Type IIB uplift of N=4, D=4 gauged supergravity

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Based on 2410.23149 with Colin Sterckx and Mario Trigiante

+ work in progress

IV GRASS-SYMBHOL Meeting

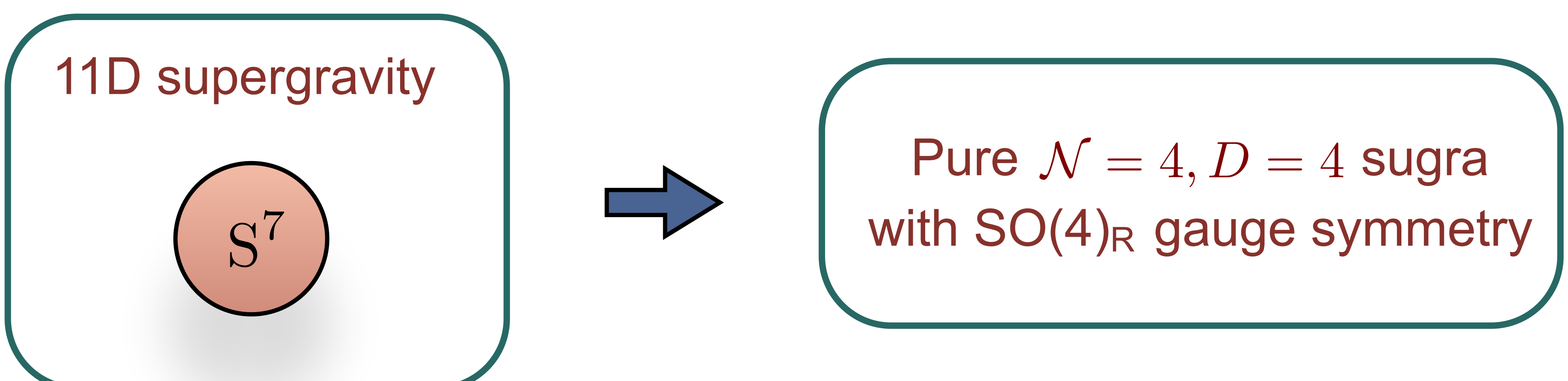
Toledo, 11-13 November 2025

Universidad de Oviedo
Universidá d'Uviéu
University of Oviedo

ICTEA
Instituto Universitario de Ciencias y Tecnologías del Espacio de Asturias

Consistent truncations of 11D / type II supergravities down to 4D have been pivotal in developing precision tests & applications of the AdS_4 / CFT_3 correspondence

Prototypical example :



[de Wit, Nicolai '82, '87]

[Cvetic, Lu, Pope '99]

$\mathcal{N}=4\,,\ D=4$ SO(4)$_R$-gauged supergravity

$$S_{\text{bos}} = \int R\star 1 - \tfrac{1}{2}\,d\xi\wedge\star d\xi - \tfrac{1}{2}\,e^{2\xi}\,d\chi\wedge\star d\chi + (4+2\cosh\xi + e^{\xi}\chi^2)\star 1$$
$$-\frac{1}{1+\chi^2 e^{2\xi}}\left(e^{\xi}F^{a}_{(2)}\wedge\star F^{a}_{(2)} + \chi\, e^{2\xi}\,F^{a}_{(2)}\wedge F^{a}_{(2)}\right) - e^{-\xi}F^{\hat a}_{(2)}\wedge\star F^{\hat a}_{(2)} + \chi\,F^{\hat a}_{(2)}\wedge F^{\hat a}_{(2)}$$

Field content : $g_{\mu\nu}$, $(A^{a}_{\mu}\,,\,A^{\hat a}_{\mu})\in\mathfrak{so}(4)_{\mathrm{R}}$, $\tau\equiv-\chi+ie^{-\xi}\in\dfrac{\mathrm{SL}(2)}{\mathrm{SO}(2)}$

$\mathcal{N}=4\,,\ D=4$ SO(4)$_R$-gauged supergravity

$$
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& -\frac{1}{1+\chi^{2}e^{2\xi}}\Big(e^{\xi}F^{a}_{(2)}\wedge\star F^{a}_{(2)}+\chi\,e^{2\xi}\,F^{a}_{(2)}\wedge F^{a}_{(2)}\Big)-e^{-\xi}F^{\hat{a}}_{(2)}\wedge\star F^{\hat{a}}_{(2)}+\chi\,F^{\hat{a}}_{(2)}\wedge F^{\hat{a}}_{(2)}
\end{aligned}
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Field content : $g_{\mu\nu}$, $(A^{a}_{\mu}\,,\,A^{\hat{a}}_{\mu})\in\mathfrak{so}(4)_{\mathrm{R}}$, $\tau\equiv-\chi+ie^{-\xi}\in\dfrac{\mathrm{SL}(2)}{\mathrm{SO}(2)}$

- **Consistent** truncation from 11D :

Abelian $\mathfrak{u}(1)_1\times\mathfrak{u}(1)_2\subset\mathfrak{so}(4)_{\mathrm{R}}$ sector (for simplicity)

$$
d\hat{F}_{(4)}=0 \qquad d\hat{F}_{(7)}=0 \qquad \text{Einstein eq.}
$$

$$
\Rightarrow
$$

$$
dF_1=0\ ,\ dF_2=0 \qquad d\tilde{F}_1=0\ ,\ d\tilde{F}_2=0 \qquad \text{EOM of }\ \tau \qquad \text{Einstein eq.}
$$

Note: $\hat{F}_{(7)}\equiv *_{11}\hat{F}_{(4)}+\tfrac{1}{2}\hat{A}_{(3)}\wedge\hat{F}_{(4)}$

