Non-Relativistic Heterotic String Theory

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based on: arXiv:2310.1916 [hep-th]
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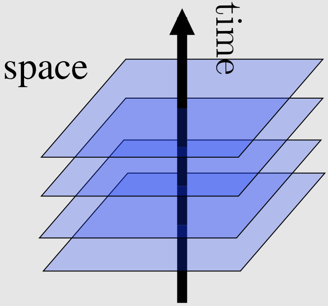
Introduction

Features & Motivations

- Non-Lorentzian theories and geometries appear in a wide range of physical contexts, from condensed matter (Quantum-Hall effect ...) to pure gravity (Near-Horizon geometry ∼ Carroll ...).
- They can be obtained with different procedures: limits, expansions, null-reduction and ...
- Limits allow us to inspect corners of Lorentzian theories



- **Non-Relativistic:** $c \to \infty$
- **Ultra-Relativistic:** $c \to 0$
- **Others:** Tensionless, Classical limiti of Quantum theories and ...

NS-NS Gravity

NS-NS Gravity Action

$$S_{\mathrm{NS-NS}} = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\left[\mathrm{R} + 4(\partial\Phi)^2 - \frac{3}{4}\mathcal{H}^2\right],$$

$$\mathcal{H}_{MNP} = \partial_{[M}B_{NP]}\,,$$

Fields

$$E_M{}^{\hat{A}}\,, B_{MN}\,, \Phi$$

Symmetries

$$\delta E_M{}^{\hat{A}} = \Lambda^{\hat{A}}{}_{\hat{B}}E_M{}^{\hat{B}}\,,$$

$$\delta B_{MN} = 2\partial_{[M}\Lambda_{N]}\,,$$

$$\delta\Phi = 0\,,$$

Index	Definition & Values
$M, N, P, \ldots$	10D Curved ,
$\hat{A}, \hat{B}, \hat{C}, \ldots$	10D Flat $(\hat{A} = \{A, a\})$,
$A, B, C\ldots$	Longitudinal Flat $A = 0, 1/\pm$,
$a, b, c\ldots$	Transverse Flat $a = 2, \ldots, 9$,

Index Splitting

Λ_{AB}	Lorentz Longitudinal
Λ_{ab}	Lorentz Transverse
Λ_{Ab}	Boost

Ansatz and Limit

Defining the Limit

Relativistic fields in terms of would-be non-relativistic fields (**Invertible** redefinitions)

$$E_M{}^A = c\,\tau_M{}^A\,,$$
$$E_M{}^a = e_M{}^a\,,$$
$$\Phi = \phi + \log c\,,$$
$$B_{MN} = \pm c^2 \tau_M{}^A \tau_N{}^B \epsilon_{AB} + b_{MN}\,,$$

Non-Relativistic Fields

$$\tau_M{}^A\,, e_M{}^a\,, b_{MN}\,, \phi$$

Parameters

$$\Lambda_{AB} = \lambda_{AB}\,, \qquad \Lambda_{ab} = \lambda_{ab}\,,$$
$$\Lambda_{Aa} = \frac{1}{c}\lambda_{Aa}\,, \qquad \Lambda_M = \lambda_M\,.$$

Taking the Limit $c \to \infty$: Results

NS-NS Action

The divergent terms coming from the R term cancel against those from $\mathcal{H}^2$ term: finite non-trivial limit.

