GRASS-SYMBHOL MEETING

The non-relativistic limit of Type II Supergravity theories

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Based on the work with J. Fernandez-Melgarejo, G. Giorgi & L. Romano

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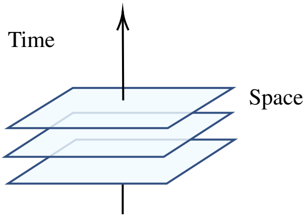
Non-Lorentzian Physics

Structure of spacetime

Spacetime is foliated and described by two degenerate metrics.



Approach

- Derivation methods: Limits, expansions, null-reduction...
- Limits allow us to inspect corners of Lorentzian theories.

Non-Lorentzian Physics

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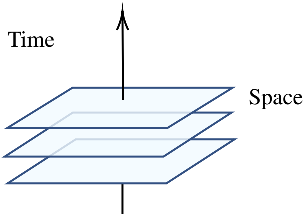

Non-Relativistic type II supergravity

- Non-relativistic limits of AdS/CFT?
- Non-relativistic quantum gravity?
- Can non-relativistic theories be used to learn more about ordinary string theory?

Non-Lorentzian Physics

Structure of spacetime

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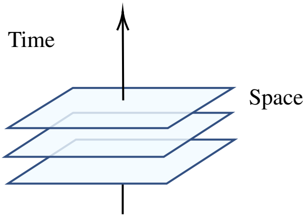
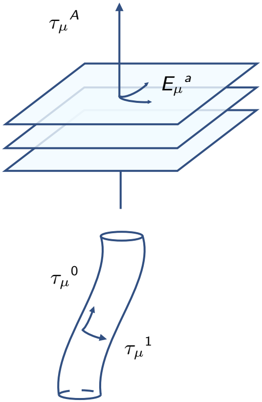


Goal

- Non-relativistic Type II supergravity
  - 10 D
  - String foliation
  - Democratic formulation
- Find solutions

1. Introduction

String foliation



Vielbein formalism

$$g_{\mu\nu} = E^{\hat{A}}{}_\mu E^{\hat{B}}{}_\nu \eta_{\hat{A}\hat{B}}$$

Index convention

$\mu, \nu, \ldots$ 10-dim curved index

$\hat{A}, \hat{B}, \ldots$ 10-dim flat index $\left(\hat{A} = \{A, a\}\right)$

$A, B, \ldots$ Longitudinal flat index $(A = 0, 1)$

$a, b, \ldots$ Transversal flat index $(a = 2, \ldots, 9)$

Type II Supergravity in Democratic Formulation

Bosonic field content

- NSNS sector: describes the geometry of the space time.

$$E_\mu{}^{\hat{A}}, B_{\mu\nu}, \Phi$$

- RR sector: All potentials!

$$C_{(2n-1)}$$

where $n \leq 5$ in massive IIA and $n = 1/2 ... 9/2$ in IIB.

Symmetries

$$\delta B = \mathrm{d}\Sigma_{(1)},$$

$$\delta C_{(2n-1)} = \mathrm{d}\Sigma \wedge e^B,$$

$$\delta E^{\hat{A}}{}_\mu = \Lambda^{\hat{A}}{}_{\hat{B}} E^{\hat{B}}{}_\mu$$

$$\delta\Phi = 0$$

Field Strengths

$$H = dB\,,$$

$$G_{(2n)} = dC_{(2n-1)} - dB \wedge C_{(2n-3)} + \mathrm{m}e^B$$

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Field Strengths

$$H = dB\,,$$

$$G_{(2n)} = dC_{(2n-1)} - dB \wedge C_{(2n-3)} + \boxed{me}^{B}$$

Stueckelberg term!

Type II Supergravity in Democratic Formulation

Bosonic action

Democratic formulation of the RR sector unifies all p-form gauge fields into a single structure. As a result, there are no Chern–Simons terms. The bosonic action is:

$$\mathcal{S}_{\mathcal{II}} = -\frac{1}{2k^2}\int d^{10}x\sqrt{-g}\left(e^{-2\Phi}\left[R + 4(\partial\Phi)^2 - \frac{1}{2\cdot 3!}H_{\mu\nu\rho}H^{\mu\nu\rho}\right]\right.$$
$$\left. - \sum_n \left(\frac{1}{4(2n)!}G_{\mu_1\cdots\mu_{(2n)}}G^{\mu_1\cdots\mu_{(2n)}}\right)\right.$$

1.Non-Relativistic Limit

Non-Relativistic Limit

Steps

1. Redefinition of the relativistic fields in terms of non relativistic ones by introducing a dimensionless parameter c.
2. Substitute the ansatze in the relativistic expression.
3. Limit $c \to \infty$

Careful!: Transformations must be finite for the limit to be well-defined.

