Massive supermembrane, type IIA massive superstring and Romans Supergravity

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Work done in collaboration with: Pablo León (PI, Canada) and Alvaro Restuccia (U. Antofagasta, Chile)
Based on: *MPGM, León, Restuccia JHEP2021, PLB2023, JHEP23*

Other collaborators: Lyonell Boulton (Herriott-Watt U, UK), Camilo Las Heras (IFT), Joselen Peña(UCN, Chile)

OUTLINE OF THE TALK

- ✓ Basics on Supermembranes.
- ✓ Massive Supermembrane
- ✓ Relation with Romans Supergravity
- ✓ Worldsheet description of the N=2 Type IIA ‘massive’ String
- ✓ Conclusions

BASICS ON SUPERMEMBRANES

- Supermembranes are M-theory elements, of 2+1D worldvolume embedded in a 11D target space. They act as sources for 11D Supergravity

$$S = T_d \int d^d\xi \left[\frac{1}{2}\sqrt{-g} g^{ij} \Pi_i{}^m \Pi_j{}^n \eta_{mn} + \frac{1}{2}\sqrt{-g} + \frac{1}{6}\varepsilon^{ijk} \Pi_i^M \Pi_j^N \Pi_k^L C_{LNM} \right]$$

Bergshoeff, Sezgin, Townsed PLB87

- The theory is not conformal. It is strongly coupled and its formulation on an arbitrary backgrounds is very much complicated.
- The quantization of Supermembrane theory, in principle could describe at least part of the microscopical d.o.f. of M-theory. It was formulated in the LCG Supermembrane on a flat spacetime, and regularized through matrix models. *De Wit, Hoppe, Nicolai NPB88*
- The regularized mass operator M2-brane has a continuum spectrum in most backgrounds -BUT NOT ALL-, hence it does not admit on such backgrounds a first quantization. The canonical example, is on M11 *De Wit, Luscher, Nicolai NPB88*
- Just the fact of compactifying the theory does not change this behaviour. *De Wit, Peeters, Plefka PLB96*
- The BFSS matrix models searched a bound state of M0/M2/M5/M9 with discrete spectrum. *Banks, Fishler, Shneker, Susskind 97*

BASICS ON SUPERMEMBRANES

- A longstanding open problem is to find the microscopic d.o.f. of M-theory. In 2002 we found the sufficient condition for discreteness of a 0+1 (BFSS) matrix model. It holds for any background. *Boulton, MPGM, Restuccia NPB 03*
- The five string theories can be obtained as kinematical limits of Supermembrane theory *Aldabe, Lasrsen PLB97*
- The eigenfunction of the groundstate with zero eigenvalue of 11D supermembrane was conjectured by DWHN to be expressed in terms of 11D supergravity. (1988 Open problem) *De Wit, Hoppe, Nicolai NPB88*
- Supermembrane compactified on toroidal backgrounds has been shown to be U-dual invariant.. *MPGM, J.M. Peña, Restuccia JHEP12*

BASICS ON SUPERMEMBRANES

- In our group we have proved the existence of the known four ‘sectors’ (backgrounds) of the supermembrane with discrete spectrum at regularized level:

Duals

- The supermembrane irreducible wrapped on toroidal backgrounds *L. Boulton, MPGM, A. Restuccia NPB 03*
- The supermembrane on a pp-wave (Dasgupta et al. (BMN matrix model) *Boulton, MPGM, Restuccia NPB 12*
- The supermembrane on a C flux background with toroidal backgrounds *MPGM, C. Las Heras, P. León, J. Peña A. Restuccia, PLB19*
- The massive supermembrane on a LCD *MPGM, P. León, A. Restuccia JHEP21*

Open problems

- Romans supergravity origin in eleven dimensions has remained an ellusive problem for long time.
- From a bottom-up approach it was conjectured that there could even no exist an ultraviolet realization in terms of M-theory (Tomasiello et al.)
- Another open problem has been to find the M-theory description of type II Gauged Supergravities.
- To find the String worldsheet description of Romans and type II Gauged supergravities
- We obtain that all of these questions are related to the nontrivial sectors of the M2-brane, i.e. M2-branes with discrete spectrum.

Summary of Recent Results

- The supermembrane irreducible wrapped on M9xT2, i.e. with central charges is formulated on a symplectic torus bundle with monodromy in SL(2,Z). *MPGM, J. Peña, I. Martín, A. Restuccia JHEP11*

- At low energies, its supergravity description it is associated to the nine classes of type II gauged supergravities with monodromies in SL(2,Z). *MPGM, J. Peña, A. Restuccia JHEP12, PRD19*

- They are duals to a supermembrane on a quantized C3 background that induces 2-form worldvolume fluxes formulated on a twisted torus bundle. *MPGM, C. Las Heras, J. Peña, P. León, A. Restuccia PLB19,JHEP20*

- There is a new type of supermembrane in ten non-compact dimensions that is the massive supermembrane *MPGM, P. León, A. Restuccia , JHEP21*

Summary of Recent Results

What are the associated strings?

