Branes, Coset Models & Supergravity

José J. Fernández Melgarejo[1]

[1]Universidad de Murcia

Based on: [2512.XXXXX] FM, Iñaki García Etxebarria, J. A. Rosabal

IV GRASS – SYMBHOL Meeting • Toledo

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN
f SéNeCa(+) Agencia de Ciencia y Tecnología Región de Murcia

Motivation 1: Higher form symmetries

Gaiotto+'14

- Continuous 0-form symmetry
  - 0-form parameter $\lambda^{(0)}$
  - $(D-1)$-form closed current $j^{(D-1)}$
  - $j^{(D-1)}$ coupled to a background $A^{(1)}$
  - Example

$$S = \int \frac{1}{2} \mathrm{d}\phi^\dagger \wedge \star \mathrm{d}\phi \,, \qquad j^{(D-1)} = i\phi^\dagger \star \mathrm{d}\phi + \text{c.c.} \,, \qquad \int A^{(1)} \wedge j^{(D-1)}$$

- Continuous q-form symmetry
  - q-form parameter $\lambda^{(q)}$
  - $(D-q-1)$-form closed current $j^{(D-q-1)}$
  - $j^{(D-q-1)}$ coupled to a background $A^{(q+1)}$
  - Example: $q=1$

$$S = \int \frac{1}{2} \mathrm{d}C^{(1)} \wedge \star \mathrm{d}C^{(1)} \,, \qquad j^{(D-2)} = \star \mathrm{d}C^{(1)} \,, \qquad \int A^{(2)} \wedge j^{(D-2)}$$

Motivation 2: Higher group symmetries

Cordova-Dumitrescu-Intriligator'18

- It involves two higher form symmetries
- Definition: 2-group

$$A^{(1)} \to A^{(1)} + \mathrm{d}\lambda^{(0)} \,,$$
$$B^{(2)} \to B^{(2)} + \mathrm{d}\lambda^{(1)} + \mathrm{d}\lambda^{(0)} \wedge A^{(1)}$$

- Definition: $(2q+2)$-group $\mathrm{U}(1)^{(q)} \times_\kappa \mathrm{U}(1)^{(2q+1)}$

$$A^{(q+1)} \to A^{(q+1)} + \mathrm{d}\lambda^{(q)} \,,$$
$$B^{(2q+2)} \to B^{(2q+2)} + \mathrm{d}\lambda^{(2q+1)} + \kappa \mathrm{d}\lambda^{(q)} \wedge A^{(q+1)}$$

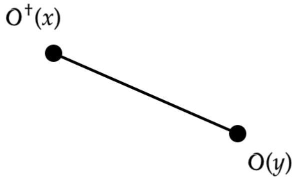
- Example: 4-group $\mathrm{U}(1)^{(2)} \times_1 \mathrm{U}(1)^{(5)}$ in 11D SUGRA ($q = 2$)

$$A^{(3)} \to A^{(3)} + \mathrm{d}\lambda^{(2)} \,,$$
$$B^{(6)} \to B^{(6)} + \mathrm{d}\lambda^{(5)} + \mathrm{d}\lambda^{(2)} \wedge A^{(3)}$$

SSB in global symmetries

- Classification of phases of matter
  - Unbroken phase. Excitations are gapped and correlation functions decay exponentially

$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y)\rangle \sim \exp\{-m|x-y|\}$$

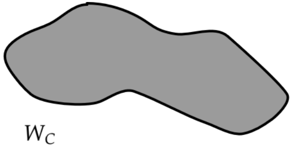


  - Broken phase. Something condenses. Correlation functions factorize and saturate at large distances

$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y)\rangle \sim \langle \mathcal{O}^\dagger\rangle\langle \mathcal{O}\rangle$$