Note: $\tilde{F}_1=|\tau|^{-2}\left(e^{-\xi}\star F_1+\chi\,F_1\right)$, $\tilde{F}_2=e^{-\xi}\star F_2-\chi\,F_2$

$\mathcal{N} = 4\,,\ D = 4$ SO(4)$_R$-gauged supergravity

$$S_{\rm bos} = \int R \star 1 - \tfrac{1}{2}\, d\xi \wedge \star d\xi - \tfrac{1}{2}\, e^{2\xi}\, d\chi \wedge \star d\chi + (4 + 2\cosh\xi + e^{\xi}\chi^2) \star 1$$
$$- \frac{1}{1+\chi^2 e^{2\xi}} \left(e^{\xi} F^{a}_{(2)} \wedge \star F^{a}_{(2)} + \chi\, e^{2\xi}\, F^{a}_{(2)} \wedge F^{a}_{(2)} \right) - e^{-\xi} F^{\hat{a}}_{(2)} \wedge \star F^{\hat{a}}_{(2)} + \chi\, F^{\hat{a}}_{(2)} \wedge F^{\hat{a}}_{(2)}$$

Field content : $g_{\mu\nu}$, $(A^{a}_{\mu}\,,\, A^{\hat{a}}_{\mu}) \in \mathfrak{so}(4)_{\rm R}$, $\tau \equiv -\chi + i e^{-\xi} \in \dfrac{\rm SL(2)}{\rm SO(2)}$

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[AdS$_4$ x S^7 : Freund, Rubin '80] $\quad \text{AdS}_4 \leftrightarrow \text{ABJM} \quad$ [M2-brane : Aharony, Bergman, Jafferis, Maldacena '08]

$\mathcal{N}=4\,,\ D=4$ SO(4)$_R$-gauged supergravity

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[AdS$_4$ x S^7 : Freund, Rubin '80] AdS$_4$ $\leftrightarrow$ ABJM [M2-brane : Aharony, Bergman, Jafferis, Maldacena '08]

[AdS$_3$ x R x S^7] Janus $\leftrightarrow$ ABJM interfaces

[D'Hoker, Ester, Gutperle, Krym '09]
[Freedman, Pufu '13]
[Bobev, Pilch, Warner '13]
[Pilch, Tyukov, Warner '15]
[Anabalón, AG, Chamorro-Burgos '22]

$\mathcal{N} = 4\,,\ D = 4$ SO(4)$_R$-gauged supergravity

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[AdS$_2$ x Σ_2 x S^7] $\text{AdS-BH} \leftrightarrow \text{ABJM on } \Sigma_2 \text{ (top. twist)}$

[Anabalón, Cacciatori, Caldarelli, Cardoso, Castro, Chimento, Chow, Compère, Cvetic, Dall' Agata, Duff, Erbin, Ferrero, Gnecchi, AG, Halmagyi, Hoxa, Hristov, Inglese, Ipiña, Karndumri, Katmadas, Klemm, Liu, Lodato, Lü, Lu, Martelli, Martinez-Acosta, Monten, Ortín, Oyarzo, Petri, Petrini, Pope, Rabbiosi, Sati, Sparks, Suh, Toldo, Tomasiello, Tran, Trigiante, Vaughan, Zaffaroni, …]

[Arav, Azzurli, Amariti, Benini, Bobev, Cabo-Bizet, Charles, Choi, Crichigno, Gauntlett, Jiao, Sparks, Hong, Hristov, Kim, Kim, Morteza Hosseini, Min, Pando Zayas, Pilch, Reys, Roberts, Rosen, Xin, Zaffaroni, …]

Take home message

> *"Any solution of pure* $\mathcal{N} = 4$, $D = 4$ $SO(4)_R$*-gauged supergravity can be oxidised to a non-geometric S-fold background of type IIB string theory."*

Outlook

- What is an S-fold?

  See works by : Arav, Berman, Bobev, Cesàro, Cheung, Fischbacher, Gauntlett, Gautason, Giambrone, Inverso, Larios, Malek, Pilch, Roberts, Rosen, Samtleben, Suh, Trigiante, van Muiden, Varela…

- Type IIB uplift of pure $\mathcal{N}=4\,,\ D=4$ gauged supergravity

- Outlook

Type IIB on $\mathrm{AdS}_5 \times \mathrm{S}^5$

R-symmetry : $SU(4)_R \sim SO(6)_R$ from S^5

[SL(2)-covariant notation $\alpha = 1, 2$ **]**

$$ds^2_{10} = ds^2_{\mathrm{AdS}_5} + g^{-2}\, d\mathring{s}^2_{\mathrm{S}^5}$$

$$\widetilde{F}_5 = 4\, g\, (1 + \star)\, \mathrm{vol}_5$$

$$m_{\alpha\beta} = \begin{pmatrix} e^{-\Phi} + e^{\Phi} C_0^2 & -e^{\Phi} C_0 \\ -e^{\Phi} C_0 & e^{\Phi} \end{pmatrix} = \begin{pmatrix} e^{-\Phi_0} & 0 \\ 0 & e^{\Phi_0} \end{pmatrix} \qquad \text{with} \qquad \Phi_0 = cst$$

$$\mathbb{B}^{\alpha} = \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} = 0$$

From now on we will set $g = 1$ and $\Phi_0 = 0$

Can we replace AdS_5 by $\mathrm{AdS}_4 \times \mathrm{S}^1_\eta$?