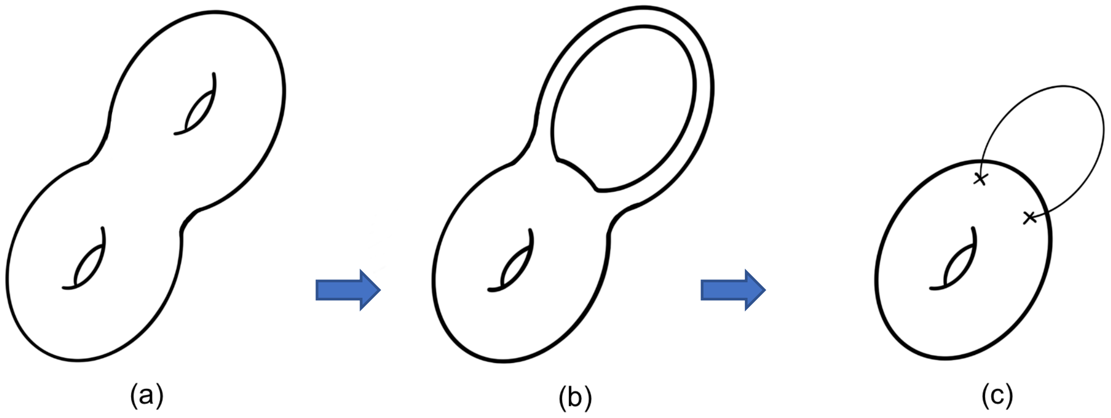
- The double dimensional reduction of the supermembrane on a constant quantized three form supergravity background: *MPGM, C. Las Heras, A. Restuccia JHEP 23;*
  - in the case of trivial monodromy it corresponds to the (p,q) strings with p,q diff from zero. No F1 or D1 can be recovered from this sector.
  - For non trivial parabolic monodromy there exists a new type IIB string on a circle, denoted as parabolic string with new tension and new symmetries.
  - Generalizations to elliptic and hyperbolic strings. (*To appear)*
- From the massive supermembrane appears a new string denoted *massive* superstring in 10D

MPGM, P.León, A. Restuccia JHEP 23;

Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Plb23

Based on previous results (MPGM+Restuccia’15, MPGM, Leon Restuccia JHEP21) we have developped a formulation of the 11D Massive M2-brane in a space with ten noncompact dimensions. We consider the worldvolume of a 11D supermembrane with genus 2 in a particular limit.



The string attached does not carry energy but fixes boundary conditions on the fields at the punctures

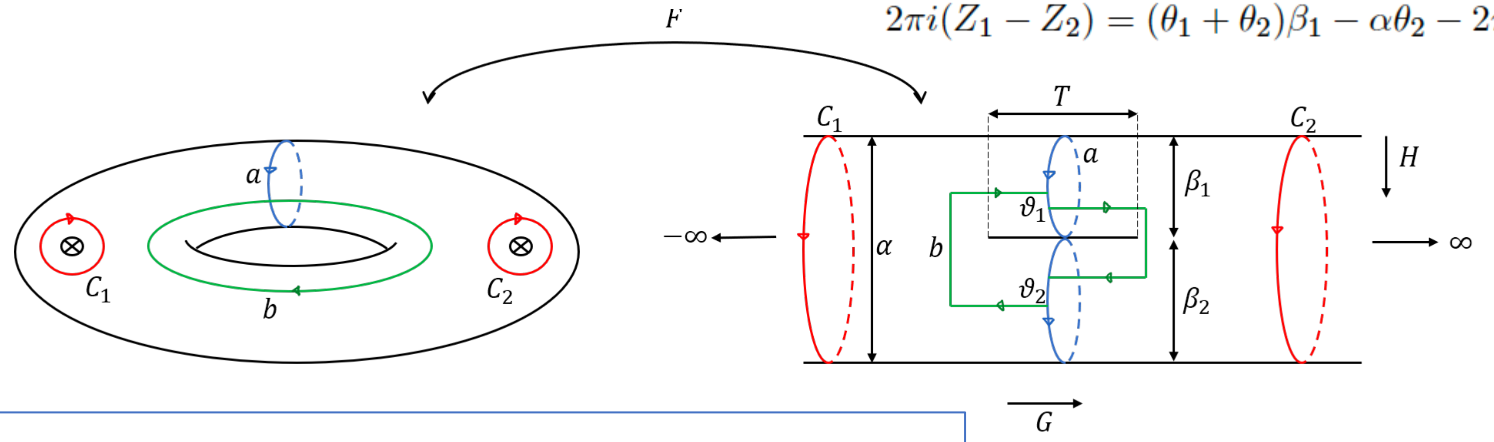
$$H = \int_{\Sigma_2 \to \tilde{\Sigma}_{1,2}} \mathcal{H} = \int_{\Sigma_{1,2}} \mathcal{H}.$$

Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21.

The target space is considered a twice puntured torus described by Light Cone Diagram (LCD) via a Mandelstam map

$$2\pi i(Z_1 - Z_2) = (\theta_1 + \theta_2)\beta_1 - \alpha\theta_2 - 2\pi i\beta_1\tau.$$



S.B Giddings, S.A. Wolpert (1987).

$$F(z) = \alpha \ln\left[\frac{\Theta_1(z - Z_1|\tau)}{\Theta_1(z - Z_2|\tau)}\right] - 2\pi i\alpha\frac{Im(Z_1 - Z_2)}{Im\tau}(z - z_0),$$

The Mandelstam map

- For simplicity, we take the minimum number of punctures to obtain a massive supermembrane. The number of punctures can be generalized.
- In String theory, LC diagrams correspond to one loop interaction diagrams. Here, this is not the interpretation.

Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21, Arxiv 230100686

$$F = G + iH.$$

G is a single-valued function but dG is harmonic due to its poles
H s a multivalued function and dH is harmonic.

Near the punctures

$$G \sim (-1)^{r+1}\alpha \ln|z - Z_r|,$$
$$H \sim (-1)^{r+1}\alpha\varphi, \quad \text{with} \quad \varphi \in (0, 2\pi) \quad (r = 1, 2).$$

Near the zeros

$$G(z) - G(P_a) \sim \frac{1}{2}Re(D(P_a)(z - P_a)^2),$$
$$H(z) - H(P_a) \sim \frac{1}{2}Im(D(P_a)(z - P_a)^2),$$

with

$$D(P_a) = \sum_{r=1}^{2}(-1)^{r+1}\left[\frac{\partial_z^2\Theta_1(P_a - z_r, \tau)}{\Theta_1(P_a - z_r, \tau)} - \left(\frac{\partial_z\Theta_1(P_a - z_r, \tau)}{\Theta_1(P_a - z_r, \tau)}\right)^2\right].$$

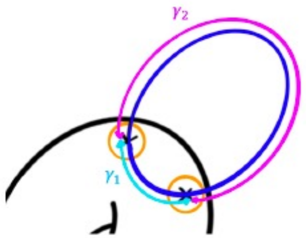
Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21

The target space is considered a twice-puntured torus described by Light Cone Diagram (LCD) via a Mandelstam map

$$\tilde{X}^m = \begin{cases} X^m(t,z,\bar{z}) & \text{over} \quad \Sigma_{1,2} \\ Y^m(t,u) & \text{over} \quad \gamma_2 \end{cases}, \qquad \tilde{\Psi} = \begin{cases} \Psi(t,z,\bar{z}) & \text{over} \quad \Sigma_{1,2} \\ \Theta(t,u) & \text{over} \quad \gamma_2 \end{cases},$$

$$\tilde{X}^r = \begin{cases} X^K(t,z,\bar{z})\delta_1^r + X^H(t,z,\bar{z})\delta_2^r & \text{over} \quad \Sigma_{1,2} \\ Y^r(t,u) & \text{over} \quad \gamma_2 \end{cases}.$$



with $\quad Y_s^m(u,t) = const, \quad Y_s^r(u,t) = const, \quad \Theta_s(u,t) = const,$

The special embedding maps are decomposed as follows

$$X^K = K + A^K, \quad X^H = mH + A^H,$$

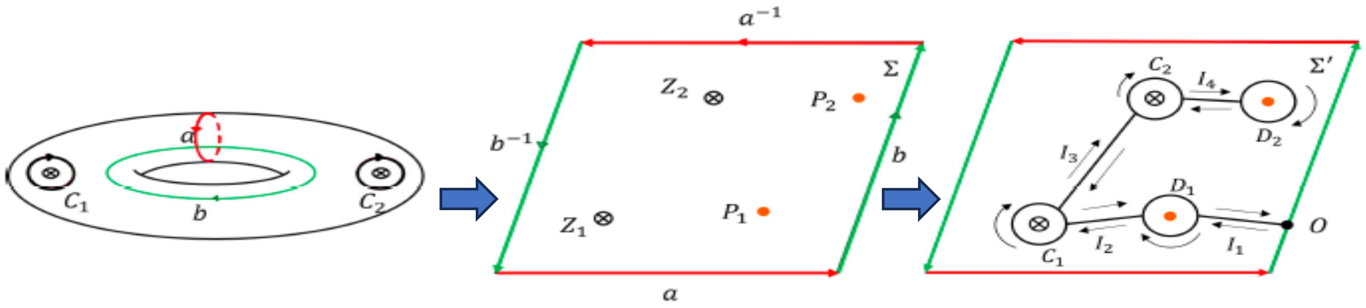
The metric on the LCD

$$ds^2 = \frac{l^2}{\cosh^4(\hat{G})} d\hat{G}^2 + dH^2 = dK^2 + \alpha^2 d\hat{H}^2,$$

$$l = \frac{kT}{\alpha}$$

Massive Supermembrane

MPGM. P. León. A. Restuccia:. JHEP21.



