Non-Abelian Symmetry Operators from Hanging Branes in $AdS_5 \times S^5$

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based on 2510.19812 with I. Bah, M. Chitoto, E. Leung

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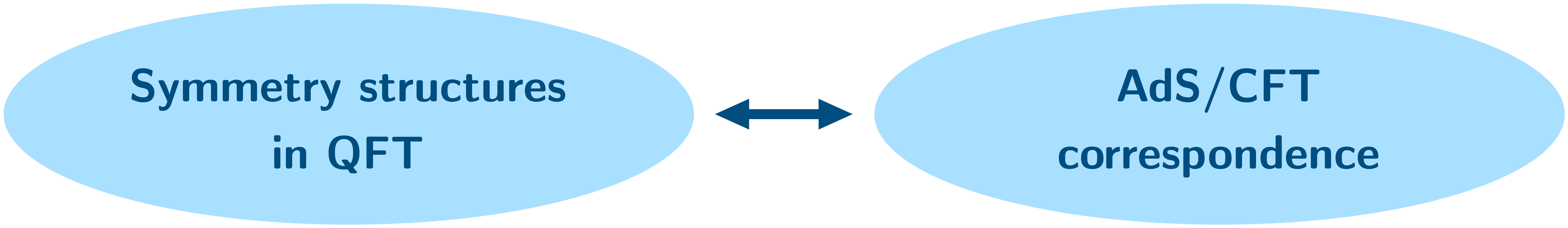
Introduction

- The AdS/CFT correspondence is a key tool in studying fundamental aspects of quantum field theories (QFTs)
- In particular, using holography we can study **symmetry structures** in QFT



- Modern language for global symmetries: sector of **topological defects** of a QFT
- Focus on this talk: ordinary continuous symmetries
  - Well-understood from the standard textbook viewpoint
  - Precise characterization in the modern language of topological defects is still lacking

Modern language for global symmetries

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*Time reversal is implemented by an anti-unitary operator

- What is the most **general** notion of **global symmetry** for a QFT?

- Usual textbook description: unitary* operators U_g with $g \in G$ a group; commute with Hamiltonian

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- This notion can be extended in many useful ways if we adopt a different point of view [Gaiotto, Kapustin, Seiberg, Willet 14]

global symmetries $\leftrightarrow$ ***extended topological defects***

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- This generalization is best understood for **finite internal symmetries**
  - Topological defects of various dimensions are described by a **higher fusion category**
  - Well-defined mathematical framework that captures interesting physics (higher-form symmetries, higher-group symmetries, non-invertible symmetries...)
- The precise mathematical framework for **continuous** symmetries is still lacking
- We revisit ordinary continuous symmetries in holography to gain possible insights

e.g. reviews [Cordova, Dumitrescu, Intriligator, Shao 22; McGreevy 22; Gomes 23; Schafer-Nameki 23; Brennan, Hong 23; Bhardwaj, Bottini, Fraser-Taliente, Gladden, Gould, Platschorre, Tillim 23; Shao 23; Carqueville, Del Zotto, Runkel 23; Simons Lectures on Categorical Symmetries 24]

Revisiting ordinary continuous symmetries

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- An ordinary continuous symmetry acts on operator insertions at points (0-form symmetry)
- Noether's theorem: **conserved current**
- The topological defect implementing the symmetry is the integral of the current on a codimension-1 slice in spacetime

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Example: $U(1)$ ***symmetry***

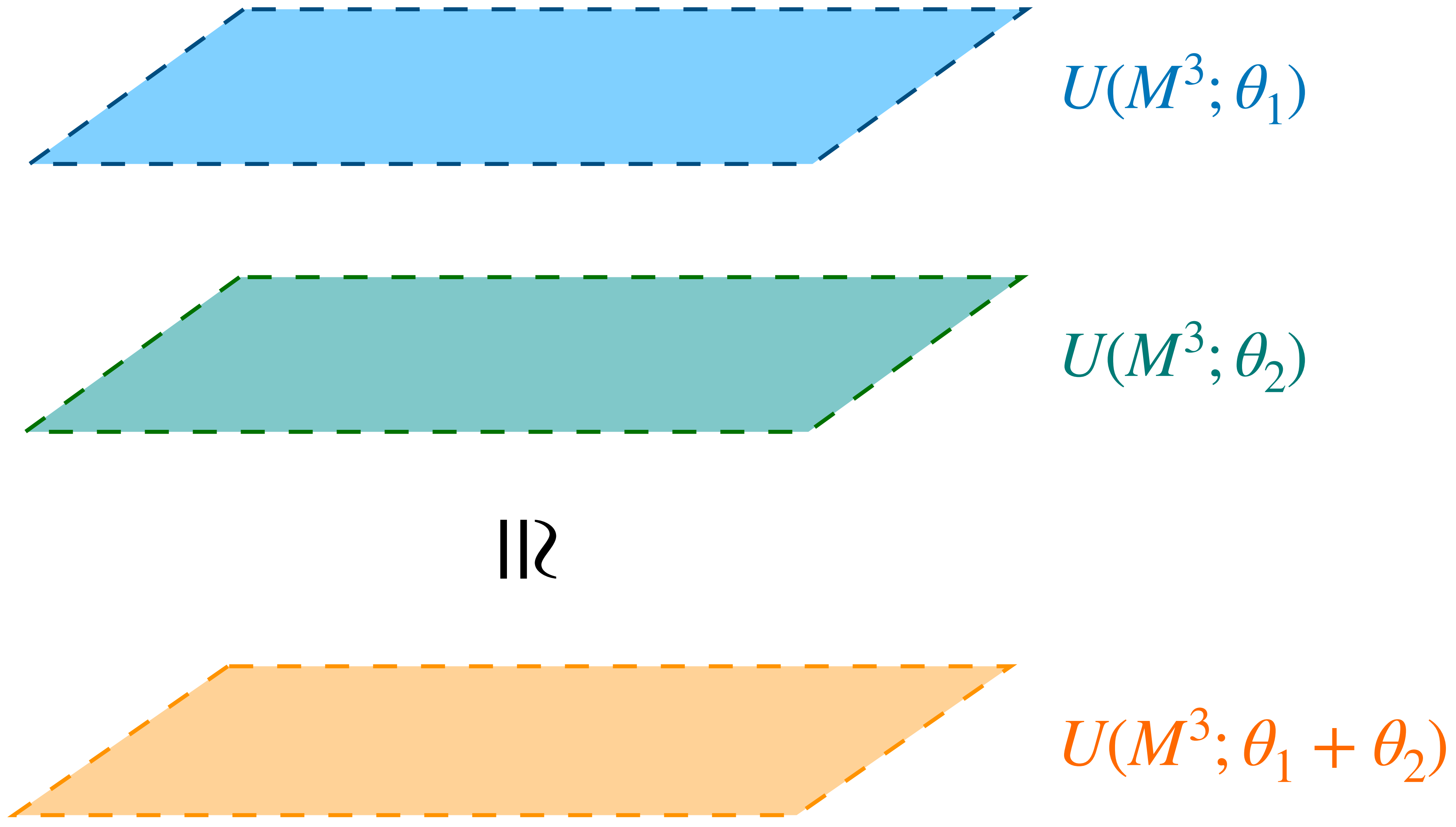
Conserved current: $\partial_\mu J^\mu = 0$ or $d * J = 0$

