Novel duality-invariant theories of electrodynamics

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Introduction and Motivation

- Electromagnetic interactions: <u>Maxwell electrodynamics</u>.

$$\mathcal{L}^{M} = -\frac{1}{4} F_{ab} F^{ab} \; ; \qquad F_{ab} = 2 \partial_{[a} A_{b]} .$$

(We work in $D=4$ flat space)

- Equations of motion (eom) and Bianchi identity:

$$d \star F = 0 \qquad ; \qquad dF = 0 .$$

Both equations are <u>linear</u> in F_{ab}.

Introduction and Motivation

- Maxwell's theory is gauge- and Lorentz-invariant.

- It features an extra special symmetry: define

$$H_{ab} = -\star F_{ab}. \implies \underset{\text{(eom)}}{dH = 0} \;;\; \underset{\text{(Bianchi)}}{dF = 0}$$

- Allow for rotations of F and H:

$$\begin{pmatrix} F' \\ H' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix}$$

- Clearly, $dF' = 0$; $dH' = 0$.

$(\star F)_{ab} = \frac{1}{2}\varepsilon_{abcd}F^{cd}$

$\varepsilon_{0123} = +1$.

Introduction and Motivation

$$\begin{pmatrix} F' \\ H' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix} \qquad (T)$$

- Clearly, $dF' = 0$; $dH' = 0$.

- For (T) to be an actual <u>symmetry</u> of eom and Bianchi, the relation between H' and F' must be the same as that of H and F:

$$F' = \cos\alpha\, F - \sin\alpha \overset{\sim\,H}{\star F} \;;\quad H' = -\sin\alpha\, F - \cos\alpha \overset{\sim\,H}{\star F}.$$

$$\Rightarrow \quad \star F' + H' = 0 \quad \Rightarrow \quad \boxed{H' = -\star F'}$$

- (T) is a <u>symmetry</u> of eom and Bianchi in Maxwell: <u>duality invariance</u>.

Introduction and Motivation

- Maxwell's theory: simplest Lorentz-, gauge- and duality-invariant theory.

  It is not the only one.

- Demanding gauge and Lorentz invariance, the theory $\mathcal{L}$:

  $\mathcal{L} = \mathcal{L}(F) \Rightarrow$ constructed from Lorentz scalars of F.

- Not only that:

  $$s = -\frac{1}{4} F_{ab} F^{ab}$$

  $$\mathcal{L} = \mathcal{L}(s, p); \qquad p = -\frac{1}{4} F_{ab} (\star F)^{ab}.$$

- Duality invariance: differential constraint on $\mathcal{L}(s,p)$ with infinite solutions.

Introduction and Motivation

- Symmetries: most powerful tool to construct physical theories
- Study of gauge-, Lorentz and duality-invariant theories beyond Maxwell

[Boillat '70, Plebański '70; Bialynicki-Birula '92; Gibbons, Rasheed '95, Gaillard, Zumino '97, '98; Chemissany, Kallosh, Ortin '12; Kruglov '15, '16; Bandos, Lechner, Sorokin, Townsend '20; Sorokin '22; Babaei-Aghbolagh, Velni, Yekta, Mohammadzadeh '22, Lechner, Marchetti, Sainaghi, Sorokin '22; Russo, Townsend '22, '23, '24 '25, ...]

- Objective of the talk: find Novel duality-invariant theories of electrodynamics

Contents of the talk

1) Modmax map and duality invariance.
2) Two new duality-invariant theories including an extra term independent of F.

Modmax map and duality invariance

- Beyond Maxwell: non-linear eom in $F \Rightarrow$ Non-linear electrodynamics (NED)

Indeed, eom and Bianchi for $\mathcal{L} = \mathcal{L}(F)$:

$$dH = 0 \qquad dF = 0$$

$$H_{ab} = 2 * \frac{\partial \mathcal{L}}{\partial F^{ab}} .$$

- Question: When is $\mathcal{L}(F)$ duality-invariant?

$$\begin{pmatrix} F' \\ H' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix} \Rightarrow \boxed{\begin{array}{l} \text{If } H_{ab} = f(F_{ab}), \\ \underline{\text{then}} \ H'_{ab} = f(F'_{ab}) \end{array}}$$

Modmax map and duality invariance

- Requiring invariance under duality rotations:

  [Bialynicki-Birula '92; Gibbons, Rasheed '95]

$$\frac{1}{2}{*F_{ab}} = \frac{\partial}{\partial F^{ab}}\left(\frac{\partial \mathcal{L}}{\partial F_{cd}} * \frac{\partial \mathcal{L}}{\partial F^{cd}}\right) \Rightarrow \frac{1}{4} F_{ab} {*F^{ab}} = \frac{\partial \mathcal{L}}{\partial F^{ab}} * \frac{\partial \mathcal{L}}{\partial F_{ab}} - \Psi, \quad (D)$$

where Ψ is independent of F. Commonly, Ψ is set to zero to recover Maxwell in weak-field limit. We will not do that.

- Since $\mathcal{L} = \mathcal{L}(s,p)$, (recall $s = -\frac{1}{4} F_{ab}F^{ab}$; $p = -\frac{1}{4} F_{ab}({*F^{ab}})$), (D) is equal to:

$$\boxed{p\left(\frac{\partial \mathcal{L}}{\partial s}\right)^2 - 2s\left(\frac{\partial \mathcal{L}}{\partial s}\right)\left(\frac{\partial \mathcal{L}}{\partial p}\right) - p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 + \Psi = p}$$

Modmax map and duality invariance

$$\boxed{p\left(\frac{\partial \mathcal{L}}{\partial s}\right)^2 - 2s\left(\frac{\partial \mathcal{L}}{\partial s}\right)\left(\frac{\partial \mathcal{L}}{\partial p}\right) - p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 + \Psi = p} \qquad (D_{sp})$$

- All along the talk, we will be interested in finding solutions of (D_{sp}), which is a quadratic first-order PDE for $\mathcal{L} = \mathcal{L}(s,p)$.
- Maxwell's $\mathcal{L} = s$ is clearly a solution. Other examples with $\Psi = 0$:

1) Born-Infeld [Born, Infeld '34].

$$\mathcal{L}^{BI} = T - \sqrt{T^2 - 2Ts - p^2}\,, \quad T \in \mathbb{R}.$$

2) ModMax [Bandos, Lechner, Sorokin, Townsend '20]

$$\mathcal{L}^{MM}_\gamma = \cosh\gamma\ s + \sinh\gamma \sqrt{s^2 + p^2}\,, \quad \gamma \in \mathbb{R}$$

Modmax map and duality invariance

- $\mathcal{L}_\gamma^{MM}$ is most general conformally and duality-invariant electrodynamics with $\psi = 0$ [Bandos, Lechner, Sorokin, Townsend '20].

Proof. (Sketch)

- let $\mathcal{L} = \mathcal{L}(s,p)$ be a conformal theory. Then $\mathcal{L}(as, ap) = a\,\mathcal{L}(s,p)$.

This translates into:

$$s\frac{\partial \mathcal{L}}{\partial s} + p\frac{\partial \mathcal{L}}{\partial p} = \mathcal{L}. \qquad (C)$$

- If $\mathcal{L}$ is duality-invariant too:

$$p\left(\frac{\partial \mathcal{L}}{\partial s}\right)^2 - 2s\frac{\partial \mathcal{L}}{\partial s}\frac{\partial \mathcal{L}}{\partial p} - p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 = p. \qquad (D_{sp})$$

- Solving in (C) for $\frac{\partial \mathcal{L}}{\partial s}$ and substituting in (D_{sp}), one gets first-order ODE

$\rightarrow$ Most general solution is $\mathcal{L}_\gamma^{MM}$. $\square$

Modmax map and duality invariance

- Let $\mathcal{L}_0 = \mathcal{L}_0(s,p)$ be a certain NED. Define the Modmax map $\mathcal{M}_\gamma$:

$$\mathcal{M}_\gamma : \mathcal{L}_0 \longrightarrow \mathcal{M}_\gamma(\mathcal{L}_0) = \mathcal{L}_0\left(\mathcal{L}_\gamma^{MM}(s,p), p\right)$$

Theorem

- If $\mathcal{L}_0$ is duality-invariant, $\mathcal{M}_\gamma(\mathcal{L}_0)$ is duality-invariant $\forall \gamma \in \mathbb{R}$.

