

Quantum state spectrum and field theory of D0-brane

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as well as on earlier Phys.Rev.D 2022, 2023, JHEP 2024 with
Unai Sarraga

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- 2 D0 brane action in (spinor) moving frame formalism and induced supergravity
- 3 (A family of candidate) Multiple D0-brane action(s)
 - Fields of mD0 system
 - Multiple D0-brane (candidate) action(s)
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Dp-branes in String Theory

- are p -branes where open (10D super)string can have its endpoints attached [[Sagnotti 1988](#), [Dai+Leigh+Polchinski 1989](#), [Hořava 1989](#)].
- Their ground states can be described by supersymmetric solitonic solutions of type II supergravity [[Duff+Lu 1982](#), ...].
- Their dynamics is described by action given by the sum of SUSY DBI and WZ functionals [[Cederwall, von Gussich, Nilsson, Westerberg, 1996](#), [Cederwall, von Gussich, Nilsson, Sundell, Westerberg 1996](#), [Aganagic, Popescu, Schwarz 96](#), 96; [Bergshoeff, Townsend 96](#)].
- They carry charges of RR gauge fields of type IIA and type IIB Supergravity (for even and odd p , respectively)).

D0-brane

- The worldvolume of Dp-brane is $d = p + 1$ -dimensional so that
- D0-brane is just massive superparticle in (D=10, $\mathcal{N} = 2$) type IIA superspace the (standard) action of which [[Bergshoeff, Townsend 96](#)] is 10D generalization of the four dimensional $\mathcal{N} = 2$ superparticle action by [de Azcárraga and Lukierski \[1982\]](#).

Quantization

- The quantization of Dp -branes and particularly D0-branes were addressed in the literature (e.g. [\[Kallosh, 1997\]](#)), mainly for contrasting it with the superstring quantization (ask me what is the difference).
- However, to our best knowledge neither the quantum state spectrum of D0-brane was analysed nor its field theory was elaborated.
- One of the aims of (the papers on which is based) this talk is to fill this gap by quantization of D0-brane in its spinor moving frame formulation [\[IB:2000\]](#).
- The choice of spinor moving frame formulation is motivated by that this allows to elaborate the so-called spinor helicity formalism for type IIA amplitudes and superamplitudes involving D0-branes.
- But even more important motivation is that in such a way we create the basis for quantization of multiple D0-brane (mD0) model from [\[IB+Unai Sarraga:2022\]](#) which is known in its spinor moving frame formulation only.
- We will also show this mD0 action and discuss briefly its symmetries and properties in this talk.
- But first let us recall what are **multiple Dp -brane or mDp** systems.

Multiple Dp-branes (mDp)

- mDp is the system of N nearly coincident Dp-branes and N^2 strings ending on different branes or on the same branes
- [E. Witten 1995]: mDp
 - carries non-Abelian gauge fields on a center of mass worldvolume;
 - at very low energy it is described by the action of $U(N)$ maximally supersymmetric Yang–Mills (SYM) theory.
- In it the $U(1)$ sector describes the center of mass motion of the multiple Dp-brane (mDp) system while the $SU(N)$ sector describes the relative motion of the mDp constituents.
- Actually, $U(1)$ SYM action decouples and can be identified as a low energy limit of gauge fixed version of the complete nonlinear action for a single Dp-brane [Cederwall, von Gussich, Nilsson, Westerberg, 1996, Cederwall, von Gussich, Nilsson, Sundell, Westerberg 1996, Aganagic, Popescu, Schwarz 96, 96; Bergshoeff, Townsend 96; IB, Sorokin, Tonin, 97].
- Then the natural problem was to find a complete action for multiple Dp-brane (mDp) system: with all expected fields and doubly supersymmetric, i.e. invariant under spacetime SUSY and κ -symmetry.

30 years of search: many interesting results

- The problem of multiple Dp -brane action has a 30 years of history and is not solved yet in its complete form.
- However, certain progress was reached and many interesting results were obtained during these years
- Let us particularly mention [Tseytlin:1997;Emparan:1998;Myers:1999;Yolanda Lozano+:2002,2005;Howe, Lindstrom, Wulff:2005,2007]
- In particular,
 - the **bosonic limit** is widely believed to be given by the Myers's 'dielectric brane' action [Myers:1999] obtained from the requirement of consistency with T-duality transformations of D-branes and background fields.
 - (Notice that this action is not Lorentz invariant).
 - A very interesting complete and supersymmetric construction on '**-1 quantization level**' was proposed in [Howe, Lindstrom, Wulff:2005,2007]. There such a dynamical system was constructed, that its quantization should reproduce the desired multiple Dp -brane (mDp) action.
 - (However, the complete realization of this step in a fool glory seems to imply the quantization of the complete interacting system of supergravity and super- Dp -brane).

mD0 and other m0-th

- A complete action (of the usual type) including fermions and invariant under spacetime SUSY and κ -symmetry (worldline susy) is known presently for
 - the system of ten-dimensional (10D) multiple 0-branes [Sorokin:2001, Panda+Sorokin:2003]
 - for 11D multiple M0-branes (mM0) or multiple M-waves [IB:2012].
 - In D=3 some $\mathcal{N} = 1$ supersymmetric multibrane actions are known: these are the non-Abelian Born-Infeld action of [Drummond, Howe and Lindstrom:2002] and non-Abelian multiwave action of [IB:2013] (the 3d $\mathcal{N} = 1$ counterpart of the mM0).
 - mD0 action in flat superspace constructed in [I.B. + Sarraga: PRD 2022] and studied in [I.B. + Sarraga: PRD 2023].
 - To be more precise, this is a family of the actions with all the known properties expected for mD0-branes in flat target superspace.
 - The generalization to curved background is not known (yet, I suppose).
 - As we have shown in [I.B. + Sarraga: PRD 2023], one of the representatives of this family of candidate mD0 actions can be obtained by dimensional reduction of 11D mM0-action from [I.B., JHEP 2012].
- Let us stress that mD0-action of [I.B. + Sarraga: PRD 2022] (as well as mM0- action of [I.B., JHEP 2012]) is known in the frame of spinor moving frame approach only.

Our mD0 and BFSS Matrix model

- The quantization of the dynamical system described by our mD0 action should result in a field theory in superspace enhanced by additional bosonic and fermionic matrix coordinates.
- Why this should be interesting in?
- Reason proceeds from the fact that the mD0-action of [I.B. + Sarraga: PRD 2023] (as well as mM0- action from [I.B., JHEP 2012]) can be considered as 10D (11D) Lorentz invariant generalization of the BFSS Matrix model [Banks, Fischler, Shenker, and Susskind, PRD 1996] .
- This latter was conjectured to provide a complete description of M-theory in certain limits (hence the name M(atrix) theory)
- and still attracts much attention in their different aspects and provides inspirations for new studies (see e.g. [Juan Maldacena, Talk at Strings 2024 at CERN, <https://www.youtube.com/watch?v=b0nMlfpZ9-0>] as well as J.Phys.A 2024, JHEP 2024].
- This allows us to conjecture that the mD0 field theory obtained by quantization of the dynamical system described by our mD0 action might give new insight in the structure and properties of String/M-theory.

In this talk,

- after describing the spinor moving frame formulation of D0-brane I will
- show the (family of) candidate mD0 actions and
- briefly discuss its properties.
- Then as a preparatory stage, to quantization of this system,
- I describe the covariant quantization of single D0-brane
- analyse D0-brane quantum state spectrum
- and describe D0-brane field theory both in a more standard form and in a formalism which will be used to describe mD0 field theory in future.
- Notice that such a formalism produces so-called on-shell superfields, one-particle counterparts of superamplitudes.

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D0-brane action in de Azcárraga–Lukierski type formulation

- Let us denote the pull-backs of supervielbein forms of flat 10D type IIA superspace $\Sigma^{(10|32)}$ to the superparticle worldline by

$$E^a = dx^a(\tau) - id\theta^{\alpha 1}(\tau)\sigma_{\alpha\beta}^a\theta^{\beta 1}(\tau) - id\theta_{\alpha}^2(\tau)\tilde{\sigma}^{a\alpha\beta}\theta_{\beta}^2(\tau) =: d\tau E_{\tau}^a, ,$$

$$E^{\alpha 1} = d\theta^{\alpha 1}(\tau) = d\tau\partial_{\tau}\theta^{\alpha 1}, \quad E_{\alpha}^2 = d\theta_{\alpha}^2(\tau) = d\tau\partial_{\tau}\theta_{\alpha}^2, \quad ,$$

where $a, b, c = 0, 1, \dots, 9$ are ten-vector indices, $\alpha, \beta, \gamma = 1, \dots, 16$ are 10D MW spinor indices and $(x^a(\tau), \theta^{\alpha 1}(\tau), \theta_{\alpha}^2(\tau))$ are bosonic vector and fermionic spinor coordinate functions of the proper time τ which define parametrically the superparticle worldline,

$$\mathcal{W}^1 \subset \Sigma^{(10|32)} : \quad x^a = x^a(\tau), \quad \theta^{\alpha 1} = \theta^{\alpha 1}(\tau), \quad \theta_{\alpha}^2 = \theta_{\alpha}^2(\tau) .$$

- These are sufficient to write the standard action for D0-brane [\[Bergshoeff+Townsend 1996\]](#)

$$S_{D0} = \frac{M}{2} \int d\tau \sqrt{E_{\tau}^a E_{a\tau}} - iM \int_{\mathcal{W}^1} (d\theta^{\alpha 1} \theta_{\alpha}^2 - \theta^{\alpha 1} d\theta_{\alpha}^2) ,$$

which is the $D=4$ generalization of the $D = 4 \mathcal{N} = 2$ action for massive superparticle [\[de Azcárraga + Lukierski 1982\]](#).