Conserved charge: $Q = \int_{t=0} J^0 d^3x$ or $Q = \int_{M^3} * J$

Topological defect: $U(M^3;\theta) = \exp\left(i\theta \int_{M^3} * J\right)$

Infinite family of topological defects with group-like fusion

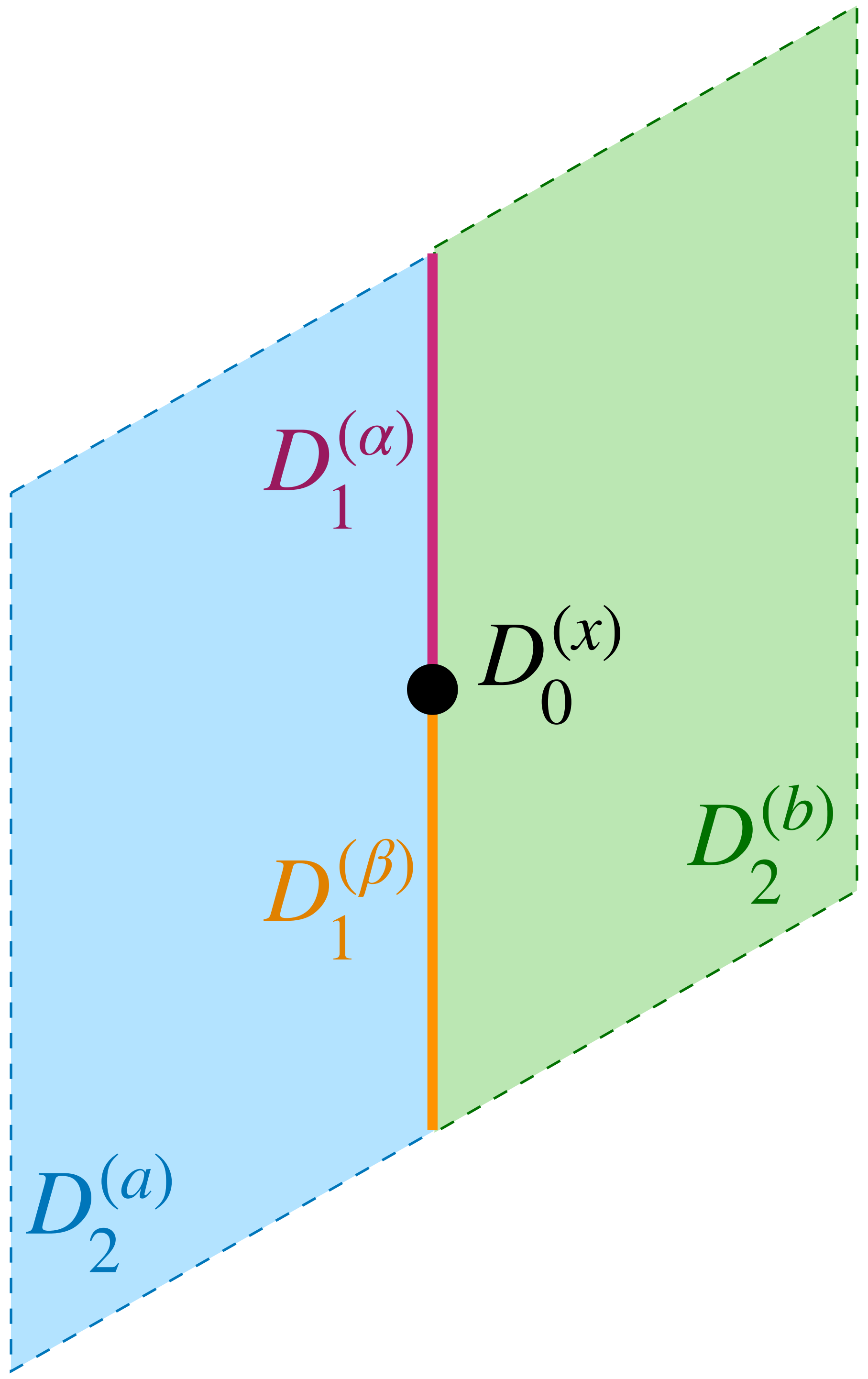
$U(M^3;\theta_1) \times U(M^3;\theta_1) = U(M^3;\theta_1+\theta_2)$