Proof

- Let $\hat{\mathcal{L}}_\gamma = \mathcal{M}_\gamma(\mathcal{L}_0)$. Let $z = \mathcal{L}_\gamma^{MM}$. We compute:

$$p\left(\frac{\partial \hat{\mathcal{L}}_\gamma}{\partial s}\right)^2 - 2s\,\frac{\partial \hat{\mathcal{L}}_\gamma}{\partial s}\frac{\partial \hat{\mathcal{L}}_\gamma}{\partial p} - p\left(\frac{\partial \hat{\mathcal{L}}_\gamma}{\partial p}\right)^2 = \overbrace{\left(p\left(\frac{\partial z}{\partial s}\right)^2 - 2s\,\frac{\partial z}{\partial s}\frac{\partial z}{\partial p} - p\left(\frac{\partial z}{\partial p}\right)^2\right)}^{\parallel\, p}\left(\frac{\partial \mathcal{L}_0}{\partial z}\right)^2$$

$$- 2\,\frac{\partial \mathcal{L}_0}{\partial z}\frac{\partial \mathcal{L}_0}{\partial s}\underbrace{\left(s\,\frac{\partial z}{\partial s} + p\,\frac{\partial z}{\partial p}\right)}_{z} - p\left(\frac{\partial \mathcal{L}_0}{\partial p}\right)^2 = p - 4$$

$\uparrow$ since $\mathcal{L}_0$ is duality-invariant

- Hence, $\hat{\mathcal{L}}_\gamma$ is duality-invariant $\forall \gamma$. $\square$

Modmax map and duality invariance

- Examples of application of ModMax map:

  1) Take Born-Infeld $\mathcal{L}^{BI}$ as seed theory:

$$\mathcal{M}_\gamma(\mathcal{L}^{BI}) = T - \sqrt{T^2 - 2T\mathcal{L}_\gamma^{MM} - p^2}\,.$$

This corresponds to the so called generalized Born-Infeld [Bandos, Lechner, Sorokin, Townsend '20].

  2) Consider duality-invariant theory:

$$\mathcal{L}^{GR} = \sqrt{\alpha \mp 2u}\,\sqrt{\kappa \pm 2v}\,;\quad 2u = \sqrt{s^2+p^2} - s\,;\quad 2v = \sqrt{s^2+p^2} + s\,.$$

for $\{\alpha, \kappa\} \in \mathbb{R}^2$. Applying the ModMax map:

$$\mathcal{M}_\gamma(\mathcal{L}^{GR}) = \sqrt{\alpha \mp 2e^{-\gamma}u}\,\sqrt{\kappa \pm 2e^{\gamma}v}\,.$$

This complete family of theories was found in [Gibbons-Rasheed '95].

Modmax map and duality invariance

$$2u = \sqrt{s^2+p^2} - s \; ; \; 2v = \sqrt{s^2+p^2} + s$$

$$g_K(b,c) = \frac{b}{2}\sqrt{2(K^2-c)-b^2} + (K^2-c)\arcsin\left(\frac{b}{\sqrt{2(K^2-c)}}\right)$$

- Examples of application of ModMax map:

3) Take the duality-invariant theory (with $\psi \neq 0$): [Gibbons, Rasheed '95]

$$\mathcal{L}_{\pm}^{GR,\psi} = g_K(\sqrt{u}+\sqrt{v},0) \pm g_K(\sqrt{u}-\sqrt{v}, \mathrm{sign}(p)\psi), \text{ for } K \in \mathbb{R}.$$

Then, $\mathcal{M}_\gamma(\mathcal{L}_{\pm}^{GR,\psi}) = g_K(e^{-\gamma/2}\sqrt{u} + e^{\gamma/2}\sqrt{v}, 0) \pm g_K(e^{-\gamma/2}\sqrt{u} - e^{\gamma/2}\sqrt{v}, \mathrm{sign}(\psi)p)$

is a novel family of duality-invariant theories.

Remarks

- ModMax map: Construction of one-parameter family of duality-invariant NEDs from a given one.

- Note: $\mathcal{M}_\gamma(\mathcal{L}_K^{MM}) = \mathcal{L}_{\gamma+K}^{MM}$. (ModMax map on ModMax only shifts the parameter).

ModMax map and causality

- Consider duality-invariant theories with $\psi = 0$.