Moving frame and spinor moving frame variables (Lorentz harmonics)

- The spinor moving frame formulation of D0-brane involves also
- a set of orthogonal and normalized vector fields $u_a^0(\tau)$ and $u_a^i(\tau)$,

$$u_a^0 u^{a0} = 1, \quad u_a^0 u^{ai} = 0, \quad u_a^i u^{aj} = -\delta^{ij},$$

which form the Lorentz group valued *moving frame matrix*

$$u_a^{(b)}(\tau) = (u_a^0, u_a^i) \in SO(1, 9)$$

- as well as 16×16 *Spin*(1, 9) valued *spinor moving frame matrix*

$$v_\alpha{}^q \in Spin(1, 9)$$

providing a kind of *square roots of the moving frame* in the sense of

$$\begin{aligned} u_a^0 \sigma_{\alpha\beta}^a &= v_\alpha{}^q v_\beta{}^q, & u_a^i \sigma_{\alpha\beta}^a &= v_\alpha{}^q \gamma_{qp}^i v_\beta{}^p \\ \Leftrightarrow & v_\alpha{}^q \tilde{\sigma}_\mu^{\alpha\beta} v_\beta{}^p &= u_\mu^0 \delta_{qp} + u_\mu^i \gamma_{qp}^i, \end{aligned}$$

where $\gamma_{qp}^i = \gamma_{pq}^i$ are $d = 9$ gamma matrices obeying $\gamma^{(i} \gamma^{j)} = \delta^{ij}$.

Moving frame and spinor moving frame variables (Lorentz harmonics)

- These constraints

$$u_a^0 \sigma_{\alpha\beta}^a = v_\alpha^q v_\beta^q, \quad u_a^i \sigma_{\alpha\beta}^a = v_\alpha^q \gamma_{qp}^i v_\beta^p \Leftrightarrow v_\alpha^q \tilde{\sigma}_\mu^{\alpha\beta} v_\beta^p = u_\mu^0 \delta_{qp} + u_\mu^i \gamma_{qp}^i$$

- which provide a massive and multi-dimensional generalization of D=4 Cartan-Penrose representation for light-like vector which is one of the basic relations of twistor approach [Penrose 1963],
- can be deduced from the Lorentz invariance of sigma-matrices

$$u_a^{(b)} \sigma_{\alpha\beta}^a = v_\alpha^q \sigma_{qp}^{(b)} v_\beta^p \text{ by choosing the representation with } \sigma_{qp}^{(b)} = (\delta_{qp}, \gamma_{qp}^i),$$

Spinor moving frame formulation of D0-brane. Action

- The spinor moving frame action of the 10D D0-brane [IB:2000] can be written with the use of just one timelike unit vector of the moving frame

$$S_{D0} = M \int_{\mathcal{W}^1} E^a u_a^0 - iM \int_{\mathcal{W}^1} (d\theta^{\alpha 1} \theta_\alpha^2 - \theta^{\alpha 1} d\theta_\alpha^2).$$

- If considered in its literal form, with $u_a^0 = u_a^0(\tau)$ obeying just $u_a^0 u^{a0} = 1$, it can be deduced from the first order form of de Azcárraga–Lukierski action $\int_{\mathcal{W}^1} \left(p_a E^a - \frac{e(\tau)}{2} (p_a p^a - M^2) \right) - iM \int_{\mathcal{W}^1} (d\theta^{\alpha 1} \theta_\alpha^2 - \theta^{\alpha 1} d\theta_\alpha^2).$

Spinor moving frame formulation of D0-brane. Action

- However, when the composite nature of the moving frame is taken into account,

$$v_{\alpha}^q \tilde{\sigma}_{\mu}^{\alpha\beta} v_{\beta}^p = u_{\mu}^0 \delta_{qp} + u_{\mu}^I \gamma_{qp}^I \quad \Rightarrow \quad u_{\mu}^0 = \frac{1}{16} v_{\alpha}^q \tilde{\sigma}_{\mu}^{\alpha\beta} v_{\beta}^q ,$$

- it becomes clear that $S_{D0} = M \int_{\mathcal{W}^1} E^a u_a^0 - iM \int_{\mathcal{W}^1} (d\theta^{\alpha 1} \theta_{\alpha}^2 - \theta^{\alpha 1} d\theta_{\alpha}^2)$ actually hides a 10D and massive generalization of the D=4 twistor-like or Ferber–Schirafuji action ($\int \lambda_A \bar{\lambda}_{\dot{A}} (dx^{\dot{A}A} - 2id\Theta^A \bar{\Theta}^{\dot{A}} + c.c.)$)

$$S_{D0} = \frac{M}{16} \int_{\mathcal{W}^1} v_{\alpha}^q v_{\beta}^q E^a \tilde{\sigma}_a^{\alpha\beta} - iM \int_{\mathcal{W}^1} (d\theta^{\alpha 1} \theta_{\alpha}^2 - \theta^{\alpha 1} d\theta_{\alpha}^2) .$$

- The advantage of the spinor moving frame action is also that it can be used, through the generalized action principle [I.B.+Sorokin+Volkov=PLB96], as a basis of constructing superembedding approach [I.B.+Pati+Sorokin+Tonin+Volkov=NPB95, Howe+Sezgin=PLB96, 97, Sorokin=Phys.Repts2000, I.B.+Sorokin=in *Handbook on QG*, 2024].
- It also allows to construct (candidate) mD0 action [I.B.+Sarraga=PRD2022].

Spinor moving frame and induced worldline supergravity

- The moving frame vectors can be used to split the bosonic supervielbein in a Lorentz invariant manner,

$$E^0 = E^a u_a^0, \quad E^i = E^a u_a^i$$

while spinor moving frame matrix $v_\alpha{}^q$ and its inverse $v_q{}^\alpha \in Spin(1, 9)$, obeying $v_q{}^\alpha v_\alpha{}^p = \delta_q^p$, allow to introduce two fermionic 1-forms with the same indices of $SO(9)$ gauge group

$$E^{q1} = d\theta^{\alpha 1} v_\alpha{}^q, \quad E_q^2 = d\theta_\alpha^2 v_q{}^\alpha.$$

- The Lagrangian one-form of the D0-brane action can be written in the form

$$\mathcal{L}_1^{D0} = ME^0 - iM(E^{q1}\theta_q^2 - \theta^{q1}E_q^2),$$

$$\text{with} \quad \theta^{q1} = \theta^{\alpha 1} v_\alpha{}^q, \quad \theta_q^2 = \theta_\alpha^2 v_q{}^\alpha.$$

Induced worldline supergravity

- But really important is that

$$E^0 \quad \text{and} \quad E^{q1} - E_q^2 = d\theta^{\alpha 1} v_\alpha{}^q - d\theta_\alpha^2 v_q{}^\alpha$$

provide us with the *induced 1d $\mathcal{N} = 16$ supergravity multiplet* for *local fermionic κ -symmetry* of the D0-brane action,

$$\begin{aligned} \delta_\kappa \theta^{1\alpha} &= \kappa^q v_q^\alpha, & \delta_\kappa \theta_\alpha^2 &= -\kappa^q v_\alpha{}^q, \\ \delta_\kappa x^\mu &= i\delta_\kappa \theta^1 \sigma^\mu \theta^1 + i\delta_\kappa \theta^2 \tilde{\sigma}^\mu \theta^2, & \delta_\kappa v_\alpha{}^q &= 0. \end{aligned}$$

Indeed: $\delta_\kappa E^0 = -2i(E^{q1} - E_q^2)\kappa^q, \quad \delta_\kappa (E^{q1} - E_q^2) = 2D\kappa^q,$

where $D\kappa^q := d\kappa^q + \frac{1}{4}\Omega^{ij}\kappa^p\gamma_{pq}^{ij}$ is the covariant derivative with induced SO(9) connection $\Omega^{ij} = u_\mu^i du^{\mu j}$.

- As the relative motion of mD0 system should be described by traceless $N \times N$ matrix fields of $d = 1$ $\mathcal{N} = 16$ SYM, $\mathbb{X}^i, \Psi_q, \mathbb{A}_0$, making its supersymmetry local by coupling to the induced supergravity allows to construct the simplest candidate mD0 action [I.B. 2018].
- More general nonlinear actions were found in [I.B. +Sarraga 2022].

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Fields of mD0: center of energy + $\mathcal{N} = 16$ $SU(N)$ SYM

- The fields used to describe mD0 system include coordinate functions, describing center of energy motion and $d = 1$ $\mathcal{N} = 16$ $SU(N)$ SYM multiplet: traceless hermitian $N \times N$ matrices
- 1d gauge field $\mathbb{A}_\tau(\tau)$, which we prefer to include in $\mathbb{A} = d\tau \mathbb{A}_\tau(\tau)$,
- nanoplet of bosonic fields $\mathbb{X}^i(\tau)$ in vector rep. of $SO(9)$
- hexadecuplet of fermionic matrix fields Ψ_q in the spinor rep. of $SO(9)$.
- We also use auxiliary fields: spinor moving frame $v_\alpha{}^q$ and bosonic matrix fields $\mathbb{P}^i(\tau)$ playing the role of momenta conjugate to $\mathbb{X}^i(\tau)$.