Einstein-Hilbert gravity

Action

$$\mathcal{S}_{EH} = \int d^{10}x \sqrt{-g}\; R$$

Ansatz

$$E_\mu{}^A = c\,\tau_\mu{}^A,$$

$$E_\mu{}^a = e_\mu{}^a,$$

$$\Lambda_{AB} = \lambda_{AB},$$

$$\Lambda_{ab} = \lambda_{ab}$$

$$\Lambda_{Aa} = c^{-1}\lambda_{Aa}$$

Finite transformation rules!

$$\delta\tau^A = \lambda^A{}_B\tau^B$$

$$\delta e^a = \lambda^a{}_b e^b + \lambda^a{}_B\tau^B$$

Substitution

$$R = -c^2\frac{1}{4}t_{abA}t^{abA} + \mathcal{O}(c^0)$$

$$t_{ab}{}^A = 2e^\mu{}_a\, e^\nu{}_b\, \partial_{[\mu}\tau_{\nu]}{}^A$$

Divergent action as $c \to \infty$

NSNS Supergravity [Bergshoeff et al, 2021]

Action

$$\mathcal{S}_{NSNS} = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\,e^{-2\Phi}\left[R + 4(\partial\Phi)^2 - \frac{1}{2\cdot 3!}H_{\mu\nu\rho}H^{\mu\nu\rho}\right]$$

Ansatz

$$\Phi = \phi + \ln c$$

$$B_{\mu\nu} = c^2\tau^A{}_\mu\tau^B{}_\nu\,\epsilon_{AB} + b_{\mu\nu}$$

$$H_{\rho\mu\nu} = 3c^2 t_{[\rho\mu}{}^A\tau_{\nu]}{}^B\epsilon_{AB} + 3\partial_{[\rho}b_{\mu\nu]}$$

$$\Sigma = \sigma$$

Finite transformation rules!

$$\delta b_{(2)} = -2\lambda^A{}_d\,e^d\wedge\tau^B\epsilon_{AB} + \mathrm{d}\sigma$$

$$\delta\phi = 0$$

NSNS Supergravity [Bergshoeff et al, 2021]

Action

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$$H_{\rho\mu\nu} = 3c^2 t_{[\rho\mu}{}^A \tau_{\nu]}{}^B \epsilon_{AB} + 3\partial_{[\rho} b_{\mu\nu]}$$

$$\Sigma = \sigma$$

Finite transformation rules!

$$-\frac{1}{2\cdot 3!}H_{\rho\mu\nu}H^{\rho\mu\nu} = \frac{1}{4}c^2 t_{abA}t^{abA} + \mathcal{O}(c^0).$$

The divergences from R cancel against those from H^2. The final NS-NS action scales as $\mathcal{O}(c^{-2})$

Type II Supergravity in Democratic Formulation

Bosonic action

$$\mathcal{S}_{II} = \mathcal{S}_{NSNS} - \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\sum_n \Big(- \frac{1}{4(2n)!} G_{\mu_1\cdots\mu_{(2n)}} G^{\mu_1\cdots\mu_{(2n)}} \Big)$$

Ansatz

$$C_{(2n-1)} = c^2 \tau_\mu{}^A \wedge \tau_\nu{}^B \wedge k_{(2n-3)}\epsilon_{AB} + k_{(2n-1)}$$

Finite transformation rules!

$$\delta k_{(2n-1)} = -\, 2\lambda^A{}_d\, e^d \wedge \tau^B \wedge \; \mathrm{k}_{(2n-3)}\epsilon_{AB} + \; \mathrm{d}\sigma \wedge e^b$$

Consistent magnetic limit

c^0 order cancels in massive IIA and IIB supergravity due to the recursive sum structure of the democratic formulation. For massless IIA, $m = 0$ after the limit.

Type II Supergravity in Democratic Formulation

Bosonic action

$$\mathcal{S}_{II} = \mathcal{S}_{NSNS} - \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\sum_n \Big(- \frac{1}{4(2n)!} G_{\mu_1\cdots\mu_{(2n)}} G^{\mu_1\cdots\mu_{(2n)}}\Big)$$

Ansatz

$$C_{(2n-1)} = c^2 \tau_\mu{}^A \wedge \tau_\nu{}^B \wedge k_{(2n-3)}\epsilon_{AB} + k_{(2n-1)}$$

Finite transformation rules!

$$\delta k_{(2n-1)} = -\,2\lambda^A{}_d\, e^d \wedge \tau^B \wedge\ \mathrm{k}_{(2n-3)}\epsilon_{AB}$$
$$+\ \mathrm{d}\sigma \wedge e^b$$

Consistent magnetic limit

No Chern-Simmons $\rightarrow$ no problem!

