

Twist Fields and Anomalies for non-Invertible Symmetries

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- Examples
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Generalized Symmetries and Charges

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$$\nabla_{\mu_1} j_p^{\mu_1 \dots \mu_p} = 0 \qquad d \star j_p = 0$$

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$$Q = \int_{\mathcal{M}_{d-p}} \star j_p$$

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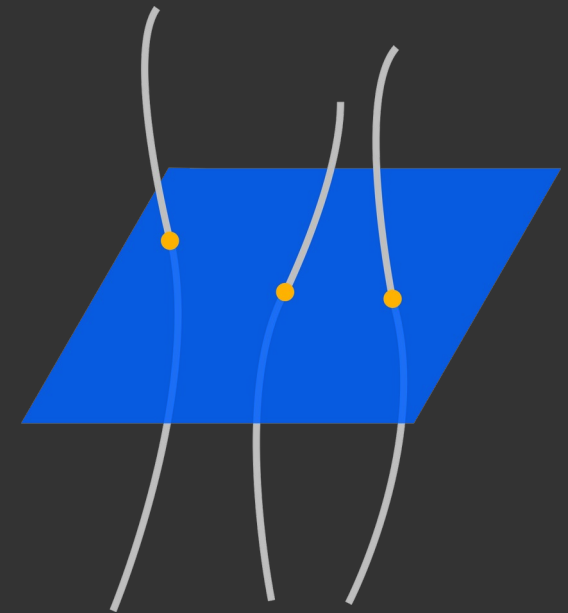
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This defines a U(1)-valued **topological operator**:

$$U_\alpha(\mathcal{M}_{d-p}) = \exp \left(i\alpha \int_{\mathcal{M}_{d-p}} \star j_p \right)$$



Gaiotto, Kapustin, Seiberg, Willett, 2015

Non-invertible Symmetries

One possible generalization is relax the group-like composition rule and allow for a sum. A **categorical symmetry**:

$$\mathcal{L}_a \times \mathcal{L}_b = \sum_c N_{ab}^c \mathcal{L}_c$$

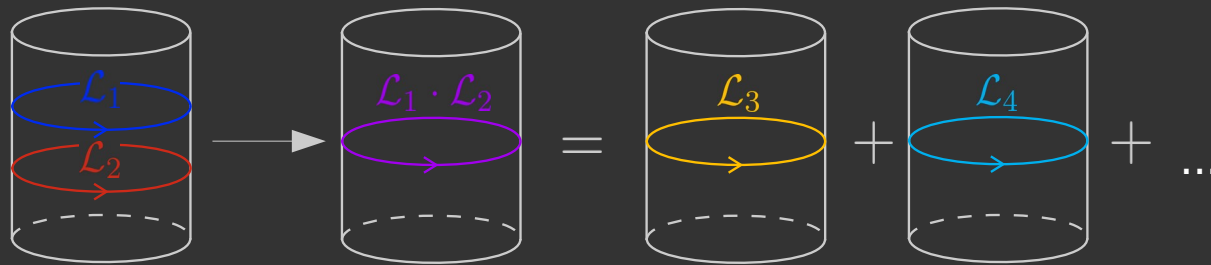
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In 2-dimensional CFTs we can have at most codimension-1 operators: **lines**. In RCFTs internal symmetries are generally represented by **Verlinde lines**:

Verlinde, 1988



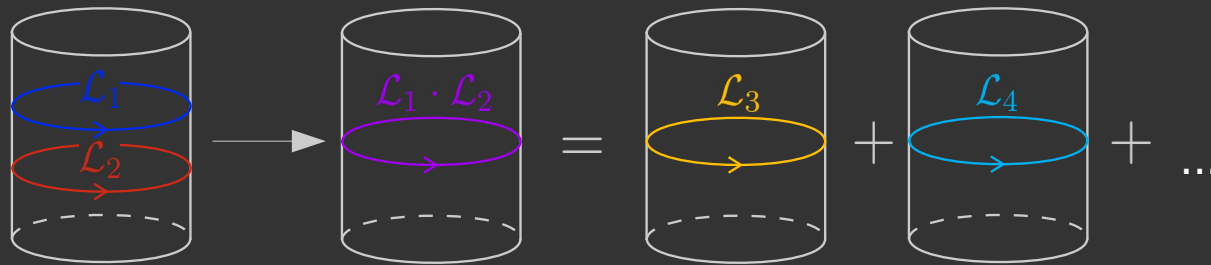
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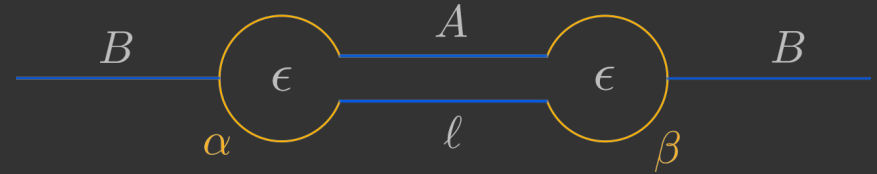
We want to compute **Entanglement Entropy** of a state in presence of these lines.