- The Lagrangian one-form for the action of $d = 1$ $\mathcal{N} = 16$ SYM is

$$\mathcal{L}_1^{SYM} = d\tau L_{SYM} = tr(-\mathbb{P}^i \nabla \mathbb{X}^i + 4i \Psi_q \nabla \Psi_q) + d\tau \mathcal{H},$$

where $\nabla \mathbb{X}^i = d\mathbb{X}^i + [\mathbb{A}, \mathbb{X}^i]$, $\nabla \Psi_q = d\Psi_q + [\mathbb{A}, \Psi_q]$ and

$$\mathcal{H} = \frac{1}{2} tr(\mathbb{P}^i \mathbb{P}^i) - \frac{1}{64} tr[\mathbb{X}^i, \mathbb{X}^j]^2 - 2 tr(\mathbb{X}^i \Psi \gamma^i \Psi)$$

is the SYM Hamiltonian including potential $\mathcal{V} = -\frac{1}{64} tr[\mathbb{X}^i, \mathbb{X}^j]^2$.

- The action $\propto \int_{W^1} \mathcal{L}_1^{SYM}$ is invariant under the rigid $d = 1$ $\mathcal{N} = 16$ SUSY

$$\delta_\varepsilon \mathbb{X}^i = 4i\varepsilon^q (\gamma^i \Psi)_q, \quad \delta_\varepsilon \mathbb{P}^i = [\varepsilon^q (\gamma^{ij} \Psi)_q, \mathbb{X}^j],$$

$$\delta_\varepsilon \Psi_q = \frac{1}{2} \varepsilon^p \gamma_{pq}^i \mathbb{P}^i - \frac{i}{16} \varepsilon^p \gamma_{pq}^{ij} [\mathbb{X}^i, \mathbb{X}^j],$$

$$\delta_\varepsilon \mathbb{A} = -d\tau \varepsilon^q \Psi_q.$$

- In the weak field limit, the complete multiple D0-brane action should reduce to the sum of the above $SU(N)$ SYM and of $U(1)$ SYM action, with later decoupled [Witten: 1995] (and can be identified with gauge fixed version of single D0-brane action).
- Supersymmetry of SYM action, as it is seen from its $U(1)$ part, is related to κ -symmetry of single D0 action.
- In the complete mD0 action this thus should become local (worldline) supersymmetry, the counterpart of kappa-symmetry of single D0-brane.
- The rigid $SU(N)$ SYM SUSY becomes local due to coupling to supergravity induced by center of energy motion.
- Now we will show the (family of) complete nonlinear action(s) constructed from these physical fields and auxiliary fields and having all the properties expected from mD0 action in flat superspace.

Our multiple D0-brane action is

$$\begin{aligned}
 S_{\text{mD0}} = & m \int_{\mathcal{W}^1} E^0 - im \int_{\mathcal{W}^1} (d\theta^1 \theta^2 - \theta^1 d\theta^2) + \\
 & + \frac{1}{\mu^6} \int_{\mathcal{W}^1} \left(\text{tr} \left(\mathbb{P}^i D \mathbb{X}^i + 4i \Psi_q D \Psi_q \right) + \frac{2}{\mathcal{M}} E^0 \mathcal{H} \right) - \frac{1}{\mu^6} \int_{\mathcal{W}^1} \frac{d\mathcal{M}(\mathcal{H})}{\mathcal{M}(\mathcal{H})} \text{tr}(\mathbb{P}^i \mathbb{X}^i) \\
 & + \frac{1}{\mu^6} \int_{\mathcal{W}^1} \frac{1}{\sqrt{2\mathcal{M}(\mathcal{H})}} (E^{1q} - E_q^2) \text{tr} \left(-4i(\gamma^i \Psi)_q \mathbb{P}^i + \frac{1}{2} (\gamma^{ij} \Psi)_q [\mathbb{X}^i, \mathbb{X}^j] \right)
 \end{aligned}$$

- where m and μ are constants of dimension of mass,
- $E^0 = E^a u_a^0$, $E^{q1} = E^{\alpha 1} v_\alpha^q = d\theta^{\alpha 1} v_\alpha^q$, $E_q^2 = E_\alpha^2 v_q^\alpha = d\theta_\alpha^2 v_q^\alpha$,
- $v_\alpha^q \in \text{Spin}(1, 9)$ obeying $u_a^0 \sigma_{\alpha\beta}^a = v_\alpha^q v_\beta^q \Rightarrow u^{a0} u_a^0 = 1$,
- The covariant derivatives use the $SU(N)$ gauge fields and $\Omega^{ij} = u^{ai} du_a^j$,
 $D\mathbb{X}^i := d\mathbb{X}^i - \Omega^{ij} \mathbb{X}^j + [\mathbb{A}, \mathbb{X}^i]$, $D\Psi_q := d\Psi_q - \frac{1}{4} \Omega^{ij} \gamma_{qp}^{ij} \Psi_p + [\mathbb{A}, \Psi_q]$,
- $\mathcal{H} = \frac{1}{2} \text{tr} \left(\mathbb{P}^i \mathbb{P}^i \right) - \frac{1}{64} \text{tr} \left[\mathbb{X}^i, \mathbb{X}^j \right]^2 - 2 \text{tr} \left(\mathbb{X}^i \Psi \gamma^i \Psi \right)$ is SYM Hamiltonian and $\mathcal{M}(\mathcal{H})$ is an arbitrary positive definite function of this.

Spacetime SUSY and local worldline SUSY (κ -symmetry)

- By construction, the action is invariant under the rigid spacetime SUSY, which acts nontrivially on the center of mass variables only,

$$\delta_\epsilon X^a = i\theta^1 \sigma^a \epsilon^1 + i\theta^2 \tilde{\sigma}^a \epsilon^2, \quad \delta_\epsilon \theta^{\alpha 1} = \epsilon^{\alpha 1}, \quad \delta_\epsilon \theta_\alpha^2 = \epsilon_\alpha^2, \\ \delta_\epsilon v_q^\alpha = 0 \Rightarrow \delta_\epsilon u_a^0 = \delta_\epsilon u_a^i = 0,$$

$$\delta_\epsilon \mathbb{X}^i = 0, \quad \delta_\epsilon \Psi_q = 0, \quad \delta_\epsilon \mathbb{P}^j = 0, \quad \delta_\epsilon \mathbb{A} = 0.$$

- Moreover, for any choice of positive definite $\mathcal{M}(\mathcal{H})$, this action is invariant under worldline supersymmetry which acts on center of mass variables as κ -symmetry of single D0-brane

$$\delta_\kappa \theta^{\alpha 1} = \kappa^q(\tau) v_q^\alpha, \quad \delta_\kappa \theta_\alpha^2 = -\kappa^q(\tau) v_\alpha^q, \\ \delta_\kappa X^a = -i\theta^1 \sigma^a \delta_\kappa \theta^1 - i\theta^2 \tilde{\sigma}^a \delta_\kappa \theta^2, \\ \delta_\kappa v_q^\alpha = 0 \Rightarrow \delta_\kappa u_a^0 = \delta_\kappa u_a^i = 0;$$

- while its action on the matrix field is more complicated:

Worldline supersymmetry transformations of the matrix fields

$$\delta_{\kappa} \mathbb{X}^i = \frac{4i}{\sqrt{\mathcal{M}}} \kappa \gamma^i \Psi + \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \delta_{\kappa} \mathcal{H} \mathbb{X}^i - \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \Delta_{\kappa} \mathcal{K} \mathbb{P}^i ,$$

$$\delta_{\kappa} \mathbb{P}^i = -\frac{1}{\sqrt{\mathcal{M}}} [\kappa \gamma^{ij} \Psi, \mathbb{X}^j] - \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \delta_{\kappa} \mathcal{H} \mathbb{P}^i +$$

$$+ \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \Delta_{\kappa} \mathcal{K} \left(\frac{1}{16} [[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] - \gamma_{pq}^i \{ \Psi_p, \Psi_q \} \right) ,$$

$$\delta_{\kappa} \Psi_q = -\frac{1}{2\sqrt{\mathcal{M}}} \left((\kappa \gamma^i)_q \mathbb{P}^i - \frac{i}{8} (\kappa \gamma^{ij})_q [\mathbb{X}^i, \mathbb{X}^j] \right) - \frac{i}{4\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \Delta_{\kappa} \mathcal{K} [(\gamma^i \Psi)_q, \mathbb{X}^i] .$$

$$\delta_{\kappa} \mathbb{A} = -\frac{2}{\mathcal{M}\sqrt{\mathcal{M}}} E^0 (\kappa^q \Psi_q) \left(\frac{1 - \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \mathcal{H}}{1 + \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \mathcal{H}} \right) + \frac{1}{\sqrt{2}\mathcal{M}} (E^{1q} - E_q^2) (\gamma^i \kappa)_q \mathbb{X}^i -$$

$$- (E^{1q} - E_q^2) \frac{1}{\mu^6} \frac{\mathcal{M}'}{\sqrt{2}\mathcal{M}^2} \left(\frac{1}{1 + \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \mathcal{H}} \right) \kappa^p \Psi_{(q} \text{tr} \left(4i(\gamma^i \Psi)_{p)} \mathbb{P}^i + \frac{5}{2} (\gamma^{ij} \Psi)_p [\mathbb{X}^i, \mathbb{X}^j] \right) .$$

