Insights on Black Hole Stability from Light Towers

Matteo Zatti

Based on [**2502.02655**], [**2505.15920**], [**2507.17857**] and work in progress

in collaboration with **A. Castellano**, D. Lüst and C. Montella.

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MAX-PLANCK-INSTITUT
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A simple question

- A BH is a solution of a gravitational EFT.
- It receives corrections due to the UV completion of the EFT.

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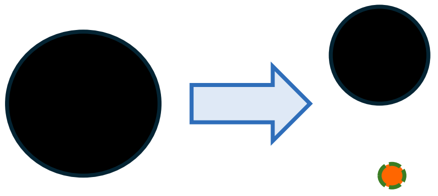
Thermodynamics

Stability

$$S = \frac{A}{4} + \dots$$



The initial ideas and questions

Thermodynamics

- **Integrating out** UV degrees of freedom we produce **higher derivative corrections** to the EFT.

At the cutoff scale, the EFT breakdown. What happens to the **black hole thermodynamics** at EFT cutoff scale?

- We study the full **infinite tower** of corrections to the entropy

Stability

- A charged BH can **discharge via pair production**.

Which massive particles make an extremal charged BH unstable?

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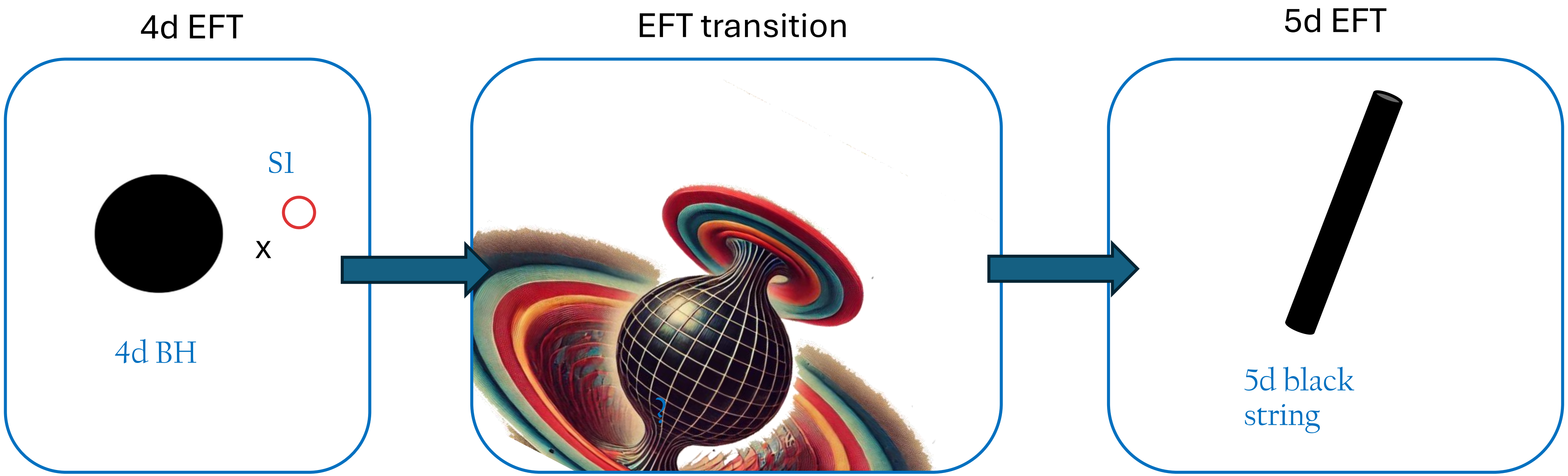
The results: 1) gluing across the cutoff scale

