

An Introduction to F-theory

Iñaki García-Etxebarria

September 29, 2013

Contents

1	Introduction and motivation	3
1.1	What is string theory	3
1.1.1	An overview of string dualities	3
1.2	What do we want from string theory	3
1.2.1	Some model building requirements	3
1.2.2	How these ingredients appear in F-theory	3
1.2.3	State of the art in F-theory model building	3
1.3	What is F-theory	3
1.3.1	Definition via type IIB	3
1.3.2	Definition via heterotic	4
1.3.3	Definition via M-theory	4
2	CY_4 compactifications of M-theory	4
2.1	The 3d theory	4
2.2	Taking the F-theory limit	4
2.3	Fluxes and chirality	4
2.4	Extra $U(1)$ factors	4
3	The spectral cover construction	4
3.1	Friedman-Morgan-Witten	5
3.2	A bit of T-branes	5
4	Type IIB perspective	5
4.1	Orientifold and D-branes	5
4.2	Sen's limit	5
4.3	String junctions	5
4.4	Flux quantization	5
5	Non-perturbative effects	6
5.1	Neutral zero modes	6
5.2	Charged zero modes	6

1 Introduction and motivation

1.1 What is string theory

String theory as a unified theory for gravitation and quantum field theory. Description of string theory as a manifold, for which we only have a patchwise description.

1.1.1 An overview of string dualities

String dualities are conjectured perfect equivalences between superficially different descriptions. Sometimes things which are very hard to compute on one of the sides of the duality become simple on the other side, so the existence of a duality can be very useful. We focus on S- and T-duality, since they enter the definition of F-theory.

1.2 What do we want from string theory

1.2.1 Some model building requirements

An introduction to effective field theory, and the MSSM. Not detailed pheno, but an explanation of what the main ingredients of a physical theory for particle physics are.

1.2.2 How these ingredients appear in F-theory

A quick heuristic explanation of how the MSSM ingredients appear in the F-theory language. This will be elaborated much more carefully later, here we want to emphasize which geometric quantities in F-theory are most interesting from the model builder's point of view.

1.2.3 State of the art in F-theory model building

Some of the most promising constructions. Bottom-up philosophy vs. global constructions. Overview of global, local and ultra-local viewpoints.

1.3 What is F-theory

1.3.1 Definition via type IIB

Here F-theory just means type IIB string theory at non-zero string coupling, perhaps with some regions intrinsically strongly coupled. The fourfold is a nice

“packaging” of the data for the varying axio-dilaton.

1.3.2 Definition via heterotic

Valid only on some circumstances, but when it is valid it can be quite illuminating. The F-theory compactification admits an equivalent (*dual*) description in terms of the heterotic string.

1.3.3 Definition via M-theory

This is the most common description, based on a dual M-theory picture on a particular limit.

2 CY_4 compactifications of M-theory

2.1 The 3d theory

A bit of supergravity for the smooth Calabi-Yau compactification of M-theory to 3 dimensions, and tadpole cancellation requirements. Explanation for why some states become light in the singular limit, and why non-abelian symmetries appear (we will work out one example of the Tate algorithm [3]).

2.2 Taking the F-theory limit

Description of the F-theory limit, explanation of why 3d is enhanced to 4d.

2.3 Fluxes and chirality

How matter appears, based on the Katz-Vafa picture [4]. The origin of chirality in the IIB picture, and then on F-theory.

2.4 Extra $U(1)$ factors

These are sometimes important and/or useful, and can be understood as a generalization of the discussion above to special cases.

3 The spectral cover construction

When a heterotic dual construction exists (the Calabi-Yau fourfold is $K3$ fibered) it is illuminating to construct the heterotic dual.

3.1 Friedman-Morgan-Witten

Geometrically, the heterotic string is compactified on the same base with the replacement $K3 \rightarrow T^2$. The spectrum of branes is obtained by the Friedman-Morgan-Witten construction [5, 6, 7].

3.2 A bit of T-branes

If there is time and interest we can review the construction of T-branes [8], which matches nicely with the spectral cover construction and highlights some subtleties in the purely geometric description.

4 Type IIB perspective

4.1 Orientifold and D-branes

An introduction to the basic objects in weakly coupled type IIB.

4.2 Sen's limit

Sen's limit [9, 10] is the basic construction allowing us to rigorously connect F-theory and type IIB. We review how this limit goes.