- This is a local version of deformation of rigid SYM supersymmetry
- where $\mathcal{M}' = \mu^6 \frac{d\mathcal{M}}{d\mathcal{H}}$ (ask me why so strange definition),

Worldline supersymmetry transformations of the matrix fields

- where $\mathcal{M}' = \mu^6 \frac{d\mathcal{M}}{d\mathcal{H}}$,

$$\delta_{\kappa} \mathcal{H} = \frac{1}{\sqrt{\mathcal{M}}} \frac{\text{tr} \left(\kappa^q \Psi_q \left([\mathbb{X}^i, \mathbb{P}^i] - 4i \{ \Psi_q, \Psi_q \} \right) \right)}{1 + \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \mathfrak{H}}$$

with

$$\mathfrak{H} := \text{tr} \left(\mathbb{P}^i \mathbb{P}^i \right) + \frac{1}{16} \text{tr} \left[\mathbb{X}^i, \mathbb{X}^j \right]^2 + 2 \text{tr} \left(\mathbb{X}^i \Psi \gamma^i \Psi \right),$$

- (cf. $\mathcal{H} = \frac{1}{2} \text{tr} \left(\mathbb{P}^i \mathbb{P}^i \right) - \frac{1}{64} \text{tr} \left[\mathbb{X}^i, \mathbb{X}^j \right]^2 - 2 \text{tr} \left(\mathbb{X}^i \Psi \gamma^i \Psi \right)$)

- and

$$\Delta_{\kappa} \mathcal{K} = \frac{1}{2\sqrt{\mathcal{M}}} \frac{\text{tr} \left(4i(\kappa \gamma^i \Psi) \mathbb{P}^i + \frac{5}{2} (\kappa \gamma^{ij} \Psi) [\mathbb{X}^i, \mathbb{X}^j] \right)}{1 + \frac{1}{\mu^6} \frac{\mathcal{M}'}{\mathcal{M}} \mathfrak{H}}.$$

- All the family of action has the properties expected for mD0 system in flat target superspace,
- but two members are special.
- The action with $\mathcal{M} = m$ is the simplest representative found in [\[I.B.2018\]](#).
- The action with

$$\mathcal{M} = \frac{m}{2} + \sqrt{\frac{m^2}{4} + \frac{\mathcal{H}}{\mu^6}}$$

can be obtained from 11D mM0 action [\[I.B.2012\]](#). We show this in [\[I.B.+Sarraga 2023\]](#)

- where we also establish an interesting correspondence between the complete relative motion Eqs. of mD0 and the Eqs. of d=1 $\mathcal{N} = 16$ SYM.
- This correspondence does not imply a gauge equivalence but establishes a relation between solutions.
- In particular, it implies that all the SUSY solutions of mD0 equations in its relative motion part coincide with supersymmetric solutions of the SYM equations.

Towards field theory of mD0

- The quantization of the dynamical system described by our mD0 action should result in a field theory in superspace enhanced by additional bosonic and fermionic matrix coordinates
- which might give new insight in the structure and properties of String/M-theory.
 - Let us recall that the reason for this hope is the fact that our mD0-action (as well as mM0- action from [\[I.B., JHEP 2012\]](#)) can be considered as 10D (11D) Lorentz invariant generalization of the BFSS Matrix model [\[Banks, Fischler, Shenker, and Susskind, PRD 1996\]](#) ,
 - which was conjectured to provide a complete description of M-theory in certain limits (hence the name M(atrix) theory).
- As our mD0 action is known only in the frame of spinor moving frame formulation,
- the natural first step to approach the problem of such quantization is to quantize 10D D0-brane in its spinor moving frame formulation.

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Dirac quantization of the constrained system

- The most commonly used method to quantize systems with constraints was proposed by Dirac on the basis of his generalized Hamiltonian approach.
- It implies the identification of the constraints from calculation of momenta canonically conjugate to coordinates as derivatives of Lagrangian with respect to associated velocities and the conservation of these constraints in the evolution.
- Then one separates constraints in the sets of the **first class**, which generate symmetries on Poisson brackets (classical analogs of commutators or anti-commutators).
- and **second class**, which can be split (in principle) on conjugated pair having non-degenerate Poisson brackets.
- In quantum theory the first class constraints are imposed as conditions on the state vectors,
- while this is impossible to do with second class constraints.

Dirac quantization of the constrained system

- The conjugate pairs of second class constraints should be either resolved explicitly (converted in the pair of equalities of the type $p_* = 0$, $q^* = f(\text{other variables})$), which allows to reduce the phase space,
- or resolved implicitly by passing to the so-called Dirac brackets,
- or converted into the first class constraint.
- This latter is done by just omitting one of the conjugate second class constraint thus converting the second in effective first class constraint.
- This latter is then imposed on the state vector.

Dirac quantization of our system

- As spinor moving frame variables are strongly constrained and the study indicates mixtures of first and second class constraints, D0-brane in spinor moving frame formulation is not easy to quantize.
- However, what help are clear group-theoretical meaning of spinor moving frame variables and
- passing to the so-called analytical coordinate basis [Sokatchev 1985, 87nk, I.B. 90].

Analytical coordinate basis (AnB)

- We are working with superspace enlarged by spinor moving frame variables (Lorentz harmonic superspace) where we can change from the original coordinate basis $\mathcal{Z}^{\mathcal{M}} = (x^\mu, \theta^{\alpha 1}, \theta_\alpha^2, v_\alpha^q)$ to

$$\mathcal{Z}_{An}^{\mathcal{M}} = (x^0, x_{An}^I, \Theta_q, \tilde{\Theta}^q, v_\alpha^q) \quad \text{with}$$

$$x^0 = x^\mu u_\mu^0, \quad x_{An}^I = x^\mu u_\mu^I - \frac{i}{2} (\theta^{q1} - \theta_q^2) \gamma_{qp}^I (\theta^{p1} + \theta_p^2),$$

$$\Theta^q = \theta^{q1} + \theta_q^2, \quad \tilde{\Theta}^q = \theta^{q1} - \theta_q^2, \quad \theta^{q1} = \theta^{\alpha 1} v_\alpha^q, \quad \theta_q^2 = \theta_\alpha^2 v_q^\alpha.$$

- The expressions for x_{An}^I is chosen in such a way that $\tilde{\Theta}^q$ disappears from the Lagrangian which reads

$$\mathcal{L}_1^{D0} = m (dx^0 - id\Theta_q \Theta_q - \frac{i}{4} \Omega^{IJ} \Theta \gamma^{IJ} \Theta - x^I \Omega^I) , \quad \Theta \gamma^{IJ} \Theta \equiv \Theta_q \gamma_{qp}^{IJ} \Theta_p$$

($\Omega^{IJ} = u_a^I du^{aJ}$, $\Omega^I = u_a^0 du^{aI}$) and includes only 16 of 32 coordinate functions,

$$\Theta_q = \theta^{1q} + \theta_q^2 = \theta^{\alpha 1} v_\alpha^q + \theta_\alpha^2 v_q^\alpha.$$

- Thus κ -symmetry is realized trivially in AnB, or one can say it is automatically gauge fixed when we pass to AnB.

Cartan forms

- The Lagrangian 1-form

$$\mathcal{L}_1^{D0} = m \left(dx^0 - id\Theta_q \Theta_q - \frac{i}{4} \Omega^{IJ} \Theta \gamma^{IJ} \Theta - x^I \Omega^I \right), \quad \Theta \gamma^{IJ} \Theta \equiv \Theta_q \gamma_{qp}^{IJ} \Theta_p$$

involves also the **SO(1,9)/SO(9) Cartan forms**: SO(9) covariant $\Omega^I = u_\mu^I du^{\mu I}$ and composite SO(9) connection $\Omega^{IJ} = u_\mu^{IJ} du^{\mu J}$.

- These provide the basis for space co-tangent to SO(1,9) parametrized by spinor frame variable v_α^q and/or moving frame vectors.
- Using these instead of $(\dot{v}_\alpha^q = dv_\alpha^q/d\tau)$ to define covariant momenta

$$\mathfrak{d}^I = \frac{\partial L}{\partial \Omega^I_\tau} = \frac{\partial \mathcal{L}_1}{\partial \Omega^I}, \quad \mathfrak{d}^{IJ} = -\frac{\partial L}{\partial \Omega^{IJ}_\tau} = -\frac{\partial \mathcal{L}_1}{\partial \Omega^{IJ}},$$

instead of canonical momenta $P_q^\alpha = \frac{\partial \mathcal{L}_1}{\partial dv_\alpha^q}$ for strongly constrained v_α^q , allows to streamline essentially and to simplify drastically the calculations with highly constrained spinor frame variables.

- One can find

$$\mathfrak{d}^I = \frac{1}{2} v_\alpha^q \gamma_{qp}^I P_p^\alpha, \quad \mathfrak{d}^{IJ} = \frac{1}{2} v_\alpha^q \gamma_{qp}^{IJ} P_p^\alpha$$

and check: they have vanishing Poisson brackets with constraints on v .