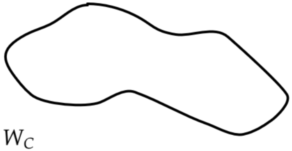
SSB in global symmetries

- For a 1-form symmetry
  - Unbroken phase. All excitations are gapped. Charged operators have an area law. Analogue of exponentially decaying correlator of local operators



$$\langle W_C \rangle \sim \exp\{-T_{p+1}\,\text{Area}[C]\}$$

  - Broken phase. Strings condense and have no tension. Charged operators develop perimeter laws at large distance. Analogue of factorized local correlators



$$\langle W_C \rangle \sim \exp\{-T_p\,\text{Perimeter}[C]\}$$

Goldstone Theorem (0-form symmetry)

Euclidean path integral

Levin-Wen '04, Hofman-Iqbal '18

- Conserved $\star j$ from 0-form symmetry and charged $\mathcal{O}$

$$\mathrm{d}\star j(x)\mathcal{O}(0) = iq\mathcal{O}(0)\,\delta^{(d)}(x)$$

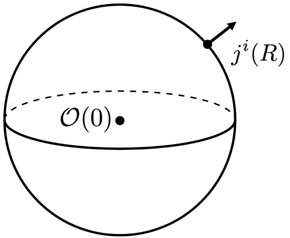
- Integrating over a 4-ball of radius R and evaluating:

$$\left\langle \left(\int_{S^3(R)} \star j \right) \mathcal{O}(0) \right\rangle = iq\langle \mathcal{O} \rangle$$

- If $\langle \mathcal{O} \rangle \neq 0$, R independence implies

$$\left\langle \star j^i(x)\mathcal{O}(0) \right\rangle \propto \frac{iqn^i}{R^3}$$

- Power-law decay $\Rightarrow$ 1 Goldstone mode



Goldstone Theorem (p-form symmetries)

- Conserved $\star j$ from a p-form symmetry with charged operator $W(C)$ over a p-dim manifold

$$\mathrm{d}\star j(x) W(C) = iq\delta_C(x) W(C)$$

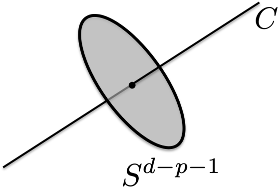
- If C is an infinite p-plane, we integrate over a $(d-p)$-ball B_{d-p}

$$\left\langle \left(\int_{S^{d-p-1}(R)} \star j \right) W(C) \right\rangle = iq \langle W(C) \rangle$$

- If $\langle W(C) \rangle \neq 0$, R independence implies

$$\langle \star j(R) W(C) \rangle \propto \frac{iqc}{R^{d-p-1}}$$

- Power-law decay $\Rightarrow$ Goldstone p-form modes



Lake'18, Hofman-Iqbal'18

EFT for Goldstone modes

Coleman-Wess-Zumino'69, Callan-Coleman-Wess-Zumino'69

- Coset construction
  - Quotient space G/H
  - Given a SSB of G, where $H \subset G$ is preserved

$$\mathrm{Lie}(G) = \{ \underbrace{V_I}_{\text{broken}} \ , \ \underbrace{Z_a}_{\text{unbroken}} \}$$

  - For $\tilde{g} \in G/H$, we can express it as $\tilde{g} = \exp\{\xi^a(x) Z_a\}$
  - ξ^a are the Goldstone modes
- For $g \in G$, $\tilde{g}$ transforms as

$$g \exp\{\xi^a Z_a\} = \exp\left\{\xi'^a Z_a\right\} h(g, \xi^a) \ , \qquad h \in H$$

- Usually, the transformation $g : \xi^a \to \xi'^a$ is complicated (nonlinear)
- How to build actions?