- In terms of $u = \frac{1}{2}(\sqrt{s^2+p^2} - s)$ and $v = \frac{1}{2}(\sqrt{s^2+p^2} + s)$, the duality-invariance condition for $\mathcal{L} = \mathcal{L}(u,v)$ reads [Gaillard, Zumino '97]:

$$\frac{\partial \mathcal{L}}{\partial u}\frac{\partial \mathcal{L}}{\partial v} = -1. \qquad (D_{uv})$$

- Resorting to [Courant, Hilbert '62], the most general solution to (D_{uv}) is:

$$\mathcal{L}[\ell(\tau)] = \ell(\tau) - \frac{2u}{\dot{\ell}(\tau)}, \qquad \tau = v + \frac{u}{\dot{\ell}(\tau)^2}, \text{ for arbitrary function } \ell.$$

- Example: (1) $\ell(\tau) = \tau$ corresponds to Maxwell theory.

  (2) $\ell(\tau) = e^{\gamma}\tau$ is ModMax with parameter γ.

Modmax map and causality

- By [Russo, Townsend '24], a theory $\mathcal{L}[\ell(\tau)]$ is causal iff:

$$\dot{\ell}(\tau) \geq 1 \quad ; \quad \ddot{\ell}(\tau) \geq 0.$$

- In terms of $\ell(\tau)$ parametrization of duality-invariant theories:

$$\mathcal{M}_\gamma(\mathcal{L}[\ell(\tau)]) = \mathcal{L}[\ell(e^\gamma \tau)] \qquad (M_\ell)$$

Theorem

If $\mathcal{L}[\ell(\tau)]$ is causal, $\mathcal{M}_\gamma[\mathcal{L}(\ell(\tau)]$ is causal for any $\gamma \geq 0$.

- Proof. It can be proven directly from (M_ℓ). $\square$.

Remarks

(1) ModMax map preserves causality.

(2) Generally: If $\dot{\ell} \geq \dot{\ell}_0 > 0$ and $\ddot{\ell} \geq 0$, then $\mathcal{M}_\gamma[\mathcal{L}(\ell(\tau))]$ is causal for $\gamma \geq -\log(\dot{\ell}_0)$.

Part II. Novel duality invariant theories with $\Psi \neq 0$.

- Remember duality-invariance condition:

$$p\left(\frac{\partial \mathcal{L}}{\partial s}\right)^2 - 2s\frac{\partial \mathcal{L}}{\partial s}\frac{\partial \mathcal{L}}{\partial p} - p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 + \Psi = p$$

- Ψ is independent of F_{ab}. Literature typically sets $\Psi = 0$ to ensure Maxwell limit for weak fields, with notable exceptions [Gibbons, Rasheed '95].

- But there are popular NEDs without Maxwell limit in weak-field regime...

- Let us find novel duality-invariant theories with non-trivial Ψ.

  1) New family of theories with Maxwell limit as $\Psi \to 0$.

  2) Generalization of Bialynicki-Birula electrodynamics.

Novel duality invariant theories with $\varphi \neq 0$.

- Interpretation of φ?
- Well, φ could be taken to be a pure integration constant.
- Other possibility: φ is built from other fields with non-trivial dynamics.
- ⚠ Caution! Promoting φ to be dynamical may break duality invariance of the entire set of equations of motion! This is due to:

$$\delta_\alpha \left(\frac{\partial \mathcal{L}}{\partial \varphi} \right) = \alpha , \qquad \text{with } \alpha \text{ infinitesimal angle.}$$

- Hence, dynamical φ could result in equation for φ not covariant under duality rotations.

Novel duality invariant theories with $\Psi \neq 0$.

- Are there acceptable choices for dynamical Ψ?

- Idea: If the equation(s) for the field(s) composing Ψ are such that $\frac{\partial \mathcal{L}}{\partial \Psi}$ always appears under differentiation... $\Rightarrow$ The equation for Ψ is invariant.

- How could we guarantee this? $\Rightarrow$ Ψ is a total derivative. Examples:

(1) $\Psi = \lambda \Box \phi$, for scalar field ϕ.

(2) $\Psi = \lambda \tilde{F}_{ab} \star \tilde{F}^{ab}$, for other field strength $\tilde{F}_{ab} = 2 \partial_{[a} \tilde{A}_{b]}$.

(3) $\Psi = \lambda (R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd})$.

(Kinetic terms are added separately)

Novel duality invariant theories with $\psi \neq 0$.

- Now, let us actually find duality-invariant NEDs with non-trivial ψ.