- To construct the Hamiltonian formalism, we first calculate the canonical and covariant momenta.
- All these results in (primary) constraints (i.e. in relations without time derivatives of phase variables):

$$p_0 - m := p_{x_0} - m \approx 0 ,$$

$$p_I := p_{x_A^I} \approx 0 ,$$

$$\tilde{\mathfrak{d}}^I = \mathfrak{d}^I + m x_A^I \approx 0 ,$$

$$\tilde{\mathfrak{d}}^{IJ} = \mathfrak{d}^{IJ} - \frac{im}{4} \Theta_q \gamma_{qp}^{IJ} \Theta_p \approx 0 ,$$

$$\mathfrak{d}_q := \mathfrak{d}_{\Theta_q} = \Pi_q^\Theta + im \Theta_q \approx 0 ,$$

Canonical Hamiltonian vanishes on the surface of these constraints.

- This set includes the pair of explicitly resolved second class constraints

$$x^I \approx -\mathfrak{d}^I / m , \quad p_I \approx 0$$

which hence can be used to reduce the phase space by removing (x^I, p_I) directions.

- The bosonic constraint (linear combination of $\tilde{\mathfrak{d}}^{IJ}$ and $\mathfrak{d}_q$)

$$\tilde{\mathfrak{d}}^{IJ} = \mathfrak{d}^{IJ} + \frac{1}{4} \Theta_q \gamma_{qp}^{IJ} \Pi_p^\Theta \approx 0$$

is of the first class. It generate $SO(9)$ gauge symmetry on the reduced phase space.

- $p_0 - m := p_{x_0} - m \approx 0$ is also of the first class. It implies that $x^0(\tau)$ can be gauged away, but we prefer to keep it (ask me why, if interested).
- Finally we have 16 imaginary fermionic second class constraints

$$\mathfrak{d}_q = \Pi_q^\Theta + i \Theta_q p_0$$

which obey the algebra

$$\{\mathfrak{d}_q, \mathfrak{d}_p\}_{PB} = -2ip_0 \delta_{qp} .$$

- Most problems for quantization come from these fermionic constraints.
- We are going to discuss the quantization in super-coordinate representation and will conventionally describe all the constraints by differential operators providing their quantum realization.

- By prescription of Dirac [Dirac:1963] we have to impose the first class constraints on the state vector.
- Imposing the generator of $SO(9)$ gauge symmetry we arrive at

$$\left(D^\mu + \frac{1}{2} \Theta_q \gamma_{qp}^\mu \frac{\partial}{\partial \Theta^q} \right) \Xi = 0, \quad D^\mu = + \frac{1}{2} v_\alpha^p \gamma_{pq}^\mu \frac{\partial}{\partial v_\alpha^q},$$

which implies that the state vector superfield Ξ is $SO(9)$ invariant.

- Imposing the last bosonic constraint $\hat{p}_0 - m$,

$$(-i\partial_{x^0} - m)\Xi = 0$$

we fix the x^0 dependence of the state vector to be exponential,

$$\Xi(x^0, v, \Theta) = e^{imx^0} \Phi(v, \Theta).$$

- The main problem is how to impose the quantum version of the fermionic constraints which can be realized as $\hat{d}_q = -iD_q$ with

$$D_q = \partial_{\Theta^q} + i\Theta_q \partial_{x^0} \quad (\Leftrightarrow \quad \partial_{\Theta^q} - m\Theta_q)$$

obeying

$$\{D_q, D_p\} = 2i\delta_{qp}\partial_{x^0} \quad (\Leftrightarrow \quad -2m\delta_{qp}).$$

Internal spinor frame and Gupta-Bleuler (conversion) method

- The problem can be solved if we could split imaginary D_q on two sets of complex conjugate and canonically conjugate constraints.
- Then we can just "omit" one of conjugate constraints and impose the second, which thus becomes first class.
- This is called Gupta-Bleuler method (first applied to quantize gauge fixed description of Maxwell Electrodynamics).
- For realizing this program we need to introduce complex structure.
- This will break $SO(9)$ symmetry. But to make such breaking a kind of spontaneous, i.e. realized with Stückelberg mechanism, we can use the counterpart of spinor moving frame variables (internal harmonics).
- First we introduce the $SO(9)$ frame described by $SO(7) \times U(1)$ covariant blocks of $SO(9)$ valued matrix

$$U_l^{(J)} = \left(U_l^{\check{J}}, U_l^{(8)}, U_l^{(9)} \right) = \left(U_l^{\check{J}}, \frac{1}{2} (U_l + \bar{U}_l), \frac{1}{2i} (U_l - \bar{U}_l) \right) \in SO(9)$$

the columns of which are two complex null and 6 orthogonal unit vectors

$$U_l U_l = 0 = \bar{U}_l \bar{U}_l, \quad U_l \bar{U}_l = 2, \quad U_l U_l^{\check{J}} = 0 = \bar{U}_l U_l^{\check{J}}, \quad U_l^{\check{J}} U_l^{\check{K}} = \delta^{\check{J}\check{K}}.$$

Internal spinor frame and Gupta-Bleuler (conversion) method

- Now let us introduce the $Spin(9)$ valued matrix

$$w_p^{(q)} = (\bar{w}_{pA}, w_p^A) \in Spin(9),$$

which provides a double covering of the vector frame matrix as an element of $SO(9)$. This implies, in particular,

$$U_l \gamma_{qp}^I = 2 \bar{w}_{qA} \bar{U}^{AB} \bar{w}_{pB}, \quad \bar{U}_l \gamma_{qp}^I = 2 w_q^A \mathcal{U}_{AB} w_p^B,$$

$$U_l^{\check{J}} \gamma_{qp}^I = i w_q^A (\sigma^{\check{J}} \bar{U})_A^B \bar{w}_{qB} + i \bar{w}_{qA} (\tilde{\sigma}^{\check{J}} \mathcal{U})^A_B w_q^B,$$

$$\bar{w}_{qA} \gamma_{qp}^I \bar{w}_{pB} = U_l \mathcal{U}_{AB}, \quad w_q^A \gamma_{qp}^I w_p^B = \bar{U}_l \bar{U}^{AB},$$

$$\bar{w}_{qA} \gamma_{qp}^I w_p^B = i U_l^{\check{J}} (\sigma^{\check{J}} \bar{U})_A^B.$$

- Here $\tilde{\sigma}^{JAB} = -\sigma_{AB}^{\check{J}} = +\sigma_{BA}^{\check{J}} = (\tilde{\sigma}^{JAB})^\dagger$ are $SO(7)$ Klebsh-Gordan coefficients and $\mathcal{U}_{AB}$ is complex symmetric unitary matrix.
- We can chose $\mathcal{U}_{AB} = \delta_{AB}$, $\bar{U}^{AB} = (\mathcal{U}_{AB})^* = \delta^{AB}$ but... (ask me why).
- Furthermore, as $Spin(9) \subset SO(16)$,

$$\bar{w}_{qA} \bar{w}_{qB} = 0 = w_q^A w_q^B, \quad \bar{w}_{qA} w_q^B = \delta_A^B, \quad \bar{w}_{qA} w_p^A + w_q^A \bar{w}_{pA} = \delta_{qp}.$$

- We can also avoid introduction of moving frame and just set

$$U_I = \delta_I^{(8)} + i\delta_I^{(9)}, \quad U_I^{\check{J}} = \delta_I^{\check{J}}$$

thus explicitly breaking $SO(9)$ down to $SO(7) \times SO(2)$.

- However, we will need to introduce anyway their factorization on products of $\bar{w}_{qA}$, w_q^A as these allow
- to split the fermionic coordinates on a pair of conjugate octuplets,

$$\Theta^A = \Theta^q w_q^A, \quad \bar{\Theta}_A = \Theta^q \bar{w}_{qA}.$$

- and also split the real constraint/derivative D_q on conjugate octuplets

$$D_A = \bar{w}_{qA} D_q = \partial_A + i\bar{\Theta}_A \partial_{x^0} = e^{-i\Theta^A \bar{\Theta}_A \partial_{x^0}} \partial_A e^{+i\Theta^A \bar{\Theta}_A \partial_{x^0}},$$

$$\bar{D}^A = w_q^A D_q = -(\bar{D}_A)^* = \bar{\partial}^A + i\Theta^A \partial_{x^0} = e^{+i\Theta^A \bar{\Theta}_A \partial_{x^0}} \bar{\partial}^A e^{-i\Theta^A \bar{\Theta}_A \partial_{x^0}},$$

which obey the algebra

$$\{D_A, D_B\} = 0, \quad \{D_A, \bar{D}^B\} = -2m\delta_A^B, \quad \{\bar{D}^A, \bar{D}^B\} = 0.$$

- With such algebra of constraints,

$$\{D_A, D_B\} = 0, \quad \{D_A, \bar{D}^B\} = -2m\delta_A^B, \quad \{\bar{D}^A, \bar{D}^B\} = 0,$$

- we can forget about constraints D_A , thus converting $\bar{D}^B$ into the first class constraints (conversion),
- and/or just impose this latter on the state vector (Gupta–Bleuler),

$$\bar{D}^A \Xi = 0.$$

- This suggests the conclusion that the state vector is just **chiral (analytical) superfield**.
- But this is not exactly the case (this is not the end of story).

