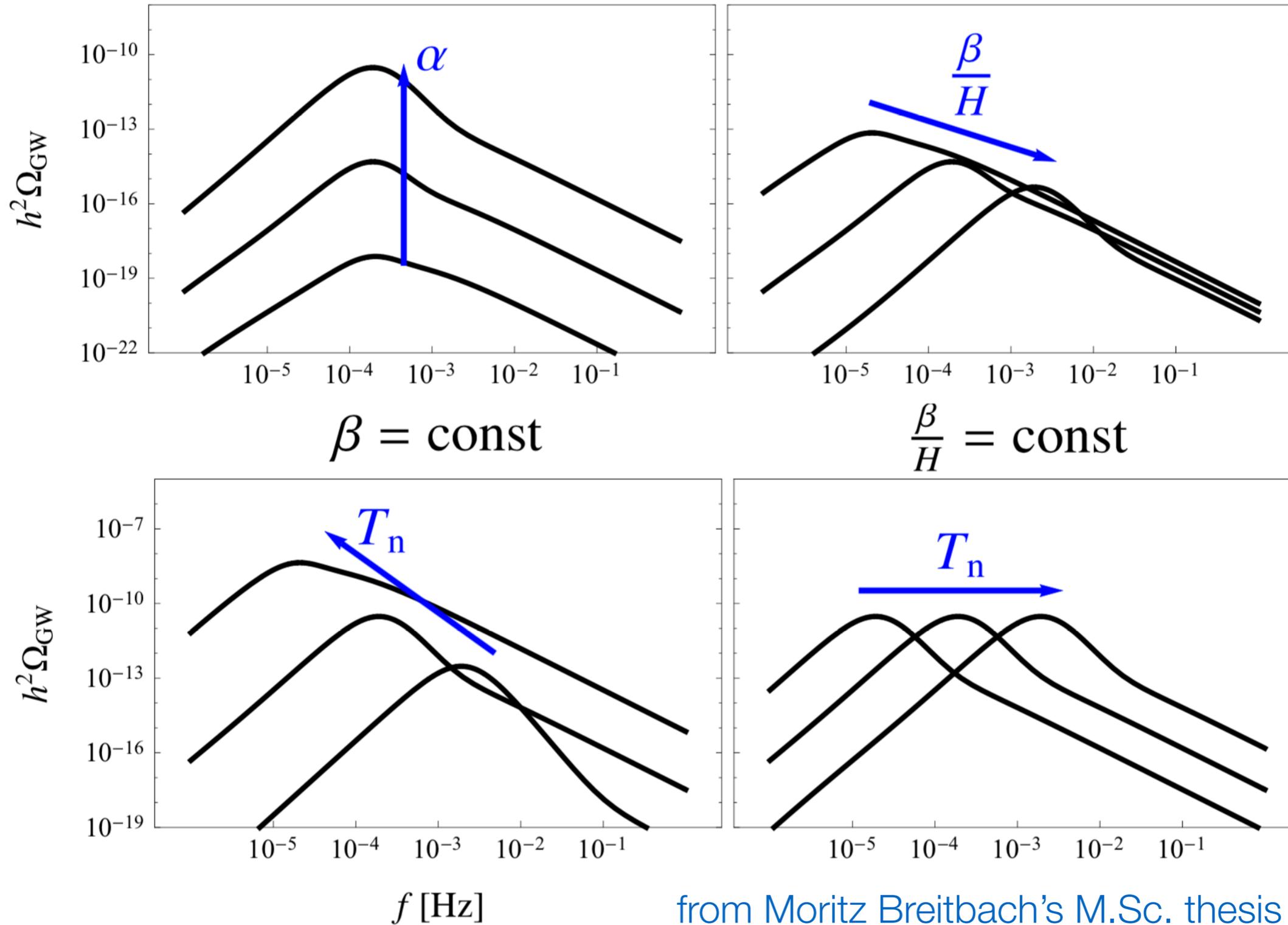
$$\alpha \equiv \frac{\epsilon}{\rho_R} = \frac{1}{\rho_R} \left( -\Delta V + T^{nuc} \frac{\partial \Delta V}{\partial T} \Big|_{T^{nuc}} \right)$$

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$$\frac{\beta}{H} = T_h^{nuc} \frac{dS_E(T)}{dT} \Big|_{T_h^{nuc}}$$