- We restrict to theories satisfying:

$$\mathcal{L}(\Theta s, \Theta p, \Theta \psi) = \Theta \mathcal{L}(s,p,\psi) \quad \text{for any function } \Theta .$$

$$\Downarrow$$

$$s \frac{\partial \mathcal{L}}{\partial s} + p \frac{\partial \mathcal{L}}{\partial p} + \psi \frac{\partial \mathcal{L}}{\partial \psi} = \mathcal{L} \qquad \text{(CI)}$$

- Just having the duality-invariance condition $\Rightarrow$ 1 PDE for 3-variable function.

- (CI) imposes a further constraint to find explicit theories. It is connected with Weyl invariance $\eta_{ab} \to \Omega^2 \eta_{ab}$ and $\psi \to \Omega^{-4} \psi$.

<u>Novel duality invariant theories with $\psi \neq 0$.</u>

- We want to solve:

$$p\left(\frac{\partial \mathcal{L}}{\partial s}\right)^2 - 2s\frac{\partial \mathcal{L}}{\partial s}\frac{\partial \mathcal{L}}{\partial p} - p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 + \psi = p \, ; \quad s\frac{\partial \mathcal{L}}{\partial s} + p\frac{\partial \mathcal{L}}{\partial p} + \psi\frac{\partial \mathcal{L}}{\partial \psi} = \mathcal{L}.$$

- Perturbatively in ψ: $\mathcal{L}(s,p,\psi) = \sum_{n=0}^{+\infty} \mathcal{L}_n(s,p)\,\psi^n$.

- We get the conditions:

$$\sum_{m=0}^{n}\left(p\frac{\partial \mathcal{L}_m}{\partial s}\frac{\partial \mathcal{L}_{n-m}}{\partial s} - 2s\frac{\partial \mathcal{L}_m}{\partial s}\frac{\partial \mathcal{L}_{n-m}}{\partial p} - p\frac{\partial \mathcal{L}_m}{\partial p}\frac{\partial \mathcal{L}_{n-m}}{\partial p}\right) = p\,\delta_{n,0} - \delta_{n,1}$$

with $n \geq 0$.

$$s\frac{\partial \mathcal{L}_n}{\partial s} + p\frac{\partial \mathcal{L}_n}{\partial p} = (1-n)\,\mathcal{L}_n$$

$\rightarrow$ Solve perturbatively from seed conformal and duality-invariant $\mathcal{L}_0$!

Novel duality invariant theories with $\varphi \neq 0$.

- Starting with $\mathcal{L}_0 = \mathcal{L}^{\gamma}_{MM}$:

$$\mathcal{L}_1 = -\frac{1}{2}\arctan\left(\frac{\mathcal{L}^{\gamma}_{MM}}{p}\right) + K_1 \quad ; \quad K_1 \in \mathbb{R},$$

$$\mathcal{L}_2 = -\frac{\mathcal{L}^{\gamma}_{MM}}{8\left((\mathcal{L}^{\gamma}_{MM})^2 + p^2\right)} + \frac{K_2}{\sqrt{(\mathcal{L}^{\gamma}_{MM})^2 + p^2}} \quad ; \quad K_2 \in \mathbb{R},$$

$$\mathcal{L}_3 = \frac{-\mathcal{L}^{\gamma}_{MM}\, p + 8K_2 p\sqrt{(\mathcal{L}^{\gamma}_{MM})^2 + p^2}}{16\left((\mathcal{L}^{\gamma}_{MM})^2 + p^2\right)} + \frac{K_3}{(\mathcal{L}^{\gamma}_{MM})^2 + p^2} \quad ; \quad K_3 \in \mathbb{R},$$

etc.

At each order, a new integration constant.

Novel duality invariant theories with $\psi \neq 0$.