Type II Supergravity in Democratic Formulation

Equations of Motion

The expressions can be combined before the limit to reduce the c dependence:

$$[V_\pm]_{Aa} = 2[G]_{Aa} \pm [B]^B{}_a \epsilon_{AB}$$

$$[P_\pm] = 2[G]_A{}^A \mp [B]_{AB}\epsilon^{AB}$$

$$[R_\pm]_{a_1\dots a_{(2n-1)}} = 2[C_{(2n-1)}]_{a_1\dots a_{(2n-1)}} \pm [C_{(2n+1)}]_{ABa_1\dots a_{(2n-1)}}\epsilon^{AB}$$

$[P_+]$ [Bergshoeff et al. 2021]

- The contributions at order c^2 and c^0 cancel. At order c^{-2}, there appears **one** Poisson-like term:

$$\epsilon^{AB} e^\mu{}_a e^\nu{}_a \partial_\mu \partial_\nu b_{AB}$$

- The cancellation at order c^0 only occurs in the presence of a dilatation symmetry

Type II Supergravity in Democratic Formulation

Equations of Motion

The expressions can be combined before the limit to reduce the c dependence:

$$[V_\pm]_{Aa} = 2[G]_{Aa} \pm [B]^{B}{}_{a}\epsilon_{AB}$$

$$[P_\pm] = 2[G]_{A}{}^{A} \mp [B]_{AB}\epsilon^{AB}$$

$$\begin{aligned}[R_\pm]_{a_1\dots a_{(2n-1)}} = & 2[C_{(2n-1)}]_{a_1\dots a_{(2n-1)}} \\ & \pm [C_{(2n+1)}]_{ABa_1\dots a_{(2n-1)}}\epsilon^{AB}\end{aligned}$$

$[R_+]$ (only present in IIB)

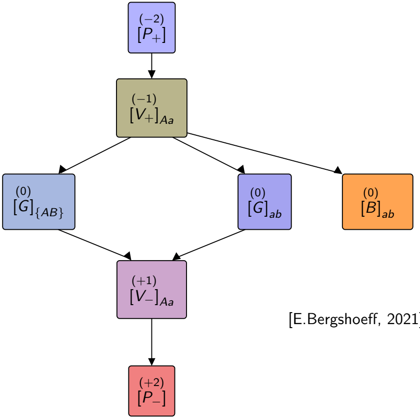
- Orders c^2 and c^0 vanish. At order c^{-2}, there is **another** Poisson-like term:

$$\epsilon^{AB} e^{\mu}{}_{a} e^{\nu}{}_{a} \partial_\mu \partial_\nu k_{AB}$$

- The presence of 2 Poisson-like terms may be led by the internal SL(2, R) symmetry of the theory.

Type II Supergravity in Democratic Formulation

Transformation under Lorentz Boosts: NS-NS sector



[E.Bergshoeff, 2021]

Type II Supergravity in Democratic Formulation

Transformation under Lorentz Boosts: R-R sector

$$
\begin{tikzcd}
\cdots \arrow[dr] & {[R^{(-2)}_{(2n-1)\,+}]_{(0|2n-1)}} \arrow[d] \arrow[dr] & \cdots \arrow[d] \\
 & {[C^{(-1)}_{(2n-1)}]_{(1|2n-2)}} \arrow[dl] \arrow[d] & {[C^{(-1)}_{(2n+1)}]_{(1|2n)}} \arrow[dl] \arrow[d] \\
{[R^{(0)}_{(2n-3)\,-}]_{(0|2n-3)}} & {[R^{(0)}_{(2n-1)\,-}]_{(0|2n-1)}} & \cdots
\end{tikzcd}
$$

3. Solutions

Solutions

Target

- Asymptotically flat solutions: recover Minkowskian geometry at space infinity.
- Dp-solutions?

$$\begin{aligned} ds^2 = & c^2 \eta_{AB}\, \tau_\mu{}^A \tau_\nu{}^B dx^\mu dx^\nu \\ & + \delta_{ab}\, e_\mu{}^a e_\nu{}^b dx^\mu dx^\nu \end{aligned}$$