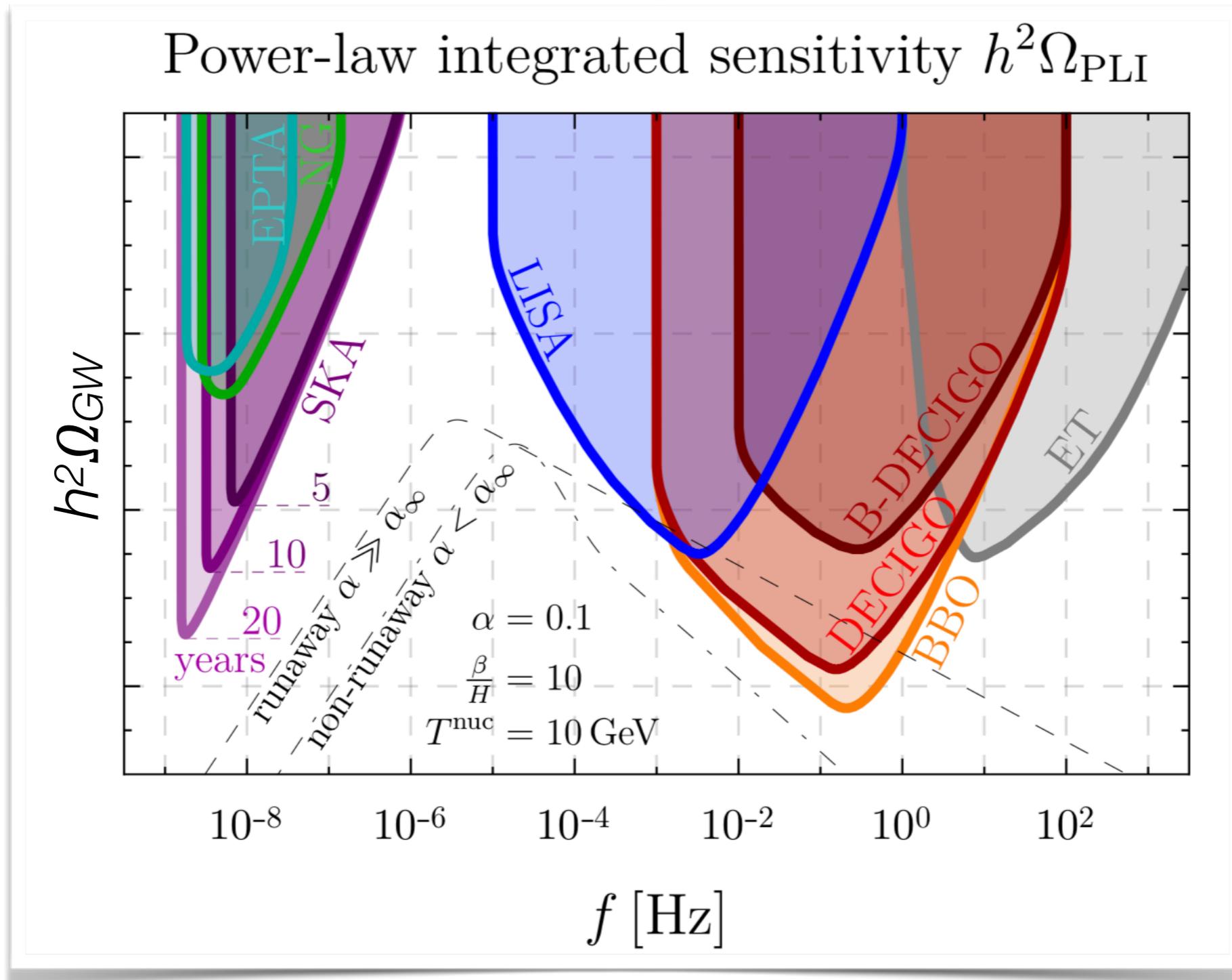
- Bubble wall velocity  $v_w$

# Parameter Dependence of GW Spectra



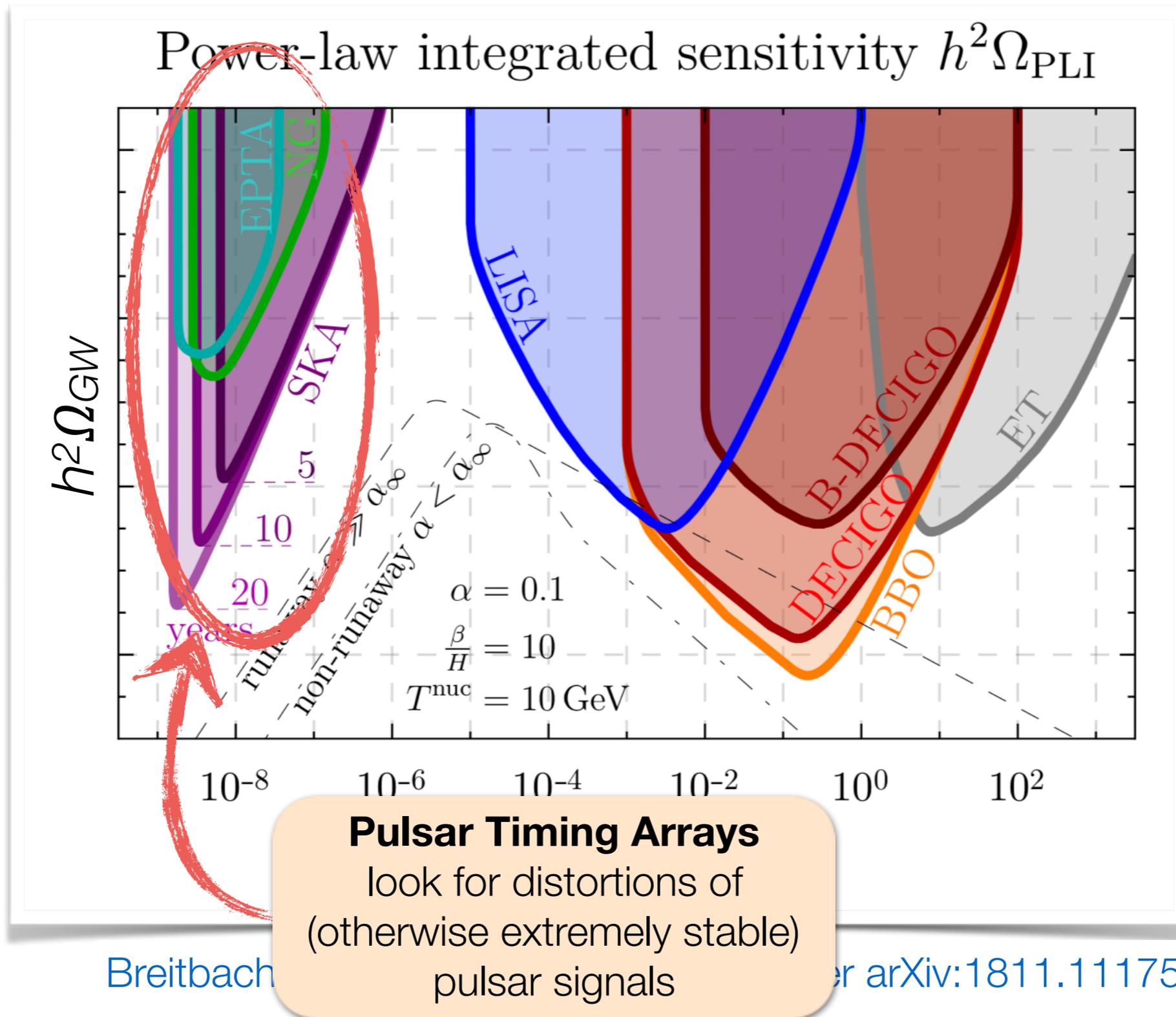
from Moritz Breitbach's M.Sc. thesis