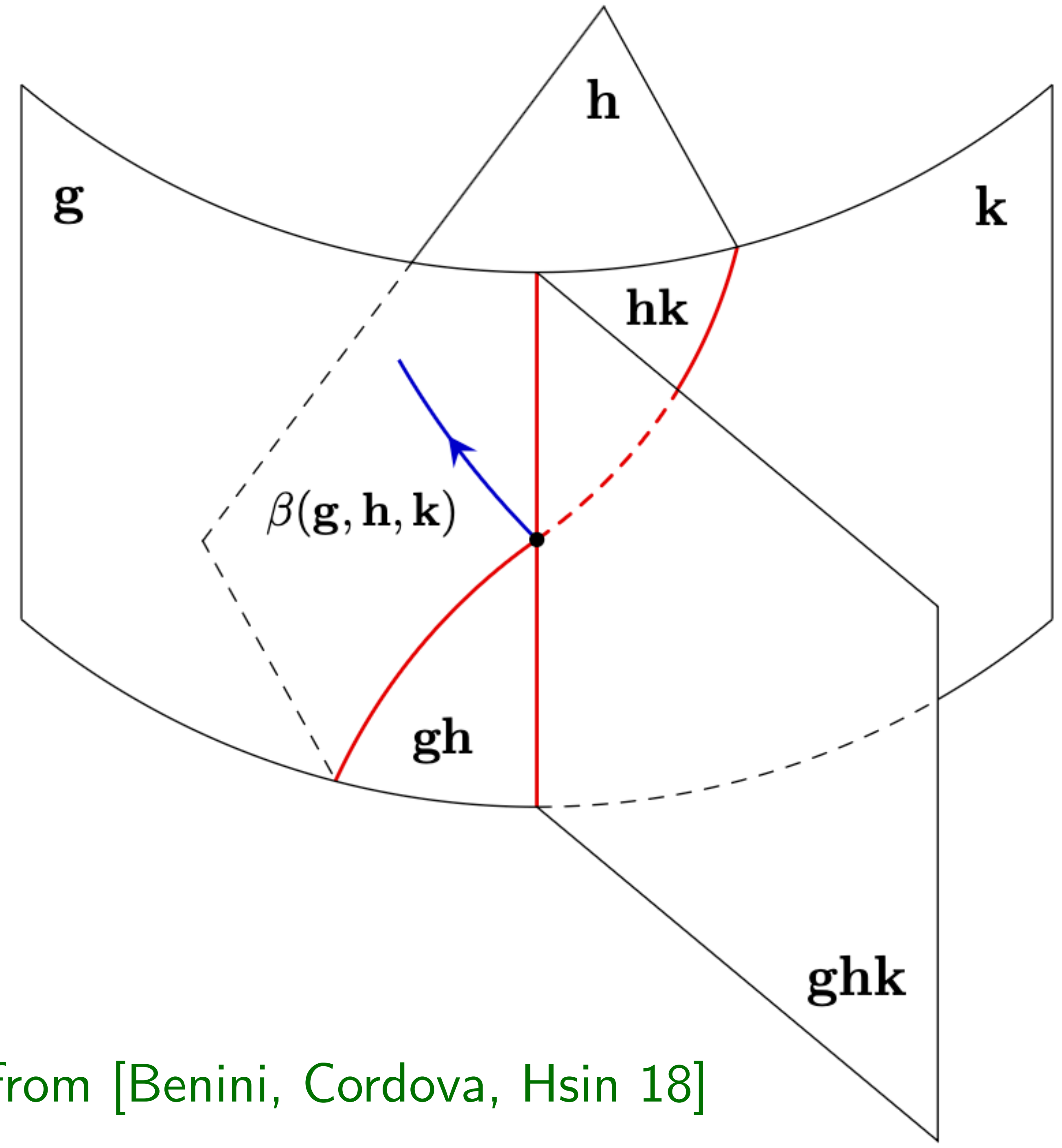


A rich variety of symmetry structures

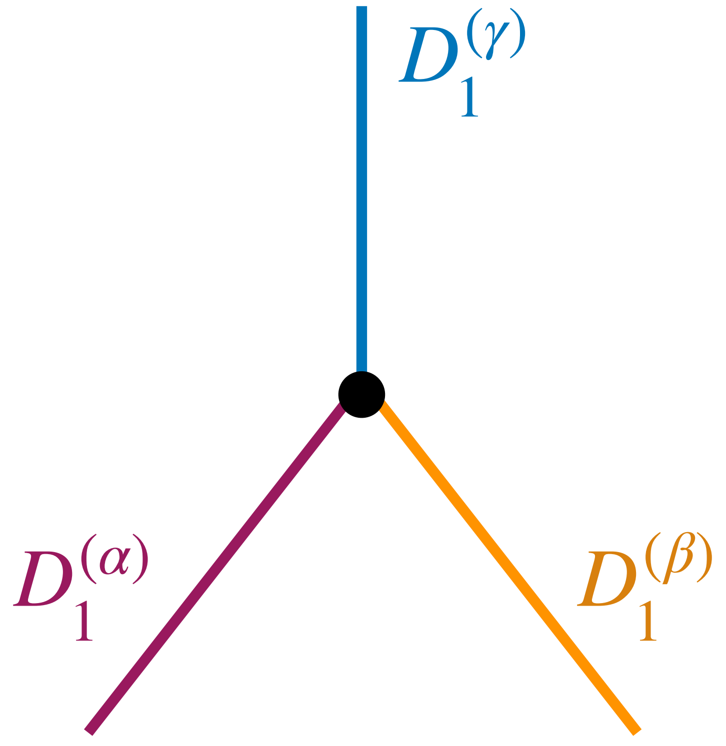
Expressing symmetries using topological defects allows for a rich variety of structures

- Topological defects of various codimensions
- Defects within defects
- Non-trivial junctions
- Fusion algebras beyond groups





from [Benini, Cordova, Hsin 18]



$$D_1^{(\alpha)} \times D_1^{(\beta)} = \sum_\gamma N_{\alpha\beta}^{\gamma} D_1^{(\gamma)}$$

These structures can have non-trivial physical implications, but they are not yet fully understood for symmetries that are not finite

Symmetries in AdS$_{d+1}$/CFT$_d$

[Witten 98; ... Harlow, Ooguri 18; ...]

- General pattern in holography

global symmetries on field theory side $\leftrightarrow$ ***gauge symmetries on AdS side***

- This applies in particular to ordinary continuous symmetries
- The conserved current on the field theory side is dual to a gauge field in AdS$_{d+1}$

J_μ ***on field theory side*** $\leftrightarrow$ A_μ ***on AdS side***

- Our focus here is to shift perspective: identify the gravity dual of the topological defect that implements the symmetry
- The dual will be an extended object (brane)
- We can build on analogous results for finite symmetries

Holographic duals of topological defects via branes

- For finite symmetries, there has been progress in understanding the gravity dual of topological defects using ordinary D-branes

  [Apruzzi, Bah, **FB**, Schafer-Nameki 22; García Etxebarria 22; Heckman, Hübner, Torres, Zhang 22; Heckman, Hübner, Torres, Yu, Zhang 22; Etheredge, García Etxebarria, Heidenreich, Rauch 23; Dierigl, Heckman, Montero, Torres 23; Bah, Leung, Waddleton 23; Apruzzi, **FB**, Gould, Schafer-Nameki 23; Cvetič, Heckman, Hübner, Torres 23; Baume, Heckman, Hübner, Torres, Turner, Yu 23; Yu 23; Del Zotto, Meynet, Moscrop 24; Hu 24; Argurio, Benini, Bertolini, Galati, Niro 24; Zhang 24; Braeger, Chakrabhavi, Heckman, Hübner 24; Franco, Yu 24; Gutperle, Li, Rathore, Roumpedakis 24; Knighton, Sriprachyakul, Vošmera 24; Fernandez-Melgarejo, Giorgi, Marques, Rosabal 24; Bergman, Mignosa 24; **FB**, Del Zotto, Minasian 24; Christensen 24; Caldararu, Pantev, Sharpe, Sung, Yu 25]

- What about topological defects for continuous symmetries?
- Recent proposals realize their holographic duals using fluxbranes, non-BPS D-branes, non-BPS Kaluza-Klein (KK) monopoles

  [García-Valdecasas 23; Cvetič, Heckman, Hübner, Torres 23; Bergman, García-Valdecasas, Mignosa, Rodriguez-Gomez 24; Waddleton 24; Cvetič, Heckman, Hübner, Murdia 25; Calvo, Mignosa, Rodriguez-Gomez 25; Najjar 25; Calvo, Mignosa, Rodriguez-Gomez 25]

- In this work we propose a **realization using BPS objects** (D5-brane and KK monopole) hanging from the AdS boundary

  building on [Calvo, Mignosa, Rodriguez-Gomez 25]

Branes vs symmetry defects vs charged defects

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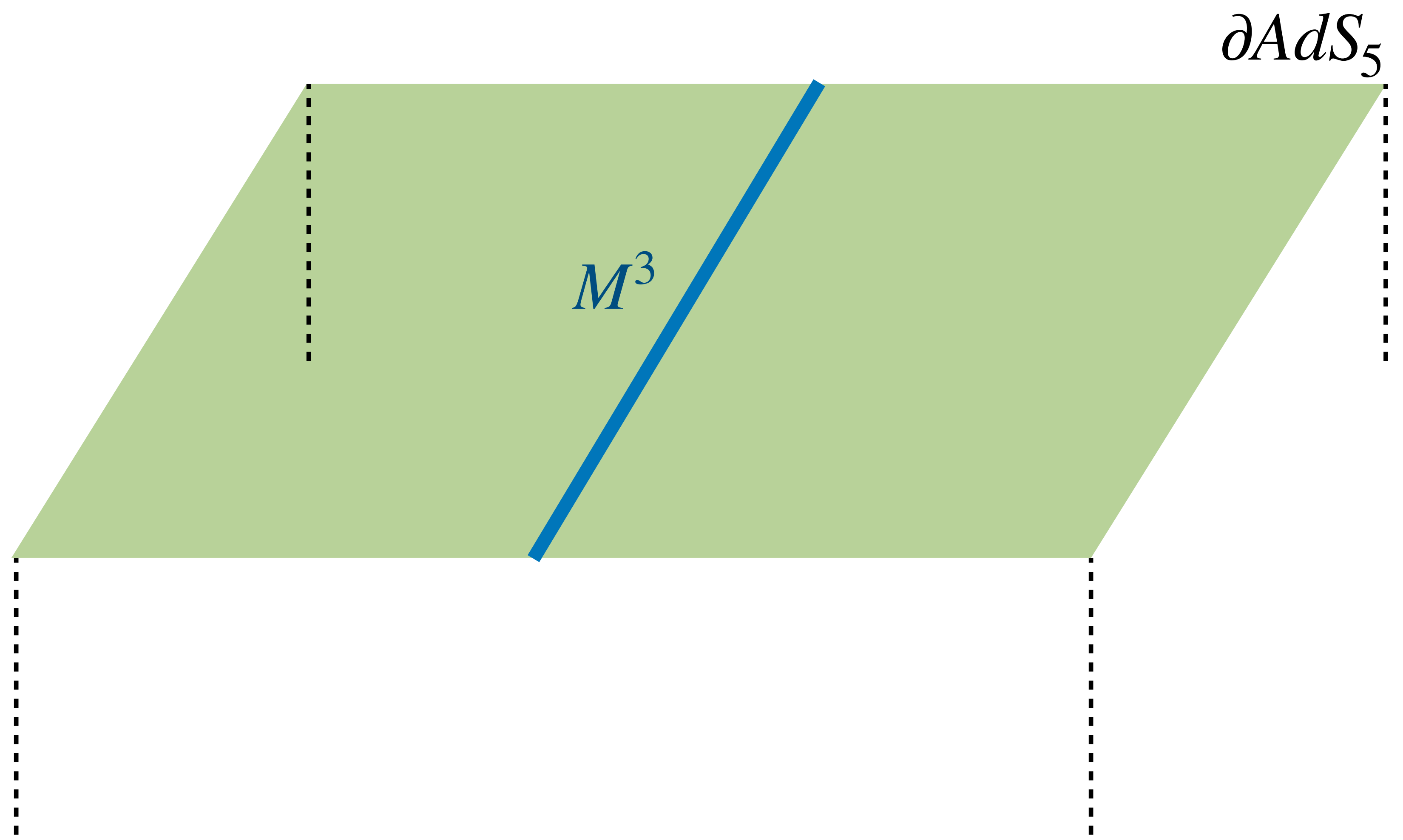
Brane realization of symmetry defects