[AG, Sterckx '19]

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[SL(2)-covariant notation $\alpha = 1, 2$ **]**

$$ds^2_{10} = \Delta^{-1}\left[\tfrac{1}{2} ds^2_{\mathrm{AdS}_4} + \tfrac{1}{2\sqrt{2}}\, d\eta^2 + \tfrac{1}{\sqrt{2}}\, d\mathring{s}^2_{\mathrm{S}^5}\right] \qquad \text{with} \qquad \Delta^{-1} = \sqrt{2}$$

$$\widetilde{F}_5 = 4(1+\star)\,\mathrm{vol}_5$$

$$m_{\alpha\beta} = \begin{pmatrix} e^{-\Phi} + e^{\Phi}\, C_0^2 & -e^{\Phi}\, C_0 \\ -e^{\Phi}\, C_0 & e^{\Phi} \end{pmatrix} = \begin{pmatrix} e^{2\eta} & 0 \\ 0 & e^{-2\eta} \end{pmatrix}$$

$$\mathbb{B}^\alpha = \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} = 0$$

Non-supersymmetric solution with **linear dilaton** profile $\Phi = -2\eta$ **!!**

Can we make sense out of S^1_η ?

[AG, Sterckx '19]

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$$m_{\alpha\beta} = \begin{pmatrix} e^{-\Phi} + e^{\Phi}\,C_0^2 & -e^{\Phi}\,C_0 \\ -e^{\Phi}\,C_0 & e^{\Phi} \end{pmatrix} = A^{-t}\,\mathfrak{m}\,A^{-1}$$

$$\mathbb{B}^\alpha = \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} = A\,\mathfrak{b} = 0 \qquad \text{with} \qquad \mathfrak{b}^\beta = 0 \qquad \text{and} \qquad \mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Rewriting in terms of a **local SL(2,R) twist** matrix $\Rightarrow$ $\boxed{A^\alpha{}_\beta(\eta) = \begin{pmatrix} e^{-\eta} & 0 \\ 0 & e^{\eta} \end{pmatrix}}$

S-fold interpretation

[Inverso, Samtleben, Trigiante '16]

- $\mathbb{R} \to \mathrm{S}^1_\eta \;\Leftrightarrow\;$ **hyperbolic** monodromy : $\quad \mathfrak{M}_{\mathrm{S}^1_\eta} = A^{-1}(\eta)\, A(\eta+T) = \begin{pmatrix} e^{-T} & 0 \\ 0 & e^{T} \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})$

[non-geometric but locally geometric backgrounds]

- Acting with a global $\mathrm{SL}(2,\mathbb{R})$ element ($k > 2$) :

$$g(k) = \begin{pmatrix} \frac{(k^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{k}{\sqrt{2}(k^2-4)^{\frac{1}{4}}} & \frac{\sqrt{2}}{(k^2-4)^{\frac{1}{4}}} \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R})$$

then

$$\boxed{\mathfrak{M}(k) = g^{-1}\, \mathfrak{M}_{\mathrm{S}^1_\eta}\, g = \begin{pmatrix} k & 1 \\ -1 & 0 \end{pmatrix} = -\mathcal{S}\mathcal{T}^k \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}}$$

CS level k in the dual S-fold CFT$_3$

[Assel, Tomasiello '18]

with $\quad T(k) = \log(k + \sqrt{k^2-4}) - \log(2) \quad$ and $\quad \mathrm{Tr}\mathfrak{M}(k) > 2.$

$\mathcal{N} = 4$ supersymmetric S-fold

[Inverso, Samtleben, Trigiante '16]

[AG, Sterckx '21]

[Giambrone, AG, Malek, Samtleben, Sterckx, Trigiante '21]

R-symmetry : $SO(4)_R \sim S^2 \times S^2$

$$ds^2_{10} = \Delta^{-1}\left[\tfrac{1}{2}ds^2_{\mathrm{AdS}_4} + d\eta^2 + d\alpha^2 + \frac{\cos^2\alpha}{2+\cos(2\alpha)}\,ds^2_{\mathrm{S}^2_1} + \frac{\sin^2\alpha}{2-\cos(2\alpha)}\,ds^2_{\mathrm{S}^2_2}\right]$$

$$\Delta^{-4} = 4 - \cos^2(2\alpha)$$

$$\widetilde{F}_5 = \Delta^4\,\sin^2(2\alpha)\,(1+\star)\left[-\tfrac{3}{2}\,\mathrm{vol}_5 + \sin(2\alpha)\,d\eta\wedge\mathrm{vol}_1\wedge\mathrm{vol}_2\right]$$

$$\mathbb{B}^\alpha = A\ \mathfrak{b} \qquad \text{with} \qquad \mathfrak{b}_1 = -2\sqrt{2}\,\frac{\cos^3\alpha}{2+\cos(2\alpha)}\,\mathrm{vol}_{\Omega_1}, \qquad \mathfrak{b}_2 = -2\sqrt{2}\,\frac{\sin^3\alpha}{2-\cos(2\alpha)}\,\mathrm{vol}_{\Omega_2}$$

$$m_{\alpha\beta} = A^{-t}\ \mathfrak{m}\ A^{-1} \qquad \text{with} \qquad \mathfrak{m}_{\gamma\delta} = \begin{pmatrix} \sqrt{\frac{2+\cos(2\alpha)}{2-\cos(2\alpha)}} & 0 \\ 0 & \sqrt{\frac{2-\cos(2\alpha)}{2+\cos(2\alpha)}} \end{pmatrix}$$

A conjecture

[Gauntlett, Varela '07]

Corollary. *Any supergravity solution with a D-dimensional AdS (or Minkowski) factor preserving $\mathcal{N}$ supersymmetries, defines a consistent truncation to the corresponding pure supergravity theory.*

[proof by Cassani, Josse, Petrini, Waldram '19]