EFT for Goldstone modes

Coleman-Wess-Zumino'69, Callan-Coleman-Wess-Zumino'69

- How to build actions?
- Lie-algebra valued Maurer-Cartan 1-form

$$\omega = \tilde{g}^{-1}\mathrm{d}\tilde{g} \equiv \omega_Z + \omega_V \equiv \omega^a Z_a + \omega^I V_I$$

- Transforms homogeneously under G

$$g : \begin{cases} \omega_Z & \to & h(x)\omega_Z h^{-1} \\ \omega_V & \to & h(x)(\omega_V + \mathrm{d})h^{-1} \end{cases}$$

- At leading order in derivative expansion:

$$\mathcal{L} \propto \mathrm{Tr}\left[\omega \wedge \star\omega\right]$$

Example: EFT for periodic scalar

- For $\theta(x)$, we consider $\Psi(x) = e^{i\theta(x)}$
- The Maurer-Cartan 1-form

$$\omega = -i\Psi^{-1}(x)\mathrm{d}\Psi(x) = \mathrm{d}\theta(x)$$

- To lowest order in derivatives,

$$S = f^2 \int \omega \wedge \star\omega = f^2 \int \mathrm{d}\theta \wedge \star\mathrm{d}\theta$$

- This is more than required!
  - Invariant under U(1) "momentum" symmetry $\theta(x) \to \theta(x) + \alpha$
  - Invariant under U(1) "winding" $(D - p - 2)$ symmetry

Example: EFT for periodic scalar

Witten95

- Understanding the "winding"

$$S = f^2 \int (\mathrm{d}\theta - \beta) \wedge \star(\mathrm{d}\theta - \beta) + 2\pi i \int \beta \wedge \mathrm{d}\eta$$

$$[\beta]: \ \star(\mathrm{d}\theta - \beta) = \pi i f^{-2} \mathrm{d}\eta$$

$$[\eta]: \qquad \mathrm{d}\beta = 0$$

  β dynamical 1-form connection
  η dynamical $(D-p-2)$ connection (Lagrange multiplier)
- Being β pure gauge:

$$\star \mathrm{d}\theta = \pi i f^{-2} \mathrm{d}\eta$$

- We say that θ and η are electromagnetic duals
- For $D = 2$, $p = 0$, this is T duality

Questions & Goals

- How about 1-form symmetries
  - Maxwell as the EFT of Goldstone bosons?
  - Coset construction?
  - Rôle of Wilson and 't Hooft operators?
- How about *p*-form symmetries?
  - Coset construction and EFT?
  - Rôle of charged objects?
- One step further: is SUGRA an EFT of Goldstone bosons?
  - Motivation: Goldstone theorem for non invertible + symmetry operators

    Iqbal-GarcíaEtxebarria'20, FM-Giorgi-Marqués-Rosabal'24
  - Bosonic sector: "generalized" electromagnetism
  - Chern Simons?
  - Higher groups?

Outline

1. Introduction
2. Maxwell as a theory of Goldstone modes
3. Generalized Maxwell for p-forms
4. 5D Maxwell-CS
5. Supergravity
6. Conclusions and prospects

Outline

1. Introduction
2. Maxwell as a theory of Goldstone modes
3. Generalized Maxwell for p-forms
4. 5D Maxwell-CS
5. Supergravity
6. Conclusions and prospects

Outline

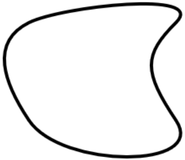
1. Introduction
2. Maxwell as a theory of Goldstone modes
3. Generalized Maxwell for p-forms
4. 5D Maxwell-CS
5. Supergravity
6. Conclusions and prospects

1-form Symmetries & Loop Space

Polyakov'79

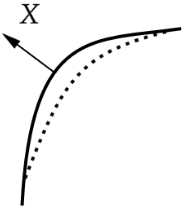
- Charged objects supported by a closed manifold Σ_1
  - The coset representative: $\Psi[\Sigma_1] = \exp i \int_{\Sigma_1} \mathrm{P}\left[A_1\right]$
  - The loop variation under a vector X is defined as

$$\delta_X \Phi[\Sigma_1] = \int_{\Sigma_1} \mathrm{d}\sigma \, \delta X^\mu(\sigma) \frac{\delta \Phi}{\delta X^\mu(\sigma)}$$



  - Maurer-Cartan 1-form

$$\omega_X(\Sigma_1) \equiv \omega(X; \Sigma_1) = -i \Psi^{-1}[\Sigma_1] \delta_X \Psi[\Sigma_1] = \int_{\Sigma_1} \iota_X \mathrm{d} A_1$$