- The solution of $\bar{D}^A \Xi(x^0, \Theta^B, \bar{\Theta}_B, v, w) = 0$ and $(-i\partial_{x^0} - m)\Xi = 0$ is

$$\begin{aligned}
e^{im(x^0 - i\Theta^A \bar{\Theta}_A)} \Xi(x^0, \Theta, \bar{\Theta}, v, w) = & \phi + \Theta^A \psi_A + \frac{1}{2} \Theta^B \Theta^A \phi_{AB} + \\
& + \frac{1}{3!} \Theta^C \Theta^B \Theta^A \psi_{ABC} + \frac{1}{4!} \Theta^D \Theta^C \Theta^B \Theta^A \psi_{ABCD} + \\
& + \frac{1}{5! 3!} \epsilon_{A_1 \dots A_5 B_1 B_2 B_3} \Theta^{A_5} \dots \Theta^{A_1} \tilde{\psi}^{B_1 B_2 B_3} + \frac{1}{6! 2!} \epsilon_{A_1 \dots A_4 B_1 B_2} \Theta^{A_6} \dots \Theta^{A_1} \tilde{\phi}^{B_1 B_2} + \\
& + \frac{1}{7!} \epsilon_{A_1 \dots A_7 B} \Theta^{A_7} \dots \Theta^{A_1} \tilde{\psi}^B + \frac{1}{8!} \epsilon_{A_1 \dots A_7 B} \Theta^{A_8} \dots \Theta^{A_1} \tilde{\phi} .
\end{aligned}$$

- This chiral superfield is similar to the so-called on-shell superfields for D=4 $\mathcal{N} = 8$ supergravity.
- As in this case, chiral superfield does not describe irreducible representation: one can reduce it by imposing

$$\tilde{\psi}^{B_1 B_2 B_3} = \propto (\psi_{B_1 B_2 B_3})^*, \quad \tilde{\phi}^{B_1 B_2} = \propto (\phi_{B_1 B_2})^*, \quad \tilde{\psi}^B = \propto (\psi_B)^*, \quad \tilde{\phi} = \propto (\phi)^*,$$

as well as the duality relation for the intermediate component,

$$\phi_{A_1 \dots A_4} = \frac{1}{4! 4!} \epsilon_{A_1 \dots A_4 B_1 B_2 B_3 B_4} \tilde{\phi}^{B_1 B_2 B_3 B_4}, \quad (\phi_{B_1 \dots B_4})^* = \tilde{\phi}^{B_1 B_2 B_3 B_4} .$$

- These relations can be collected in one **superfield duality equations**

$$D_{A_1} \dots D_{A_4} \Xi = \frac{1}{4! 4!} \epsilon_{A_1 \dots A_4 B_1 B_2 B_3 B_4} \bar{D}^{B_1} \dots \bar{D}^{B_4} \Xi, \quad (\Xi)^* = \Xi.$$

- The imposing of the additional condition is motivated by requirement that the state vector should be described by an irrep of SUSY
- (cf. the paradigm that elementary particle is described by irreducible representation of the Poincaré group).
- Thus we have reduced the field content of the state vector superfield to **bosonic**

$$\begin{aligned} \phi(V, W, \bar{W}), \quad \phi_{AB}(V, W, \bar{W}) &= -\phi_{BA}(V, W, \bar{W}), \\ \phi_{A_1 \dots A_4} &= \frac{1}{4! 4!} \epsilon_{A_1 \dots A_4 B_1 B_2 B_3 B_4} (\phi_{B_1 \dots B_4})^* \end{aligned}$$

and **fermionic spin-tensors of SO(7)**

$$\psi_A(V, W, \bar{W}), \quad \psi_{ABC}(V, W, \bar{W}) = \psi_{[ABC]}(V, W, \bar{W}).$$

- These describe the **quantum state spectrum of D0-brane**.
- But what is the supermultiplet described by these?
- We will show: this is the massive counterpart of 10D type IIA SUGRA.

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- To show that the above obtained quantum state spectrum of D0 brane is described by massive counterpart of type IIA SUGRA, we should
-
- know the linearized equations of this,
- solve these and
- show that their general solution can be expressed in terms of the above set of $SO(7)$ spin tensors.
-
- To our best knowledge such a massive counterpart of IIA supergravity was not elaborated before.
- (Notice that this cannot be neither Romans massive SUGRA nor Howe–Lambert–West massive SG model; ask me why).
- As the usual (massless) type IIA supergravity can be obtained by “zero mode” dimensional reduction of 11D SUGRA, the natural way to search for its massive counterpart is to consider a nontrivial “massive” mode of such dimensional reduction.

Massive type IIA SG from 11D SG. Fermionic story

- In 11D RS equation $\Gamma_{\underline{\alpha}\underline{\beta}}^{\underline{\mu}\underline{\nu}\underline{\rho}} \partial_{\underline{\nu}} \underline{\Psi}_{\underline{\rho}}^{\underline{\beta}} = 0$ we make SO(1,9) invariant splitting

$$\underline{x}^{\underline{\nu}} = (x^{\nu}, y^*), \quad \underline{\partial}_{\underline{\nu}} = (\partial_{\nu}, \partial_*),$$

$$\underline{\Psi}_{\underline{\mu}}^{\underline{\alpha}} = \begin{pmatrix} \underline{\Psi}_{\mu}^{\alpha 1}(x, y^*) & \underline{\Lambda}^{\alpha 2}(x, y^*) \\ \underline{\Psi}_{\mu\alpha}^2(x, y^*) & \underline{\Lambda}_{\alpha}^1(x, y^*) \end{pmatrix},$$

- and use the ansatz

$$\underline{\Psi}_{\mu}^{\alpha 1}(x, y^*) = \Psi_{\mu}^{\alpha 1}(x) \cos(my^*), \quad \underline{\Lambda}_{\alpha}^1(x, y^*) = \Lambda_{\alpha}^1(x) \cos(my^*),$$

$$\underline{\Psi}_{\mu\alpha}^2(x, y^*) = \Psi_{\mu\alpha}^2(x) \sin(my^*), \quad \underline{\Lambda}^{\alpha 2}(x, y^*) = \Lambda^{\alpha 2}(x) \sin(my^*).$$

- Then local supersymmetry allows to fix the gauge $\Lambda_{\alpha}^1 = 0 = \Lambda^{\alpha 2}$ and to obtain the following split form of the gauge fixed equations

$$\begin{aligned} \not{\partial}_{\alpha\beta} \Psi_{\mu}^{\beta 1} &= m \Psi_{\mu\alpha}^2, & \tilde{\not{\partial}}^{\alpha\beta} \Psi_{\mu\beta}^2 &= -m \Psi_{\mu}^{\alpha 1}, \\ \sigma_{\alpha\beta}^{\mu} \Psi_{\mu}^{\beta 1} &= 0, & \tilde{\sigma}^{\mu\alpha\beta} \Psi_{\mu\beta}^2 &= 0, \\ \partial^{\mu} \Psi_{\mu}^{\alpha 1} &= 0, & \partial^{\mu} \Psi_{\mu\alpha}^2 &= 0. \end{aligned}$$

Solution of the dimensionally reduced RS equations. SO(9) invariant form

- Passing to the momentum representation and using the spinor moving frame method with $p_\mu = mu_\mu^0$, we solve these equations by

$$\Psi_\mu^{\alpha 1} = v_q^\alpha u_\mu^I \Psi_q^I, \quad \Psi_{\mu\alpha}^2 = v_\alpha^q u_\mu^I \Psi_q^I,$$

with gamma-traceless SO(9) vector-tensor Ψ_q^I ,

$$\gamma_{pq}^I \Psi_q^I = 0.$$

- Notice that the gauge fixed version of the original massless 11D equation in momentum representation and with the use of 11D spinor moving frame method with $\underline{p}_\mu = \rho^\# \underline{u}_\mu^-$ is also solved

$$\underline{\Psi}_\mu^\alpha = \underline{u}_\mu^I v_q^\alpha \underline{\Psi}_q^I$$

in terms of similar gamma-traceless SO(9) vector-tensor, $\gamma_{pq}^I \underline{\Psi}_q^I = 0$, but dependent on 11D moving frame variables (parametrizing 11D celestial sphere),

$$\underline{\Psi}_q^I = \underline{\Psi}_q^I(\rho^\# \underline{u}_\mu^-) \quad \text{vs} \quad \Psi_q^I = \Psi_q^I(u_\mu^0).$$

Dimensional reduction of graviton equation

- As the linearized Riemann tensor reads $R_{\underline{\mu}\underline{\nu}\underline{\rho}\underline{\sigma}} = 2\partial_{[\underline{\mu}}\partial_{[\underline{\rho}}h_{\underline{\sigma}]\underline{\nu}]}$, the linearized Einstein equation obeyed by 11D graviton is

$$R^{\underline{\nu}}_{\underline{\mu}_1\underline{\nu}\underline{\mu}_2} = 0 \quad \Rightarrow \quad \square h_{\underline{\mu}_1\underline{\mu}_2} - \partial_{\underline{\mu}_1}\partial^{\underline{\nu}}h_{\underline{\nu}\underline{\mu}_2} - \partial_{\underline{\mu}_2}\partial^{\underline{\nu}}h_{\underline{\nu}\underline{\mu}_1} + \partial_{\underline{\mu}_1}\partial_{\underline{\mu}_2}h^{\underline{\nu}}_{\underline{\nu}} = 0.$$

- We can fix the gauge in which these equation splits into

$$\square h_{\underline{\mu}\underline{\nu}} = 0, \quad \partial^{\underline{\mu}}h_{\underline{\mu}\underline{\nu}} = 0, \quad h^{\underline{\mu}}_{\underline{\mu}} = 0,$$

which in momentum representation is solved by moving frame method with $\underline{p}_{\underline{\mu}} = \rho^{\#}u_{\underline{\mu}}^{-}$ by

$$h_{\underline{\mu}\underline{\nu}} = \underline{u}_{\underline{\mu}}^I \underline{u}_{\underline{\nu}}^J \underline{h}^{IJ}, \quad \underline{h}^{IJ} = \underline{h}^{JI}, \quad \underline{h}^{II} = 0.$$