Thermodynamics

Stability

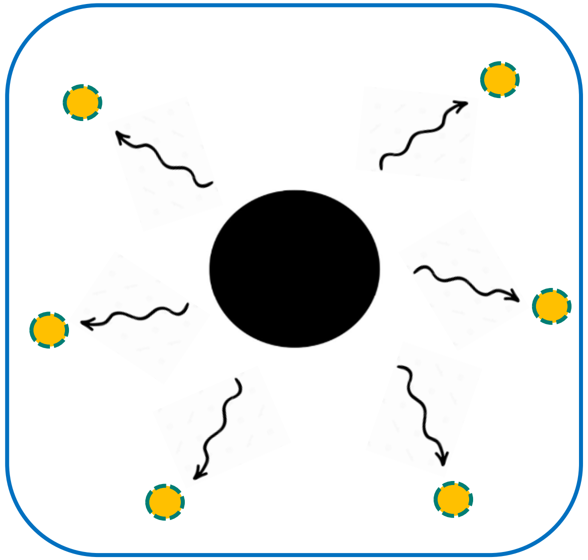


The results: 2) threshold for quantum decay

Thermodynamics

Stability

4d EXT BH



$(m, q_{ee}, q_{me}, spin)$

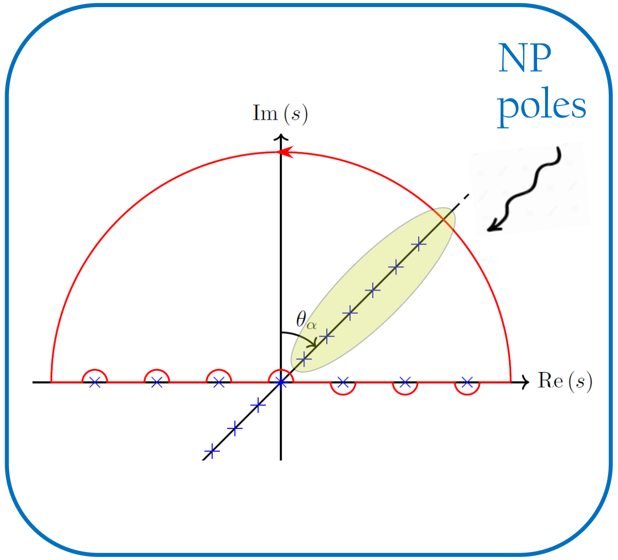
$$q_{ee}^2 \geq R^2 m^2 + C_{spin}\,(2\, q_{me} \; - C_{spin})$$

The results: 3) an interesting correspondence

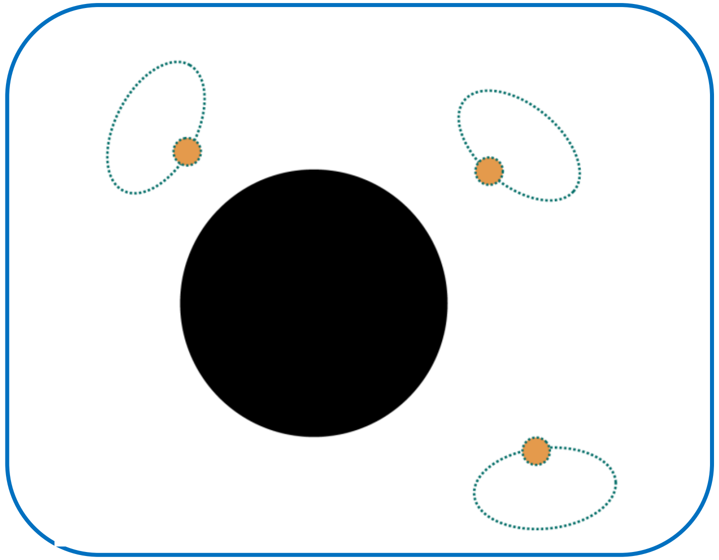
Thermodynamics

Stability

NP corrections to Entropy



Pair production of
virtual particles



Outline

Part I:	Non perturbative (NP) corrections to BH entropy
Part II:	Schwinger pair production (SPP)
Part III:	NP = Virtual SPP ?
Part IV:	Conclusion

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Remarks

- An **EFT** is not UV-complete, is valid up to a **cut-off scale** Λ.
- We can easily produce an EFT **integrating out massive fields**.
- Examples: Euler-Heisenberg theory, Kaluza-Klein theory,…

Consider a free scalar in $d+1$ dimensions. We compactify the direction
$z \sim z + 2\,\pi\,R$

$$\phi(x,z) = \sum_n e^{i\,n\frac{z}{R}}\,\phi^{(n)}(x)$$

The Fourier modes are massive fields in d dimensions

$$\left(\Delta + m_n^2\right)\phi^{(n)} = 0 \qquad\qquad m_n = n\,R^{-1}$$

We can extract and EFT for the massless mode integrating out the massive fields. It is valid up to $M_{kk} = R^{-1}$

Remarks

- Quantum effects can be incorporated as **higher derivative corrections** to gravitational EFTs

$$S_{\mathrm{EFT},\,d} \supset \frac{1}{2\kappa_d^2}\int \mathrm{d}^d x\sqrt{-g}\left(\mathcal{R} - 2\Lambda_{\mathrm{c.c.}} + \sum_{n>2}\frac{\mathrm{O}_n(\mathcal{R})}{\Lambda_{\mathrm{QG}}^{n-2}}\right) + \int \mathrm{d}^d x\sqrt{-g}\sum_{n>2}\frac{\mathrm{O}_n(\mathcal{R})}{M^{n-d}}$$