- Maurer-Cartan (Bianchi-like)

$$\delta_X \omega_Y(\Sigma_1) - \delta_Y \omega_X(\Sigma_1) - \omega_{[X,Y]}(\Sigma_1) = 0$$

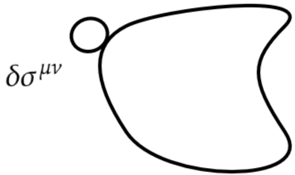
Area & Loop Derivatives

Makeenko-Migdal'79, Migdal'80, Iqbal-McGreevy'21

- An equivalent approach: area derivative

$$\Psi[C \cup \delta C] = \Psi[C] + \sigma^{\mu\nu}(\delta C)\frac{\delta \Psi[C]}{\delta \sigma^{\mu\nu}(\lambda)} \,,$$

$$\sigma^{\mu\nu}(\delta C) = \frac{1}{2}\oint_{\delta C} \mathrm{d}X^{\mu} X^{\nu}$$



- Example: $\phi[C] \equiv \oint_C \mathrm{d}X^{\mu} A_{\mu}$

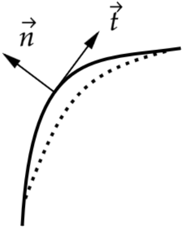
$$\begin{aligned}\phi[C \cup \delta C] - \phi[C] &= \oint_{\delta C} \mathrm{d}X^{\mu} A_{\mu} \\ &= \oint_{\delta C} \mathrm{d}X^{\mu}\left(A(X_0)_{\mu} + (X_0 - X)^{\nu}\partial_{\nu}A_{\mu}(X_0) + \cdots\right)\end{aligned}$$

which implies

$$\frac{\delta \phi[C]}{\delta \sigma^{\mu\nu}(\lambda)} = 2\partial_{[\mu}A_{\nu]}(X_0(\lambda))$$

Area & Loop Derivatives

- Equivalence with Polyakov



$$\frac{\delta\Psi[C]}{\delta X^\mu(s)} = \frac{1}{2}\frac{\mathrm{d}X^\nu}{\mathrm{d}s}\frac{\delta\Psi[C]}{\delta\sigma^{\mu\nu}(s)}$$

- A generic action

$$S[\Psi] = \mathcal{N}\int[\mathrm{d}C]\oint_C \mathrm{d}s\sqrt{h}\frac{\delta\Psi^\dagger[C]}{\delta\sigma_{\mu\nu}(s)}\frac{\delta\Psi[C]}{\delta\sigma^{\mu\nu}(s)} + V(\Psi^\dagger[C]\Psi[C])$$

- For $V = 0$ and $\Psi[C] = \exp i\int_C \mathrm{P}\,[A]$,

$$S[\Psi] = \mathcal{N}\int[\mathrm{d}C]\oint_C \mathrm{d}s\frac{\delta\Psi^\dagger[C]}{\delta\sigma_{\mu\nu}(s)}\frac{\delta\Psi[C]}{\delta\sigma^{\mu\nu}(s)} = -\frac{1}{2}\int_{\Sigma_d}\mathrm{d}A\wedge\star\mathrm{d}A$$

Outline

Area derivative for p-forms

Hidaka-Kawana'23

- Consider $\Psi[C_p] = \exp i \int_{C_p} A_p$
- Variation of $\Psi[C_p]$ for an arbitrary change δC_p,

$$\delta\Psi[C_p] = \int \mathrm{d}^p\xi \sqrt{h}\delta X^\mu(\xi)\frac{\delta\Psi[C_p]}{\delta X^\mu(\xi)} = \frac{1}{(p+1)!}\sigma^{\mu_1\cdots\mu_{p+1}}(\delta C_p)\frac{\delta\Psi[C_p]}{\delta\sigma^{\mu_1\cdots\mu_{p+1}}(\xi)} ,$$

$$\sigma^{\mu_1\cdots\mu_{p+1}}(\delta C_p) = \int_{\delta C_p} \delta X^{[\mu_1}\mathrm{d}X^{\mu_2} \wedge \cdots \wedge \mathrm{d}X^{\mu_{p+1}]}$$