A consistent truncation of type IIB supergravity on $\mathrm{S}^1_\eta \times \mathrm{S}^5$ **must** exist down to pure $\mathcal{N} = 4\,,\ D = 4$ SO(4)$_\mathrm{R}$-gauged supergravity

Field content : $g_{\mu\nu}$, $(A^a_\mu\,,\ A^{\hat{a}}_\mu) \in \mathfrak{so}(4)_\mathrm{R}$, $\tau \equiv -\chi + i e^{-\xi} \in \dfrac{\mathrm{SL}(2)}{\mathrm{SO}(2)}$

Reminder : M-theory embedding on the 7-sphere [Cvetic, Lu, Pope '99]

Bad news: There is no *standard* group-theoretic procedure to work out the IIB embedding

[*i.e.* STU-model from M-theory on the 7-sphere]

Good news: *Generalised Geometry* and *Exceptional Field Theory* *come to rescue !*

[AG, Sterckx, Trigiante '24]

A recipe

[AG, Sterckx, Trigiante '24]

1. An N=4 solution of (maximal) D=4 supergravity defines a generalised $SU(4)_R$ structure

2. That $SU(4)_R$ structure has a specific embedding inside the exceptional $E_{7(7)}$ symmetry group of Exceptional Field Theory (ExFT)

3. One can then fluctuate ExFT about the N=4 solution in a way that preserves the generalised $SU(4)_R$ structure

4. The dynamics of the fluctuations is described by the pure N=4, D=4 supergravity

Proof along the lines of [Malek '17] & [Cassani, Josse, Petrini, Waldram '19]

Type IIB embedding (I)

Note : Abelian $\mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2 \subset \mathfrak{so}(4)_{\mathrm{R}}$ sector (for simplicity)

The S^5 is embedded in $\mathbb{R}^6$ using embedding coordinates

$$
\begin{array}{llll}
Y_1 = \cos\alpha\, \cos\theta_1 & , & Y_4 = \sin\alpha\, \cos\theta_2 & , \\
Y_2 = \cos\alpha\, \sin\theta_1\, \sin\varphi_1 & , & Y_5 = \sin\alpha\, \sin\theta_2\, \sin\varphi_2 & , \\
Y_3 = \cos\alpha\, \sin\theta_1\, \cos\varphi_1 & , & Y_6 = \sin\alpha\, \sin\theta_2\, \cos\varphi_2 & ,
\end{array}
$$

satisfying $\sum Y_m^2 = 1$. It proves convenient to introduce the (τ,α)-dependent functions

$$
\begin{aligned}
f_1 &= 1 + 2\, e^{\xi}\, |\tau|^2 \cos^2\alpha \ , \\
f_2 &= 1 + 2\, e^{\xi}\, \sin^2\alpha \ ,
\end{aligned}
$$

as well as the non-singular warping factor

$$
\Delta^{-4} = f_1\, f_2 \ .
$$

The ten-dimensional metric takes the form

$$
ds_{10}^2 = \Delta^{-1} \left(\tfrac{1}{2}\, ds_4^2 + g_{mn} Dy^m Dy^n \right) \ ,
$$

where $y^m = (\eta,\, \alpha,\, \theta_i,\, \varphi_i)$ denotes the internal coordinates and $D \equiv d + A^{\mathrm{KK}}$ becomes covariantised with respect to the Kaluza–Klein (KK) vector

$$
A^{\mathrm{KK}} \equiv A_\mu{}^m dx^\mu \otimes \partial_m = A_1\, \partial_{\varphi_1} + A_2\, \partial_{\varphi_2} \ .
$$

The metric g_{mn} on the internal $\mathrm{S}^1 \times \mathrm{S}^5$ is given by

$$
g_{mn}\, dy^m dy^n = d\eta^2 + d\alpha^2 + \frac{\cos^2\alpha}{f_1} ds^2_{\mathrm{S}_1^2} + \frac{\sin^2\alpha}{f_2} ds^2_{\mathrm{S}_2^2} \ ,
$$

with

$$
ds^2_{\mathrm{S}_i^2} = d\theta_i^2 + \sin^2\theta_i\, d\varphi_i{}^2
$$

being the line element of the 2-spheres S_i^2 of unit radius.

The type IIB dilaton Φ and the RR axion C_0 are given by

$$
e^{\Phi} = \Delta^2\, e^{-2\eta}\, e^{\xi}\, f_0 \ , \quad C_0 = e^{2\eta}\, \chi\, \cos(2\alpha)\, f_0{}^{-1},
$$

with

$$
f_0 = 1 + 2\, e^{-\xi} \cos^2\alpha \ .
$$

The two-form potentials $\mathbb{B}^\alpha = (\mathbb{B}^1, \mathbb{B}^2) = (B_2, C_2)$ have a decomposition à la KK of the form

$$
\mathbb{B}^\alpha = e^{(-1)^\alpha \eta}\, \mathfrak{b}^\alpha_{(0)} + \mathfrak{b}^\alpha_{(1)} \ ,
$$

involving four-dimensional scalar contributions

$$
\begin{aligned}
\mathfrak{b}^1_{(0)} &= -\sqrt{2}\, \chi\, e^{\xi} \cos\alpha\ \widetilde{\mathrm{vol}}_1 - \sqrt{2}\,(1 + e^{\xi}) \sin\alpha\ \widetilde{\mathrm{vol}}_2 \ , \\
\mathfrak{b}^2_{(0)} &= \sqrt{2}\,(1 + e^{\xi}|\tau|^2) \cos\alpha\ \widetilde{\mathrm{vol}}_1 - \sqrt{2}\, \chi\, e^{\xi} \sin\alpha\, \widetilde{\mathrm{vol}}_2 \ ,
\end{aligned}
$$