4.3 String junctions

Given Sen's limit we can discuss the weakly coupled picture of non-perturbative enhancements, namely string junctions, following Zwiebach and friends ([11] and follow-ups). If nobody picks the topic for discussion on the last week, and time allowing, we will proceed to discuss the Higgs branch of F-theory [12].

4.4 Flux quantization

If there is time and nobody discusses it during the last week, we will review Freed-Witten anomaly cancellation on branes, and its manifestation in F-theory as G_4 flux quantization, following the works [13, 14].

5 Non-perturbative effects

We discuss euclidean branes and corrections to the superpotential, starting with Witten's criterion [15], and then discussing modern refinements [16, 17].

5.1 Neutral zero modes

These are the modes for which Witten gave a criterion in [15]. They are characterized by having no charge with respect to the brane gauge groups.

5.2 Charged zero modes

Charged zero modes have charge with respect to the gauge groups, and they generally induce charged couplings in the low energy effective action. They are still not completely understood in F-theory, but we can discuss some aspects that follow from the weakly coupled limit, as in [18, 16].

References

- [1] S. Katz, D. R. Morrison, S. Schafer-Nameki, and J. Sully, *Tate's algorithm and F-theory*, *JHEP* **1108** (2011) 094, [[arXiv:1106.3854](#)].
- [2] H. Skarke, *Nonperturbative gauge groups and local mirror symmetry*, *JHEP* **0111** (2001) 013, [[hep-th/0109164](#)].
- [3] M. Bershadsky, K. A. Intriligator, S. Kachru, D. R. Morrison, V. Sadov, et. al., *Geometric singularities and enhanced gauge symmetries*, *Nucl.Phys.* **B481** (1996) 215–252, [[hep-th/9605200](#)].
- [4] S. H. Katz and C. Vafa, *Matter from geometry*, *Nucl.Phys.* **B497** (1997) 146–154, [[hep-th/9606086](#)].
- [5] R. Friedman, J. Morgan, and E. Witten, *Vector bundles and F theory*, *Commun.Math.Phys.* **187** (1997) 679–743, [[hep-th/9701162](#)].
- [6] R. Friedman, J. W. Morgan, and E. Witten, *Vector bundles over elliptic fibrations*, [[alg-geom/9709029](#)].
- [7] R. Friedman, J. W. Morgan, and E. Witten, *Principal G bundles over elliptic curves*, *Math.Res.Lett.* **5** (1998) 97–118, [[alg-geom/9707004](#)].

- [8] S. Cecotti, C. Cordova, J. J. Heckman, and C. Vafa, *T-Branes and Monodromy*, *JHEP* **1107** (2011) 030, [[arXiv:1010.5780](#)].
- [9] A. Sen, *F theory and orientifolds*, *Nucl.Phys.* **B475** (1996) 562–578, [[hep-th/9605150](#)].
- [10] A. Sen, *Orientifold limit of F theory vacua*, *Phys.Rev.* **D55** (1997) 7345–7349, [[hep-th/9702165](#)].
- [11] O. DeWolfe and B. Zwiebach, *String junctions for arbitrary Lie algebra representations*, *Nucl.Phys.* **B541** (1999) 509–565, [[hep-th/9804210](#)].
- [12] A. Grassi, J. Halverson, and J. L. Shaneson, *Matter From Geometry Without Resolution*, [arXiv:1306.1832](#).
- [13] A. Collinucci and R. Savelli, *On Flux Quantization in F-Theory*, *JHEP* **1202** (2012) 015, [[arXiv:1011.6388](#)].
- [14] A. Collinucci and R. Savelli, *On Flux Quantization in F-Theory II: Unitary and Symplectic Gauge Groups*, *JHEP* **1208** (2012) 094, [[arXiv:1203.4542](#)].
- [15] E. Witten, *Nonperturbative superpotentials in string theory*, *Nucl.Phys.* **B474** (1996) 343–360, [[hep-th/9604030](#)].
- [16] M. Bianchi, A. Collinucci, and L. Martucci, *Magnetized E3-brane instantons in F-theory*, *JHEP* **1112** (2011) 045, [[arXiv:1107.3732](#)].
- [17] M. Kerstan and T. Weigand, *Fluxed M5-instantons in F-theory*, *Nucl.Phys.* **B864** (2012) 597–639, [[arXiv:1205.4720](#)].
- [18] R. Blumenhagen, A. Collinucci, and B. Jurke, *On Instanton Effects in F-theory*, *JHEP* **1008** (2010) 079, [[arXiv:1002.1894](#)].