- Brane sits at $z = 0$ (conformal boundary)
- Effective tension scales as $T \sim z^{-p}$ $(p > 0)$
- Gives rise to topological defect in dual QFT

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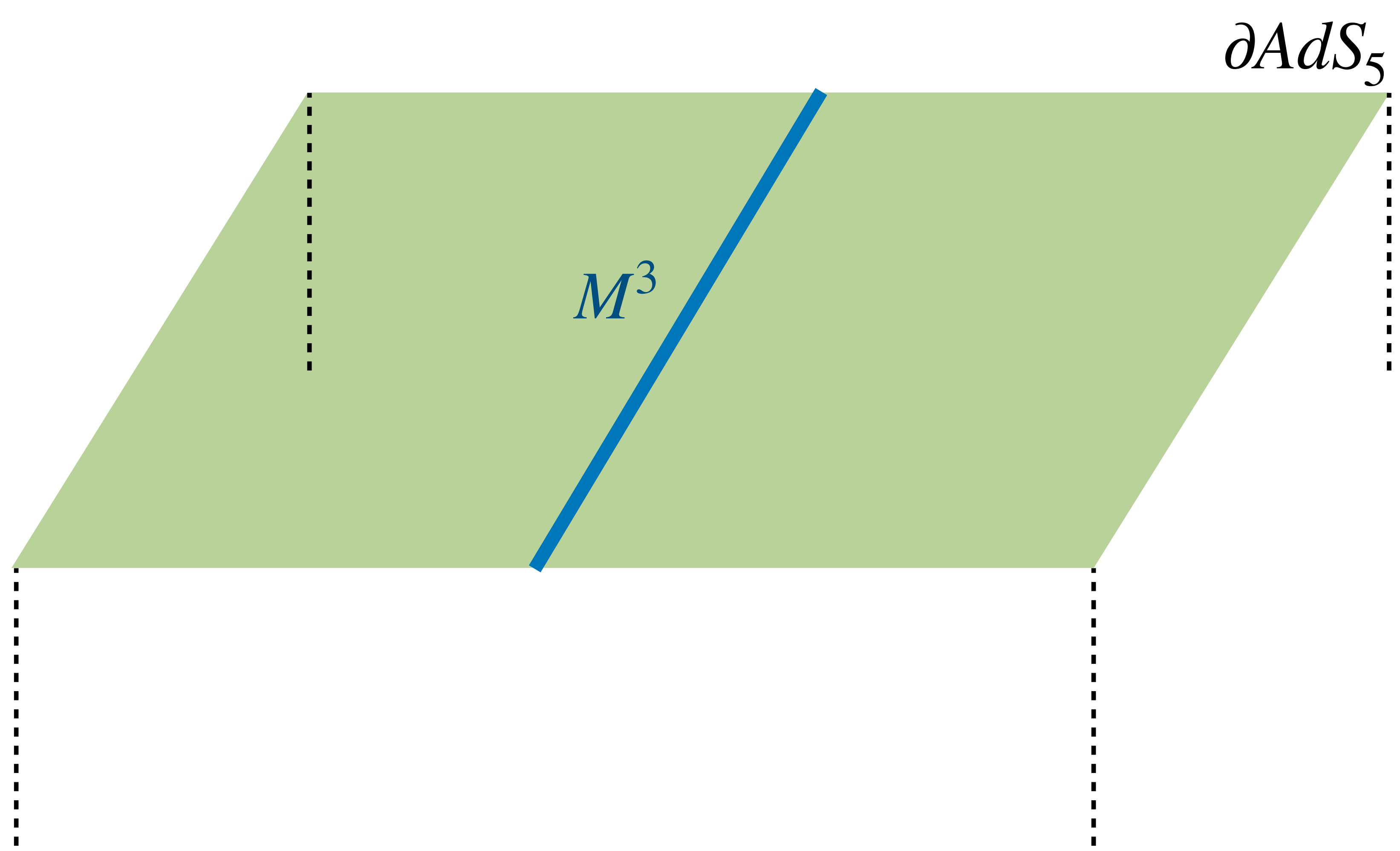

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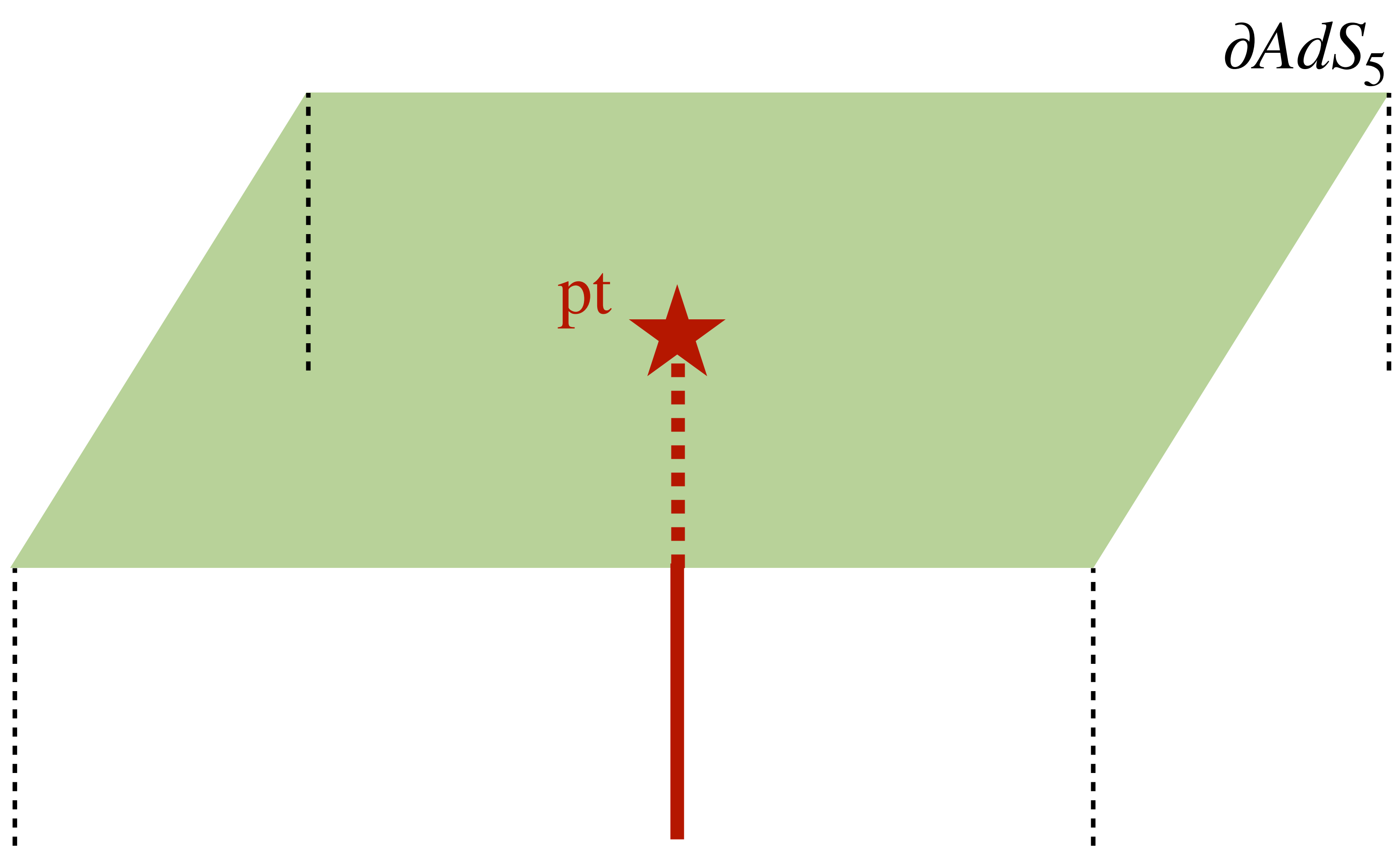