- Several scales appears in the EFT, associated with an **EFT breakdown**

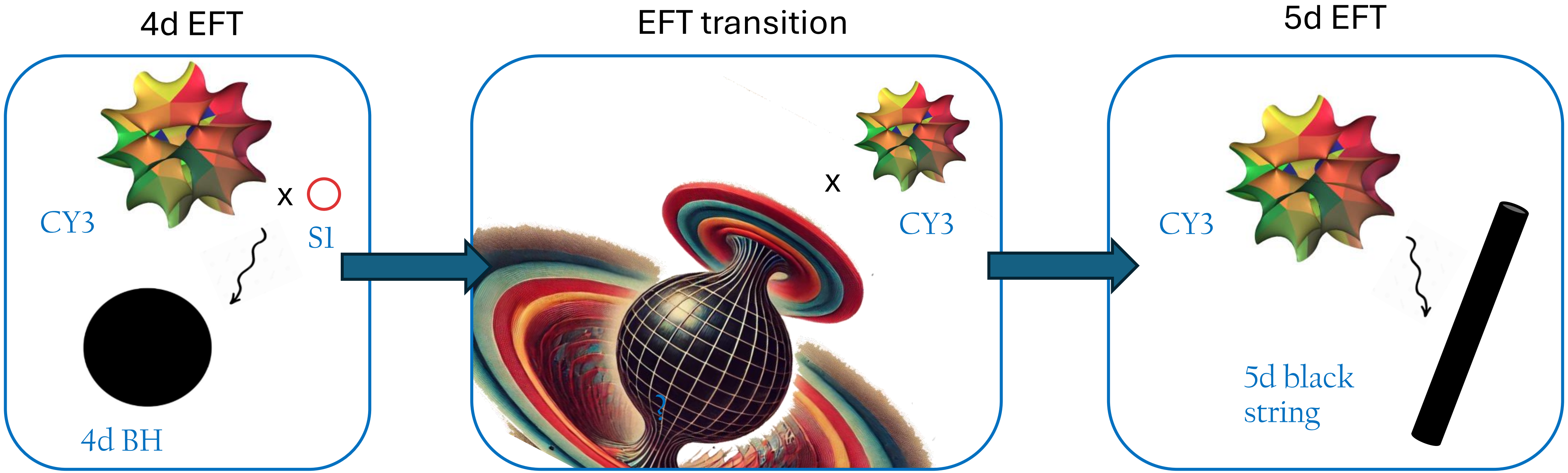
- In the case of a **Kaluza-Klein scale** the theory organizes in a higher dimensional EFT

What does a **protected** and **stable black hole** see at the **EFT transition**?

Result 1: Beyond the EFT cutoff

We study **(non)perturbative corrections** to **CY BPS black holes** in the **large volume approximation** due to **D0 branes**. The corrections to the **BH entropy glue** a 4d BH and a 5d black object. They are finite at the **EFT transitions**.

[Castellano, M.Z. ‘25]



Sketch derivation

$4d\ \ N = 2\ \ SUGRA$

(IIA on CY3)

$Higher\ Derivative\ F - terms$

$$\mathcal{L}_{\text{h.d.}} \supset \sum_{g\geq 1} \int \mathrm{d}^4\theta\, \mathcal{F}_g(\mathcal{X}^A)\ \left(\mathcal{W}^{ij}\mathcal{W}_{ij}\right)^g\ +\ \text{h.c.}$$

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$Wald\ Entropy$

$$S_{BH} = \frac{A}{4} + \sum_{g\geq 0} S^{(2g)}\ \alpha^{2g}$$

$$\alpha \overset{\text{hor}}{=} \frac{r_{D0}}{r_{BH}}$$

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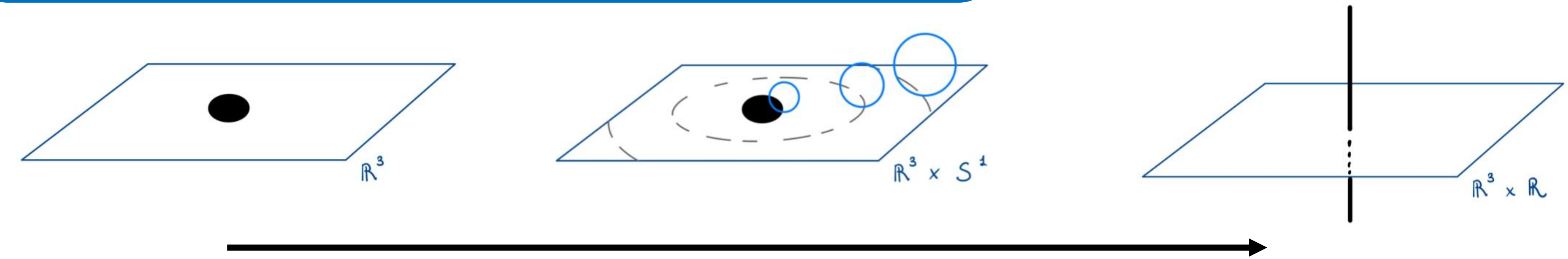


BH in 4d $\quad \alpha \ll 1$

$r_h \sim r_{D0}$

$\alpha \sim 1$

$\alpha \gg 1$ Black string in 5d

More details

- Quantum-corrected (Wald) **entropy** [Lopes-Cardoso, Wit, Mohaupt '99]

$$\mathcal{S}_{\mathrm{BH}} = \pi \left[|\mathcal{Z}|^2 + 4\mathrm{Im}\, \left(\Upsilon \partial_\Upsilon F(Y,\Upsilon) \right) \right]$$

Generalized BH central charge

Deviation Area-law

- The generalized prepotential for a **generic CY**

$$F(Y,\Upsilon) = \frac{D_{abc} Y^a Y^b Y^c}{Y^0} + d_a \frac{Y^a}{Y^0} \Upsilon + G(Y^0,\Upsilon) + \mathcal{O}\left(e^{2\pi i z^a}\right)$$

Tree level

g = 1 corrections

(loop corrections in 5d)

Higher-genus corrections

(4d loop corrections)