Transformations

$$\delta\tau_M{}^A = \lambda^A{}_B \tau_M{}^B\,,$$
$$\delta e_M{}^a = \lambda^a{}_b e_M{}^b + \lambda^a{}_A \tau_M{}^A\,,$$
$$\delta\phi = 0\,,$$
$$\delta b_{MN} = 2\partial_{[M}\lambda_{N]} \mp 2\epsilon_{AB}\lambda^A{}_a \tau_{[M}{}^B e_{N]}{}^a$$

[Bergshoeff, Lahnsteiner, Romano, Rosseel, and Şimşek 2021]

Ten-Dimensional Minimal Supergravity

10D Minimal Supergravity

The minimal $\mathcal{N} = 1$ supersymmetric theory whose bosonic sector is that of NS-NS gravity $(E_M{}^{\hat{A}}\,, B_{MN}\,, \Phi\,, \Psi_\mu\,, \Lambda)$

Non-Relativistic Fields

$$\tau_M{}^A\,, e_M{}^a\,, b_{MN}\,, \phi\,, \psi_{\mu\pm}\,, \lambda_\pm$$

Non-Relativistic Ten-Dimensional Minimal Supergravity

The Limit $c \to \infty$

- Transformation Rules ,
- Equations of Motion .
- Action (Pseudo-Action)

[Bergshoeff, Lahnsteiner, Romano, Rosseel, and Simsek 2021]

Main Features

- Geometric Constraint DSNC$^-$,
- Emergent Local Dilatation Symmetry ,
- Two Emergent Super-Conformal Symmetries .
- Shorter Multiplet

What's Next?

Ten-Dimensional Minimal Supergravity

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The minimal $\mathcal{N} = 1$ supersymmetric theory whose bosonic sector is that of NS-NS gravity $({E_M}^{\hat{A}}\,, B_{MN}\,, \Phi\,, \Psi_\mu\,, \Lambda)$

Non-Relativistic Fields

$${\tau_M}^A\,, {e_M}^a\,, b_{MN}\,, \phi\,, \psi_{\mu\pm}\,, \lambda_\pm$$

Non-Relativistic Ten-Dimensional Minimal Supergravity

The Limit $c \to \infty$

- Transformation Rules ,
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[Bergshoeff, Lahnsteiner, Romano, Rosseel, and Simsek 2021]

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What's Next?

Adding a Vector Multiplet (Heterotic Supergravity)

Heterotic Gravity

Heterotic Lagrangian

$$S_{\text{heterotic}} = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\left[\mathrm{R} + 4(\partial\Phi)^2 - \frac{3}{4}\mathcal{H}^2 + \frac{1}{2}F_{MN\,I}F^{MN\,I}\right],$$

$$F^I_{MN} = 2\partial_{[M}V^I_{N]} - \sqrt{2}\mathsf{g}\,f_{KL}{}^I V^K_M V^L_N\,,$$

$$\mathcal{H}_{MNP} = \partial_{[M}B_{NP]} - V^I_{[M}F_{NP]I} - \frac{\sqrt{2}}{3}\mathsf{g}\,f_{IJK}V^I_{[M}V^J_N V^K_{P]}\,,$$

Index	Definition & Values
$M, N, P, \ldots$	10D Curved $(M = \{x, \mu\})$,
$\mu, \nu, \rho, \ldots$	9D Curved ,
x	Isometry Direction ,
$\hat{A}, \hat{B}, \hat{C}, \ldots$	10D Flat $(\hat{A} = \{A, a\})$,
$A, B, C\ldots$	Longitudinal Flat $A = 0, 1/\pm$,
$a, b, c\ldots$	Transverse Flat $a = 2, \ldots, 9$,
I, J, K, L	Yang-Mills (YM) .

Fields

$$E_M{}^{\hat{A}}\,, B_{MN}\,, \Phi\,, V^I_M$$

Symmetries

$$\delta E_M{}^{\hat{A}} = \Lambda^{\hat{A}}{}_{\hat{B}}E_M{}^{\hat{B}}\,,$$

$$\delta B_{MN} = 2\partial_{[M}\Lambda_{N]} + 2V^I_{[M}\partial_{N]}\Lambda_I\,,$$

$$\delta V^I_M = \partial_M\Lambda^I + \sqrt{2}\mathsf{g}\,f_{JK}{}^I\Lambda^J V^K_M\,.$$