- Brane extends in the AdS_5 radial direction z
- Gives rise to non-topological defect in dual QFT
- Can link with other branes in the bulk: realizes defect charged under symmetries

Case study: $AdS_5 \times S^5$

- Well-studied example of AdS/CFT correspondence

$$\textit{Type IIB string theory on } AdS_5 \times S^5 \quad \leftrightarrow \quad \textit{4d } \mathcal{N} = 4 \; SU(N) \textit{ super Yang-Mills}$$

- Holographic dictionary ($L \equiv$ AdS radius = sphere radius; $G_5 \equiv$ RR 5-form field strength)

$$L^4 = 4\pi g_s N(\alpha')^2 \qquad g_{\rm YM}^2 = 2\pi g_s \qquad \int_{S^5} G_5 = N$$

- We focus on the **R-symmetry** of the 4d field theory
- On the gravity side, it corresponds to the $SO(6)$ **isometry** group of S^5
- ***Goal***: identify the gravity duals of the 3d topological defects that implement the R-symmetry of 4d SYM

Low-energy analysis

Low-energy analysis

- To gain intuition, we start with an analysis based on the AdS_5 low-energy **effective action**
- **Consistent truncation**: Type IIB supergravity on S^5 gives 5d maximal gauged $SO(6)$ supergravity
  - metric $g_{\mu\nu}$
  - $SO(6)$ gauge fields $A_\mu^{ab} = -A_\mu^{ba}$, $a, b = 1,\ldots,6$
  - 42 real scalars
  - 12 two-forms
  - Fermions

[Cvetič, Lu, Pope, Sadrzadeh, Tran 00; Pilch, Warner 00; Cassani, Dall'Agata, Faedo 10; Liu, Szepietowski, Zhao 10; Gauntlett, Varela 10; Baguet, Hohm, Samtleben 15]

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- We only need the terms in the effective action that contain the $SO(6)$ gauge fields

$$S = \int_{AdS_5} \left[-\frac{1}{4}\,\tau\, F_{ab} \wedge *F^{ab} - k\,\mathrm{CS}_5 \right] \qquad d\mathrm{CS}_5 = \frac{1}{384\pi^2}\epsilon_{a_1\ldots a_6} F^{a_1a_2} \wedge F^{a_3a_4} \wedge F^{a_5a_6}$$

- $SO(6)$ field strength ${F_{\mu\nu}}^{ab} = 2\partial_{\mu}{A_{\nu]}}^{ab} + 2{A_{[\mu}}^{ac}{A_{\nu]c}}^{b}$

$SO(6)$ gauge coupling: $\tau = \dfrac{N^2}{8\pi^2 L}$ Chern-Simons level: $k = N^2 - 1$

Symmetry defects from Gauss' law constraints

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- Let us analyze the AdS_5 effective action in a **Hamiltonian formalism** where "time" in the radial direction z of AdS_5

[Witten 98; Belov, Moore 04]

- The action does not contain "time" derivatives of the "temporal" components: they are Lagrange multipliers
- They impose the **Gauss' law constraints** $\mathbf{G}_{ab} = 0$ where

$$\mathbf{G}_{ab} = \frac{1}{2} \tau D * F_{ab} + \frac{1}{128\pi^2} k \, \epsilon_{abc_1 \dots c_4} F^{c_1 c_2} \wedge F^{c_3 c_4} \qquad \text{pulled back to a 4d slice at constant } z = z_0$$

- $SO(6)$ covariant exterior derivative $D * F_{ab} = d * F_{ab} + 2A_{[a|c} \wedge *F^{c}{}_{b]}$

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1) Gauss' law constraints generate bulk gauge transformations

$\int_{z=z_0} \theta^{ab}(x)\mathbf{G}_{ab}$ generates gauge transformation with parameter $\theta^{ab}(x)$: $\delta A^{ab} = D\theta^{ab}$

2) Gauge transformations that are non-trivial at the conformal boundary of AdS_5 correspond to global symmetry transformations

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- From *1)* and *2)* we consider $\mathbf{G}_{ab}$ in the limit $z_0 \to 0$. We use Dirichlet boundary conditions $A^{ab}|_{z=0} = 0 \quad \Rightarrow \quad \mathbf{G}_{ab} = \frac{1}{2}\,\tau\, d * F_{ab}$
- On the slice $z = z_0$, we select a region B^4 with $\partial B^4 = M^3$.
  We specialize $\theta^{ab}(x)$ to nonzero and constant inside B^4, and zero outside B^4 $\quad \Rightarrow \int_{z=z_0} \theta^{ab}(x)\mathbf{G}_{ab} = \int_{M^3} \frac{1}{2}\,\theta^{ab}\,\tau\, * F_{ab}$

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3d topological defect implementing the symmetry : $U(\theta, M^3) = \exp\left(i\displaystyle\int_{M^3}\frac{1}{2}\,\theta^{ab}\,\tau\,*F_{ab}\right)$

Towards a 10d origin

3d topological defect implementing the symmetry : $\quad U(\theta, M^3) = \exp\left(i \int_{M^3} \frac{1}{2} \theta^{ab} \tau * F_{ab} \right)$

- We would like to identify an extended object in Type IIB that can give rise to this object at low-energies
- Preliminary question: what is the 10d origin of the $SO(6)$ gauge coupling τ?
- It receives **two contributions**: from 10d Einstein-Hilbert term, and from the kinetic term for the G_5 flux

10d origin of the gauge coupling τ

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[Cvetič, Lu, Pope, Sadrzadeh, Tran 00; Barnes, Gorbatov, Intriligator, Wright 05; Benvenuti, Pando Zayas, Tachikawa 06]

$SO(6)$ gauge fields in the 10d metric

10d metric $\quad ds^2_{10} = ds^2_{AdS_5} + L^2 \delta_{ab} Dy^a Dy^b$

S^5 parametrized by y^a with $\delta_{ab} y^a y^b = 1$

$D_\mu y^a = \partial_\mu y^a + A_\mu{}^a{}_b y^b$

The reduction of the 10d Einstein-Hilbert term yields,
among others, a Yang-Mills term for the $SO(6)$ gauge fields

$$\tau_{\text{EH}} = \frac{N^2}{24\pi^2 L}$$

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$SO(6)$ *gauge fields in the 5-form flux*

$G_5 = \mathscr{G}_5 + *_{10} \mathscr{G}_5$

$\mathscr{G}_5 = N V_5 + \frac{1}{4\pi} N F^{ab} \wedge \omega_{ab}$

$V_5 = \frac{1}{\pi^3 5!} \epsilon_{a_1 \dots a_6} y^{a_1} Dy^{a_2} \wedge Dy^{a_3} \wedge Dy^{a_4} \wedge Dy^{a_5} \wedge Dy^{a_6}$

$\omega_{ab} = \frac{1}{2\pi^2 3!} \epsilon_{abc_1 \dots c_4} y^{c_1} Dy^{c_2} \wedge Dy^{c_3} \wedge Dy^{c_4}$

The kinetic term for G_5 in the Type IIB pseudo-action is another source of Yang-Mills term for the $SO(6)$ gauge fields

$$\tau_{\rm flux} = \frac{N^2}{12\pi^2 L}$$

Strategy: combining D5-brane and KK monopole

- The fact that τ originates from two distinct contributions suggests a strategy to identify the extended object that realizes the $SO(6)$ symmetry defects
- We will consider a bound state of a D5-brane and a KK monopole

$$\text{D5-brane} \leftrightarrow \text{flux contribution} \qquad \text{KK monopole} \leftrightarrow \text{EH contribution}$$

- Both these objects are going to be **"hanging"** from the conformal boundary of AdS_5
- We study the effective action on their worldvolumes
- We show that it reproduces the expression from the low-energy analysis

Branes hanging from the conformal boundary

[Calvo, Mignosa, Rodriguez-Gomez 25;
Bah, **FB**, Chitoto, Leung 25]