- **Leading** quantum corrections

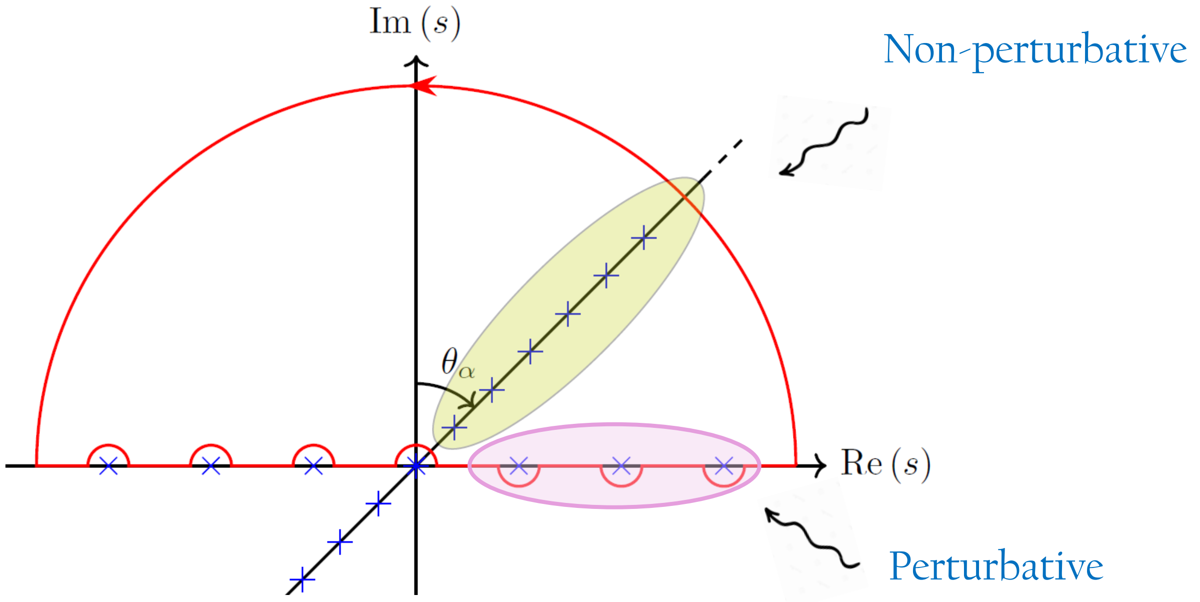
$$G(Y^0,\Upsilon) = -\frac{i}{2(2\pi)^3} \chi_E(X_3) (Y^0)^2 \sum_{g=0,2,3,\dots} c^3_{g-1}\, \alpha^{2g} + \dots$$

More details

- The **Gopakumar-Vafa** representation of the topological free energy [Gopakumar, Vafa '98]

$$G(Y^0, \Upsilon) = \frac{i}{2(2\pi)^3}\,\chi_E(X_3)\,(Y^0)^2\,\mathcal{I}(\alpha) \qquad \mathcal{I}(\alpha) = \frac{\alpha^2}{4}\sum_{n\in\mathbb{Z}}\int_{0^+}^{\infty}\frac{\mathrm{d}s}{s}\,\frac{1}{\sinh^2(\pi n\alpha s)}\,e^{-4\pi^2 n^2 i s}$$

- We can write $I(\alpha)$ as a **contour integral**



More details

- In the case of the **black string**

$$\mathcal{I}^{(p)}(\alpha) = \alpha^2 \sum_{n=1}^{\infty} n\,\mathrm{Li}_1\left(e^{-\alpha n}\right)$$

$$\mathcal{I}^{(np)}(\alpha) \;=\; -2\pi i\alpha \sum_{n=1}^{\infty}\left(n\,\mathrm{Li}_1\left(e^{-\frac{4\pi^2 n}{\alpha}}\right) + \frac{\alpha}{4\pi^2}\,\mathrm{Li}_2\left(e^{-\frac{4\pi^2 n}{\alpha}}\right)\right)$$