Symmetry Resolved EE

Entanglement entropy measures quantum correlations between a region A and its complementary B:

$$\rho_A = \text{Tr}_{\mathcal{H}_B} |\psi\rangle \langle\psi|$$

$$S_A = \lim_{n \rightarrow 1} \frac{1}{1-n} \log \text{Tr} \rho_A^n$$

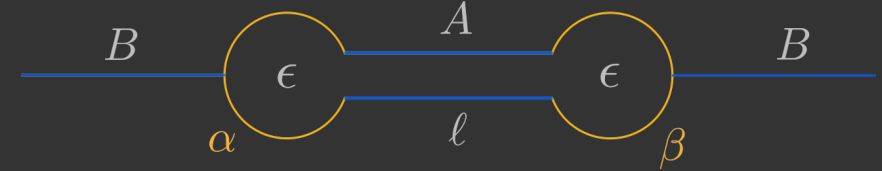


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In presence of a global symmetry, **Symmetry Resolved Entanglement Entropy** accounts for how EE is distributed among the charged sectors of the symmetry.

Goldstein and Sela, 2018

$$\mathcal{Q} = \mathcal{Q}_A \otimes \mathbb{1}_B + \mathbb{1}_A \otimes \mathcal{Q}_B$$

$$\rho_A = \bigoplus_r [P_r \rho_A] = \sum_r p_A[r] \rho_A[r]$$

$$S_A[r] = \lim_{n \rightarrow 1} \frac{1}{1-n} \log \text{Tr} \rho_A^n[r]$$

$$\rho_{A,\mathcal{Q}} =$$



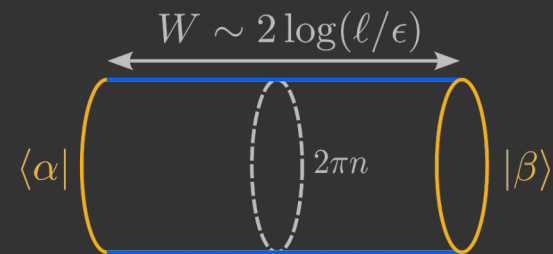
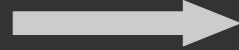
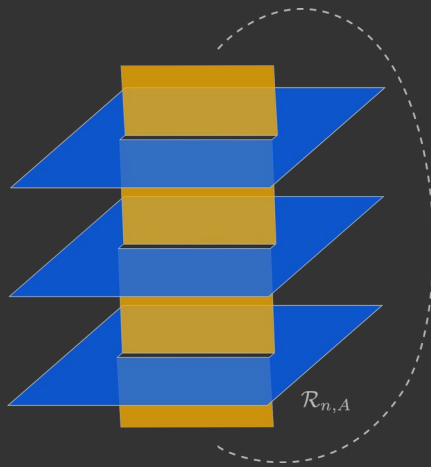
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The Replica Trick

We can interpret the replica trick in two ways:

Glue the geometry: **Worldsheet approach**



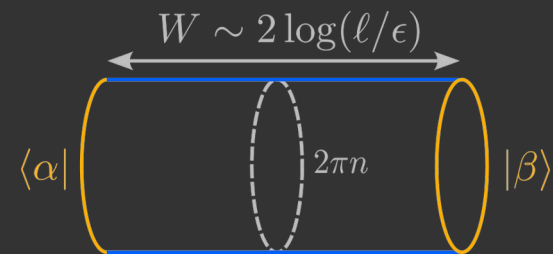
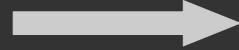
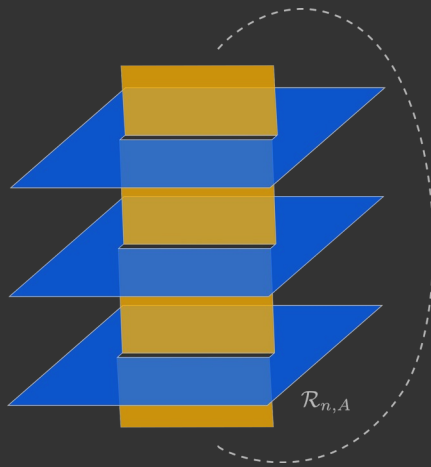
Boundary Conformal Field Theory