The Integration region

$$
\begin{aligned}
H =& \frac{(l\alpha T_{M2} m)^2}{2P_0^+} + \frac{1}{2P_0^+}\lim_{\epsilon\to 0}\int_{\Sigma'} d\sigma^2\sqrt{W}\Bigg[\left(\frac{P_m}{\sqrt{W}}\right)^2 + \left(\frac{P_K}{\sqrt{W}}\right)^2 + \left(\frac{P_H}{\sqrt{W}}\right)^2 \\
&+ T_{M2}^2\Big(\frac{1}{2}\{X^m,X^n\}^2 + m^2\{X^m,H\}^2 + \{X^m,K\}^2 + 2\{X^m,K\}\{X^m,A^K\} \\
&+ \{X^m,A^K\}^2 + 2m\{X^m,H\}\{X^m,A^H\} + \{X^m,A^H\}^2 + 2m\{H,A^K\}\{A^H,A^K\} \\
&+ m^2\{H,A^K\}^2 + 2\{A^H,K\}\{A^H,A^K\} + \{A^H,A^K\}^2 + \{K,A^H\}^2 + \{K,A^K\}^2 \\
&+ \{H,A^H\}^2\Big) - 2P_0^+T_{M2}(\bar\Psi\Gamma^-\Gamma_m\{X^m,\Psi\} + \bar\Psi\Gamma^-\Gamma_K\{A^K,\Psi\} + \bar\Psi\Gamma^-\Gamma_H\{A^H,\Psi\} \\
&+ \bar\Psi\Gamma^-\Gamma_K\{K,\Psi\} + \bar\Psi\Gamma^-\Gamma_H\{H,\Psi\})\Bigg]. \qquad (3.26)
\end{aligned}
$$

It can be shown that it satisfies the suficiency criteria for discreteness of the spectrum

Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21,

Constraints: There are five first class constraints: a local APD constraint and four global APD constraints, the new ones are associated with curves between the singular points. At String theory level they generate the constraint and restrict the configurations values at the singularities.

Subject to local and global APD

$$df = 0. \qquad \zeta_1 = \int_a f = 0, \quad \zeta_2 = \int_b f = 0, \qquad \zeta_3 = \int_{C_1} f = 0 \quad \zeta_4 = \int_{\gamma_1} f = 0,$$

With

$$f \equiv \left(\frac{P_K}{\sqrt{W}} dX^K + \frac{P_H}{\sqrt{W}} dX^H + \frac{P_m}{\sqrt{W}} dX^m + \bar{\Psi}\Gamma^- d\Psi \right),$$

Properties of the Massive Supermembrane

MPGM, P. León, A. Restuccia:, JHEP21,

- It appears a topological term from the mass terms contribution of the massive supermembrane

  It can be seen as the uplift to ten non-compact dimensions of the central charge condition

$$\lim_{\epsilon\to 0}\int_{\Sigma'} dK\wedge d\hat{H}\frac{\alpha\, m^2}{4}\{K,\hat{H}\}^2 = 2\pi\alpha m^2 k\hat{T}$$

- It contains nonvanishing mass terms

$$\begin{cases} (\partial_K X^m)^2 + (\partial_{\hat{H}} X^m)^2 \neq 0\,, \\ (\partial_K A^K)^2 + (\partial_{\hat{H}} A^K)^2 \neq 0\,, \\ (\partial_K A^H)^2 + (\partial_{\hat{H}} A^H)^2 \neq 0\,. \end{cases}$$

- The spectrum of the theory satisfies the sufficient condition for discreteness, once a proper regularization is provided.

- Due to the singularities only half of the susys are preserved and the M2-brane theory is N=1

Massive Supergravity

E. Bergshoeff, Y. Lozano, T. Ortin

The 11D uplift of massive supergravity is

$$S = \frac{1}{\kappa}\int d^{11}x\sqrt{(-g)}\left[R - \frac{1}{2\times 4!}F(4)^2 - \frac{1}{8}m^2|k^2|^2\right] + \frac{1}{(144)^2}\frac{\epsilon}{\sqrt{-g}}\left(2^4\partial C\partial CC + 3^2 m\partial CC(i_kC)^2 + \frac{9}{20}m^2C(i_kC)^4\right),$$

It admits as a source a wrapped M9 brane in 11D which under dimensional reduction along the isometry it becomes a D8-branes which are the source of 10D Romans supergravity. To couple a ten form potential associated with the M9-brane it is necessary to promote the mass parameter m into a field M(x)

$$\tilde{S} = \frac{1}{11!k}\int d^{11}x\epsilon^{\mu_1..\mu_{11}}\partial_{[\mu_1}A^{(10)}_{\mu_2...\mu_{11}]}. \qquad \mathcal{L}_kM = \mathcal{L}_kA^{(10)} = 0,$$

Equations of motion with respect to Ar lead →

$$M = \begin{cases} 0 & \text{si} \quad r=0 \\ \bar{m} & \text{si} \quad r>0 \end{cases}. \qquad \bar{m} = \kappa T_{M9};$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP23

The target space metric M9x LCD for the massive M2-brane contains singularities associated with the presence of M-brane sources. They are of two types of singularities: the punctures and the zeros. We want to compare the curvature of the M2-brane background singularities with those generated by the M9-brane.