- The action

$$S[\Psi] = \mathcal{N}_p \int [\mathrm{d}C_p] \oint_{C_p} \mathrm{d}^p\xi \frac{\delta\Psi^\dagger[C_p]}{\delta\sigma_{\mu_1\cdots\mu_{p+1}}(\xi)}\frac{\delta\Psi[C]}{\delta\sigma^{\mu_1\cdots\mu_{p+1}}(\xi)} = -\frac{1}{2}\int_{\Sigma_d} \mathrm{d}A_p \wedge \star\mathrm{d}A_p$$

EFT for p- and q-forms

- Coset construction

$$\Psi[\Sigma_p, \Sigma_q] = \exp i \left\{ \int_{\Sigma_p} A_p + \int_{\Sigma_q} A_q \right\}$$

- Maurer-Cartan 1-form

$$\omega_X(\Sigma_p, \Sigma_q) = \int_{\Sigma_p} \iota_X \mathrm{d}A_p + \int_{\Sigma_q} \iota_X \mathrm{d}A_q$$

- Maurer-Cartan equations

$$\int_{\Sigma_p} \iota_X \mathrm{d}^2 A_p + \int_{\Sigma_q} \iota_X \mathrm{d}^2 A_q = 0 \qquad \Rightarrow \qquad \mathrm{d}^2 A_p = \mathrm{d}^2 A_q = 0$$

- If $q = d - p - 2$ and $\star \mathrm{d}A_p = \mathrm{d}A_q$

$$\mathrm{d}^2 A_p = 0 \, , \qquad \mathrm{d} \star \mathrm{d}A_p = 0$$

- Other: read off the Maurer-Cartan components: $\mathrm{d}A_p$ and $\mathrm{d}A_q$

Outline

5D Maxwell-CS

- Action $S = \int \frac{1}{2} \mathrm{d}A \wedge \star \mathrm{d}A + \frac{\kappa}{3} A \wedge \mathrm{d}A \wedge \mathrm{d}A$
- Democratic formulation

$$\begin{aligned} S_{\mathrm{dem}} = \int & \alpha_1 \mathrm{d}A \wedge \star \mathrm{d}A + \alpha_2 (\mathrm{d}B - A \wedge \mathrm{d}A) \wedge \star (\mathrm{d}B - A \wedge \mathrm{d}A) \\ & + \frac{2\alpha_1 - 4\alpha_2}{3} A \wedge \mathrm{d}A \wedge \mathrm{d}A \end{aligned}$$

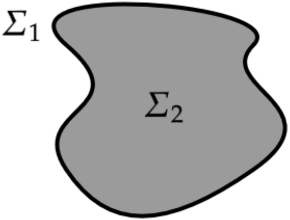
- Higher group: $\delta B = \mathrm{d}\lambda_1 + \mathrm{d}\lambda_0 \wedge A$
- Coset construction (charged objects)

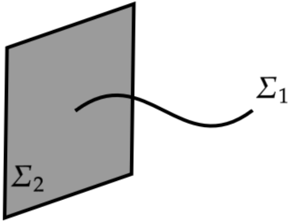
$$\Psi[\Sigma_1, \Sigma_2] = \exp i \left\{ \int_{\Sigma_1} A + \int_{\Sigma_2} B \right\}$$

- String theory: objects localized on the brane?
- Why? Ψ only depends on Σ_1 and Σ_2

5D Maxwell-CS

- What can we do with a Wilson line (Σ_1) and a 't Hooft string (Σ_2)?
  - $\partial\Sigma_2 = \Sigma_1$
  - $\partial\Sigma_1 \subseteq \Sigma_2$ with $\partial\Sigma_2 = \emptyset$
  - Others?