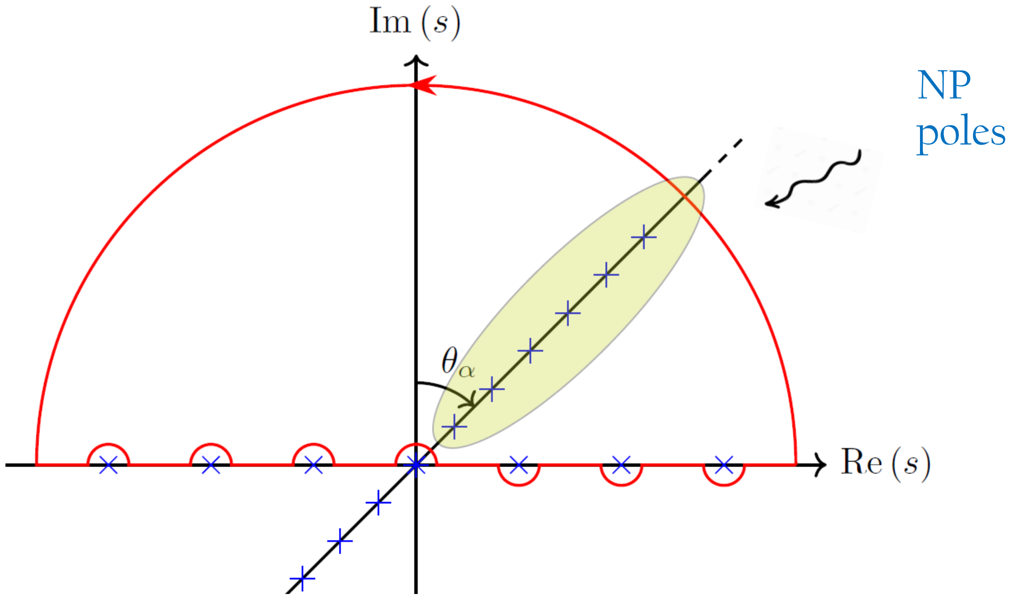
- The **quantum corrected entropy**

$$\mathcal{S}_{\mathrm{BH}} \;=\; 2\pi\sqrt{\frac{1}{6}|\hat{q}_0|\left(\mathcal{K}_{abc}p^a p^b p^c + c_{2,\,a}p^a\right)}\left(1 - \frac{\chi_E(X_3)\,Y^0\,\alpha^2}{(2\pi)^3|\hat{q}_0|}\sum_{n=1}^{\infty} n^2\,\mathrm{Li}_0\left(e^{-\alpha n}\right)\right)^{-1/2}$$
$$+\frac{\chi_E(X_3)}{4\pi^2}(Y^0)^2\alpha^2\left(\sum_{n=1}^{\infty} n\,\mathrm{Li}_1\left(e^{-\alpha n}\right) + (Y^0)^{-1}\sum_{n=1}^{\infty} n^2\,\mathrm{Li}_0\left(e^{-\alpha n}\right)\right)$$

Are black strings special?

- Correction for more general CY BHs are **finite**! [Castellano, Lüst, Montella, **M.Z.** ‘25]

- The **structure** of the **NP corrections** depends on the charges we turn on!



Outline

Part I: Non perturbative (NP) corrections to BH entropy

Part II: Schwinger pair production (SPP)

Part III: NP = Virtual SPP ?

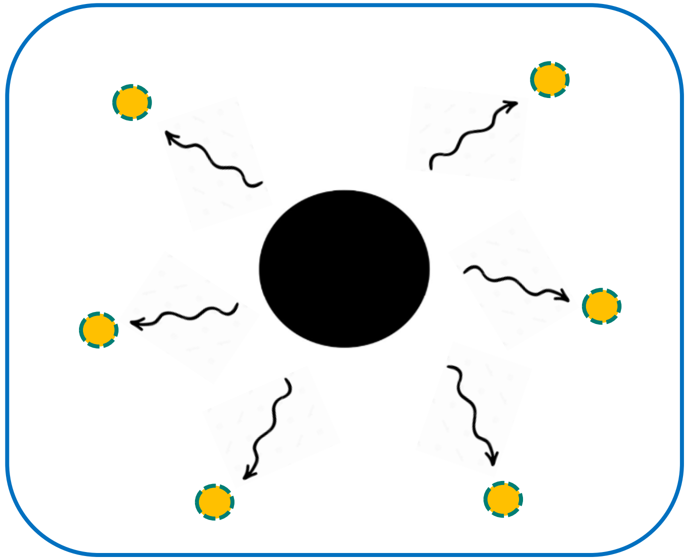
Part IV: Conclusion

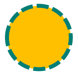
Result 2: quantum decay threshold

We study which condition a **dyon** must satisfy to be **pair produced**. We work in $\boldsymbol{AdS_2} \times \boldsymbol{S^2}$ with constant electric and magnetic fields.

[Castellano, Montella, **M.Z.** to appear]

4d EXT BH





$(m, q_{ee}, q_{me}, spin)$

$$q_{ee}^2 \geq R^2 m^2 + \mathrm{C}_{spin}\,(2\, q_{me} \; - \mathrm{C}_{spin})$$

Remarks

- **Integrating** out a charged scalar in minimal QED

$$\int \mathcal{D}A\,\mathcal{D}\phi\,\mathcal{D}\phi^*\, e^{-S^{\mathrm{E}}_{\mathrm{QED}}[A,\phi]} = \int \mathcal{D}A\, e^{-\Gamma_{\mathrm{E}}[A]}$$

- Leading order correction is the **1-loop determinant** of the **kinetic operator**

$$\Delta\Gamma_{\mathrm{E}}[A] = -\int_{\epsilon_{\mathrm{uv}}}^{\infty} \frac{ds}{s}\, e^{-sm^2}\, \mathrm{Tr}\left(e^{-sH_{\mathrm{E}}}\right)$$

- The system is **unstable** when

$$\mathrm{Im}\,(\Delta\,\Gamma_{\mathrm{E}}) \neq 0$$

Sketch derivation

- Consider a **particle** with charges $(q_A{}', p^{A}{}')$ moving in (charged) $\boldsymbol{AdS_2} \times \boldsymbol{S^2}$

$$S_{wl} \supset \int_{\Sigma_2} p^{A\prime} G_A - q_A{}' F^A \equiv -\frac{q_{ee}}{R^2}\int \omega_{AdS_2} + \frac{q_{me}}{R^2}\int \omega_{S^2}$$