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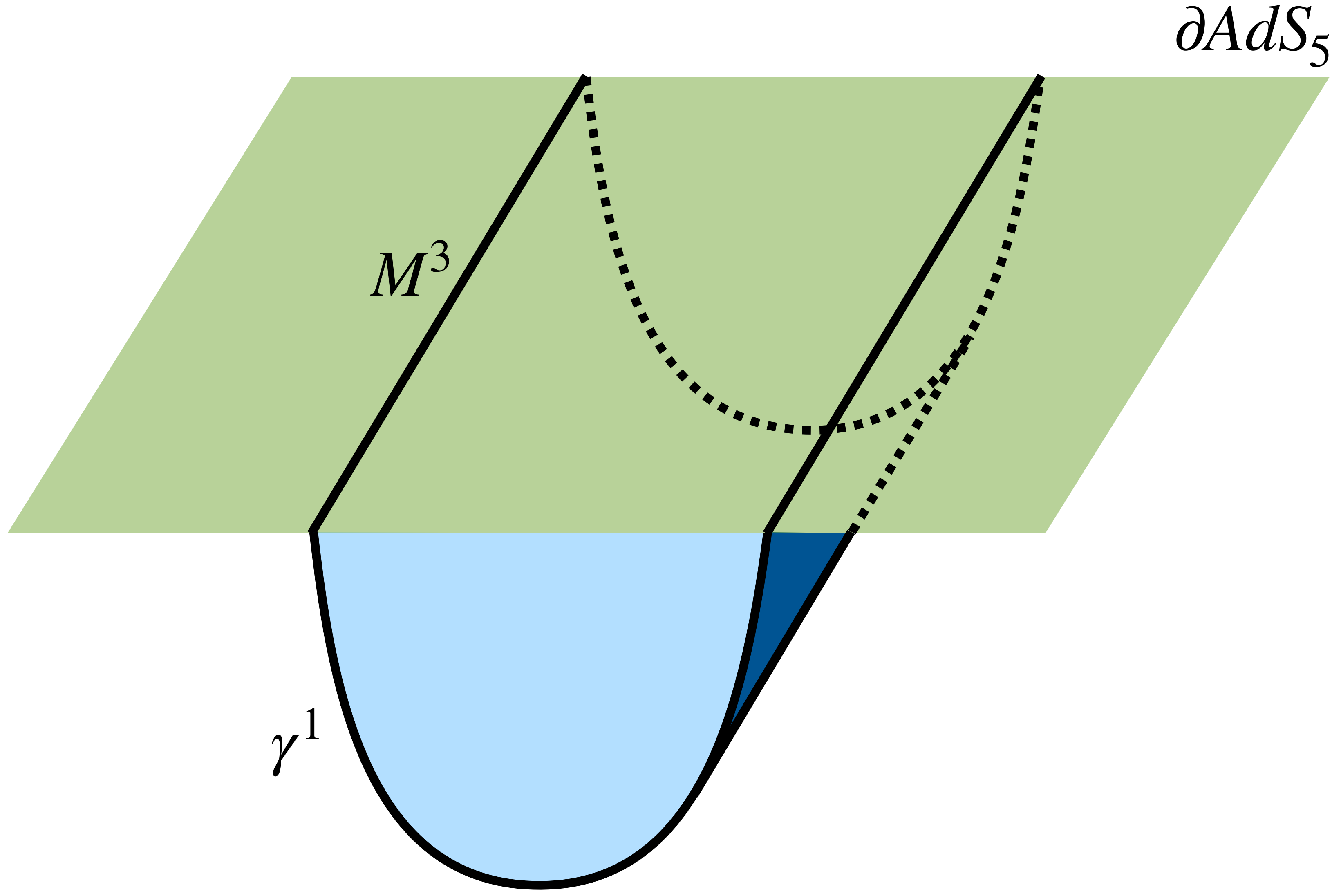
- For definiteness, let us focus on the D5-brane
- The equations of motion that follow from DBI + WZ action allow for "hanging configurations"
- 6d worldvolume : $M^3 \times \gamma^1 \times \Sigma^2$



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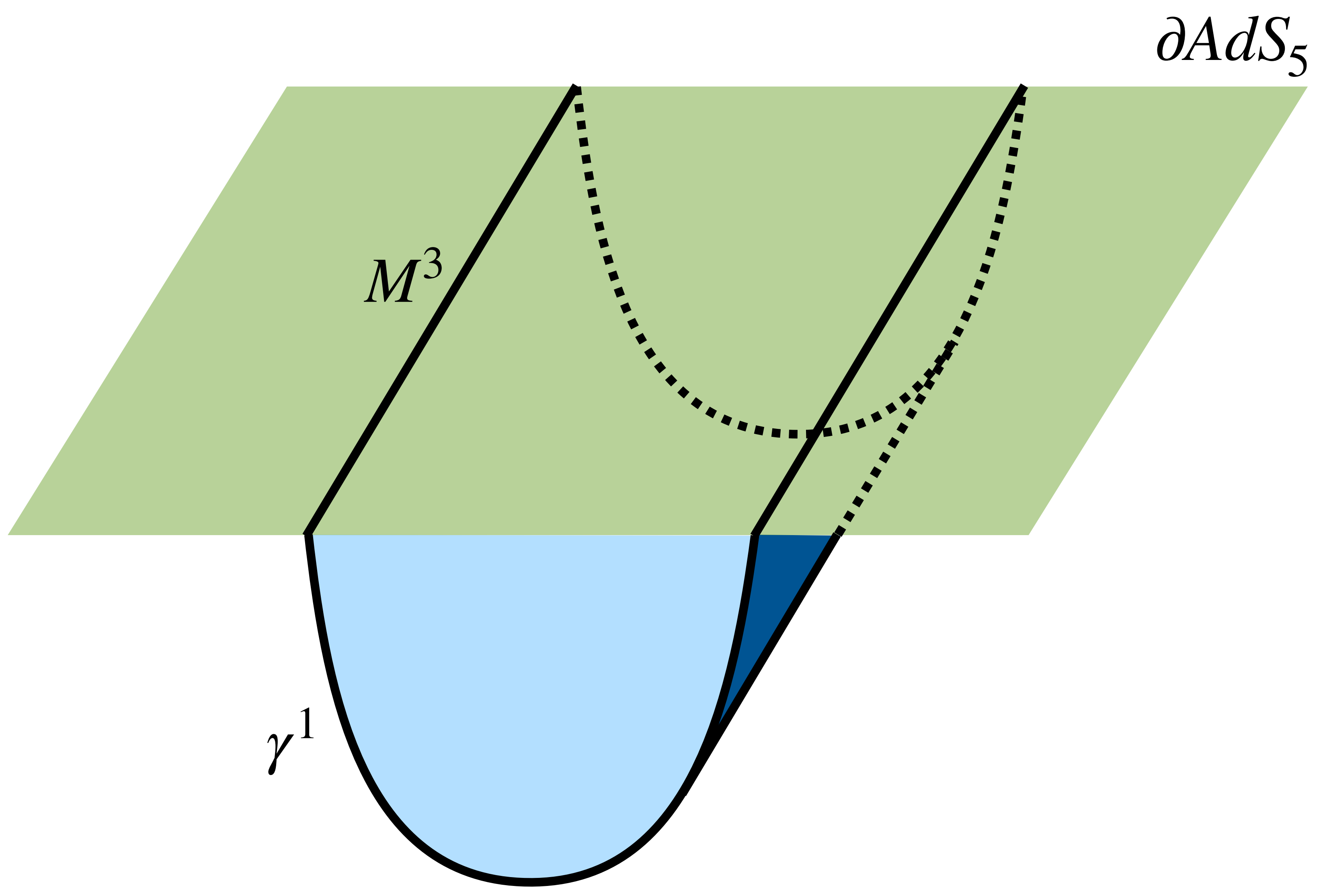

