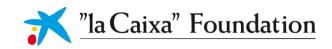
Stringy compactifications with non-trivial torsional homology

Matteo Zatti Meeting of the Coordinated Projects – Avila 16/11/2023







Our Starting Point

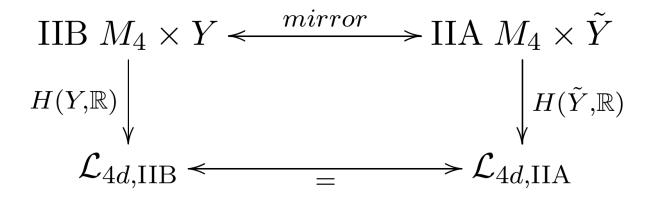
• To perform a dimensional reduction we expand the higher dimension fields in a basis of forms

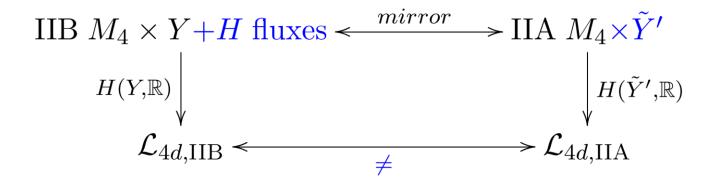
• It might be non-consistent to expand using only the internal space harmonics

• Observed for SU(3) structures with fluxes

<u>An Interesting Example 1/2</u>

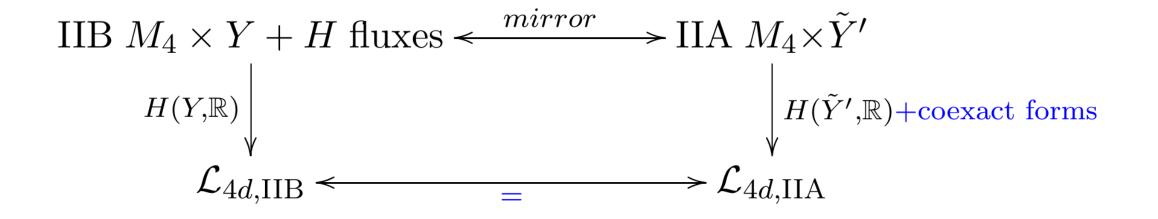
[Gurrieri, Louis, Micu, Waldram 03]





An Interesting Example 2/2

[Gurrieri, Louis, Micu, Waldram 03]



• A basis of coexact forms is often postulated but not described explicitly for SU(3) structures

Representatives of torsion cohomology can be related in a natural way to this basis

In a Nutshell

• Torsion Cohomology classes represented with delta forms

• EFTs may have access to the smeared delta forms

The Linking Numbers of calibrated torsional cycles can be computed via the smeared delta forms

[Gonzalo C., Fernando M., M.Z. 23]

What is torsion (co)homology?

 $H_{\text{free}}(X_n,\mathbb{Z})$

- Counts # Cycles
- Not trivial

Intersection numbers

 $H_{tor}(X_n,\mathbb{Z})$

- Counts # Torsion Cycles
- Trivial with N windings
- Linking numbers

Smeared delta forms

• Bump Delta forms = Poincare Duals of a cycle

$$\int_{\Pi_p} \omega_p = \int_{X_n} \omega_p \wedge \delta(\Pi_p)$$

• Expansion in a basis of smooth forms

$$\delta(\Pi_p) = \sum_i c_i \, b_{n-p}^i \qquad c_i = \int_{\Pi_p} \star b_{n-p}^i$$

• Smearing

$$\delta^{\rm sm}(\Pi_p) = \sum_{\lambda_i \ll \ell_s m_{kk}} c_i \, b_{n-p}^i$$

Smeared Linking Number

• Working definition [Horowitz, Srednicki 90]

$$L(\Pi_{n-p-1},\Pi_p) = \int_{X_n} d^{-1}\delta(\Pi_{n-p-1}) \wedge \delta(\Pi_p)$$

• Expansion

$$\delta(\Pi_p) = \sum_i c_i b_{n-p}^i \qquad \delta(\Pi_{n-p-1}) = \sum_i e_i \frac{1}{\lambda_i} d \star b_{n-p}^i \quad \Rightarrow \quad L = \sum_i \frac{c_i e_i}{\lambda_i}$$

• Smearing

$$L^{\rm sm}(\Pi_{n-p-1},\Pi_p) = \sum_{\lambda_i \ll \ell_s m_{kk}} \frac{c_i e_i}{\lambda_i}$$

The Proposal

The Smeared Linking Numbers of calibrated torsional cycles is equal to the Exact Linking Number

$$L = L^{SM}$$

Based on:

- Explicit examples (twisted torus)
- Supersymmetric EFTs structures (4d Aharanov-Bohm strings)

EFT Perspective

- 1. The $\delta^{\rm SM}$ enter in the EFTs as massive modes
 - Form a basis for dimensional reduction
- 2. The \mathbb{Z}_n factors of $H(X_n, \mathbb{Z})$ produce \mathbb{Z}_n gauge symmetries via Stuckelberg like Lagrangians
 - It is possible to extract L^{SM} from the Lagrangian
 - If $L = L^{SM}$ we can extract topological information!