Kusuki, Murciano, Ooguri, Pal, 2023
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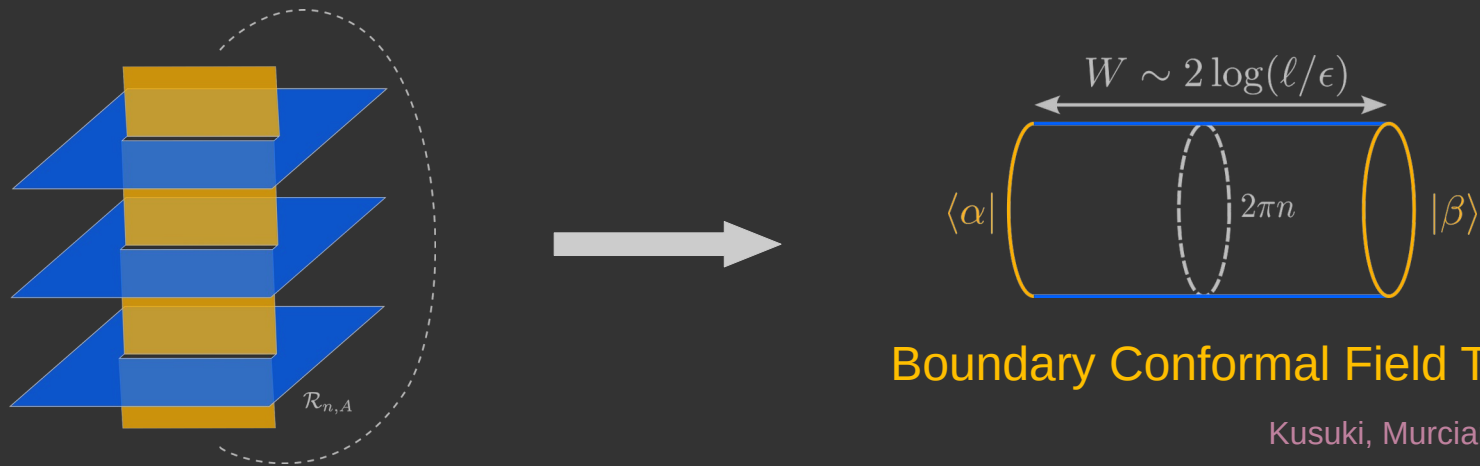
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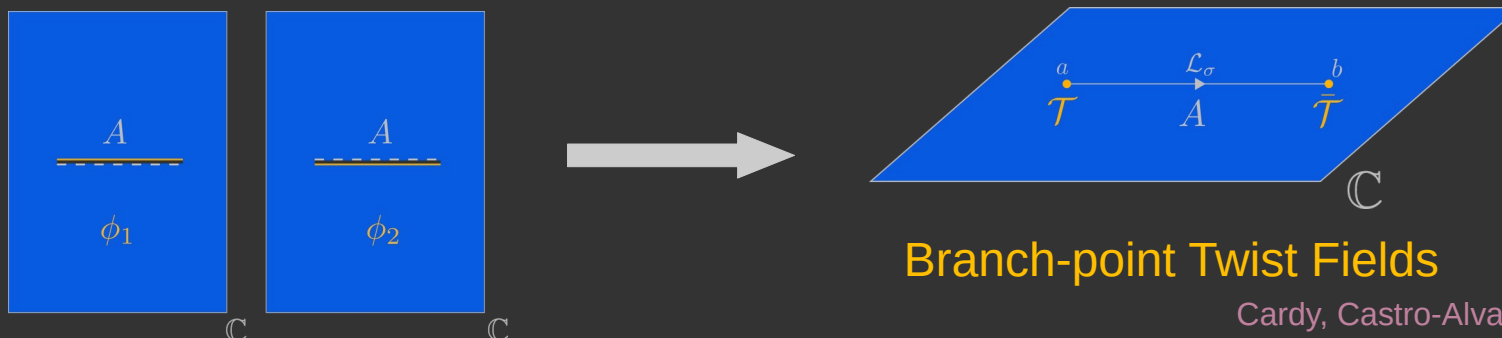
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Glue the field content: **Target Space approach**