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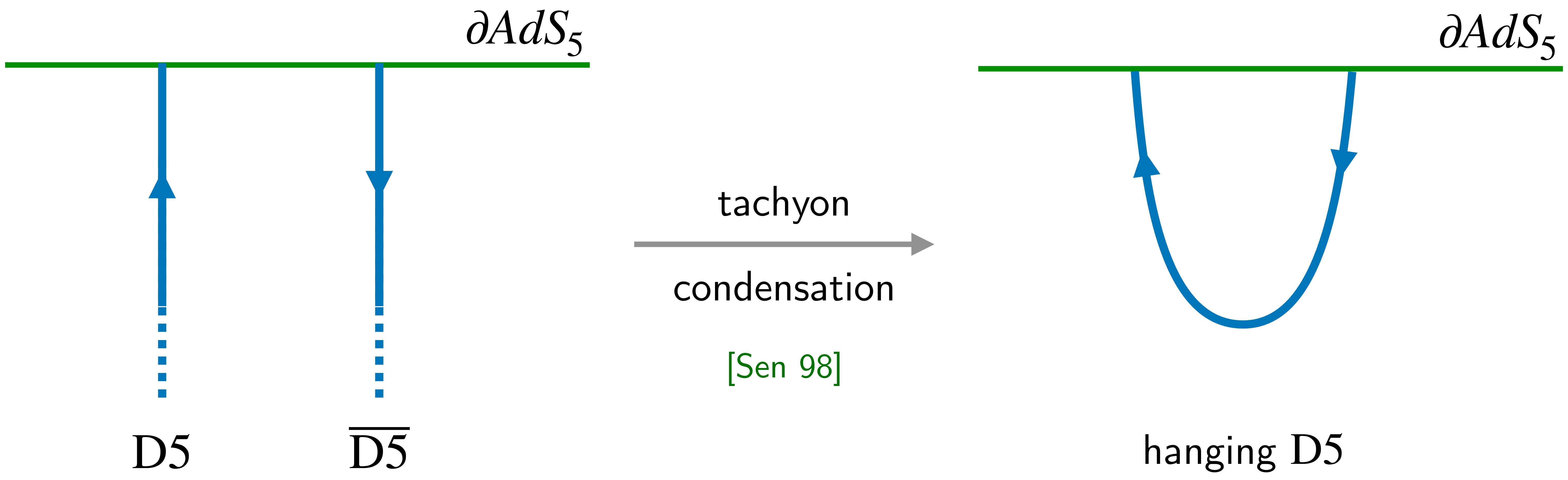
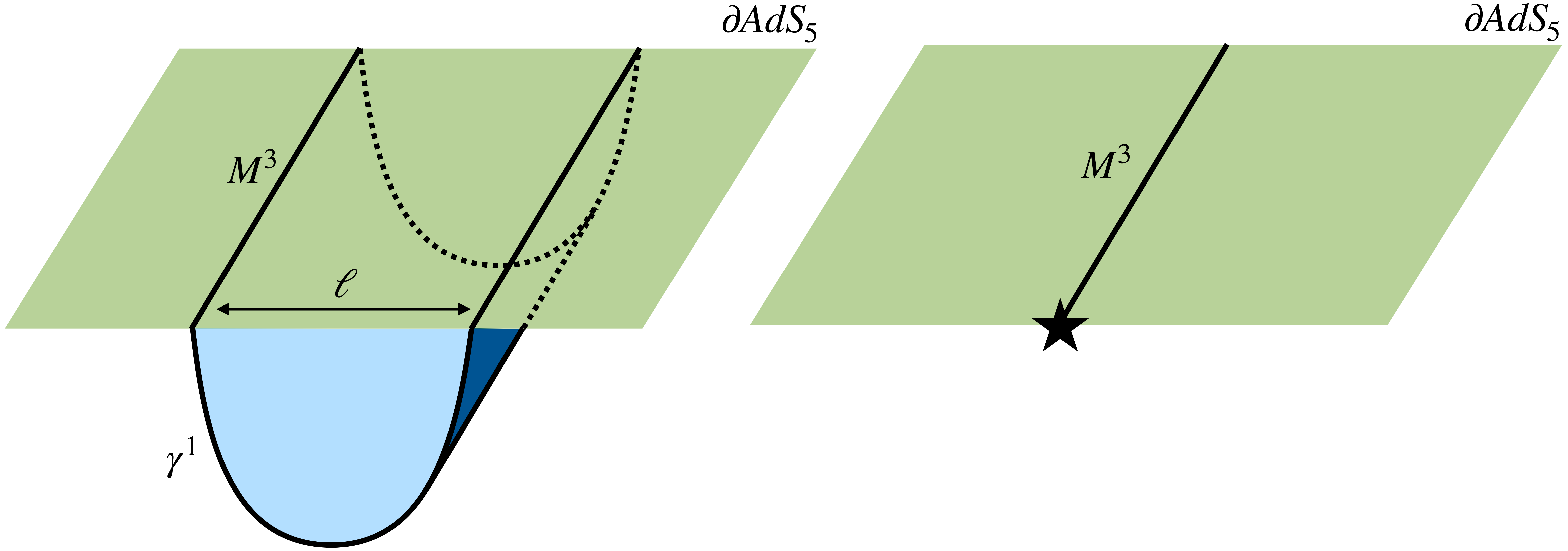
- The equations of motion that follow from DBI + WZ action allow for "hanging configurations"
- 6d worldvolume : $M^3 \times \gamma^1 \times \Sigma^2$
- We can regard the hanging brane as the final product of a **partial annihilation** between a D5-brane and an anti D5-brane anchored to the boundary



Branes hanging from the conformal boundary

[Calvo, Mignosa, Rodriguez-Gomez 25; Bah, **FB**, Chitoto, Leung 25]

- The width ℓ of the arc γ^1 is a modulus
- We consider the limit $\ell \to 0$
- We get an object living on $M^3 \subset \partial AdS_5$



- Claim: the resulting object realizes the symmetry defect

Topological WZ couplings on the D5-brane

- WZ action on a D5-brane

$$S_{\mathrm{WZ}} = \int_{W^6} e^{f_2 - B_2} \sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}} \sum_p C_p = \int_{W^6} C_6 + f_2 \wedge C_4 + \dots \qquad f_2 = da_1$$

Chan-Paton $U(1)$ **gauge field** on D5-brane

- We focus on the second term and integrate by parts: $S_{\mathrm{WZ}} \supset \int_{\gamma^1 \times M^3 \times \Sigma^2} a_1 \wedge G_5$

- We turn on a non-trivial profile for a_1 along γ^1. The relevant terms factorize

$$\int_{\gamma^1 \times M^3 \times \Sigma^2} a_1 \wedge G_5 = \left(\int_{\gamma^1} a_1 \right) \left(\int_{M^3 \times \Sigma^2} G_5 \right)$$

First ingredient: profile for a_1

- To have a solution to the DBI + WZ equations of motion of the hanging brane, a_1 must be flat

$$da_1 = 0$$

- However we can still have a nontrivial profile for a_1

flat connection satisfying $$\int_{\gamma^1} a_1 = \alpha_{\mathrm{D5}}$$

- The constant parameter $\alpha_{\mathrm{D5}} \sim \alpha_{\mathrm{D5}} + 2\pi$ is a modulus of the hanging brane configuration, can be tuned at will

Second ingredient: integrating G_5

- The relevant piece of G_5 comes from $*_{10}\,\mathcal{G}_5 \sim *\,F_{ab} \wedge *\omega^{ab} + \ldots$

$$\int_{M^3\times\Sigma^2} G_5 = \frac{1}{2}\left(\int_{M^3} *\,F_{ab}\right)\ \left(\int_{\Sigma^2} *\,\omega^{ab}\right)$$

- The internal piece depends on the choice of 2d submanifold $\Sigma^2 \subset S^5$

$$\int_{\Sigma^2} *\,\omega_{ab} = \tau_{\text{flux}}\, m_{ab}$$

- The constant parameters $m_{ab} = m_{-ba}$ are integers and determine the choice of $\Sigma^2 \subset S^5$