The metric of the LCD around the singularities

$$ds^2 \to l^2 \frac{|du_r|^2}{|u_r|^2}, \quad u_r = c_r|z - Z_r| \left(\frac{z - Z_r}{\bar{z} - \bar{Z}_r}\right)^{i\alpha/2l}$$

Near the punctures the curvature behaves as $R \to \hat{\delta}^2(|u_r|),$

$$\int_0^{2\pi} d\theta_r \int_0^U d|u_r| \frac{2|u_r|}{l^2} \hat{\delta}^2(|u_r|)\phi(|u_r|) = 2\pi \int_0^U d|u_r| \frac{2|u_r|}{l^2} \hat{\delta}^2(|u_r|)\phi(|u_r|) = -\phi(0).$$

Near the zeros the curvature behaves as $R \to \tilde{\delta}^2(|\tilde{u}_a|),$

$$\int_0^{2\pi} d\tilde{\theta}_a \int_0^{\tilde{U}} d|\tilde{u}_a| \frac{2\tilde{c}^4}{l^2|D(P_a)|^2|\tilde{u}_a|^3} \tilde{\delta}^2(|\tilde{u}_a|)\tilde{\phi}(|\tilde{u}_a|) = 2\pi \int_0^{\tilde{U}} d|\tilde{u}_a| \frac{2\tilde{c}^4}{l^2|D(P_a)|^2|\tilde{u}_a|^3} \tilde{\delta}^2(|\tilde{u}_a|)\tilde{\phi}(|\tilde{u}_a|) = \tilde{\phi}(0).$$

Connection with Romans Supergravity

The M9-brane supergravity solution

$$ds^2_{M9-brane} = -H(y)^{-p/3}(dt^2 - dx_8^2) + H(y)^{-10p/3-2}dy^2 + H(y)^{5p/3}dz^2,$$

with p, d constants and m the Romans supergravity mass. The coordinate z represents the dimension along the isometry

$$H(y) = d \pm \frac{\bar{m}}{p}|y|,$$

The wrapped M9-brane tension after KK reduction along the isometry is Tension of the D8-brane

$$\frac{1}{16\pi G_N^{(11)}}\left(\int dz\right)\bar{m} = T_{M9},$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP23

The curvature scalar of the M9-brane metric near $y=0$, behaves as $R_{M_9} \to \bar{\delta}^2(y),$

$$\int dz \int dy \frac{4}{3} H^{10p/3+1} \frac{\bar{m}^2}{16\pi G_N^{(11)} T_{M9}} \bar{\delta}^2(y)\bar{\phi}(y) = \int dy \frac{12}{9} H^{10p/3+1} \bar{m}\bar{\delta}^2(y)\bar{\phi}(y) = \bar{\psi}(0).$$

We search for values of d and p in which delta's functions coincide. This gives us a relation between the Romans mass and supermembrane parameters

- **At the zeros,** $d = 0$ and $p = -6/5$

$$\bar{m}_a = \frac{1}{3}\left(\frac{5}{2}\right)^{3/2} \frac{l|D(P_a)|}{\sqrt{\pi}\tilde{c}_a^2}, \quad l = k\hat{T}.$$

$$\frac{\alpha \bar{m}_a}{8G_N^{(11)}} = T^a_{M9}. \quad \xrightarrow{(\alpha \to 0)} \quad \frac{\bar{m}_a}{8G_N^{(10)}} = T^a_{M8}.$$

Connection with Romans Supergravity

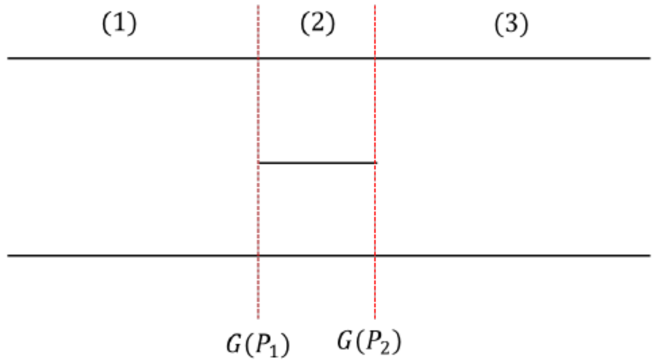
MPGM, P. León, A. Restuccia:, JHEP23

- **At the punctures,** $|p| << 1 \qquad d = 0,$ ⇨ $\frac{4}{3} H^{10p/3+1} \bar{m} = \frac{4\bar{m}^2 |y|}{2p} \left(1 + \frac{10}{3} p \ln\left(\frac{\bar{m}y}{p}\right) + \cdots \right).$

The match with this particular solution of M9-brane requires corrections in p

$$\frac{4\bar{m}^2}{2p} = \frac{2}{l^2} + (\text{corrections in p}).$$

The picture is that the massive M2-brane lives on M9x LCD. At efective level is consistent with sugra with M9-branes, i.e. massive sugra



N=2 Type IIA massive String

MPGM, P. León, A. Restuccia, Accepted for JHEP23

An interesting question, even in this simple background is the associated CFT description of the massive superstring. We perform a double dimensional reduction. Supersymmetry and the constraints restrict completely the dependence on K and H

$$X^m = \hat{X}^m(t,K), \quad A^K = \hat{A}^K(t,K), \quad A^H = \hat{A}^H(t,K), \quad \Psi = \hat{\Psi}(t,K).$$