- Now we use the splitting and dimensional reduction ansatz

$$h_{\underline{\mu}\underline{\nu}}(\underline{x}) = \begin{pmatrix} \underline{h}_{\mu\nu} & \underline{A}_{\nu} \\ \underline{A}_{\mu} & \underline{h} \end{pmatrix}, \quad \underline{h}_{\mu\nu}(x, y^*) = h_{\mu\nu}(x) \cos my^*,$$

$$\underline{A}_{\mu}(x, y^*) = A_{\mu}(x) \sin my^*, \quad \underline{h}(x, y^*) = h(x) \cos my^*.$$

Massive graviton equations and its solution

- With this ansatz and fixing by diffeomorphisms the gauge $A_\mu = 0$, we find the gauge fixed form the dimensionally reduced 11D linearized Einstein equations, which are equations for massive graviton

$$(\square + m^2)h_{\mu\nu}(x) = 0, \quad \partial^\mu h_{\mu\nu}(x) = 0, \quad h_\mu{}^\mu = 0.$$

- In momentum representation, using moving frame method with $p_\mu = mu_\mu^0$, we can solve these equations in terms of symmetric traceless SO(9) tensor

$$h_{\mu\nu} = u_\mu^I u_\nu^J h^{IJ}, \quad h^{IJ} = h^{JI}, \quad h^I{}_I = 0.$$

- So, like in 11D case above, the solution is expressed in terms of symmetric and traceless SO(9) tensor of second rank, but now this tensor field depends on 10D vector frame variables,

$$h^{IJ} = h^{IJ}(u_\mu^0) \quad \text{vs} \quad \underline{h}^{IJ}(\rho^\# u_\mu^-).$$

Dimensional reduction of 3-form gauge field. SO(9) invariant solution

- Similarly, one can show that the gauge fixed version of the Maxwell equation for 3-form gauge field

$$\partial^\mu F_{\underline{\mu}\underline{\nu}\underline{\rho}\underline{\sigma}} = 0, \quad F_{\underline{\mu}\underline{\nu}\underline{\rho}\underline{\sigma}} = 4\partial_{[\underline{\mu}} A_{\underline{\nu}\underline{\rho}\underline{\sigma}]}$$

splits into Klein-Gordon equation and divergence-free conditions

$$\square A_{\underline{\mu}\underline{\nu}\underline{\rho}} = 0, \quad \partial^\rho A_{\underline{\mu}\underline{\nu}\underline{\rho}} = 0,$$

which in momentum representations, after using the residual gauge invariance, are solved by

$$A_{\underline{\mu}\underline{\nu}\underline{\rho}} = u_\mu^I u_\nu^J u_\rho^K A_{IJK},$$

in terms of antisymmetric SO(9) tensor $A_{IJK} = A_{[IJK]}(\rho^\#, u_\mu^-)$.

- The dimensionally reduced gauge fixed equations for 3-form gauge field are given by

$$(\square + m^2)A_{\mu\nu\rho} = 0, \quad \partial^\rho A_{\mu\nu\rho} = 0,$$

- which are solved by $A_{\mu\nu\rho} = u_\mu^I u_\nu^J u_\rho^K A_{IJK}$ in terms of a similar antisymmetric SO(9) tensor $A_{IJK} = A_{[IJK]}(u_\mu^0)$.

D0-brane field theory: SO(9) vector-spinor and SO(7) fermionic spin-tensors

- Now the problem is to show that the parameters of solution of linearized massive (counterpart of) type A supergravity equations,
- this is to say, the components of fermionic gamma traceless Ψ_q^I , bosonic symmetric traceless h^{IJ} and bosonic totally antisymmetric A_{IJK} ,
- are in one-to-one correspondence with SO(7) spin-tensors which were shown to describe quantum state spectrum of D0-brane.
- This is indeed the case.
- For fermionic and bosonic field the one-to-one correspondence is describe by

$$\Psi_q^I = \frac{1}{2} U_I w_q^A \psi_A + \frac{1}{2} \bar{U}_I \bar{w}_{qA} \bar{\psi}^A + U_I^{\check{J}} \bar{w}_{qA} \sigma_{BC}^{\check{J}} \bar{\psi}^{ABC} + U_I^{\check{J}} w_q^A \tilde{\sigma}^{\check{J}BC} \psi_{ABC}$$

$$(128 = 8 + 8 + 56 + 56) .$$

- $\tilde{\sigma}^{\check{I}AB} = -\tilde{\sigma}^{\check{I}BA}$ and $\sigma_{AB}^{\check{I}} = (\tilde{\sigma}^{\check{I}BA})^*$ are SO(7) counterpart of Pauli matrices,
- $\mathcal{U}_{AB} = \mathcal{U}_{BA}$ and its inverse $\bar{\mathcal{U}}^{AB} = (\mathcal{U}_{AB})^*$ parametrize $SU(8)/SO(7)$ coset and play the role of SO(7) charge conjugation matrix,

D0-brane field theory: SO(9) tensors in terms of SO(7) spin-tensors

$$\begin{aligned}
 h_{IJ} = & U_I U_J \phi + \bar{U}_I \bar{U}_J \bar{\phi} + U_{[I}^{\check{I}} U_{J]} \tilde{\sigma}^{\check{I}AB} \phi_{AB} + U_{[I}^{\check{I}} U_{J]} \sigma_{AB}^{\check{I}} \bar{\phi}^{AB} + \\
 & + U_I^{\check{I}} U_J^{\check{J}} \tilde{t}_{\check{I}\check{J}}^{ABCD} \phi_{ABCD} - U_I \bar{U}_J \tilde{t}_{\check{I}\check{J}}^{ABCD} \phi_{ABCD} , \\
 & (44 = 1 + 1 + 7 + 7 + 28) ,
 \end{aligned}$$

$$\begin{aligned}
 A_{IJK} = & U_{[I}^{\check{I}} U_J \bar{U}_{K]} i \tilde{t}_{\check{I}}^{ABCD} \phi_{ABCD} + U_{[I}^{\check{I}} U_J^{\check{J}} U_{K]}^{\check{K}} i \tilde{t}_{\check{I}\check{J}\check{K}}^{ABCD} \phi_{ABCD} + \\
 & + U_{[I}^{\check{I}} U_J^{\check{J}} U_{K]} (\tilde{\sigma}^{\check{J}\check{K}} \bar{\mathcal{U}})^{AB} \phi_{AB} + U_{[I}^{\check{I}} U_J^{\check{J}} \bar{U}_{K]} (\sigma^{\check{J}\check{K}} \mathcal{U})_{AB} \bar{\phi}^{AB} , \\
 & (84 = 7 + 35 + 21 + 21) .
 \end{aligned}$$

- $\tilde{\sigma}^{\check{I}AB} = -\tilde{\sigma}^{\check{I}BA}$ and $\sigma_{AB}^{\check{I}} = (\tilde{\sigma}^{\check{I}BA})^*$ are SO(7) counterpart of Pauli matrices,
- $\mathcal{U}_{AB} = \mathcal{U}_{BA}$ and $\bar{\mathcal{U}}^{AB} = (\mathcal{U}_{AB})^* = (\mathcal{U}^{-1})^{AB}$ play the role of SO(7) charge conjugation matrix,
- $t_{ABCD}^{\check{I}\check{J}} = (\sigma^{[\check{I}\check{K}} \mathcal{U})_{[AB} (\sigma^{\check{J}\check{K}} \mathcal{U})_{CD]} + (\sigma^{[\check{I}})_{[AB} (\sigma^{\check{J}})_{CD]} = \frac{1}{4!} \epsilon^{ABCDEFGH} t_{EFGH}^{\check{I}\check{J}}$,
- $\tilde{t}_{\check{I}\check{J}\check{K}}^{ABCD} = (\tilde{\sigma}^{[\check{I}\check{J}} \bar{\mathcal{U}})^{[AB} \tilde{\sigma}^{CD]} \check{K}] = (t_{ABCD}^{\check{I}\check{J}\check{K}})^* = -\frac{1}{4!} \epsilon^{ABCDEFGH} t_{EFGH}^{\check{I}\check{J}\check{K}}$
- $\tilde{t}_{\check{I}}^{ABCD} = (\tilde{\sigma}^{\check{I}\check{K}} \bar{\mathcal{U}})^{[AB} \tilde{\sigma}^{CD]} \check{K} = (t_{ABCD}^{\check{I}})^* = \frac{1}{4!} \epsilon^{ABCDEFGH} t_{EFGH}^{\check{I}}$.