- The solution of the corresponding **Landau problem** for a **spin 0** boson

$$\mathbf{S}^2: \quad \rho = 2\left(q_{me} + n + \frac{1}{2}\right) \qquad \mathcal{E} = \frac{1}{2R^2}\left(n + \frac{1}{2} + \frac{n(n+1)}{2q_{me}}\right)$$

$$\mathbf{AdS}^2: \quad \rho = \frac{V_{\mathrm{AdS}}}{2\pi R^2}\frac{\lambda \sinh(2\pi\lambda)}{\cosh(2\pi\lambda) + \cosh(2\pi q_{ee})} \qquad \mathcal{E} = \frac{1}{2R^2}\left(\lambda^2 + \frac{1}{4} - q_{ee}^2\right)$$

- The **1-loop determinant**

$$\log \mathcal{Z}_\phi = -\frac{V_{\mathrm{AdS}}}{2\pi R^2}\sum_{n\geq 0}\int_{\mathbb{R}+i\delta_t} dt\, \delta_n^{\mathbb{S}^2}\, W_B^{\mathrm{AdS}_2}(t)\, \frac{e^{-\sqrt{\Delta_n^2(\epsilon^2+t^2)}}}{\sqrt{t^2+\epsilon^2}}$$

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Encodes information about stability

$$\Delta_n^2 = (\mathrm{m}^2 - \mathrm{q}_{\mathrm{ee}}^2 - \mathrm{q}_{\mathrm{me}}^2) + \left(\mathrm{n} + \mathrm{q}_{\mathrm{me}} + \frac{1}{2}\right)^2$$

C_{spin}

- One can show

$$\boxed{\mathrm{Im}\log Z_\phi = 0} \quad \Longleftrightarrow \quad \boxed{\Delta_0^2 \geq 0}$$

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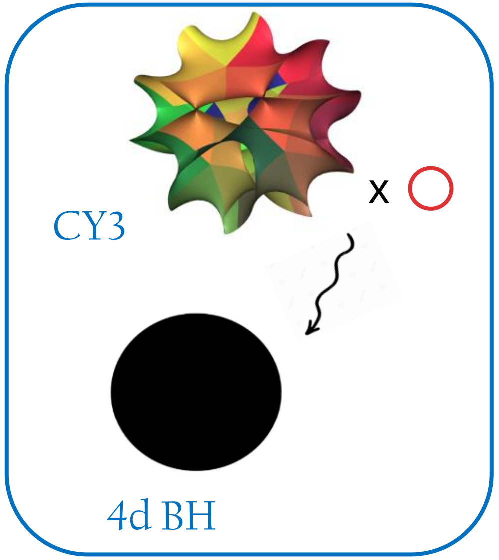
Motivations

NP corrections to **CY BHs** from a **D0 tower** have an interesting **structure**

[Castellano, M.Z. ‘25]
[Castellano, Lüst, Montella, M.Z. ‘25]