# Gravitational Wave Observatories

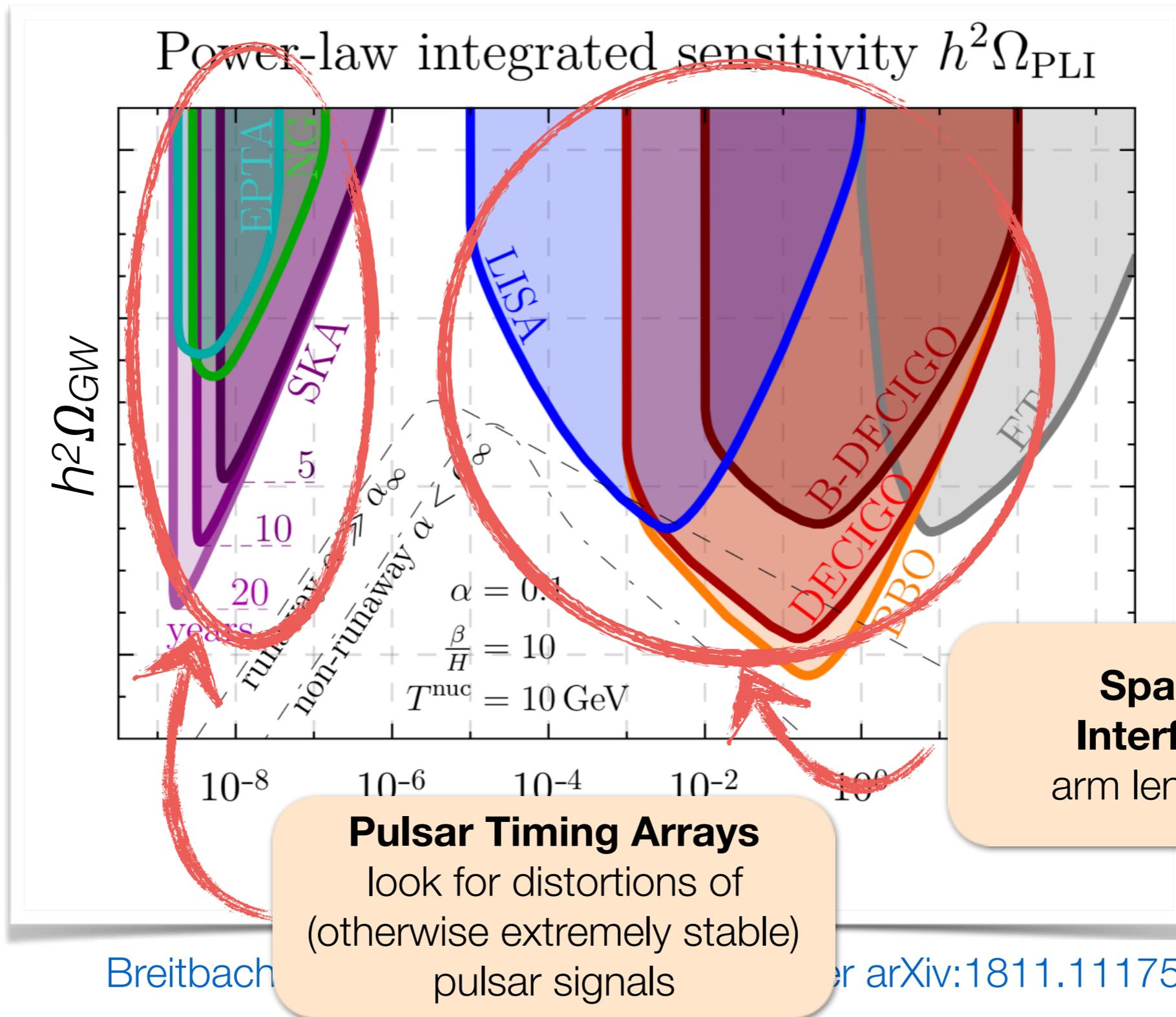


Breitbach JK Madge Opferkuch Schwaller arXiv:1811.11175

# Gravitational Wave Observatories

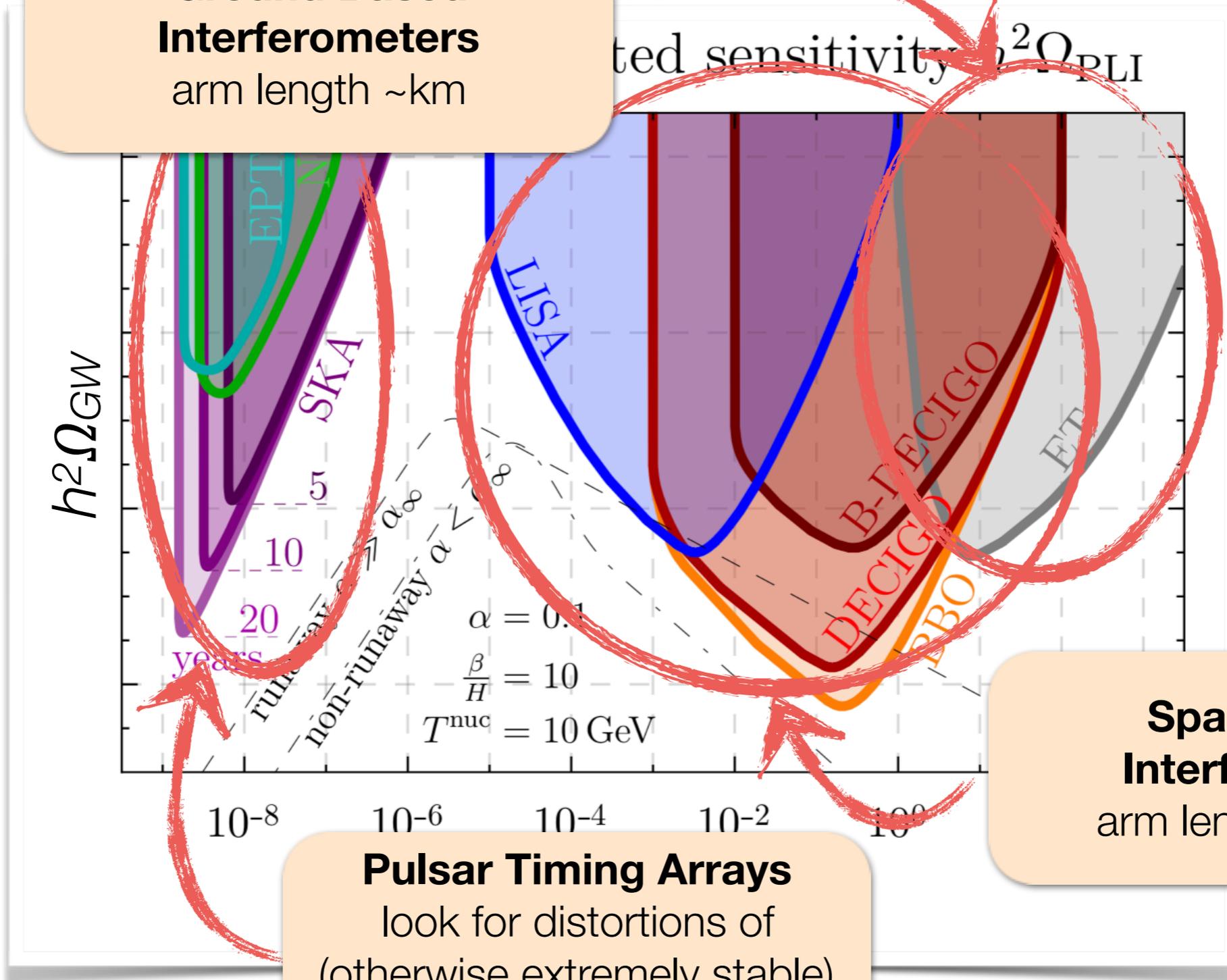