The Ansatz

Defining the Limit

Relativistic fields in terms of would-be non-relativistic fields (**Invertible** redefinitions)

$$
\begin{aligned}
E_M{}^- &= c\, \tau_M{}^- \,, \\
E_M{}^+ &= -c^3 \frac{v_-^2}{2}\, \tau_M{}^- + c\, \tau_M{}^+ \,, \\
E_M{}^a &= e_M{}^a \,, \\
\Phi &= \phi + \log c \,, \\
V_M^I &= c^2 \tau_M{}^- v_-^I + \tau_M{}^+ v_+^I + e_M{}^a v_a^I \,, \\
B_{MN} &= c^2 \tau_M{}^A \tau_N{}^B \epsilon_{AB} (1 + v_{+-}) + \\
&\quad + 2 c^2 \tau_{[M}{}^- e_{N]}{}^a v_{-a} + b_{MN} \,,
\end{aligned}
$$

Main Features

- $E_M{}^+$ leading power is c^3 and is proportional to v_-^I ,
- B_{MN} ansatz is modified ,
- Reduces to usual ansatz setting YM field to zero

Non-Relativistic Fields

$$\tau_M{}^A \,, e_M{}^a \,, b_{MN} \,, \phi \,, v_+^I \,, v_-^I \,, v_a^I$$

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B_{MN} &= c^2 \tau_M{}^A \tau_N{}^B \epsilon_{AB}(1+v_{+-}) + \\
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$$\tau_M{}^A\,,\, e_M{}^a\,,\, b_{MN}\,,\, \phi\,,\, v_+^I\,,\, v_-^I\,,\, v_a^I$$

Applications

Transformations | **Heterotic Action** | **T-Duality**

Divergences of the Heterotic Action

Heterotic Action

$$S_{\text{heterotic}} = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\left[\mathrm{R} + 4(\partial\Phi)^2 - \frac{3}{4}\mathcal{H}^2 + \frac{1}{2}F_{MN\,I}F^{MN\,I}\right],$$

Remark

The “finite order” is already fixed by the limit without YM fields.

Divergences!

Potentially diverging terms appear at order c^4 and c^2.

Expansion

$$\mathrm{R} = c^4\overset{(4)}{\mathrm{R}} + c^2\overset{(2)}{\mathrm{R}} + \overset{(0)}{\mathrm{R}} + \frac{1}{c^2}\overset{(-2)}{\mathrm{R}},$$

$$\mathcal{H}_{MNP} = c^4\overset{(4)}{\mathcal{H}}_{MNP} + c^2\overset{(2)}{\mathcal{H}}_{MNP} + \overset{(0)}{\mathcal{H}}_{MNP}\,,$$

$$F^I_{MN} = c^2\overset{(2)}{F}{}^I_{MN} + \overset{(0)}{F}{}^I_{MN}\,,$$

Order c^4

$$F_{MN\,I}F^{MN\,I} = -4c^4\overset{(4)}{\mathrm{R}} + \mathcal{O}(c^2)\,,$$

$$\mathcal{H}_{MNP}\mathcal{H}^{MNP} = -\frac{4}{3}c^4\overset{(4)}{\mathrm{R}} + \mathcal{O}(c^2)\,,$$

Divergences of the Heterotic Action

Heterotic Action

$$S_{\text{heterotic}} = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,e^{-2\Phi}\left[\mathrm{R} + 4(\partial\Phi)^2 - \frac{3}{4}\mathcal{H}^2 + \frac{1}{2}F_{MN\,I}F^{MN\,I}\right],$$

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Order c^4

$$F_{MN\,I}F^{MN\,I} = -4c^4\overset{(4)}{\mathrm{R}} + \mathcal{O}(c^2)\,,$$

$$\mathcal{H}_{MNP}\mathcal{H}^{MNP} = -\frac{4}{3}c^4\overset{(4)}{\mathrm{R}} + \mathcal{O}(c^2)\,,$$