- Could we resum the whole perturbative solution somehow?
- Let's explore this if all integration constants $K_i = 0$.
- Set first $\gamma = 0$ (any γ may fixed by ModMax map). Full resummation is possible!

$$\mathcal{L}_{\psi M} = \frac{S}{\sqrt{2}} t_+ + \frac{P}{\sqrt{2}} t_- - \frac{\psi}{2} t_\psi$$

$$t_+ = \sqrt{1 - \frac{\psi p}{s^2+p^2} + \sqrt{\frac{s^2+(\psi-p)^2}{s^2+p^2}}}\, ; \quad t_- = \frac{1}{\psi s}\sqrt{\psi^2 s^2 \left[\frac{\psi p}{s^2+p^2} - 1 + \sqrt{\frac{s^2+(\psi-p)^2}{s^2+p^2}}\right]}\, ;$$

$$t_\psi = \arctan\left(\frac{S}{P}\right) + 2\arctan\left(\frac{t_-}{\sqrt{2}+t_+}\right).$$

Novel duality invariant theories with $\gamma \neq 0$.

- As it turns out, $\mathcal{L}_{4M}$ is always real-valued (although not analytic in $s=p=0$) and satisfies:

$$\lim_{\gamma \to 0} \mathcal{L}_{4M} = s .$$

- $\mathcal{L}_{4M}$ represents a new duality-invariant NED with non-trivial γ with Maxwell limit as $\gamma \to 0$.

- Using ModMax map, one can construct one-parameter generalization of $\mathcal{L}_{4M}$:

$$M_\gamma(\mathcal{L}_{4M}) = \mathcal{L}_{4M}(\mathcal{L}_\gamma^{MM}, p, \gamma),$$

such that $\lim_{\gamma \to 0} M_\gamma(\mathcal{L}_{4M}) = \mathcal{L}_\gamma^{MM}$.

Novel duality invariant theories with $\psi \neq 0$.

- Let us find a different instance of duality-invariant NED with $\psi \neq 0$.
- Are there theories independent of s? $\Rightarrow$ Most general fixed point of ModMax map.
- $\mathcal{L} = \mathcal{L}(p, \psi)$, so that the duality invariance condition becomes:

$$-p\left(\frac{\partial \mathcal{L}}{\partial p}\right)^2 + \psi = p$$

- This equation can be exactly solved:

$$\mathcal{L}_{gBB} = \sqrt{(\psi - p)\,p} + \psi \arctan\left[\sqrt{\frac{p}{\psi - p}}\right] + h(\psi)$$

for arbitrary h (it must be linear for $\mathcal{L}_{gBB}$ to be homogeneous).

- Generalization of Bialynicki-Birula theory $\mathcal{L}_{BB} = i|p|$ [Bialynicki-Birula '92].

$$\left(\lim_{\psi \to 0} \mathcal{L}_{gBB} = \mathcal{L}_{BB}\right)$$

Conclusions

- We have found novel duality-invariant NEDs:

- ModMax map: generates one-parameter family of duality-invariant theories from a given one.

  $\rightarrow$ It preserves causality.

- Solved directly the duality-invariance condition with non-trivial γ.

  (1) Theory $\mathcal{L}_{\gamma M}$ with Maxwell limit $\lim_{\gamma \to 0} \mathcal{L}_{\gamma M} = S$.

  (2) Theory $\mathcal{L}_{gBB}$ with Bialynicki-Birula limit $\lim_{\gamma \to 0} \mathcal{L}_{gBB} = \mathcal{L}_{BB}$.

Future Directions

- Physical properties preserved by ModMax map?
- Hamiltonian formalism for duality-invariant theories with non-trivial Ψ.
- Examining $\mathcal{L}_{4M}$ in more detail. Explore possible dynamical choices for Ψ.
- Same for $\mathcal{L}_{gBB}$. What dynamics does this theory give for the gauge field?
- Explore other duality-invariant NEDs with $\Psi \neq 0$. Fully analytic theories?

¡Muchas gracias!

"Todo lo mudará la edad ligera, por no hacer mudanza en su costumbre"

[Garcilaso de la Vega, 1535].

Introduction and Motivation

- Why a gauge- and Lorentz-invariant theory is $\mathcal{L}=\mathcal{L}(s,p)$? Note:

$$F_{ac}(*F)_b{}^c = -p\, g_{ab} \quad ; \quad (*F)_{ac}(*F)_b{}^c = F_{ac}F_b{}^c + 2s\, g_{ab}, \qquad (f)$$

- By an induction procedure:

a) Use (f) to encode $*F$ in p, s and terms without $*F$.

b) Use "Schouten-like" identities $F^{a_1}{}_{[a_1}\cdots F^{a_n}{}_{a_n]}=0$ for $n>4$.

Any invariant with odd # of F's vanishes

Any invariant with even # of F's is pure function of s and p.