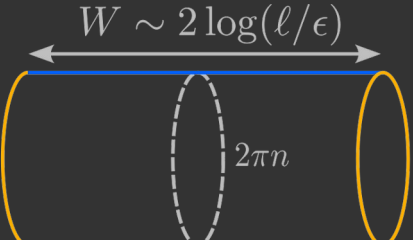


Branch-point Twist Fields

Cardy, Castro-Alvaredo, Doyon, 2007
 Goldstein, Sela, 2018

BCFT vs Twist Fields

In the **Worldsheet** approach the partition function can be written as an **amplitude**:

$$\text{Tr}[\rho_A^n] = \langle \alpha | \left(\text{Cylinder with width } W \sim 2 \log(\ell/\epsilon) \text{ and length } 2\pi n \right) | \beta \rangle = \text{Tr}[q^n (L_0 - \frac{c}{24})] = \langle \alpha | \tilde{q}^{\frac{1}{n}} (L_0 - \frac{c}{24}) | \beta \rangle$$


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With the geometric quantities:

$$q = e^{-2\pi^2/W} \qquad \tilde{q} = e^{-2W}$$

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Which, at leading order in $\epsilon \ll \ell$ gives the result for **entanglement entropy**:

$$S_A = \frac{c}{3} \log \frac{\ell}{\epsilon} + \log \langle \alpha | 0 \rangle + \log \langle 0 | \beta \rangle$$

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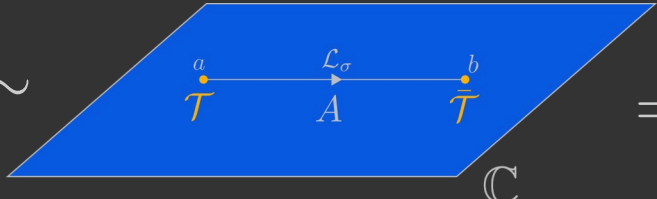
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Affleck-Ludwig boundary
entropy

Affleck and Ludwig, 1991

In the **Target Space** approach the partition function takes the form of a **conformal correlator**:

Cardy, Castro-Alvaredo, Doyon, 2007

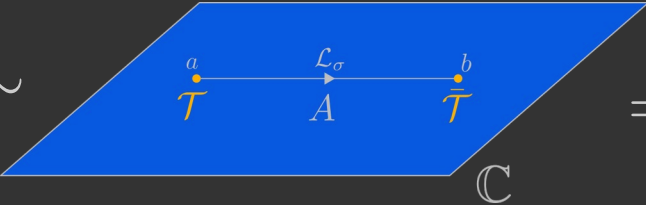
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Branch-point twist fields are scalar fields of the same conformal dimension

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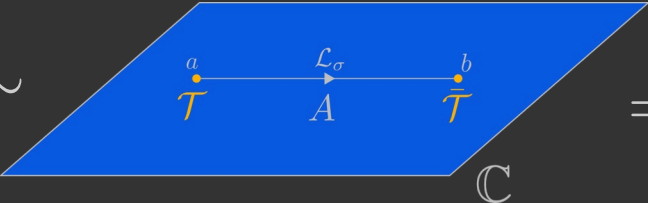
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This gives the usual result for **entanglement entropy**

$$S_A^{(n)} = \frac{c}{6} \frac{n+1}{n} \log \frac{\ell}{\epsilon} \xrightarrow{n \rightarrow 1} \frac{c}{3} \log \frac{\ell}{\epsilon}$$

Twist Fields for SREE

Symmetry resolve involves computing **charged moments**:

Goldstein, Sela, 2018

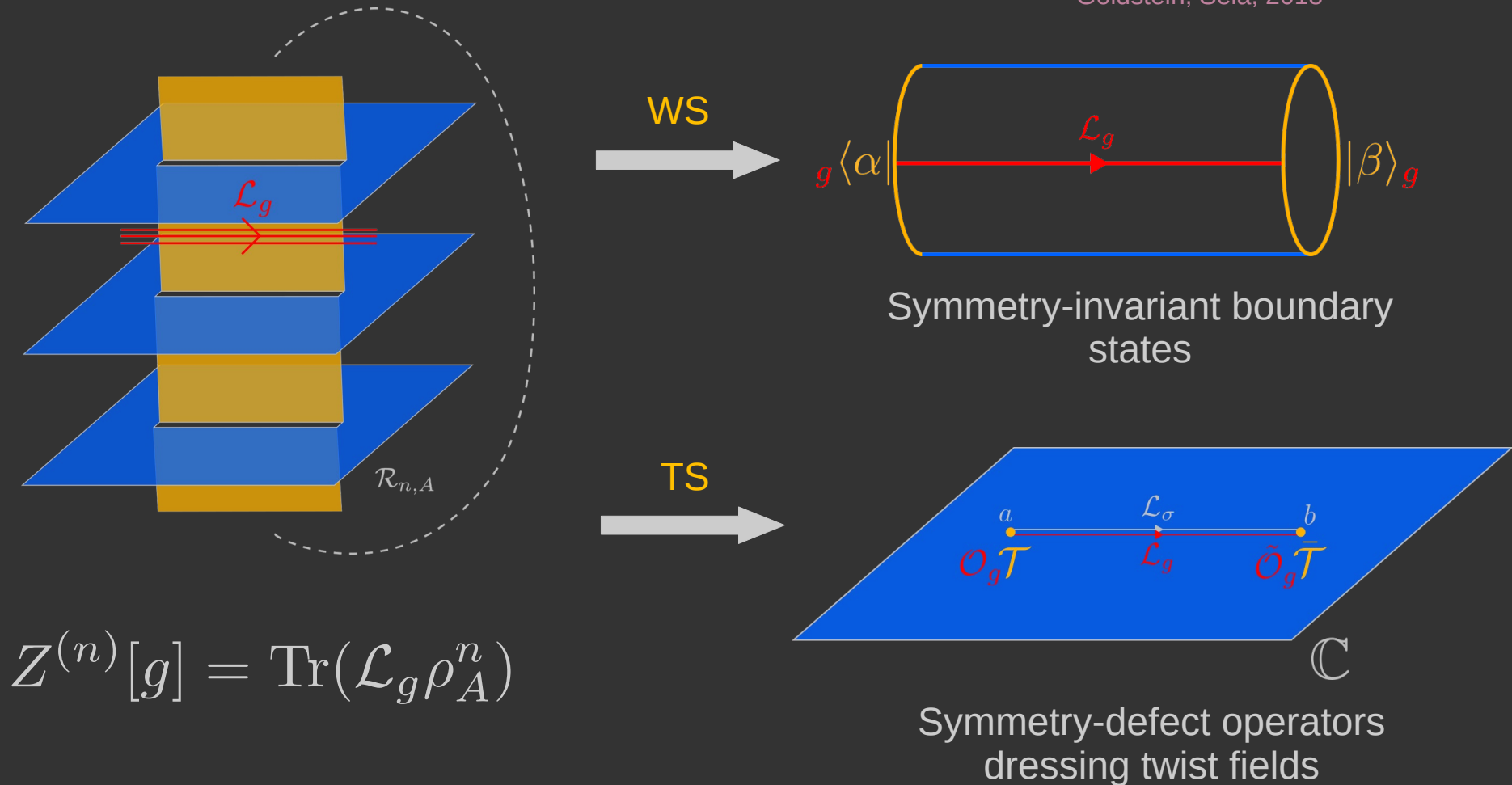


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Anyonic Sectors

For a categorical symmetry, generalized **charges** are associated with the **irreps of the tube Algebra** $Tub(\mathcal{C})$. From the **SymmTFT** picture, these irreps are in one to one correspondence with **anyons of the Drienfield Center**.