given in terms of *fibered* 2-sphere volumes

$$
\begin{aligned}
\widetilde{\mathrm{vol}}_1 &= \cos^2\alpha\, f_1{}^{-1} \sin\theta_1\, d\theta_1 \wedge D\varphi_1 \ , \\
\widetilde{\mathrm{vol}}_2 &= \sin^2\alpha\, f_2{}^{-1} \sin\theta_2\, d\theta_2 \wedge D\varphi_2 \ ,
\end{aligned}
$$

and one-form contributions

$$
\mathfrak{b}^1_{(1)} = \tfrac{1}{\sqrt{2}}\, \tilde{A}_1 \wedge d(e^{-\eta}\, Y_1) + \tfrac{1}{\sqrt{2}}\, A_2 \wedge d(e^{-\eta}\, Y_4) \ ,
$$

$$
\mathfrak{b}^2_{(1)} = -\tfrac{1}{\sqrt{2}}\, A_1 \wedge d(e^{\eta}\, Y_1) - \tfrac{1}{\sqrt{2}}\, \tilde{A}_2 \wedge d(e^{\eta}\, Y_4) \ .
$$

Type IIB embedding (II)

Note : Abelian $\mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2 \subset \mathfrak{so}(4)_{\mathrm{R}}$ sector (for simplicity)

The field-strength $\widetilde{F}_5 = dC_4 + \frac{1}{2}\,\epsilon_{\alpha\beta}\,\mathbb{B}^\alpha \wedge \mathbb{H}^\beta$ takes the form

$$\begin{aligned}\widetilde{F}_5 &= (1+\star)\Big[\Big((2+V)\sin(2\alpha)\,d\eta + 2\big((2+e^\xi)\cos(2\alpha) + V\cos^2\alpha\big)d\alpha\Big) \wedge \widetilde{\mathrm{vol}}_1 \wedge \widetilde{\mathrm{vol}}_2 \\ &+ \sin(2\alpha)\,\big(e^{2\xi}\,\chi\,d\chi - d\xi\big) \wedge \widetilde{\mathrm{vol}}_1 \wedge \widetilde{\mathrm{vol}}_2 + F_1 \wedge v_1 + \tilde{F}_1 \wedge \tilde{v}_1 + F_2 \wedge v_2 + \tilde{F}_2 \wedge \tilde{v}_2\Big]\ ,\end{aligned}$$

in terms of the quantities

$$v_1 = \chi\,e^\xi\,Y^1\,e^{-\eta}\,d(e^\eta\cos\alpha) \wedge \widetilde{\mathrm{vol}}_1 \quad , \quad v_2 = \chi\,e^\xi\,Y^4\,e^\eta\,d(e^{-\eta}\sin\alpha) \wedge \widetilde{\mathrm{vol}}_2\ ,$$

and

$$\begin{aligned}\tilde{v}_1 &= -(1+e^\xi|\tau|^2)Y^1 e^\eta d(e^{-\eta}\cos\alpha) \wedge \widetilde{\mathrm{vol}}_1 - \tfrac{1}{2}\,d\left[(Y^2)^2 + (Y^3)^2\right] \wedge D\varphi_1 \wedge d\eta\ , \\ \tilde{v}_2 &= (1+e^\xi)Y^4 e^{-\eta} d(e^\eta\sin\alpha) \wedge \widetilde{\mathrm{vol}}_2 - \tfrac{1}{2}\,d\left[(Y^5)^2 + (Y^6)^2\right] \wedge D\varphi_2 \wedge d\eta\ .\end{aligned}$$