5D Maxwell-CS

- Gauge invariance for $\partial\Sigma_1 \subseteq \Sigma_2$ with $\partial\Sigma_2 = \emptyset$

$$\delta\left(\int_{\Sigma_1} A + \int_{\Sigma_2} B\right) = \int_{\partial\Sigma_1} \lambda_0$$

- We propose a localized field

$$\int_{\Sigma_2} B - \mathrm{d}\theta \wedge A \qquad \delta\theta = \lambda_0$$

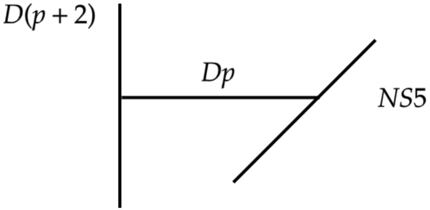
- This requires $\delta B = \mathrm{d}\lambda_1 + \mathrm{d}\lambda_0 \wedge A$ but it's not enough! We need

$$\mathrm{d}\mathrm{d}\theta = -\delta_2(\partial\Sigma_1)$$

$$\delta\left(\int_{\Sigma_1} A + \int_{\Sigma_2} B - \mathrm{d}\theta \wedge A\right) = \int_{\partial\Sigma_1} \lambda_0 - \int_{\Sigma_2} \mathrm{d}\theta \wedge \mathrm{d}\lambda_0 = 0$$

- This was studied in string theory! Witten'95, Hanany-Witten'96

Hanany–Witten & Freed–Witten



- In type IIB

D5	x	x	x				x	x	x	
NS5	x	x	x	x	x	x				
D3	x	x	x							x

Hanany–Witten & Freed–Witten

- Here we require $\mathrm{d}\mathcal{F} = H + \delta_3(\partial\Sigma_4)$ at the boundary of the D3
- The presence of the NS5 is crucial, as it requires $\oint H \neq 0$!
- Freed-Witten: branes in the presence of nontrivial H are anomalous
- Anomaly cancels by making another brane end on it Maldacena-Moore-Seiberg'01

$$[H] = \cancel{[W]} + [\delta] \tag{1}$$

- Punchline: The HW brane creation is required to cancel the FW anomaly. Conversely, the FW condition is required to explain the HW brane creation.
- Generalization to M theory Witten'99

"This reduces to (1) *in the appropriate situation, and I suspect it holds in general"*

Coset for Maxwell-CS

- From Freed-Witten,

$$[F_2] = [\delta_2] \quad \Rightarrow \quad F_2 = \mathrm{d}C_1 + \delta_2(\partial\Sigma_1)$$

- Coset construction

$$\Psi[\Sigma_1, \Sigma_2] = \exp i \left(\int_{\Sigma_1} A + \int_{\Sigma_2} B - C_1 \wedge A \right)$$

- C_1 not entirely determined
  - If $C_1 = \mathrm{d}\theta + A$ is gauge invariant (like $\mathcal{F}$) then we have to prescribe $\delta_X \theta$
  - If C_1 is gauge invariant (like B) then we have to choose a section of the bundle and integrate over the gauge transformations of C_1 (or, equivalently, over $\mathrm{d}\theta$)
- Inherent integration over $\mathrm{d}\theta$ makes the coset noninvertible

Coset for Maxwell-CS

- We can do the path integral over θ and average over all the gauge transformations

$$\Psi_{\text{eff}}[\Sigma_1, \Sigma_2] = \int [D\theta] \Psi[\Sigma_1, \Sigma_2]$$

- To do so, we have to do it legally, including every single term involving θ

$$\Psi_{\text{eff}}[\Sigma_1, \Sigma_2] = \int [D\theta] \Psi[\Sigma_1, \Sigma_2] \exp\left\{ i \int_{\Sigma_2} (d\theta - A) \wedge \star(d\theta - A) \right\}$$

- The Σ_2 integral can be written as

Witten'96

$$\Psi_{\text{eff}}[\Sigma_2] = \exp\left\{ i \int_{\Sigma_2} B \right\} \int [Da] \exp\left\{ -ik \int_{\Sigma_2 \times \mathbb{R}^+} a \wedge da \right\}, \qquad a|_{t=0} = A$$