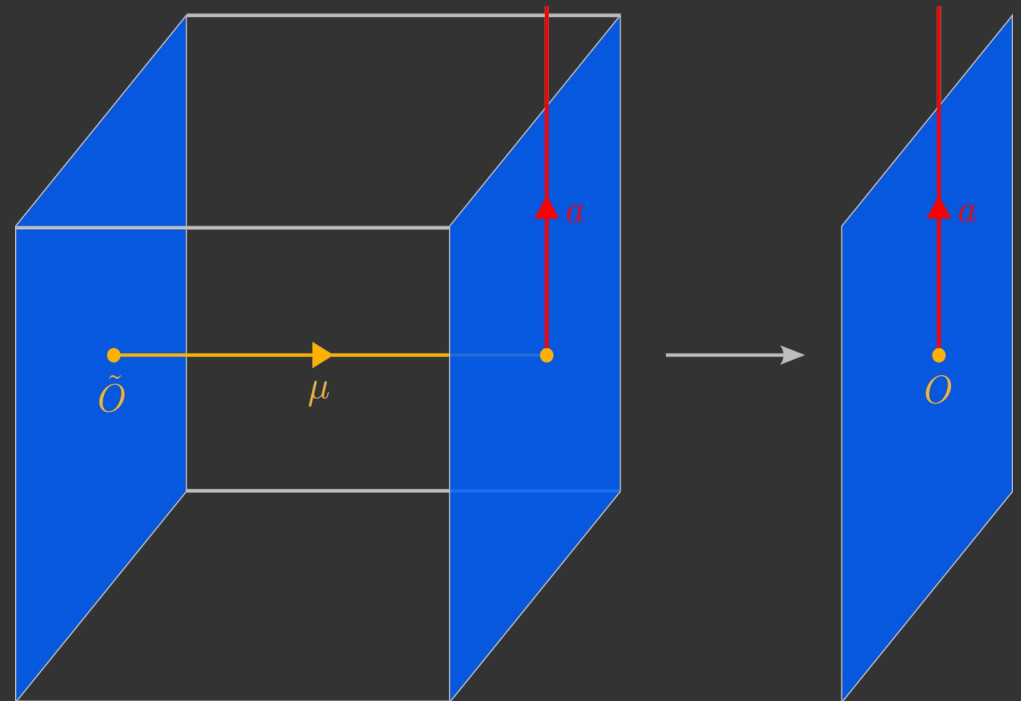
Freed, Moore, Teleman, 2022

In diagonal RCFTs

$$\mathcal{D}(\mathcal{C}) = \mathcal{C} \boxtimes \mathcal{C}$$

$$\mu = c \otimes d$$

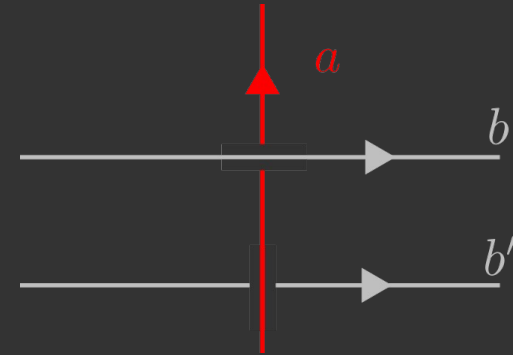
Charged sectors = Anyonic Sectors



The **projectors into anyonic sectors** can be written in terms of the 2-dimensional category:

Lin, Okada, Seifnashri, Tachikawa, 2022

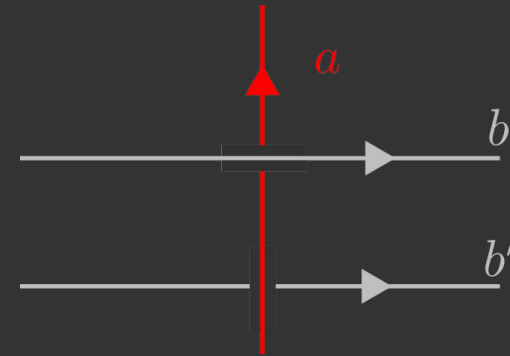
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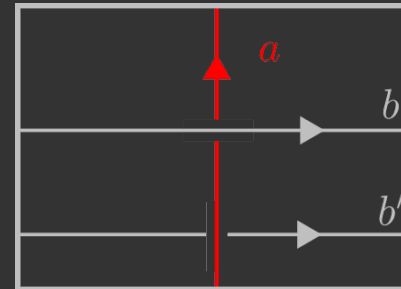
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Introducing the projectors on the torus partition function we can simplify the expressions using the modular data of the RCFT and the fusion rules:

$$\text{Tr}[P^\mu \rho] = \sum_{a,b,b'} S_{1c} S_{bc}^* S_{1d} S_{b'd}^*$$

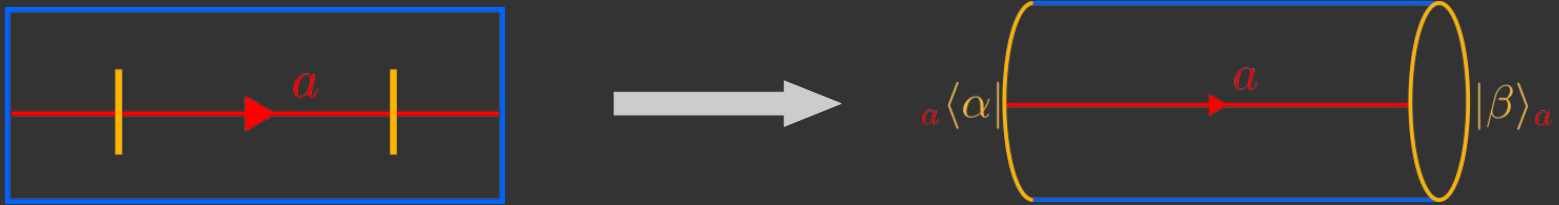


$$\sim \sum_a \frac{N_{ad}^c}{|\mathcal{C}|} \frac{d_c d_d}{d_a}$$



The BCFT Approach

To compute entanglement entropy we have to cut the interval:

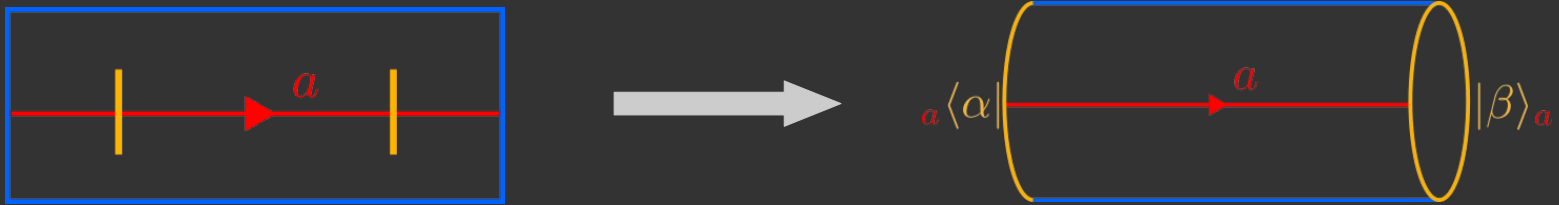


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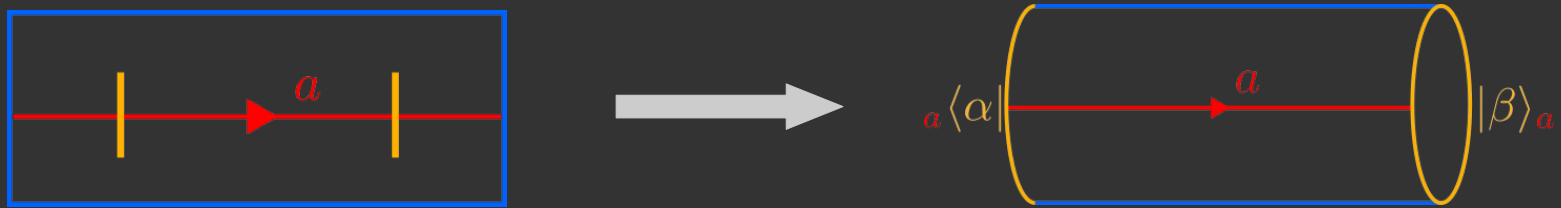


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$$\mathcal{L}_a |\mathcal{B}\rangle = \langle \mathcal{L}_a \rangle |\mathcal{B}\rangle$$

Strongly symmetric

$$\mathcal{L}_a |\mathcal{B}\rangle = |\mathcal{B}\rangle + \dots$$

Weakly symmetric

If there are not such conditions we cannot symmetry resolve. This is a signature of an **Anomaly**.

Each **twisted sector** contributes as:

$$\frac{d_\mu \langle \mu, a \rangle}{d_a |\mathcal{C}|} g_\alpha^a \bar{g}_\beta^a \tilde{q}^{\frac{1}{n}} \left(h_0^a - \frac{c}{24} \right)$$

$$d_\mu \langle \mu, a \rangle = N_{ad}^c d_d d_c$$

$$g_\alpha^a = {}_a \langle \alpha | 0 \rangle_a$$

And h_a^0 is the conformal weight of the lowest primary in the twisted sector, being the smallest $h_0^{\mathbb{1}} = 0$ the identity operator. $\longrightarrow$ **Sum over a is dominated by $\mathbb{1}$**

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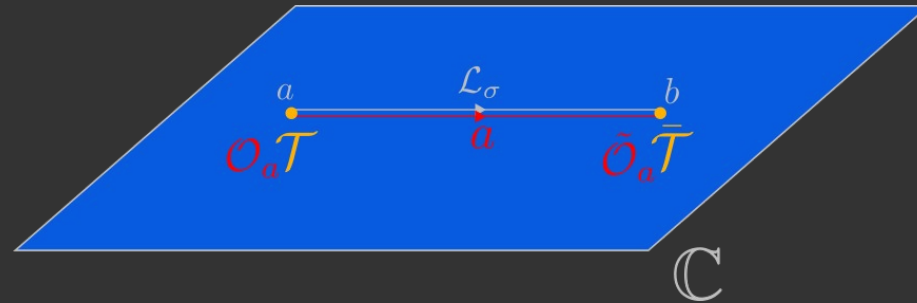
Das, Molina-Vilaplana, PSB, 2024

For abelian group-like symmetries

$$d_\mu \langle \mu, \mathbb{1} \rangle = 1 \longrightarrow \text{Entanglement equipartition}$$

Target Space Approach

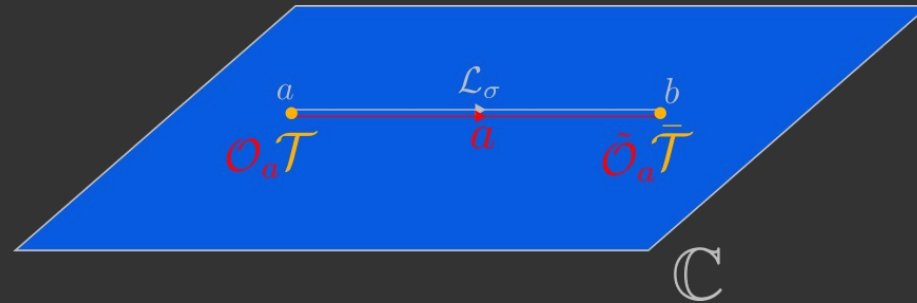
In the target space approach we have to **dress** the branch-point twist field with a defect operator.