# Gravitational Wave Observatories



# Gravitational Wave Observatories

**Ground Based Interferometers**  
arm length ~km



**Space Based Interferometers**  
arm length  $\sim 10^6$  km

**Pulsar Timing Arrays**  
look for distortions of  
(otherwise extremely stable)  
pulsar signals

Breitbach et al. arXiv:1811.11175



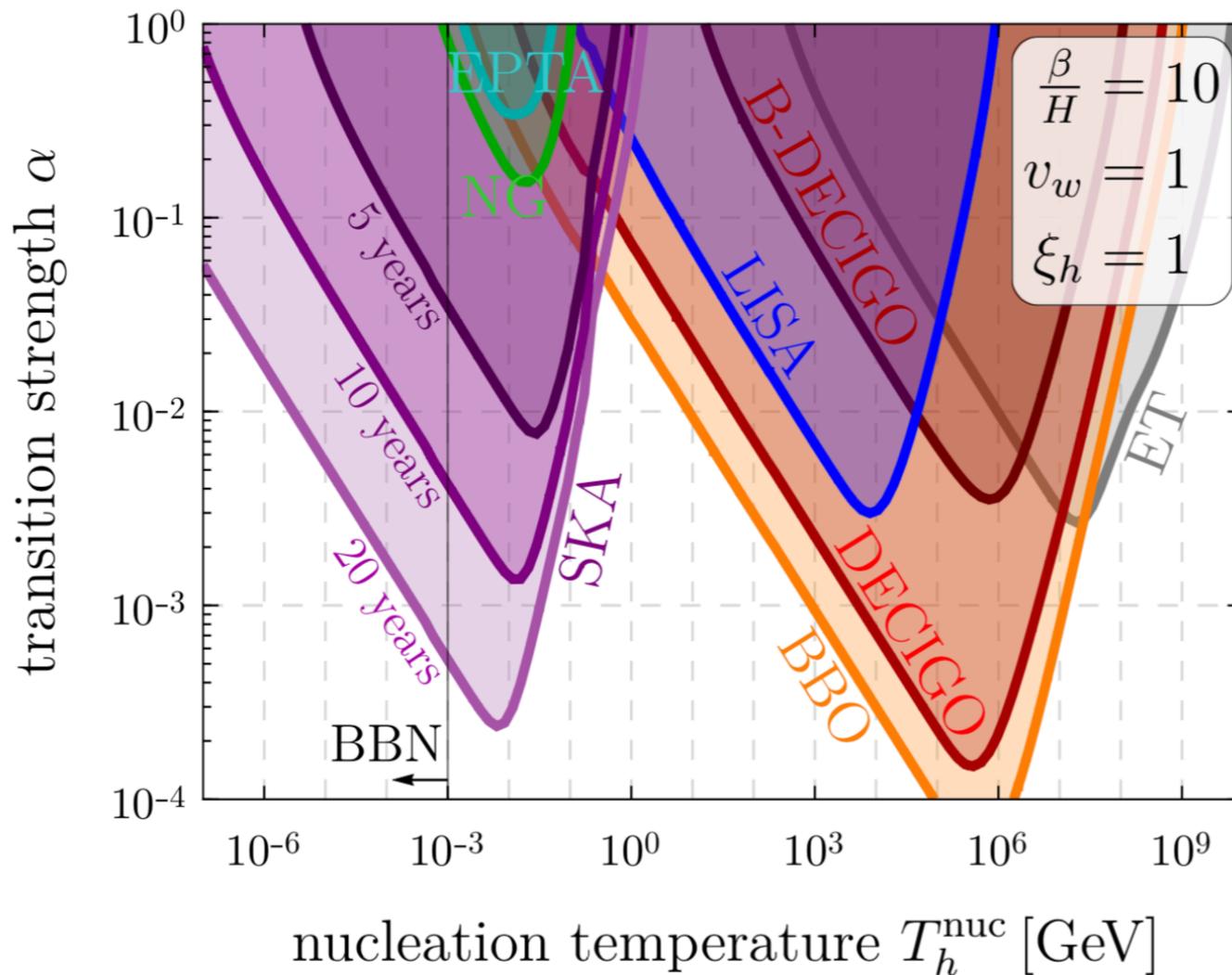
# GWs from Hidden Sector Phase Transitions