<u>A Simple Example:</u> $\tilde{T}^6 = \tilde{T}^3 \times T^3$

 Realized in type 2A as half-flat manifold mirror dual to T⁶ in type 2B with electric NS flux [Gurrieri, Louis, Micu, Waldram 03]

$$ds_{\tilde{T}^3}^2 = \delta_{ab} e^a \otimes e^b$$

$$e^{1} = r_{1}dx^{1} \qquad e^{2} = r_{2}dx^{2} \qquad e^{3} = r_{3}(dx^{3} + Nx^{2}dx^{1})$$
$$\Pi_{2}^{\text{tor}}, \Pi_{3}^{\text{tor}} \implies \Pi_{1}^{\text{tor}} \times \Pi_{1}, \Pi_{1}^{\text{tor}} \times \Pi_{2}$$

$$L(\Pi_1^{\text{tor}}, \Pi_1^{\text{tor}}) = L^{\text{sm}}(\Pi_1^{\text{tor}}, \Pi_1^{\text{tor}})$$

Step 0: we consider a 3-dim manifold such that

$$\xi$$
 killing, $\star d\xi = f\xi$, $\xi^2 = 1$, $f \in \mathbb{R}$

<u>Step 1</u>: we consider a basis of scalars such that

$$\{\phi_i\} \text{ scalars}, \qquad \Delta \phi_i = \sigma_i^2 \phi_i, \qquad \mathcal{L}_{\xi} \phi_i = i \mu_i \phi_i, \qquad \sigma_i^2, \mu_i \in \mathbb{R}$$
$$\Rightarrow \quad \phi_i = e^{i \mu_i \theta} \tilde{\phi}_i, \qquad \mu_i \in \mathbb{Z}, \quad \theta \sim \theta + 2\pi$$

Step 2: we build a basis of 1-forms

• We define

$$A_i = d\phi_i, \qquad B_i = \star d(\phi_i \xi), \quad C_i = \star dB_i$$

• Closed under * d

$$\star dB_i = C_i, \qquad \star dC_i = \sigma_i^2 B_i + fC_i$$

• Diagonalize $\Delta D_i^{\pm} = (\lambda_i^{\pm})^2 D_i^{\pm}$

$$D_i^{\pm} = \left(\frac{1}{2} \pm \frac{f}{2\sqrt{f^2 + 4\sigma_i^2}}\right) C_i \pm \frac{\sigma_i^2}{\sqrt{f^2 + 4\sigma_i^2}} B_i \,, \qquad (\lambda_i^{\pm})^2 = \sigma_i^2 + \frac{f^2}{2} \pm \frac{f}{2}\sqrt{f^2 + 4\sigma_i^2}$$

<u>Step 3</u>: Replacing in the Linking number

$$L(\Pi_{1},\tilde{\Pi}_{1}) = \frac{1}{\lambda_{0}}K_{0}\tilde{K}_{0} + \sum_{i,\mu=0} \left[\frac{K_{i}^{+}\tilde{K}_{i}^{+}}{\lambda_{i}^{+}} + \frac{K_{i}^{-}\tilde{K}_{i}^{-}}{\lambda_{i}^{-}}\right] + \sum_{i,\mu\neq0} \left[\frac{K_{i}^{+}\tilde{K}_{i}^{+}}{\lambda_{i}^{+}} + \frac{K_{i}^{-}\tilde{K}_{i}^{-}}{\lambda_{i}^{-}}\right]$$

With

$$K_i^{\pm} = \int_{\Pi_1} D_i^{\pm}, \qquad K_0 = \int_{\Pi_1} D^0$$

blue and red term cancels

$$L^{\rm sm} \equiv L = \frac{1}{N}$$

Conclusions

- Non-Renormalization theorem, verified in examples, conjectured to be valid in calibrated SUSY EFT. Evidences based on how torsion leaks into the EFTs.
- Prescription to build a basis for dimensional reduction in presence of massive modes

Our findings suggest that torsion in Cohomology may result in measurable physics in the EFT