These are fields on the **twisted Hilbert space** of the line $\mathcal{O}_a \in \mathcal{H}_a$. Both of them have the same conformal dimension and is the lowest in that Hilbert space. They have to be **scalar fields**.

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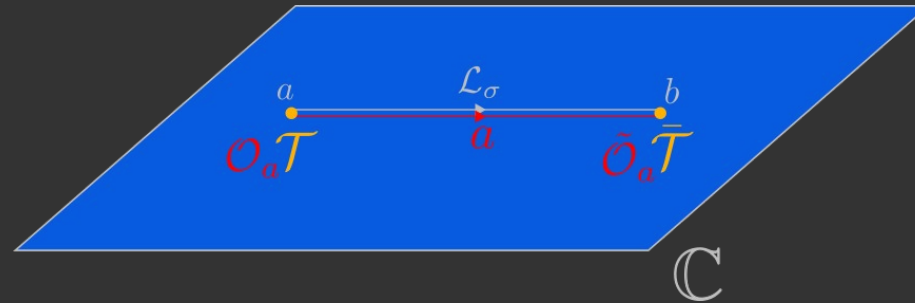
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$$\langle \phi(z_1, \bar{z}_1) \phi(z_2, \bar{z}_2) \rangle = \frac{1}{|z_{12}|^{2\Delta_\phi}} e^{2i\theta s_\phi}$$

The correlator picks an imaginary part if $s_\phi \neq 0$, and so thus SREE. If there are no scalar defect fields symmetry resolution cannot be performed. This is a signature of an **Anomaly**.

Each **twisted sector** then contributes as:

$$\frac{d_\mu \langle \mu, a \rangle}{d_a |\mathcal{C}|} \langle \mathcal{T}_a \tilde{\mathcal{T}}_a \rangle_{\mathbb{C}} \sim \frac{d_\mu \langle \mu, a \rangle}{d_a |\mathcal{C}|} \left(\frac{\ell}{\epsilon} \right)^{-2\Delta_{\mathcal{T}} - 2\frac{\Delta_{\mathcal{O}_a}}{n}} \quad \Delta_{\mathcal{O}_a} = 2h_0^a$$

The sum over a is again dominated by the identity sector, giving the **familiar result**:

$$S_A[\mu] = \frac{c}{3} \log \frac{\ell}{\epsilon} + \log \frac{d_\mu \langle \mu, \mathbb{1} \rangle}{|\mathcal{C}|}$$

Which is the same as the one obtained with the BCFT approach appart from the Affleck-Ludwig boundary entropy.

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Twist Fields

$$\mathcal{H}_W = \{\alpha_{\frac{3}{80}, \frac{3}{80}}, \beta_{\frac{3}{5}, \frac{3}{5}}, \gamma_{\frac{1}{10}, \frac{1}{10}}, \dots\}$$

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$$S_A[\mu] = \frac{c}{3} \log \frac{\ell}{\epsilon} + \log \frac{d_\mu \langle \mu, \mathbb{1} \rangle}{|\mathcal{C}|} + \log g_\alpha + \log \bar{g}_\beta$$

μ	$d_\mu \langle \mu, \mathbb{1} \rangle$
$\mathbb{1} \otimes \mathbb{1}$	1
$\mathbb{1} \otimes W$	1
$W \otimes \mathbb{1}$	φ
$W \otimes W$	φ

The Ising Category

The Ising Category contains three elements

$$\eta \times \eta = \mathbb{1} \quad N \times N = \mathbb{1} + \eta \quad \eta \times N = N \times \eta = N$$

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TABLE OF CONTENTS

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- We have generalized **composite branch-point twist fields** for categorical symmetries and computed SREE both in the worldsheet approach using BCFT and in the target space approach, obtaining the **same result**.
- Even though one can insert the flux, not every dressed operator or boundary is suitable to symmetry resolve. One must be careful with **anomalies**.
- **Entanglement equipartition is broken** by categorical symmetries. Even for **non-abelian** symmetries we believe the origin is categorical.
- There has been some **controversy** in the literature regarding SREE for categorical symmetry regarding the BCFT approach. We hope to clarify confusions presenting the parallel twist field approach.

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THANK YOU!