- Varying $\Psi[\Sigma_2]$ w.r.t. the surface along X:

$$\delta_X \Psi_{\text{eff}}[\Sigma_2] = i \Psi_{\text{eff}}[\Sigma_2] \int_{\Sigma_2} \mathrm{P}\left[\iota_X (dB - A \wedge dA) \right]$$

An action for Maxwell-CS

- Maurer-Cartan 1-form

$$\omega_X(\Sigma_1, \Sigma_2) = \int_{\Sigma_1} \mathrm{P}\left[\iota_X \mathrm{d}A\right] + \epsilon_p \int_{\Sigma_2} \mathrm{P}\left[\iota_X(\mathrm{d}B - A \wedge \mathrm{d}A)\right]$$

- Maurer-Cartan equation
    - Fixed gauge transformations $\Rightarrow$ Field strengths $\Rightarrow$ Bianchi's
    - Bianchi's + duality relations = EOMs
- Effective action
    - Democratic action + duality relations

$$S[\Psi] = \mathcal{N} \int [\mathrm{D}\Sigma_1][\mathrm{D}\Sigma_2]\, \omega(\Sigma_{1,2}) \wedge \star\omega(\Sigma_{1,2})$$

Outline

Supergravity (bosonic sector)

- 11D Supergravity
  - M2 branes ending on M5 branes
  - CS coefficient entirely fixed by SUSY
  - CS origin: anomaly inflow arising from the self-dual 2-form in the M5
- Type II
  - H flux induced by NS5
    - Freed-Witten & Hanany-Witten
  - IIA CS origin: anomaly inflow arising from the self-dual 2-form in the NS5
  - IIB CS origin: anomaly inflow arising from the self-dual 2-form in the KKM?

Outline

1. Introduction
2. Maxwell as a theory of Goldstone modes
3. Generalized Maxwell for p-forms
4. 5D Maxwell-CS
5. Supergravity
6. Conclusions and prospects

Conclusions

1. Higher form and and higher group symmetries in supergravity
2. Spontaneous symmetry breaking of higher form symmetries
3. EFT for Goldstone modes and the coset construction (Maurer-Cartan)
4. Loop space and area derivative
   - Maxwell as the EFT of Goldstone modes for SSB of 1-form symmetry
5. Generalization to higher p-forms
   - Generalized electromagnetism with no higher group
   - Coset $\Psi[\Sigma_p]$

Conclusions

6. Maxwell-CS
   - We have built a coset $\Psi[\Sigma_1, \Sigma_2]$ with good properties
   - Manifolds with boundaries enter the game
   - Charged objects restricted by Freed-Witten anomalies
   - Hanany-Witten: branes ending on branes
   - Noninvertibility still around ($\int [\mathrm{D}\theta]$)
   - Effective description
     - Maurer-Cartan eq + duality relations = EOMs
     - Maurer-Cartan components + duality relations = EOMs
7. 11D SUGRA: straightforward extension of Maxwell-CS
8. Type II
   - D-branes: Maurer-Cartan eq's for RR fields
   - NS5: Necessary for the EOM of B_2

Prospects

- Finish the paper!
- Other applications: Axion-Maxwell (type IIA analogue)
- Loop space: Hodge operator and other formal problems?
- Supersymmetry? Goldstinos?
- How are they related? CS = higher group = Witten effect?
- Green-Schwarz mechanism in this set-up? Orientifolds?
- Spacetime symmetries and Goldstone modes
- Higher derivatives? No assumption taken until duality relation

Prospects

- Finish the paper!
- Other applications: Axion-Maxwell (type IIA analogue)
- Loop space: Hodge operator and other formal problems?
- Supersymmetry? Goldstinos?
- How are they related? CS = higher group = Witten effect?
- Green-Schwarz mechanism in this set-up? Orientifolds?
- Spacetime symmetries and Goldstone modes
- Higher derivatives? No assumption taken until duality relation

Thanks!