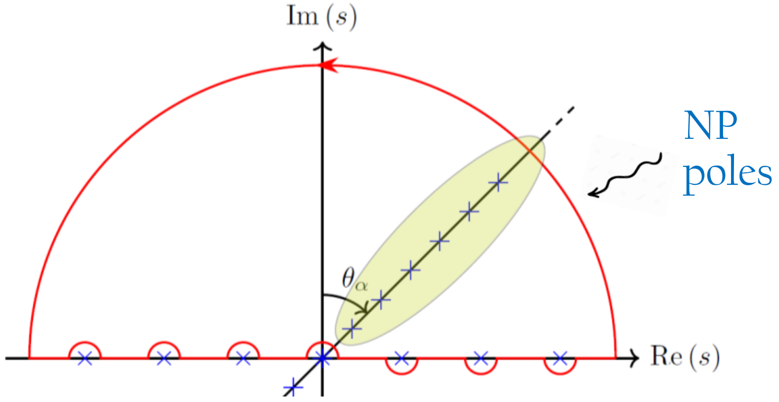
4d N=2



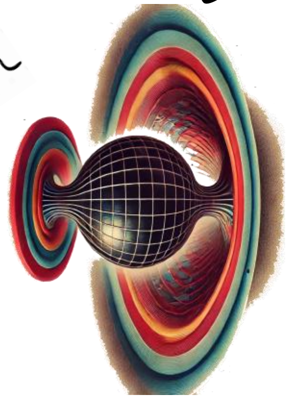
✗ D0-D2-D4: $q_{me} = 0$

✗ D2-D6: $q_{ee} = 0$

✓ D0-D2-D4-D6: $\tilde{m} > |q_{ee}|$



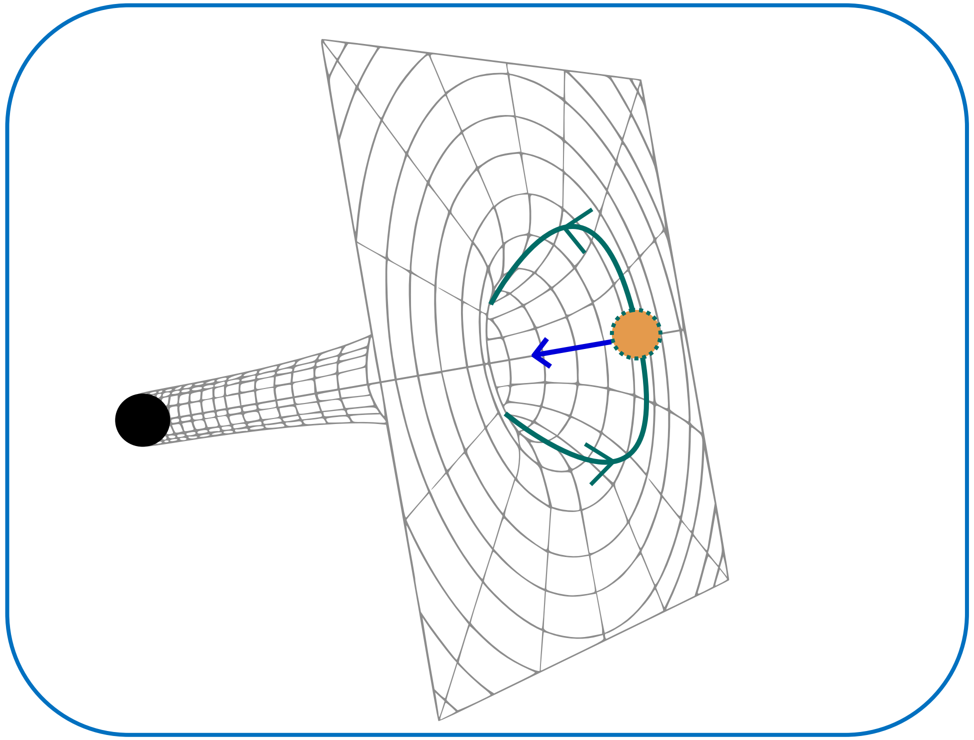
Black string
in 5d N=1



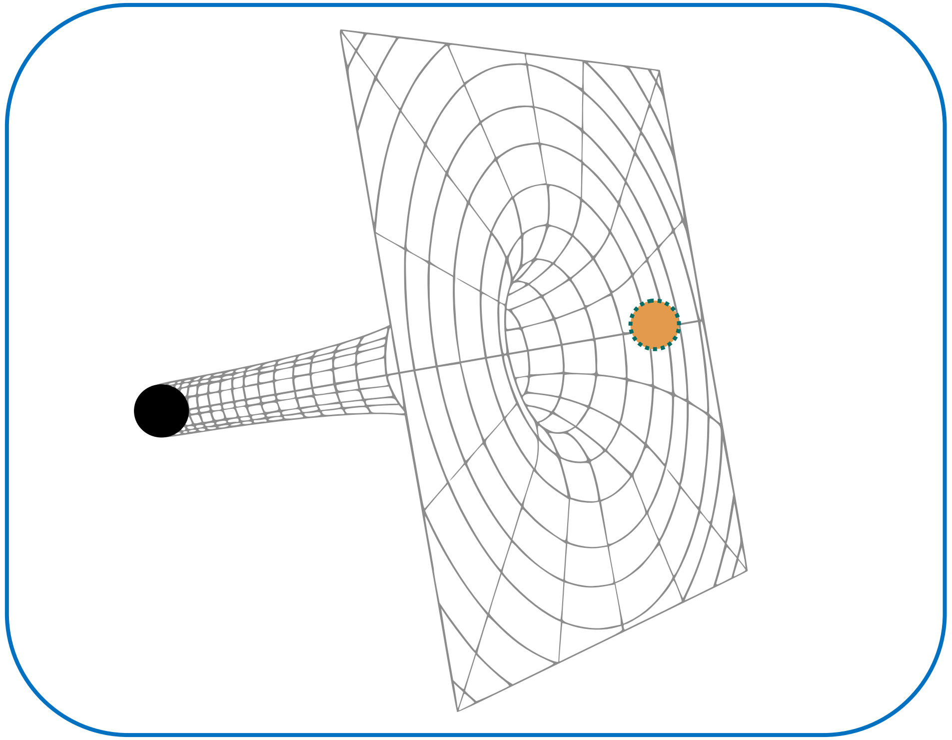
Taub-NUT
in 5d N=1

Motivations

Not Aligned / Attraction



Aligned / No force



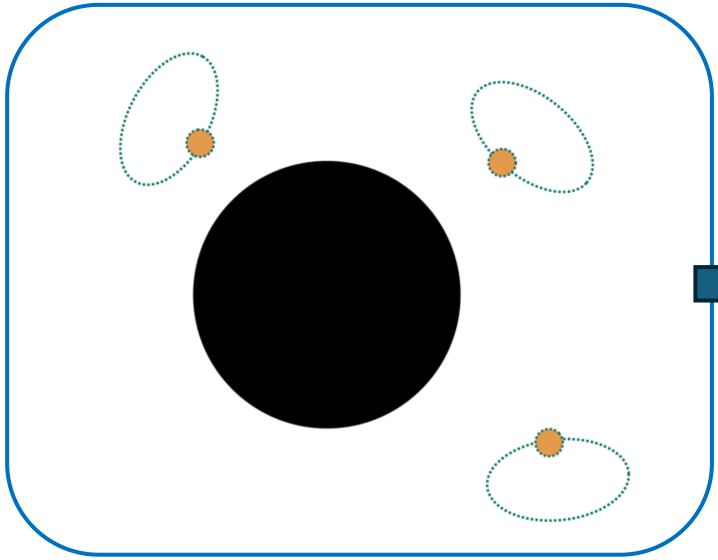
$q_{ee} = 0$ does not have a classical interpretation!