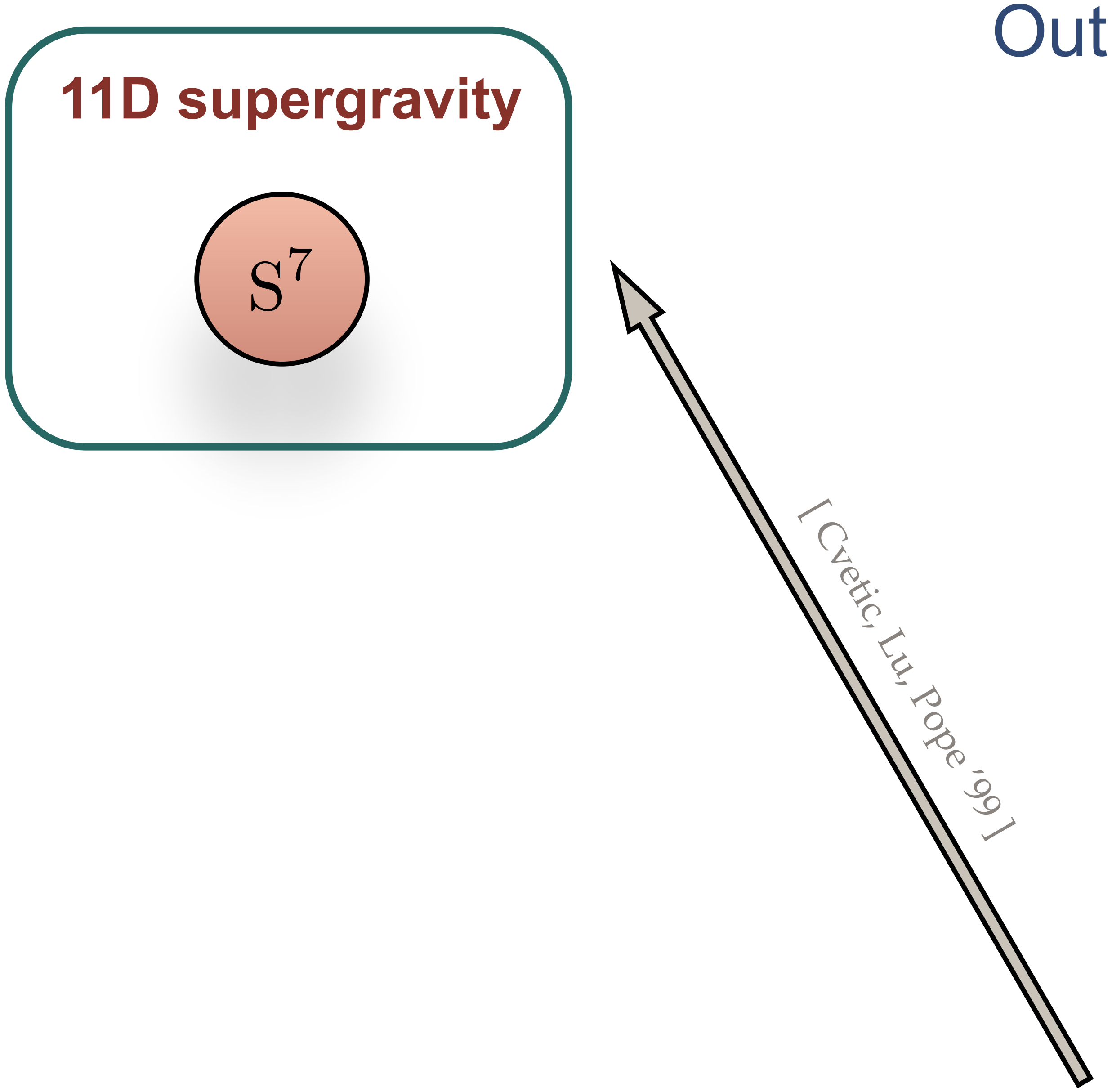
10D self-duality of $\widetilde{F}_5$ $\Longleftrightarrow$ 4D *twisted* self-duality of $\big(F_1, F_2\,;\,\tilde{F}_1, \tilde{F}_2\big)$

Outlook

$\mathcal{N} = 4\,,\ D = 4$ SO(4)$_R$-gauged supergravity

$$
\begin{aligned}
S_{\text{bos}} \quad = \quad & \int R \star 1 - \tfrac{1}{2}\, d\xi \wedge \star d\xi - \tfrac{1}{2}\, e^{2\xi}\, d\chi \wedge \star d\chi + (4 + 2\cosh\xi + e^{\xi}\chi^2) \star 1 \\
& - \frac{1}{1+\chi^2 e^{2\xi}} \left(e^{\xi} F^{a}_{(2)} \wedge \star F^{a}_{(2)} + \chi\, e^{2\xi}\, F^{a}_{(2)} \wedge F^{a}_{(2)} \right) - \, e^{-\xi} F^{\hat{a}}_{(2)} \wedge \star F^{\hat{a}}_{(2)} + \chi\, F^{\hat{a}}_{(2)} \wedge F^{\hat{a}}_{(2)}