D1-D3' intersection [Lambert, 2024]

Dp-branes are p-dimensional surfaces. Intersections combine harmonic functions with power $1/2$ in transverse directions and power $-1/2$ in worldvolume directions.

	t	y	$z_1 \ldots z_3$	$x_1 \ldots x_5$
D1	x	x		
D3	x		$x \cdots x$	

$$ds^2 = {H_{D_1}}^{-\frac{1}{2}} {H_{D_3}}^{-\frac{1}{2}} dt^2 - {H_{D_1}}^{-\frac{1}{2}} {H_{D_3}}^{\frac{1}{2}} dy^2$$
$$- {H_{D_1}}^{\frac{1}{2}} {H_{D_3}}^{-\frac{1}{2}} d\vec{z}_p{}^2 - {H_{D_1}}^{\frac{1}{2}} {H_{D_3}}^{\frac{1}{2}} d\vec{x}_{8-p}{}^2$$

$$e^{-2\Phi} = {H_{D_1}}^{\frac{1}{2}}$$

$$C_{tz^1z^2z^3} = \left({H_{D_3}}^{-1} - 1\right)$$

$$C_{ty} = {H_{D_1}}^{-1}$$

$$H_{D_1,D_3} = 1 + \frac{h_{D_1,D_3}}{|\vec{x}_{8-p}|^{6-p}}$$

D1-D3' intersection [Lambert, 2024]

The charge density from D1 is spread through the other directions → H_{D_1} can be considered to be constant:

$$H_{D_1} = c^{-4}$$

	t	y	$z_1 \ldots z_3$	$x_1 \ldots x_5$
D1	x	x	$\sim \cdots \sim$	$\sim \cdots \sim$
D3	x		$x \cdots x$	

$$ds^2 = c^2 \left(H_{D3}{}^{-\frac{1}{2}} dt^2 - H_{D3}{}^{\frac{1}{2}} dy^2 \right)$$

$$- c^{-2} \left(H_{D3}{}^{-\frac{1}{2}} \, d\vec{z}_p{}^2 + H_{D3}{}^{\frac{1}{2}} \, d\vec{x}_{8-p}{}^2 \right)$$

$$e^{-2\Phi} = c^{-2} e^{-2\phi}$$

$$C_{tz^1z^2z^3} = \left(H_{D_3}{}^{-1} - 1 \right)$$

$$C_{ty} = c^4$$

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D3	x		$x \cdots x$	

$$ds^2 = c^2 \left(H_{D3}{}^{-\frac{1}{2}} dt^2 - H_{D3}{}^{\frac{1}{2}} dy^2 \right)$$

$$- c^{-2} \left(H_{D3}{}^{-\frac{1}{2}} \, d\vec{z}_p{}^2 + H_{D3}{}^{\frac{1}{2}} \, d\vec{x}_{8-p}{}^2 \right)$$

$$e^{-2\Phi} = c^{-2} e^{-2\phi}$$

$$C_{tz^1z^2z^3} = \left(H_{D_3}{}^{-1} - 1 \right)$$

$$C_{ty} = c^4$$

Not a solution of our theory!

F1-Dp intersection

For Dp-F1 intersections, the harmonic function H_{F_1} scales with power $+1$ in transverse directions and -1 in worldvolume directions.

	t	y	$z_1 \dots z_p$	$x_1 \dots x_{8-p}$
F1	x	x		
Dp	x		$x \cdots x$	

$$ds^2 = {H_{Dp}}^{-\frac{1}{2}} {H_F}^{-1} dt^2 - {H_{Dp}}^{\frac{1}{2}} {H_F}^{-1} dy^2$$

$$- {H_{Dp}}^{-\frac{1}{2}} d\vec{z}_p{}^2 - {H_{Dp}}^{\frac{1}{2}} d\vec{x}_{8-p}{}^2$$

$$e^{-2\Phi} = e^{-2\phi} {H_{Dp}}^{\frac{(p-3)}{2}} H_{F1},$$

$$C^{(p+1)}{}_{tz^1 \dots z^p} = e^{-\phi}\left({H_{Dp}}^{-1} - 1\right),$$

$$B_{ty} = {H_{F1}}^{-1} - 1,$$

$$H_{Dp,\,F1} = 1 + \frac{h_{Dp,\,F1}}{|\vec{x}_{8-p}|^{6-p}}.$$

F1-Dp intersection

H_{F_1} can be completely smeared:

$$H_{F_1} = c^{-2}$$

	t	y	$z_1 \dots z_p$	$x_1 \dots x_{8-p}$
F1	x	x	$\sim \cdots \sim$	$\sim \cdots \sim$
Dp	x		$x \cdots x$	

$$ds^2 = c^2 \left(H_{Dp}{}^{-\frac{1}{2}} dt^2 - H_{Dp}{}^{\frac{1}{2}} dy^2 \right) - H_{Dp}{}^{-\frac{1}{2}} d\vec{z}_p{}^2 - H_{Dp}{}^{\frac{1}{2}} d\vec{x}_{8-p}{}^2,$$

$$e^{-2\Phi} = c^{-2} e^{-2\phi} H_{Dp}{}^{\frac{(p-3)}{2}}$$

$$C^{(p+1)}{}_{tz^1 \dots z^p} = e^{-\phi} \left(H_{Dp}{}^{-1} - 1 \right)$$

$$B_{ty} = c^2 - 1.$$

Solution to Non-Relativistic Type II Supergravity

Matching the previous result with ds^2 and imposing vielbein orthonormality reveals a solution.

	t	y	$z_1 \dots z_p$	$x_1 \dots x_{8-p}$
F1	x	x	$\sim \cdots \sim$	$\sim \cdots \sim$
Dp	x		$x \cdots x$	

$$\tau_t{}^0 = H_{D_p}{}^{-1/4}$$

$$\tau_y{}^1 = H_{D_p}{}^{1/4}$$

$$e_{z_1}{}^2 = \cdots = e_{z_p}{}^{p+1} = H_{D_p}{}^{-1/4}$$

$$e_{x_1}{}^{p+2} = \cdots = e_{x_{8-p}}{}^{10} = H_{D_p}{}^{1/4}$$

$$e^{-2\Phi} = c^{-2} e^{-2\phi} H_{Dp}{}^{\frac{(p-3)}{2}}$$

$$C^{(p+1)}{}_{tz^1 \dots z^p} = e^{-\phi}\left(H_{Dp}{}^{-1} - 1\right)$$

$$B_{ty} = c^2 - 1$$

Case D3

Substituting this solution into the equations of motion with no prior assumptions on H_{D_3} leads to:

$$\sum_{x_i} \partial_{x_i} \partial_{x_i} H_{D_3} = 0$$

	t	y	$z_1 \dots z_3$	$x_1 \dots x_5$
F1	x	x	$\sim \cdots \sim$	$\sim \cdots \sim$
Dp	x		$x \cdots x$	

$$\tau_t{}^0 = H_{D_3}{}^{-1/4}$$

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$$e_{x_1}{}^{p+2} = \cdots = e_{x_{8-p}}{}^{10} = H_{D_3}{}^{1/4}$$

$$e^{-2\Phi} = c^{-2} e^{-2\phi}$$

$$C_{tz^1 z_2 z_3} = e^{-\phi}\left(H_{D3}{}^{-1} - 1\right)$$

$$B_{ty} = c^2 - 1$$

4. Conclusions & future prospects

Conclusions and future prospects

Conclusions

- ✓ Consistent magnetic limit in democratic (massive) IIA and IIB supergravity.
  - No lagrange multipliers
  - IIB supergravity: two Poisson equations
  - Independent transformations under boosts
- ✓ Dp-solutions

Next steps

- Other solutions
  - Near Horizon limit
  - Other asymptotic limits
- Fermions

Thank you!

Type II Supergravity in Democratic Formulation

Equations of motion can be combined before the limit to reduce the dependence on c. Let us consider two equations, $[\mathrm{X}] = 0$ and $[\mathrm{Y}] = 0$, with the same leading order terms in the c expansion.

$$\left.\begin{aligned} [X] &= c^n \overset{(n)}{[X]} + c^{n-2} \overset{(n-2)}{[X]} + \mathcal{O}(c^{n-4}) \\ [Y] &= c^n \overset{(n)}{[X]} + c^{n-2} \overset{(n-2)}{[Y]} + \mathcal{O}(c^{n-4}) \end{aligned}\right\} \quad \begin{aligned} [X] + [Y] &= 2c^n \overset{(n)}{[X]} + c^{n-2}\left(\overset{(n-2)}{[X]} + \overset{(n-2)}{[Y]}\right) + \mathcal{O}(c^{n-4}) \\ [X] - [Y] &= c^{n-2}\left(\overset{(n-2)}{[X]} - \overset{(n-2)}{[Y]}\right) + \mathcal{O}(c^{n-4}) \end{aligned}$$