## Important Plot Twist

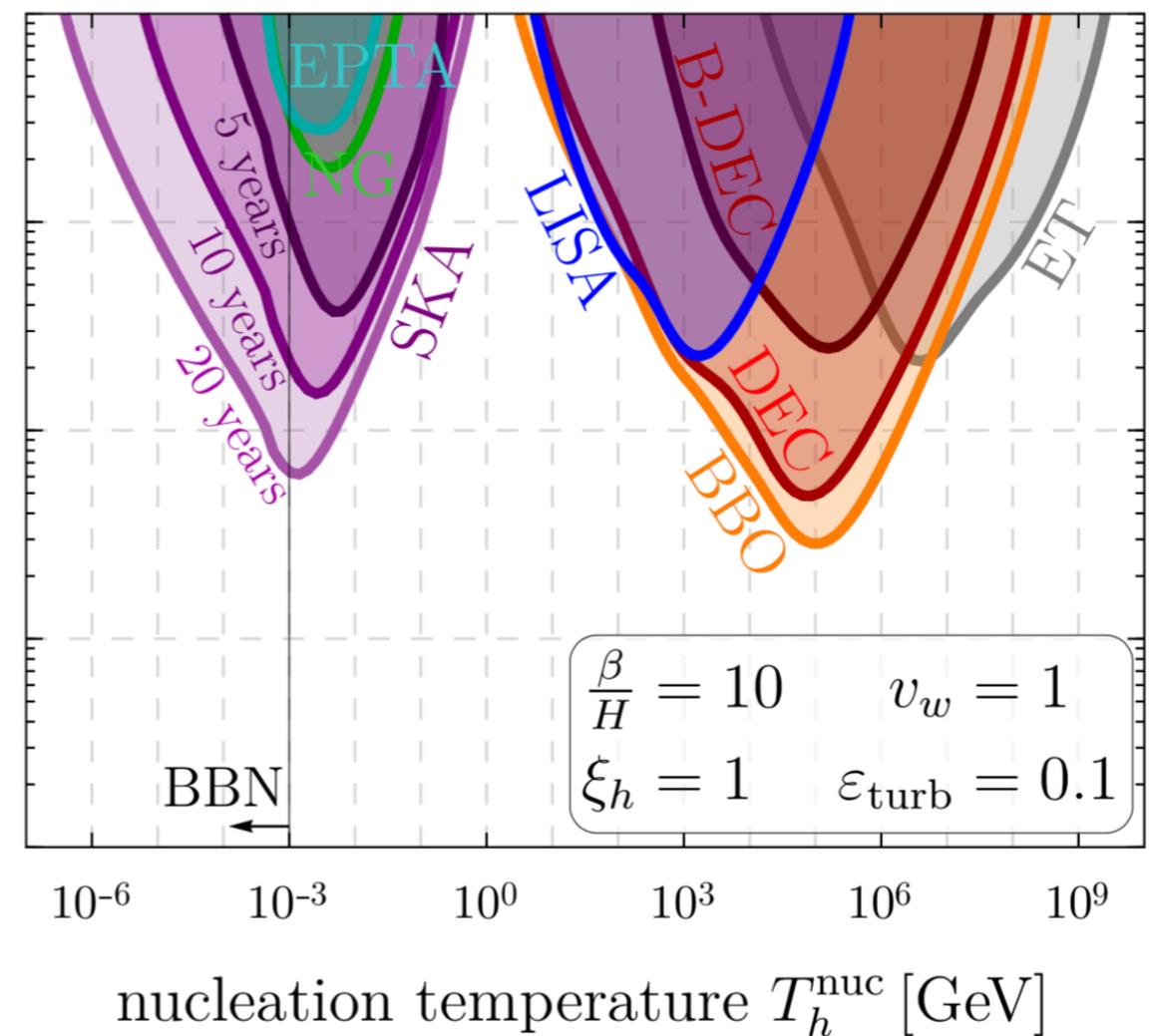
- hidden sector may have **different temperature** than visible sector
- parameterized by temperature ratio  $\xi_h$

