Instituto de Física Teórica UAM-CSIC

5-dimensional geometry of 4d static Kaluza-Klein black holes

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in collaboration with P.Meesen, T.Ortín, M.Zatti

Carmen Gómez-Fayrén

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Motivation

Thermodynamics of 4d KK black holes from 5d point of view

- Does the 4d event horizon imply the existence of 5d one?
- Will it also be a Killing horizon in 5d?
- To which Killing vector is it associated?
- How is the 5d surface gravity?

Outline

1. Kaluza-Klein theory
2. 5d extension of 4d Killing vector
3. Geometric interpretation
4. 5d surface gravity

Kaluza-Klein basics

We consider pure Einstein gravity in 5 dimensions

$$(\hat{g}_{\hat{\mu}\hat{\nu}}) \to (g_{\mu\nu}, A_{\mu}, k), \text{ with } \hat{\mu} = (\mu, \underline{z})$$

$$ds^2_{(5)} = \hat{g}_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} = ds^2_{(4)} - k^2 (dz + A)^2 \,, \qquad z \sim z + 2\pi\ell$$

- z-independent metric $\to$ isometry by $\hat{k} = \partial_{\underline{z}}$
- 5d geodesics $\to$ 4d geodesics + conservation P_z

getting 4-dimensional electrically charged black hole solution

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Uplift 4d Killing horizon

5-dimensional geometries of 4d static BH

- Spacelike Killing vector $\rightarrow$ $\hat{k} = \partial_{\underline{z}}$
- Timelike Killing vector $\rightarrow$ $l = \partial_t$ $\rightarrow$ 4d Killing horizon

5-dimensional event horizon

$$5\text{d } \mathcal{H} = 4\text{d } \mathcal{H} \times S^1$$

$$l^2 = l^\mu \hat{g}_{\mu\nu} l^\nu = -k^2 \left(\iota_l A\right)^2 \neq 0 \;\; at \;\; \mathcal{H}$$

5d Killing horizon

Analogous to 4d characterization of $\mathcal{KH}$ $\rightarrow$ extension of $l \equiv \hat{l}$

$$\hat{l}^2 = \hat{l}^{\hat{\mu}} \hat{g}_{\hat{\mu}\hat{\nu}} \hat{l}^{\hat{\nu}} = 0 \quad at \quad \mathcal{KH} \tag{1}$$

5d extension of l

Assuming the form $\rightarrow$ $\hat{l} = l + f\hat{k}$

[C.G., P.Meesen, T.Ortín, M.Zatti, 23]

- From Eq. (1) $\rightarrow$ $(f + \iota_l A)\,|_{\mathcal{KH}} = 0$
- To be Killing vector of $\hat{g}_{\hat{\mu}\hat{\nu}}$ $\rightarrow$ $\mathcal{L}_l A_\mu + \partial_\mu f = 0$

Emergence of some formulas

$$\mathcal{L}_I A + df = \iota_I F + d\left(\iota_I A + f\right) = 0, \text{ with } P_I \equiv \iota_I A + f$$

Momentum map equation

$$\iota_I F_E + dP_{E\,I} = 0$$

Gauge-covariant Lie derivative

$$\mathbb{L}_I A_E \equiv \iota_I F_E + dP_{E\,I} = \mathcal{L}_I A_E - \delta_{\chi_I} = 0$$

with a "compensating gauge transformation" parameter $\chi_I \equiv \iota_I A_E - P_{E\,I}$

[Z. Elgood, P.Meesen, T.Ortín, 20]

5-dimensional extension of l

Finally

$$\hat{l} = l - k_{\infty}^{1/2} \left(\iota_l A_E - \bar{P}_{E_l} \right) \hat{k}$$

5-dimensional Killing horizon

5d $\mathcal{KH}$ = 4d $\mathcal{KH} \times S^1$

[C.G., P.Meesen, T.Ortín, M.Zatti, 23]

Outline

Geometric interpretation

On the Killing horizon $\rightarrow$ $\hat{l} = l - k_\infty^{1/2}\Omega\hat{k}$, with $\Omega = \iota_l A_E|_{\mathcal{KH}}$

since $\bar{P}_{E_l}|_{\mathcal{KH}} = 0$

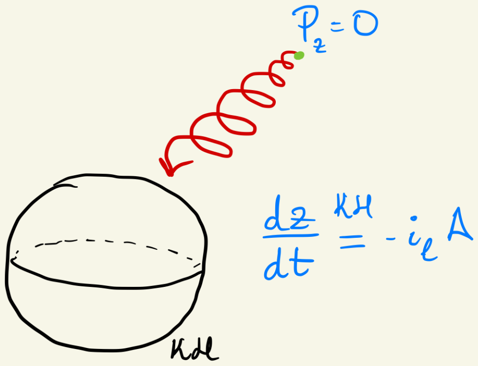
$\exists$ ambiguity in Ω $\rightarrow$ $A_E = 0$ at spatial infinity

Geometric interpretation

$$\Omega = \Phi$$

$\sim$ angular velocity in rotating black holes

Physical interpretation



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Surface gravity

Surface gravity of 5d Killing horizon

- 4d surface gravity $\rightarrow$ $\nabla_\mu l^2|_{\mathcal{KH}} = -2\kappa l_\mu$
- 1-form dual to $\hat{l}$ $\rightarrow$ $\hat{l}_{\hat{\mu}} dx^{\hat{\mu}}|_{\mathcal{KH}} = l_\mu dx^\mu|_{\mathcal{KH}}$

$$\hat{\nabla}_\mu \hat{l}^2 = \nabla_\mu l^2$$

Both surface gravities coincide

Summary

5d geometry of 4d static, Kaluza-Klein black holes

- Existence of 5d event horizon
- Uplifted 4d Killing vector $l \rightarrow$ 5d Killing vector $\hat{l}$
- 5d Killing horizon
- Interpretation of 4d electrostatic potential on the horizon
- 5d surface gravity coincides with 4d surface gravity

Future directions

- Magnetic case
- 5d with matter

Thank you!!

Enjoy dinner!!