D0-brane field theory: linearized equations

- Thus we have shown that the quantum state spectrum of D0 brane, originally described by chiral (analytical) on-shell superfield,
- is actually given by massive (counterpart of) type IIA supergravity multiplet.
- The standard form of equations of this supermultiplet are given by 10D Fierz-Pauli equation

$$(\square + m^2)h_{\mu\nu} - 2\partial^\sigma \partial_{(\mu} h_{\nu)\sigma} + \partial_\mu \partial_\nu h_\rho{}^\rho + \eta_{\mu\nu} [\partial^\sigma \partial^\rho h_{\rho\sigma} - (\square + m^2)h_\rho{}^\rho] = 0,$$

- massive 3-form gauge field equations

$$\partial^\mu F_{\mu\nu\rho\sigma} + m^2 A_{\nu\rho\sigma} = 0 .$$

- and type IIA generalization of massive gravitino equations

$$\sigma^{\mu\nu\rho} \partial_\nu \Psi_\rho^1 = -m \sigma^{\mu\nu} \Psi_\nu^2 , \quad \tilde{\sigma}^{\mu\nu\rho} \partial_\nu \Psi_\rho^2 = m \tilde{\sigma}^{\mu\nu} \Psi_\nu^1 .$$

D0-brane field theory: linearized action

- These equations can be obtained from the following Lagrangian

$$\begin{aligned}
 L = & -h^{\mu\nu}(\square + m^2)h_{\mu\nu} + 2h^{\mu\nu}\partial^\sigma\partial_\mu h_{\nu\sigma} - \\
 & - h^{\mu\nu}\partial_\mu\partial_\nu h_\rho{}^\rho - h_\sigma{}^\sigma\partial^\sigma\partial^\rho h_{\rho\sigma} + h_\sigma{}^\sigma(\square + m^2)h_\rho{}^\rho + \\
 & + \frac{1}{4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - m^2\frac{1}{3!}A_{\mu\nu\rho}A^{\mu\nu\rho} + \\
 & + i\Psi_\mu^1\sigma^{\mu\nu\rho}\partial_\nu\Psi_\rho^1 + i\Psi_\mu^2\tilde{\sigma}^{\mu\nu\rho}\partial_\nu\Psi_\rho^2 + 2mi\Psi_\mu^1\sigma^{\mu\nu}\Psi_\nu^2
 \end{aligned}$$

- which is invariant under the following supersymmetry transformations:

$$\begin{aligned}
 \delta h_{\mu\nu} = & -2i\Psi_{(\mu}^1\sigma_{\nu)}\epsilon^1 - 2i\bar{\Psi}_{(\mu}^2\tilde{\sigma}_{\nu)}\epsilon^2 - \frac{2i}{m}\partial_{(\mu}\Psi_{\nu)}^1\epsilon^2 + \frac{2i}{m}\partial_{(\mu}\Psi_{\nu)}^2\epsilon^1, \\
 \delta A_{\mu\nu\rho} = & 3i\Psi_{[\mu}^1\sigma_{\nu\rho]}\epsilon^2 - 3i\Psi_{[\mu}^2\tilde{\sigma}_{\nu\rho]}\epsilon^1 - \frac{6i}{m}\partial_{[\mu}\Psi_{\nu}^1\sigma_{\rho]}\epsilon^1 - \frac{6i}{m}\partial_{[\mu}\Psi_{\nu}^2\tilde{\sigma}_{\rho]}\epsilon^2, \\
 & \dots
 \end{aligned}$$

D0-brane field theory: linearized action

- This Lagrangian is invariant under the following SUSY transformations:

$$\delta h_{\mu\nu} = -2i\psi_{(\mu}^1 \sigma_{\nu)} \epsilon^1 - 2i\bar{\psi}_{(\mu}^2 \tilde{\sigma}_{\nu)} \epsilon^2 - \frac{2i}{m} \partial_{(\mu} \psi_{\nu)}^1 \epsilon^2 + \frac{2i}{m} \partial_{(\mu} \bar{\psi}_{\nu)}^2 \epsilon^1,$$

$$\delta A_{\mu\nu\rho} = 3i\psi_{[\mu}^1 \sigma_{\nu\rho]} \epsilon^2 - 3i\bar{\psi}_{[\mu}^2 \tilde{\sigma}_{\nu\rho]} \epsilon^1 - \frac{6i}{m} \partial_{[\mu} \psi_{\nu}^1 \sigma_{\rho]} \epsilon^1 - \frac{6i}{m} \partial_{[\mu} \bar{\psi}_{\nu}^2 \tilde{\sigma}_{\rho]} \epsilon^2,$$

$$\begin{aligned} \delta \psi_{\mu}^1 &= -2\tilde{\sigma}^{\nu\rho} \epsilon^1 \partial_{\nu} h_{\mu\rho} + 2m\tilde{\sigma}^{\nu} \epsilon^2 h_{\mu\nu} - \\ &\quad - \frac{1}{9} \left[\left(\delta_{\mu}^{[\nu} \tilde{\sigma}^{\rho\tau\lambda]} - \frac{1}{8} \tilde{\sigma}_{\mu}^{\nu\rho\tau\lambda} \right) F_{\nu\rho\tau\lambda} - \partial_{\mu} A_{\rho\tau\lambda} \tilde{\sigma}^{\rho\tau\lambda} \right] \epsilon^2 - \\ &\quad - \frac{1}{18} \left(\frac{1}{4m} \partial_{\mu} F_{[4]} \tilde{\sigma}^{[4]} + mA_{[3]} \tilde{\sigma}_{\mu}^{[3]} - 6mA_{\mu[2]} \tilde{\sigma}^{[2]} \right) \epsilon^1, \end{aligned}$$

$$\begin{aligned} \delta \bar{\psi}_{\mu}^2 &= 2\sigma^{\nu\rho} \epsilon^2 \partial_{\nu} h_{\mu\rho} + 2m\sigma^{\nu} \epsilon^1 h_{\mu\nu} - \\ &\quad - \frac{1}{9} \left[\left(\delta_{\mu}^{[\nu} \sigma^{\rho\tau\lambda]} - \frac{1}{8} \sigma_{\mu}^{\nu\rho\tau\lambda} \right) F_{\nu\rho\tau\lambda} - \partial_{\mu} A_{\rho\tau\lambda} \sigma^{\rho\tau\lambda} \right] \epsilon^1 + \\ &\quad + \frac{1}{18} \left(\frac{1}{4m} \partial_{\mu} F_{[4]} \sigma^{[4]} + mA_{[3]} \sigma_{\mu}^{[3]} - 6mA_{\mu[2]} \sigma^{[2]} \right) \epsilon^2. \end{aligned}$$

Outline

- 1 Introduction
- 2 D0 brane action in (spinor) moving frame formalism and induced supergravity
- 3 (A family of candidate) Multiple D0-brane action(s)
 - Fields of mD0 system
 - Multiple D0-brane (candidate) action(s)
- 4 D0-brane Hamiltonian mechanics and quantization
- 5 D0-brane quantum state spectrum and field theory
- 6 **Conclusion and Discussion**
 - Conclusion
 - Outlook

Conclusion: mD0 action

- In this talk I have shown the family of candidate for complete supersymmetric action of the system of N nearly coincident D0-branes (mD0 system) in flat ten dimensional type IIA superspace.
- Particular representative of this family can be obtained by dimensional reduction of mM0–action.
- As the model can be considered as 10D Lorentz covariant version of BFSS matrix model, the basis of M(atrix) theory, its quantization resulting in obtaining the mD0 field theory might shed new light on the structure and properties of String/M-thery.

D0 quantization

- As these mD0 actions are known in the frame of spinor moving frame formulation only, it is natural to begin with quantization of single D0-brane in its spinor moving frame formulations.
- Curiously, to our best knowledge the D0-brane quantization and its field theory had not been elaborated before; its quantum state spectrum had not been discussed.
- We have done covariant quantization of D0-brane and have shown that its quantum state spectrum is given by massive counterpart of type IIA supergravity.
- We have derived linearized equations of this supergravity by dimensional reduction of 11D supergravity, solved them and wrote solutions in terms of SO(7) spin-tensors describing quantum state spectrum of D0 in the quantization scheme which we have used.
- Many interesting technical problems appeared and were solved on the way.

Outlook

- Many interesting problems is to be solved.
- The next step towards field theory of mD0 system is to quantize the model from [I.B.2018] which is member of our family of candidate mD0 actions [I.B.+Sarraga=2022] with $\mathcal{M}(\mathcal{H}) = m = \text{const.}$
- Notice that massive type IIA supergravity gives rise to a higher spin model in D=4, which folklore relates to problems with causality.
- However, one can expect that this problem, if appears, is to be resolved in a complete type IIA string theory, including D0–branes.
- Actually the spinor moving frame formulation is providing us with a basis of the spinor helicity formalism which can be used to study amplitudes involving D0–branes and type IIA supergraviton (quantum state of massless type IIA superparticle and also massless supestring modes).
- The quantization of mD0 action, beginning the simplest $\mathcal{M} = m$ representative of the family, will be the subject of forthcoming papers and talks.
- Also the search for higher p mD p actions and mD0 and mM0 actions in supergravity superspaces is on the way.
- ...

THE END!

THANK YOU FOR YOUR ATTENTION!

Outline

7 Additional slides

Infrared limit of mM2 and so on

- The infrared fixed points of the system of N M2-branes is believed to be described by [Bagger–Lambert–Gustavsson \(BLG\)](#) model [2006,2007] for $N=2$ and
- by [Aharony–Bergman–Jafferis–Maldacena \(ABJM\)](#) model [2008] for $N \geq 2$.
- The infrared fixed point of multiple M5-brane system is given by an enigmatic $D = 6$ $(2, 0)$ superconformal theory;
- in 2010 it was conjectured [[Douglas:2010](#); [Lambert, Papageorgakis, Schmidt-Sommerfeld:2010](#)] that this can be described by $D = 5$ SYM model.