# Dependence on Hidden Sector Temperature

Runaway bubbles with  $\alpha \gg \alpha_\infty$

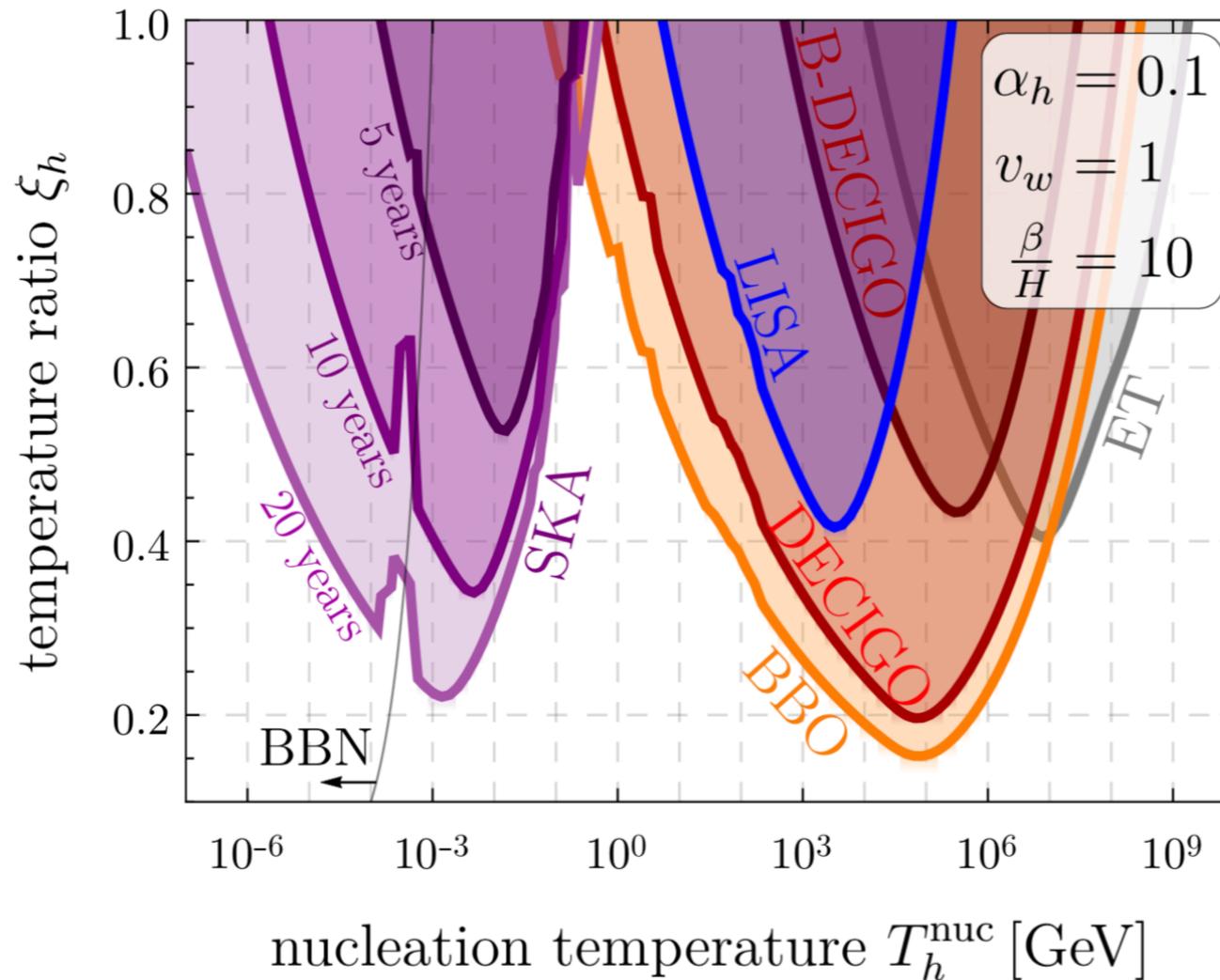


Non-runaway bubbles ( $\alpha < \alpha_\infty$ )