Total contribution from the hanging D5-brane

- Combining all ingredients we get the expression

$$U_{\mathrm{D5}}(\alpha_{\mathrm{D5}}, m; M^3) = \exp\left(\frac{1}{2} i \, \alpha_{\mathrm{D5}} \, \tau_{\mathrm{flux}} \, m^{ab} \int_{M^3} * F_{ab} \right)$$

- This has the correct functional form, but instead of the total gauge coupling τ we find the flux contribution τ_{flux} only

- The KK monopole contribution provides the missing piece

Contribution from hanging KK monopole

[Bergshoeff, Janssen, Ortin 98; Bergshoeff, Eyras, Lozano 98; Eyras, Janssen, Lozano 98]

- Brief reminder on some salient features of BPS KK monopole in Type IIB
  - 6d worldvolume, 4 transverse directions
  - One of the transverse directions must be an **isometry**
  - Worldvolume fields:
    - Non-compact scalars X^μ (embedding of worldvolume in 10d ambient space)
    - **Two compact scalars φ_0 and $\widetilde{\varphi}_0$**
    - One chiral 2-form
- We can consider a KK monopole with the same worldvolume $M^3 \times \gamma^1 \times \Sigma^2$ as the hanging D5-brane

Contribution from hanging KK monopole

- The KK monopole effective action contains a rich set of topological WZ-like couplings
- In particular one finds a coupling of the form

$$\int_{\gamma^1 \times M^3 \times \Sigma^2} \frac{1}{2} \left(\varphi_0 \, d\widetilde{\varphi}_0 - \widetilde{\varphi}_0 \, d\varphi_0 \right) \wedge G_5$$

[Eyras, Janssen, Lozano 98]

- We can apply the same strategy as in the D5-brane case. The analog of the profile for a_1 is now

$$\int_{\gamma^1} \frac{1}{2} \left(\varphi_0 \, d\widetilde{\varphi}_0 - \widetilde{\varphi}_0 \, d\varphi_0 \right) = \alpha_{\mathrm{KK}}$$

Final result

Final result

- We combine the D5-brane and KK monopole contributions
- We demand $$\alpha_{\mathrm{D5}} = 2\alpha_{\mathrm{KK}} =: \alpha$$
- The total result from the D5-KK system is the expected quantity

$$U_{\mathrm{D5-KK}}(\alpha, m; M^3) = \exp\left(\frac{1}{2} i \, \alpha \, \tau \, m^{ab} \int_{M^3} * F_{ab}\right) \qquad \text{cfr.} \qquad U(\theta, M^3) = \exp\left(i \int_{M^3} \frac{1}{2} \tau \, \theta^{ab} * F_{ab}\right)$$

$$\theta^{ab} = \alpha m^{ab}$$

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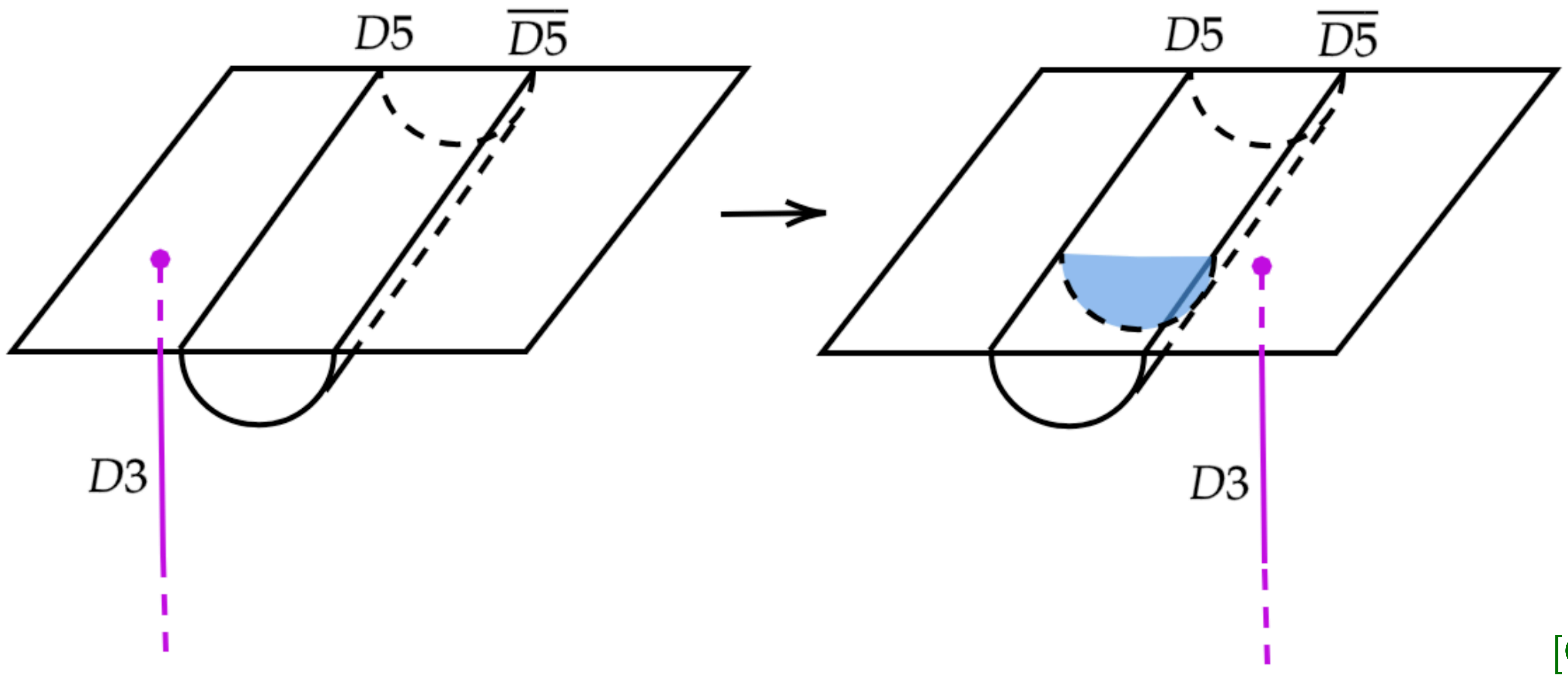
- Heuristically:
  - The choice of Σ^2 corresponds to the choice of m^{ab}: which **"direction"** in the Lie algebra $\mathfrak{so}(6)$
  - The parameter α governs the **"magnitude"** of the symmetry transformation
- The symmetry defect $U_{\mathrm{D5-KK}}(\alpha, m; M^3)$ implements the transformation by the $SO(6)$ element

$$g = \exp\left(\frac{1}{2}\, \alpha\, m^{ab}\, T^{(\mathrm{vec})}_{ab}\right) \qquad\qquad (T^{(\mathrm{vec})}_{ab})^c{}_d = \delta^c_a \delta_{bd} - \delta^c_b \delta_{ad}$$

Further supporting evidence

- In our paper we also study Wilson lines for the 5d $SO(6)$ gauge fields. We realize them using wrapped D3-branes
- We reproduce the expected interplay between Wilson lines and symmetry defects
- This can be seen using a **Hanany-Witten transition**

D3-brane crosses D5-brane $\Rightarrow$ F1-string is created [Hanany, Witten 96]



[Calvo, Mignosa, Rodriguez-Gomez 25]

Conclusions and outlook

- The modern language to describe symmetries puts **topological defects** at the center
- Precise framework for topological defects of **continuous symmetries** is lacking. We turn to AdS/CFT to draw lessons
- Case study: $AdS_5 \times S^5$
  - Topological defects implementing the $SO(6)$ global symmetry of 4d $\mathcal{N} = 4$ SYM can be realized by **hanging D5-KK**

Outlook

- Study more systematically the D5-KK system (e.g. relation between α_{D5} and α_{KK})
- Generalize the construction to other holographic setups (e.g. $AdS_5 \times$ Sasaki-Einstein; $AdS_4 \times S^7$; ...)
- Explore continuous symmetries beyond ordinary group-like symmetries

Thank you!