Existence of the Limit

Both at order c^4 and c^2 the divergent terms cancel leaving us with a finite limit

Boost

Parameters Ansatz

The parameters should be redefined in such a way to cancel divergences

$$\Lambda_{a+} = \frac{1}{c}\lambda_{a+}(1+v_{+-})\,, \qquad \Lambda_{a-} = \frac{1}{c}\lambda_{a-} + \frac{c}{2}\lambda_{a+}v_-^2(1+v_{+-})\,.$$

Non-Relativistic Boost

$$\begin{aligned}
\delta\,\tau_M{}^- &= 0\,,\\
\delta\,\tau_M{}^+ &= \frac{\lambda_-{}^a v_{a-}}{1+v_{+-}}\tau_M{}^- - e_M{}^a\lambda_{+a}v_-^2(1+v_{+-})\,,\\
\delta\,e_M{}^a &= -\lambda_-{}^a\tau_M{}^- - \lambda_+{}^a\tau_M{}^+(1+v_{+-})\,,\\
\delta\,b_{MN} &= 2\tau_{[M}{}^A e_{N]}{}^a\lambda_{Aa} + \dots\,,
\end{aligned}$$

$$\begin{aligned}
\delta\,v_+^I &= \lambda_+{}^a(1+v_{+-})v_a^I\,,\\
\delta\,v_-^I &= 0\,,\\
\delta\,v_a^I &= \lambda_{+a}(1+v_{+-})(v_-^I + v_-^2 v_+^I)\,,
\end{aligned}$$

Remarks

- String Galileian transformations restored setting to zero the YM fields ,
- Longitudinal and Transverse vielbein are mixed, $\tau_M{}^-$ is invariant
- v_-^I is boost invariant ,

Non-Abelian Gauge Transformations

Parameters Ansatz

We consider the trivial redefinition

$$\Lambda^I = \sigma^I \,,$$

Other redefinition could also be possible ...

Non-Relativistic YM transformations

$$\delta\, \tau_M{}^- = 0\,,$$
$$\delta\, e_M{}^a = 0\,,$$
$$\delta\, \tau_M{}^+ = \frac{v_{-I}\partial_-\sigma^I}{1+v_{+-}}\tau_M{}^- \,,$$
$$\delta\, b_{MN} = 2\tau_{[M}{}^+ e_{N]}{}^a \left(v_{+I}\partial_a\sigma^I - v_{aI}\partial_+\sigma^I\right) + \ldots\,,$$

$$\delta\, v_+^I = \partial_+\sigma^I + \sqrt{2}\mathrm{g} f_{JK}{}^I \sigma^J v_+^K \,,$$
$$\delta\, v_-^I = \sqrt{2}\mathrm{g} f_{JK}{}^I \sigma^J v_-^K \,,$$
$$\delta\, v_a^I = \partial_a\sigma^I + \sqrt{2}\mathrm{g} f_{JK}{}^I \sigma^J v_a^K \,,$$

Remarks

- v_-^I transforms no longer as a gauge field,
- $\tau_M{}^+$ is not invariant,
- b_{MN} transforms under YM,
- The transformations rules define a Lie algebra

Relativistic T-Duality

T-Duality Rules

$$\tilde{G}_{xx} = \frac{1}{G_{xx}} \,, \qquad \tilde{G}_{x\mu} = \frac{G_{x\mu}}{G_{xx}} \,, \qquad \tilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{G_{x\mu}G_{\nu x}}{G_{xx}^2} \,,$$

$$\tilde{B}_{\mu\nu} = B_{\mu\nu} + \frac{G_{x[\mu}G_{\nu]x}}{G_{xx}} \,, \qquad \tilde{B}_{x\mu} = \frac{G_{x\mu} - B_{x\mu}}{G_{xx}} \,, \qquad \tilde{\Phi} = \Phi - \frac{1}{2}\log|G_{xx}| \,,$$

$$\tilde{V}_x^I = -\frac{V_x^I}{G_{xx}} \,, \qquad \tilde{V}_\mu^I = V_\mu^I - \frac{V_x^I G_{x\mu}}{G_{xx}} \,,$$

Generalized Metric

$$G_{MN} = g_{MN} + B_{MN} - V_{MI}V_N^I$$

The Isometry Direction

- The geometric field are not invariant under non-Abelian gauge transformations
- How to define the nature of the isometry direction?