The constraints forces the string to be closed and $\hat{A}^H = constant.$

The Hamiltonian of the N=2 closed type IIA superstring in ten noncompact dimensions is:

$$H_s = \frac{2l^2\tilde{T}_s}{\pi^2} + \frac{1}{4\pi\tilde{T}_s}\int_0^\pi d\theta\left([\hat{P}^2_{\hat{m}} + \tilde{T}^2_s(\partial_\theta \hat{X}^{\hat{m}})^2 - \frac{i}{\pi}\tilde{T}_s\chi^\dagger\rho^0\rho^1\partial_\theta\chi]\right)$$

$$\tilde{T}_s = 2\pi^2 m T_s.$$
$$T_s = \alpha T_{M_2}.$$

N=2 Type IIA massive String

MPGM, P. León, A. Restuccia, Accepted for JHEP23

- Expanding the maps in terms of oscillators

$$M^2 \equiv 4\pi\tilde{T}_s H_s - \sum_{\hat{m}} (P_0^{\hat{m}})^2 = \frac{2l^2\tilde{T}_s^2}{\pi} + 8\pi\tilde{T}_s(N+\bar{N}).$$

$$N = N_B + N_F,$$
$$\bar{N} = \bar{N}_B + \bar{N}_F,$$

- It satisfies the level matching constraint

$$N - \bar{N} = 0.$$

- The tesion of the massive string becomes modified by the integer m associated with the map H
- It contains a topological term that encodes the information on the singularities and the zeros of the LCD. Is associated with the presence of a Romans mass at effective level.
- At effective level this term is related to the presence of a cosmological constant of Romans supergravity.

$$\bar{m}_a = \frac{1}{3}\left(\frac{5}{2}\right)^{3/2} \frac{l|D(P_a)|}{\sqrt{\pi}\tilde{c}_a^2}, \quad l = k\hat{T}.$$

Conclusions

- We obtain a formulation of a supermembrane formulated on a twice punctured torus or equivalently on M9x LCD. Its Hamiltonian is discrete and it contains mass terms for all of the fields, so it can be considered a first quantized theory.

- We analyze its connection at effective level with Romans supergravity:

  1-It is realization of Hull's conjecture, according to which the uplift of M-theory on a torus with parabolic monodromy by decompactification must describe the UV uplift of Romans supergravity.
  2- The singularities of the worldvolume and the background imply the existence of δ^2 -singularities associated with the presence of M9-branes with an isometry in 11D. M9-branes are related to the uplift at 11D of Romans supergravity.
  3-At the level of the superalgebra, we have proven the existence of a nonvanishing central charge Z_{+M} associated with the presence of M9-branes.
  4- For the case of the M9-brane found by Bergshoeff et al, and Sato , we are able to obtain the Romans mass in terms of the supermembrane parameters at the zeros of the LCD

Conclusions

5- By comparing the curvature scalars associated to the singular points of the twice punctured torus and those of the M9-brane we are able to express the Romans mass in terms of the supermembrane parameters. It also contains a nontrivial topological term associated at effective level with the presence of a cosmological constant.

The complete matching requires to include more complicated backgrounds. In the punctures the M9-brane solutions are not those considered here. A complete solution requires to obtain the M2-brane in the supergravity background with M9-brane sbut this si a very complicated problema.

String worldsheet description:

After double dimensional reduction, *taking care of the singularities,* we obtain a N=2, type IIA closed superstring that contains a topological term due to the nontrivial background related to the Romans mass, and a modified tension. It satisfies the usual level matching constraint. We denote it as a massive string.

Thanks!

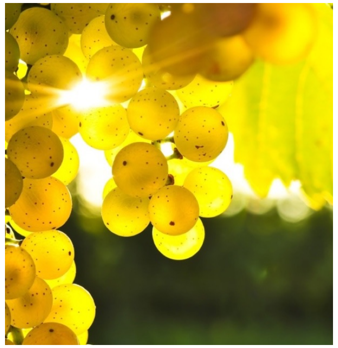


Foto tomada del sitio web https://www.spain.info/es/region/la-rioja/

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP23

The dimensions of the singularities are consistent with the presence of wrapped M9-branes. Furthermore at the level of the algebra we obtain a Z0m central charge that was conjectured to be dual to the M9-branes.

The metric describing the target space M9x LCD that we are considering is

$$ds^2 = 2dX^+dX^- + \delta_{mn}dx^m dx^n + \frac{T^2}{cosh^4(G)}dG^2 + dH^2, \quad K \equiv T\tanh(G)$$

The metric of the LCD around the singularities

$$ds^2 = l^2 d\hat{G}^2 + dH^2 = dK^2 + \alpha^2 d\hat{H}^2$$

⬇

$$ds_2^2 \sim \frac{T^2}{r^2}dr^2 + \alpha^2 d\theta^2$$

One of the solutions to Romans supergravity explored by Bergshoeff at’99 and Sato’2000 for the

$$X^i = \xi^i, \quad i = 0, 1, .., 8. \qquad A^{(10)}_{0..,8\theta} = A^{(10)}_{0..,8\theta}(r),$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP23

We compare the type of singularities associated with the curvature of both metrics in order to fix Romans supergravity mass parameter in terms of the massive supermembrane parameters.
The Romans mass parameter in 11D is associated with the presence of an M9-brane

$$ds^2_{M9-brane} = -H(y)^{-p/3}(dt^2 - dx_8^2) + H(y)^{-10p/3-2}dy^2 + H(y)^{5p/3}dz^2,$$

$$H(y) = d \pm \frac{\bar{m}}{p}|y|,$$

The massive M2-brane near the punctures

$$ds^2 \to l^2 \frac{|du_r|^2}{|u_r|^2}, \quad u_r = c_r|z - Z_r|\left(\frac{z - Z_r}{\bar{z} - \bar{Z}_r}\right)^{i\alpha/2l}$$