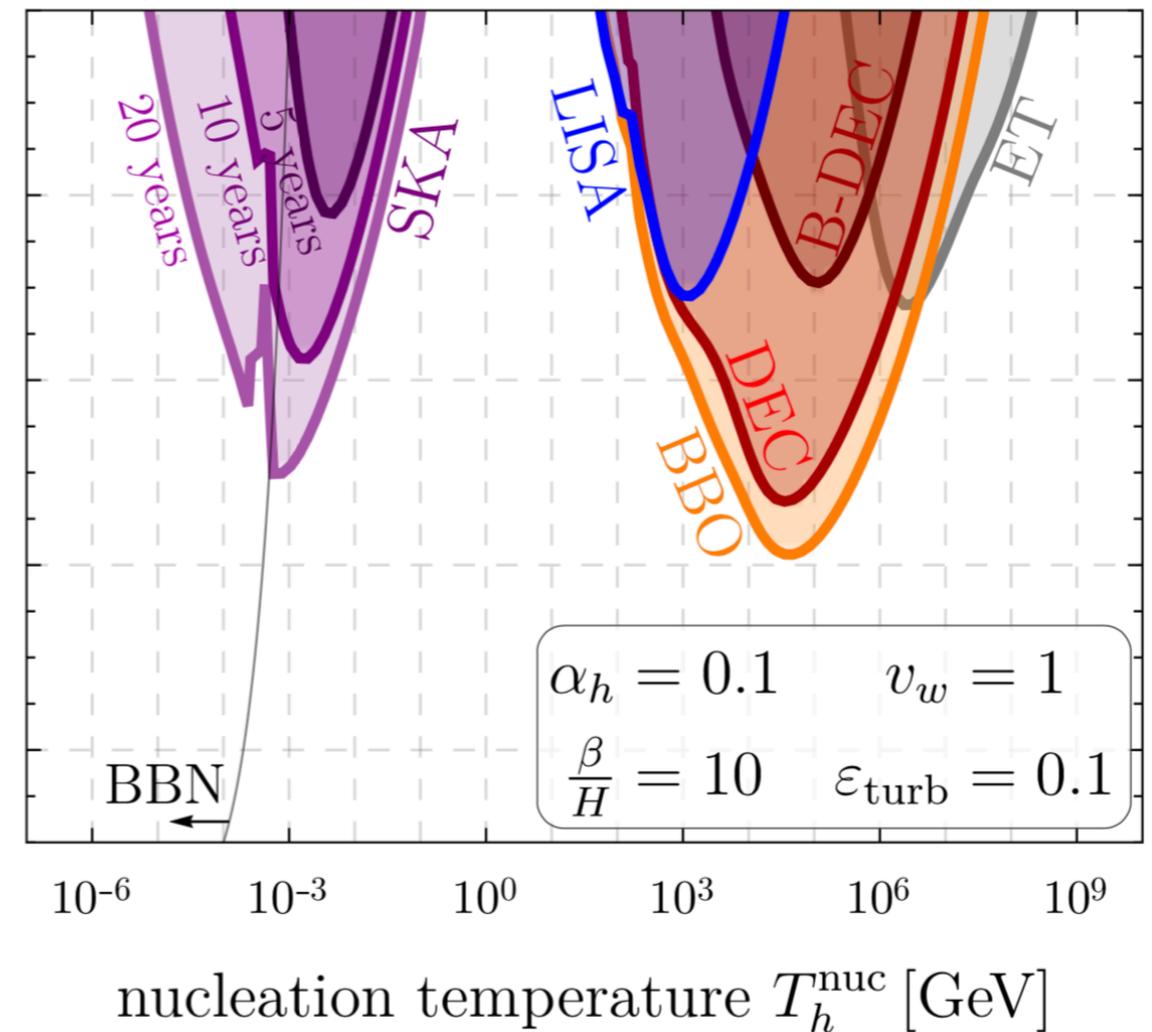


# Dependence on Hidden Sector Temperature

Runaway bubbles with  $\alpha \gg \alpha_\infty$



Non-runaway bubbles ( $\alpha < \alpha_\infty$ )



# What is Needed for a Strong Phase Transition?

## ☑ In practice

- difficult to realize sufficiently strong 1<sup>st</sup> order phase transitions (participating particles must be large fraction of total radiation density)
- easier at lower energies (pulsar timing arrays!)
- but strong constraints from BBN