The Geometry

Redefinition

$$\hat{\tau}_M{}^- = \tau_M{}^- \,, \qquad \hat{\tau}^M{}_- = \tau^M{}_- \,,$$

$$\hat{\tau}_M{}^+ = \tau_M{}^+(1 + v_{+-}) + e_M{}^a v_{a-} \,, \qquad \hat{\tau}^M{}_+ = \frac{1}{1 + v_{+-}}\tau^M{}_+ \,,$$

$$\hat{e}_M{}^a = e_M{}^a \,, \qquad \hat{e}^M{}_a = e^M{}_a - \frac{v_{a-}}{1 + v_{+-}}\tau^M{}_+ \,.$$

The hatted Vierbeine and their inverses satisfy the usual inverse relations of Newton-Cartan Geometry.

Transformation Rules

$$\delta\,\hat{\tau}_M{}^- = 0\,, \qquad \delta\,\hat{\tau}_M{}^+ = v^I_- \partial_M \sigma_I \,, \qquad \delta\,\hat{e}_M{}^a = -\lambda_A{}^a \hat{\tau}_M{}^A + \lambda_+{}^a \hat{e}_M{}^b v_{b-} \,,$$

Longitudinal Spatial Isometry Direction

The longitudinal spatial isometry direction can then be defined by imposing the following conditions:

$$\hat{\tau}_x{}^0 = 0\,, \qquad \hat{\tau}_x{}^1 \neq 0\,, \qquad \hat{e}_x{}^a = 0\,.$$

These conditions break half of the boost transformations, i.e. $\lambda_{1a} = 0$.

Non-Relativistic Longitudinal T-Duality

$$\tilde{G}_{xx} = 0\,,$$

$$\tilde{G}_{x\mu} = \frac{\hat{\tau}_\mu{}^-}{\hat{\tau}_x{}^-}\,,$$

$$\tilde{G}_{x\mu} = \frac{\hat{\tau}_\mu{}^+}{\hat{\tau}_x{}^+}\,,$$

$$\tilde{G}_{\mu\nu} = h_{\mu\nu} + b_{\mu\nu} + \dots\,,$$

$$\tilde{\Phi} = \phi - \frac{1}{2}\log|\hat{\tau}_{xx}|\,,$$

$$\tilde{g}_{xx} = \frac{v_-^2\,(\hat{\tau}_x{}^-)^2}{\hat{\tau}_{xx}^2}\,,$$

$$\tilde{V}_x^I = -\frac{\hat{\tau}_x{}^- v_-^I}{\hat{\tau}_{xx}}\,,$$

Remarks & Open Questions

- In absence of the YM fields the longitudinal isometry is mapped to a null isometry direction ,
- In presence of YM field the null isometry direction is in the generalized metric ,
- Could also transverse and lightlike T-duality be obtained? ,
- What are the compactification patterns related to the T-Duality rules?
- Geometry ...

Conclusion and Outlook

Conclusion

- The **NR Limit** of Heterotic Gravity exists and requires a deformation of the usual ansatz for the metric and Kalb-Ramond 2-form
- **NR Heterotic Geometry** shows intriguing aspects: the longitudinal vielbein is not gauge invariant, one of the vector field is no longer a gauge field

Outlook

- NR Heterotic Supergravity
- Sigma Model
- T-Duality and Solutions

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Thank You!