Result 3: NP corrections to the entropy as (virtual) SPP?

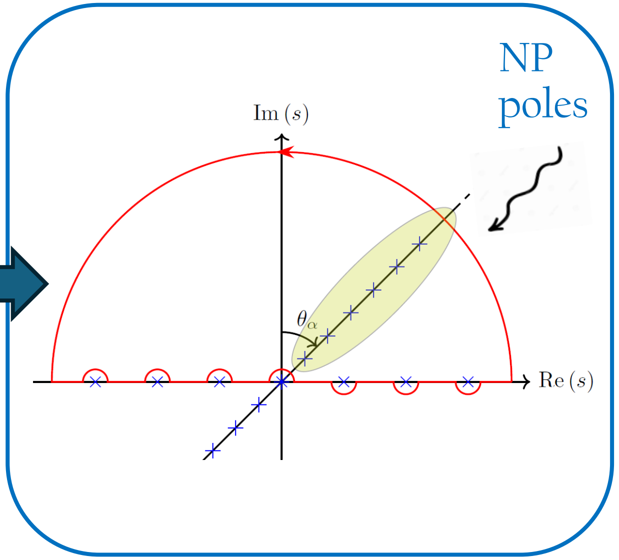
We study **BPS** particles **(virtual) pair production** in $AdS_2 \times S^2$. We obtain the same corrections we got integrating out particles in Minkowski and evaluating the prepotential at the BH horizon.

[Castellano, Montella, **M.Z.** to appear]

Pair production of virtual particles



NP corrections to Entropy



Sketch derivation

We compute the 1-loop determinant in **integrating out particles** in $AdS_2 \times S^2$

$$\log \mathcal{Z} = -\int_{\epsilon}^{\infty} \frac{d\tau}{\tau} \mathrm{Tr}\left[e^{-\tau\left(-\mathcal{D}^2_{\mathrm{AdS}_2\times S^2}+m^2\right)}\right] \propto \int dt\, du\; f^{\mathrm{AdS}_2}_{B,F}(t)\; f^{\mathbb{S}^2}_{B,F}(u)\; I_\Delta(t,u)$$

In the case of a $\boldsymbol{N=2}$ **matter multiplets** and a **BPS black hole** background, the calculation gives

$$\log \mathcal{Z} = \frac{V_{\mathrm{AdS}}}{4\pi R^2}\left[4q_{ee}q_{me}\tan^{-1}\left(\frac{q_{ee}}{q_{me}}\right) + \Re\int_0^{\infty} \frac{d\tau}{\tau}\frac{e^{i\,\tau\,\bar{Z}_{\mathrm{BH}}\,Z}}{\sinh^2\left(\frac{\tau}{2}\right)}\right]$$

Same as Minkowski topological free energy evaluated at the BH horizon

We compare with the Wald entropy via

$$S_{BH} = \log \mathcal{Z} - i q_A \phi^A$$

Open question

Why the two approaches match?

Outline

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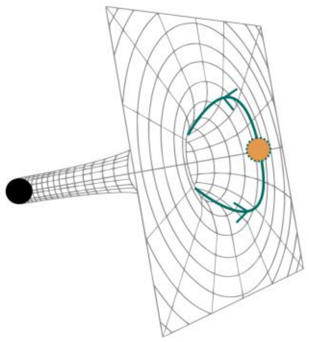
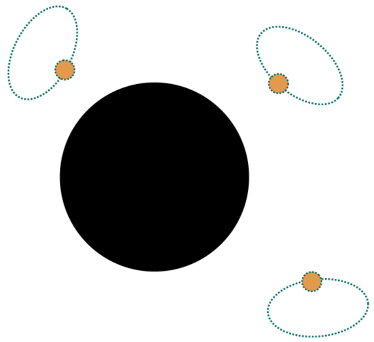
Summary

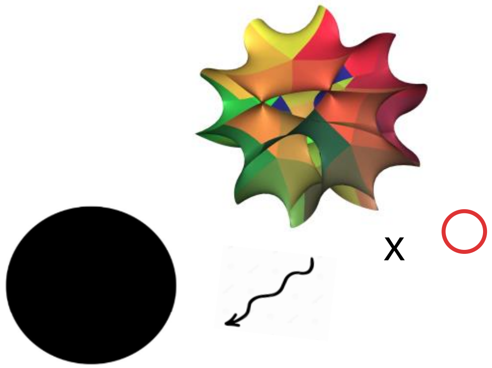
- We **glued** explicitly a 4d BH with a 5d black string **across** the **EFT transition**

- We showed when **EXT black holes** are **unstable** under **pair production** of dyonic particles

- We related **multiple descriptions**: cancellation of forces, pair production, NP correction to CY black holes…







Outlook [WIP]

- **Resummation** of higher derivative corrections in **modified gravities**?

- **Small BHs** with NP corrections

- Understand the fate of NP effects in the general case: revisit the **Gopakumar-Vafa** computation in **AdS2xS2**

- Instability of **more vacua**: de Sitter, black branes, …

Thank you for the attention!