\end{aligned}
$$

Outlook



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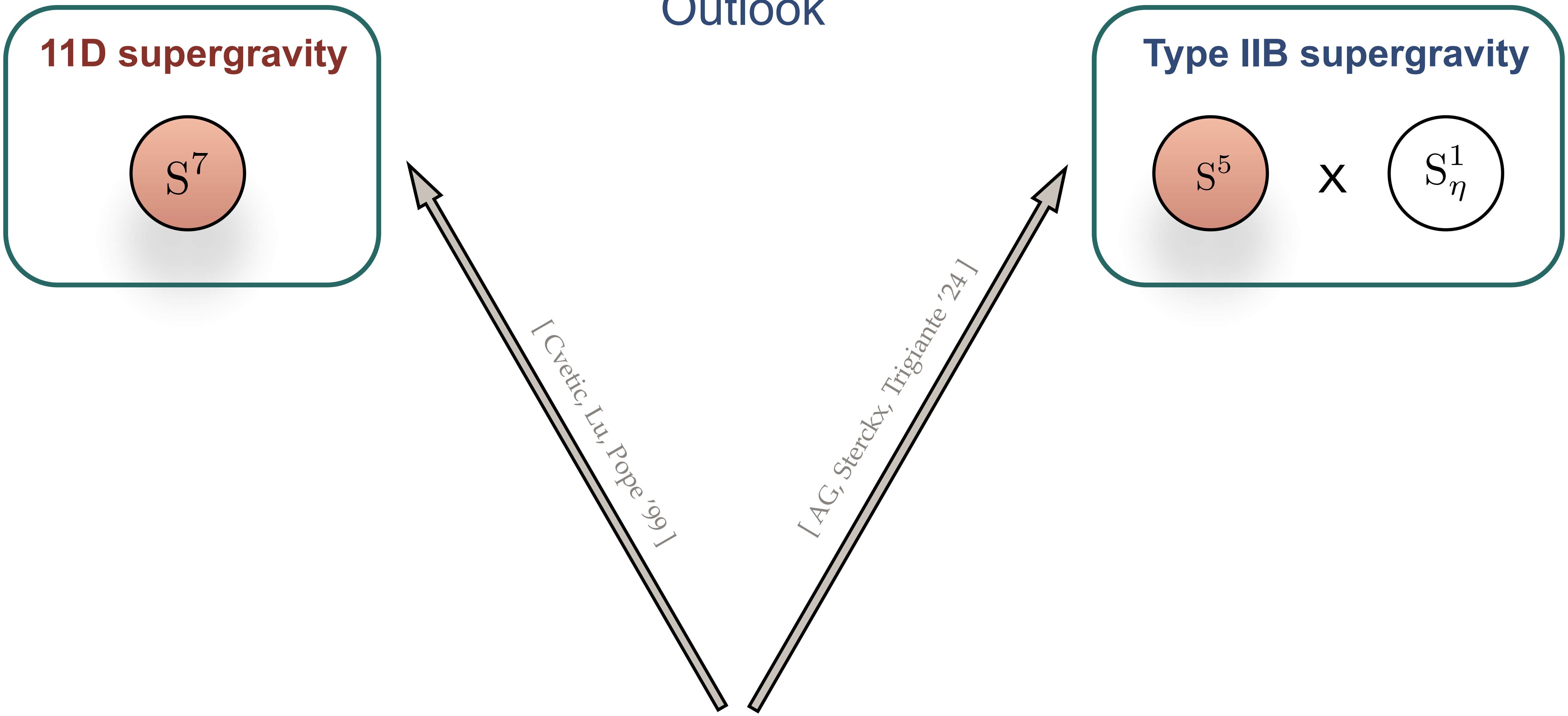
\begin{aligned}
S_{\text{bos}} \;=\; & \int R \star 1 - \tfrac{1}{2}\, d\xi \wedge \star d\xi - \tfrac{1}{2}\, e^{2\xi}\, d\chi \wedge \star d\chi + (4 + 2\cosh\xi + e^{\xi}\chi^2) \star 1 \\
& - \frac{1}{1+\chi^2 e^{2\xi}} \left(e^{\xi} F^{a}_{(2)} \wedge \star F^{a}_{(2)} + \chi\, e^{2\xi}\, F^{a}_{(2)} \wedge F^{a}_{(2)} \right) - e^{-\xi} F^{\hat a}_{(2)} \wedge \star F^{\hat a}_{(2)} + \chi\, F^{\hat a}_{(2)} \wedge F^{\hat a}_{(2)}
\end{aligned}
$$