The tension of the M9-brane is defined in terms of the Romans mass

$$\frac{1}{16\pi G_N^{(11)}}\left(\int dz\right)\bar{m} = T_{M9},$$

By comparing the curvature of the metric of the M2-brane around the zeros with that of the M9-brane, the Romans mass gets fixed to

$$\bar{m}_a = \frac{1}{3}\left(\frac{5}{2}\right)^{3/2}\frac{l|D(P_a)|}{\sqrt{\pi}\tilde{c}_a^2}, \quad l = k\hat{T}.$$

Connection with Romans Supergravity

MPGM, P. León, A. Restuccia:, JHEP23.

Equations of motion with respect to Ar lead $\partial_r M = \kappa T_{M9}\delta(r) = \kappa T_{M9} r\hat{\delta}(r),$

$$M = \begin{cases} 0 & \text{si} \quad r = 0 \\ \bar{m} & \text{si} \quad r > 0 \end{cases} . \qquad \bar{m} = \kappa T_{M9} ;$$

Massive supergravity metric coupled to a M9-brane is

$$ds^2 = B^p(-dt^2 + dx^2_{(8)}) + B^\beta dr^2 + B^{-5p}dz^2, \quad p > 0$$

An approximation to this metric occurs when $\beta = 10p - 2 \text{ y } c = 0,$

Both metrics agree when $$ds^2 \sim -dt^2 + dx^2_{(8)} + \frac{9p^2}{\bar{m}^2 r^2}dr^2 + dz^2, \qquad T \equiv \frac{3p}{\bar{m}}$$

The topological term of the Massive supermembrane is directly related in this approximation to the Cosmological term of the Romans supergravity uplift to 11D.

Parabolic (p,q) string in 9D

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NonTrivial Monodromy case: By double dimensional reduction of the supersymmetric case the parabolic string mass operator is obtained

$$M^2_{C_q} = \left(\frac{n}{R_B}\right)^2 + (2\pi R_B \widehat{m}_8 T_{C_q})^2 + 4\pi T_{C_q}(N_L + N_R) - \frac{2P^0_- T^{1/6}_{C_q} R_B^{-2/3} n k_+}{|\widehat{\lambda}^T C_q|^{1/6}},$$

where the tension becomes modified

$$T_{C_q} \equiv |\widehat{\lambda}^T C_q| T_c, \quad \widehat{\lambda}^T = \frac{m_1}{(\mathrm{Im}(\lambda_0))^{1/2}} \begin{pmatrix} -1 & \lambda_0 \end{pmatrix}$$

Symmetries

The coinvariant Cq contains different p-charge. Inherited from the M2-brane with monodromy, there is a residual parabolic symmetry in SL(2,Q) that moves the charges Q inside the same equivalence class, leaving invariant the mass operator. This is the origin of parabolic gauge symmetry at effective level, in analogy with gauge invariance.

$$Q' = \Lambda Q,$$
$$\lambda'_0 = \lambda_0 + \frac{\mathbb{Z}}{q},$$

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Symmetries

Symmetries between different class of Parabolic strings become restricted to a parabolic subgroup of SL(2,Q). They are inherited from the supermembrane with parabolic monodromy

$$C_{q_2} = \widetilde{\Lambda} C_{q_1} \quad \Longrightarrow$$

$$Q' = \widetilde{\Lambda} Q,$$

$$\lambda_0' = \frac{\left(1 + \frac{\mathbb{Z}_2}{q_2}\beta\right)\lambda_0 + \left(-\frac{\mathbb{Z}_1}{q_1} + \frac{\mathbb{Z}_2}{q_2}\left(1 - \frac{\mathbb{Z}_1}{q_1}\beta\right)\right)}{\beta\lambda_0 + 1 - \frac{\mathbb{Z}_1}{q_1}\beta}$$

They leave invariant the Mass operator. This symmetry os the analogue of the SL(2,Z) in the Schwarz (p,q) string.

Moduli and properties of the Mandelstam map

- Characterization of the LCD moduli in terms of the Mandelstam map

$$\int_{C_r} dF = (-1)^{r+1} 2\pi i\alpha, \quad \int_a dF = 2\pi i\beta_1, \quad \int_b dF = \frac{i}{2\pi}(\beta_1\theta_1 - \beta_2\theta_2), \quad T \equiv \int_{P_1}^{P_2} dG.$$

Properties of the functions

$$G(z+1) - G(z) = G(z+\tau) - G(z) = 0$$

$$H(z+1) - H(z) = 2\pi\alpha\frac{Im(Z_2 - Z_1)}{Im(\tau)},$$

$$H(z+\tau) - H(z) = \frac{2\pi\alpha Im((Z_2 - Z_1)\bar{\tau})}{Im(\tau)}.$$

M2-brane bundle classification

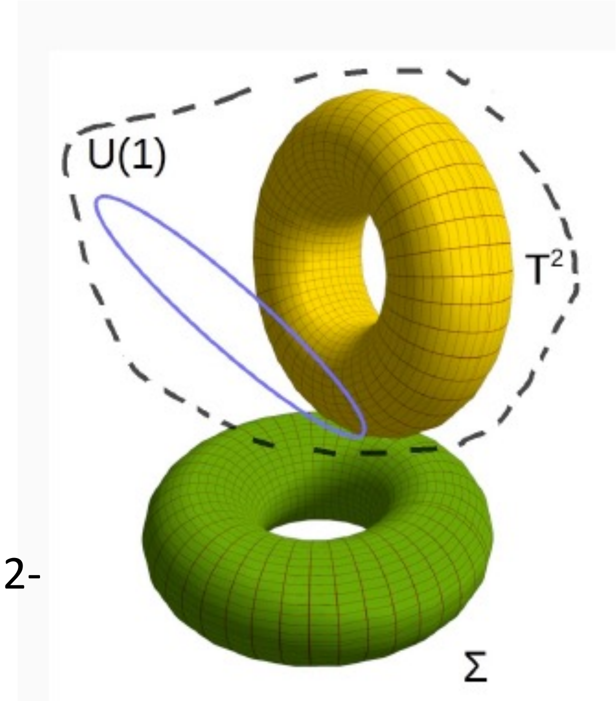
From the worldvolume picture the embedding of the M2-brane with C fluxes on a M9xT2 target space can be seen as a M2-brane on a twisted torus bundle with monodromy in SL(2,Z).

$$\mathcal{M}_G : \Pi_1(\Sigma) \to \Pi_0(Symp(T^2)) = SL(2,Z).$$

The inequivalent classes of symplectic torus bundle are classified by $H^2(\Sigma, Z^2_\rho)$
There is a 1:1 correspondence with the inequivalent classes of coinvariants

$$C_F = \left\{Q + \mathcal{M}_g\widehat{Q} - \widehat{Q}\right\}, \quad C_B = \left\{W + \mathcal{M}_g^*\widehat{W} - \widehat{W}\right\},$$

with $Q = \begin{pmatrix} p \\ q \end{pmatrix}$ and $W = \begin{pmatrix} l_1 \\ m_1 \end{pmatrix}$



The M2-brane with C fluxes on a symplectic torus bundle with monodromy defines a M2-brane on a twisted torus bundle

$$\mathbb{T}^3_W \equiv T^2_{U(1)} \to E' \to \Sigma\,,$$

(p,q) string in 9D

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Schwarz obtain the formulation of (p,q)-strings. He conjectured they had its origin in the M2-brane on M9xT2.

We extend these results to the supersymemtric case of the M2-brane with C fluxes. We analize two cases: Trivial monodromy and nontrivial monodromy

$$M^2 = \beta^2 M^2_{(p,q)} \qquad T_{(p,q)} = \frac{|q\lambda_0 - p|}{(\mathrm{Im}(\lambda_0))^{1/2}} T_c \qquad \tau = \lambda_0, \quad \beta^2 = \frac{T A_{T^2}^{1/2}}{T_c}, \quad R_B^2 = (T A_{T^2}^{3/2} T_c)^{-1}$$

Trivial Monodromy case: By double dimensional reduction of the supersymmetric case

$$M^2_{(p,q)} = \left(\frac{n}{R_B}\right)^2 + (2\pi R_B m T_{(p,q)})^2 + 4\pi T_{(p,q)}(N_L + N_R) + 2P^0_- T_c^{1/6} R_B^{-2/3} k_+$$

- **Important point**: We demonstrate that only the supermembrane with C fluxes (with nontrivial central charge) is able to generate string bound states (p,q). Vanishing central charge implies fundamental strings only (1,0)

LCG M2-brane on 11D bosonic curved Backgrounds

In the Light Cone Gauge $X^+(\xi) = X^+(0) + \tau$ so that $\partial_i X^+ = \delta_{i0}$, and $\gamma^+\theta = 0$. *B de Wit, Peeters, Plefka 98*

$$
\begin{aligned}
H \;=\; \int \mathrm{d}^2\sigma \Big\{ \frac{G_{+-}}{P_- - C_-} \Big[\tfrac{1}{2}\Big(P_a - C_a - \frac{P_- - C_-}{G_{+-}} G_{a+}\Big)^2 + \tfrac{1}{4}(\varepsilon^{rs}\,\partial_r X^a\,\partial_s X^b)^2 \Big] \\
- \frac{P_- - C_-}{2\,G_{+-}} G_{++} - C_+ - C_{+-} + c^r\phi_r \Big\}\,. \qquad (2.20)
\end{aligned}
$$

With

$$
\begin{aligned}
C_a &= -\varepsilon^{rs}\partial_r X^- \partial_s X^b\, C_{-ab} + \tfrac{1}{2}\varepsilon^{rs}\partial_r X^b \partial_s X^c\, C_{abc} \\
C_\pm &= \tfrac{1}{2}\varepsilon^{rs}\partial_r X^a \partial_s X^b\, C_{\pm ab}\,, \\
C_{+-} &= \varepsilon^{rs}\partial_r X^- \partial_s X^a\, C_{+-a}\,.
\end{aligned}
$$

Subject to the residual DPA constraint

$$\phi_r = P_a\,\partial_r X^a + P_-\,\partial_r X^- \approx 0$$