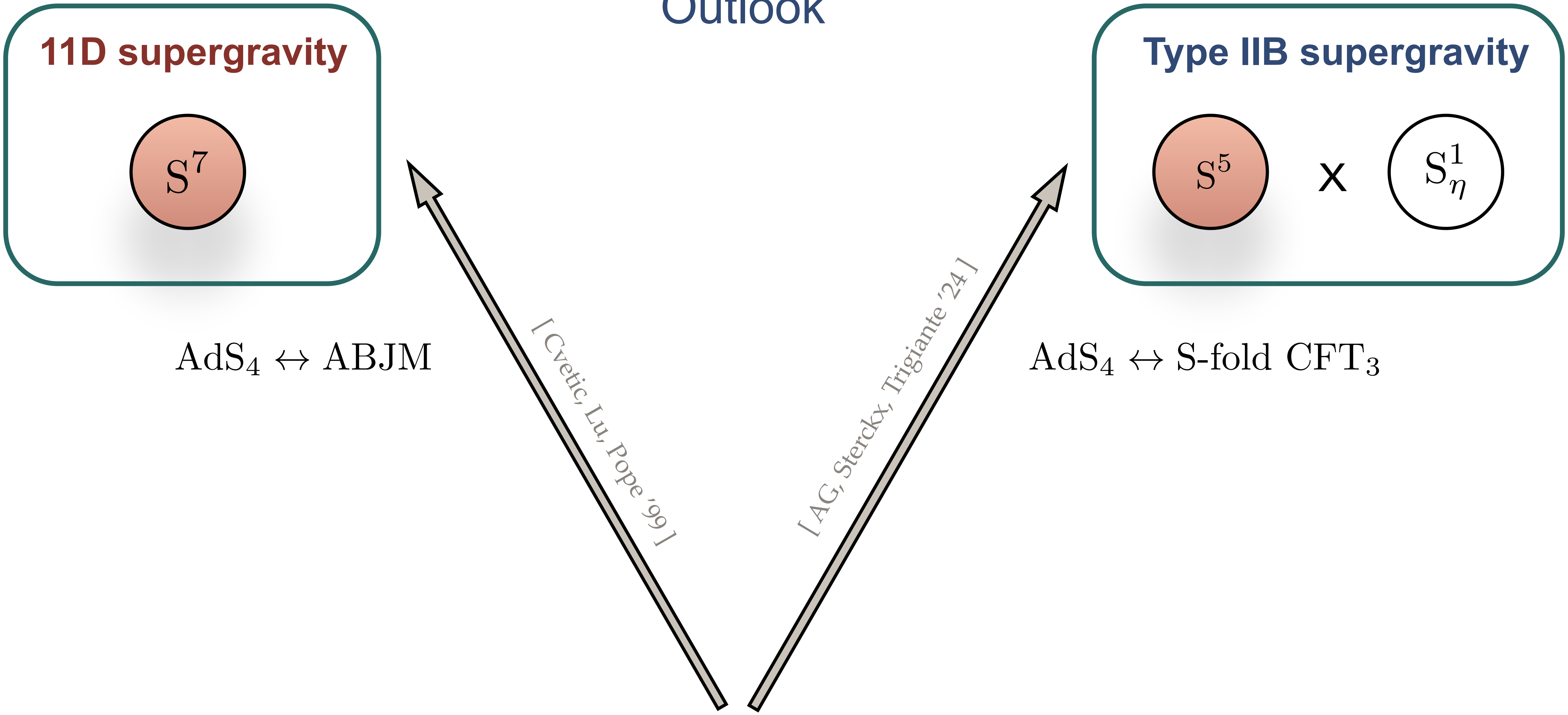
Outlook



$\mathcal{N}=4\,,\ D=4$ SO(4)$_R$-gauged supergravity

$$
\begin{aligned}
S_{\text{bos}} \;=\; & \int R\star 1-\tfrac{1}{2}\,d\xi\wedge\star d\xi-\tfrac{1}{2}\,e^{2\xi}\,d\chi\wedge\star d\chi+\left(4+2\cosh\xi+e^{\xi}\chi^{2}\right)\star 1\\
& -\frac{1}{1+\chi^{2}e^{2\xi}}\left(e^{\xi}F^{a}_{(2)}\wedge\star F^{a}_{(2)}+\chi\,e^{2\xi}\,F^{a}_{(2)}\wedge F^{a}_{(2)}\right)-e^{-\xi}F^{\hat{a}}_{(2)}\wedge\star F^{\hat{a}}_{(2)}+\chi\,F^{\hat{a}}_{(2)}\wedge F^{\hat{a}}_{(2)}
\end{aligned}
$$

Outlook



$\mathcal{N}=4\,,\ D=4$ $SO(4)_R$-gauged supergravity

$$
\begin{aligned}
S_{\text{bos}} \;=\; \int & R\star 1 - \tfrac{1}{2}\,d\xi\wedge\star d\xi - \tfrac{1}{2}\,e^{2\xi}\,d\chi\wedge\star d\chi + (4+2\cosh\xi + e^{\xi}\chi^2)\star 1 \\
& - \frac{1}{1+\chi^2 e^{2\xi}}\left(e^{\xi}F^{a}_{(2)}\wedge\star F^{a}_{(2)} + \chi\, e^{2\xi}\,F^{a}_{(2)}\wedge F^{a}_{(2)}\right) - e^{-\xi}F^{\hat a}_{(2)}\wedge\star F^{\hat a}_{(2)} + \chi\,F^{\hat a}_{(2)}\wedge F^{\hat a}_{(2)}
\end{aligned}
$$

Outlook

11D supergravity

S^7

$\mathrm{AdS}_4 \leftrightarrow \mathrm{ABJM}$

$\mathrm{Janus} \leftrightarrow \mathrm{ABJM\ interfaces}$

[Cvetic, Lu, Pope '99]

[AG, Sterckx, Trigiante '24]

Type IIB supergravity

$S^5 \times S^1_\eta$

$\mathrm{AdS}_4 \leftrightarrow \mathrm{S\text{-}fold\ CFT}_3$

$\mathrm{Janus} \leftrightarrow \mathrm{S\text{-}fold\ CFT}_3\ \mathrm{interfaces}$

$\mathcal{N} = 4\,,\ D = 4$ SO(4)$_R$-gauged supergravity

$$S_{\text{bos}} \;=\; \int R \star 1 - \tfrac{1}{2}\, d\xi \wedge \star d\xi - \tfrac{1}{2}\, e^{2\xi}\, d\chi \wedge \star d\chi + (4 + 2\cosh\xi + e^{\xi}\chi^2) \star 1$$
$$- \frac{1}{1+\chi^2 e^{2\xi}} \left(e^{\xi} F^{a}_{(2)} \wedge \star F^{a}_{(2)} + \chi\, e^{2\xi}\, F^{a}_{(2)} \wedge F^{a}_{(2)} \right) - e^{-\xi} F^{\hat a}_{(2)} \wedge \star F^{\hat a}_{(2)} + \chi\, F^{\hat a}_{(2)} \wedge F^{\hat a}_{(2)}$$

Outlook

11D supergravity

S^7

$AdS_4 \leftrightarrow$ ABJM

Janus $\leftrightarrow$ ABJM interfaces

AdS-BH $\leftrightarrow$ ABJM on Σ_2 (top. twist)

e.g. S^2 or H^2, spindles, ...

[Cvetic, Lu, Pope '99]

Type IIB supergravity

$S^5 \times S^1_\eta$

$AdS_4 \leftrightarrow$ S-fold CFT_3

Janus $\leftrightarrow$ S-fold CFT_3 interfaces

AdS-BH $\leftrightarrow$ S-fold CFT_3 on Σ_2 (top. twist)

e.g. S^2 or H^2, spindles, ...

[AG, Sterckx, Trigiante '24]

$\mathcal{N}=4\,,\ D=4$ $SO(4)_R$-gauged supergravity

$$
\begin{aligned}
S_{\text{bos}} \;=\; \int & R\star 1 - \tfrac{1}{2}\, d\xi\wedge\star d\xi - \tfrac{1}{2}\, e^{2\xi}\, d\chi\wedge\star d\chi + (4+2\cosh\xi + e^{\xi}\chi^2)\star 1 \\
& - \frac{1}{1+\chi^2 e^{2\xi}}\left(e^{\xi} F^{a}_{(2)}\wedge\star F^{a}_{(2)} + \chi\, e^{2\xi}\, F^{a}_{(2)}\wedge F^{a}_{(2)}\right) - e^{-\xi} F^{\hat{a}}_{(2)}\wedge\star F^{\hat{a}}_{(2)} + \chi\, F^{\hat{a}}_{(2)}\wedge F^{\hat{a}}_{(2)}
\end{aligned}
$$

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Sujet : Acceptance LY18367D Guarino
Date : Wed, 15 Jan 2025 19:45:49 +0000
De : prd@aps.org
Répondre à : prd@aps.org

Report of the Referee -- LY18367D/Guarino

This paper establishes very non-trivial new results that could be important for the AdS/CFT correspondence. Since it is also well written I recommend this paper for publication in PRD in its present form.

gracias !

thanks !